# A Study on Non-Blind Algorithm of Subarray Signal Processing for Desired Signal Estimation

Kwan Hyeong Lee

*Dept. of Communication Engineering, Daejin University,*
*1007 Hoguk-ro, Pocheon, Gyeonggi-do, Korea*
*khlee@daejin.ac.kr*

***Abstract***

*In this paper, a DOA estimation method using subarray and spatial smooth is proposed. The DOA is then estimated based on the computation of the phase shift between the reference signal and its phase shifted signal. To significantly improve the robustness of DOA estimation and of beamforming and to estimate azimuth angle in multipath mobile channel environment, we developed techniques for applying spatial smooth subarray method to arrays of linear geometry. We developed method for applying spatial smooth subarray to arrays of arbitrary geometry. We study the cause of ambiguities in a multiple signal environment and find the necessary and sufficient conditions for an array manifold to be ambiguity free. We show that can be the application of our method MUSIC and adaptive beamforming method. The estimation results are verified by simulations.*

***Keywords:*** *Subarray, Algorithm, Covariance, Correlation, Weight*

## 1. Introduction

The sensor array signal processing is used in several application areas such as radar, sonar, seismology, wireless communication and biomedical signal processing [1]. In array signal processing, direction of arrival (DOA) estimation of multiple narrowband signals is a classic problem. Recently, as the mobile communication technology advances, sensor array signal processing emerges as potential technology to improve the spectral efficiency [2]. Adaptive signal processing has been applied to a variety of areas, ranging from bio-medical application to telecommunications networks [3]. Such adaptive filters can self- adjust to the characteristics of an incoming signal without external intervention. This is similar to a closed-loop control system such as a thermostatically controlled central heating system. Well-known applications of adaptive signal processing include: cancelling the maternal heart beat when performing fetal electrocardiography, noise cancellation in speech signals, echo cancellation in long distance telephone circuits, and adaptive equalization in mobile communication systems [4]. These adaptive filters are all based upon time domain filtering, often utilizing familiar finite impulse response or infinite impulse response filter architectures, where the filter taps are computed according to some criteria and the incoming signal characteristics [5]. Over time a number of applications have been developed where spatial filtering is desirable. The spatial filtering principle is demonstrated by considering the alignment of a array antenna, where misalignment either results in no signal, or reception of a distant unwanted transmitter. Both array antennas receive the interfering and wanted signals, but since the antennas are spatially separated the two signals can be distinguished using a suitable filter [6]. This is because the system directs a null towards a specific signal and under there conditions the null is directed towards the wanted signal. There are several techniques that can reduce this effect such as injecting additional noise, or using a specially modified adaptive algorithm. The robust desing of an adaptive array system is a multi-disciplinary process, where component technologies include: signal processing, transceiver design, array design, antenna element design, and signal propagation characteristics.

Much of the work in sensor array signal processing had focused on methods for high resolution DOA estimation and optimum adaptive beamforming. These methods include the well-known MUSIC and ESPRIT for DOA estimation [8]. These methods lead to an acceptable DOA estimation if the number of signal sources is less than the number of antenna elements. These are called by subspace method. In the case where the total number of interfering and target signal sources is larger than the number of antenna elements, only sources is larger than the number of antenna elements, only some of the DOA of the signals can be properly estimated [3]. In MUSIC method, the DOA are determined by finding the direction for which their antenna response vectors lead to peaks in the MUSIC spectrum formed by the eigenvectors of the noise subspace [9].

However, an important drawback of these techniques is the severe degradation of the estimation accuracy in DOA estimation or signal cancellation in adaptive beamforming in the presence of highly correlated or coherent signal.

Research has been carried out in developing algorithms for coherent interference using arrays of arbitrary geometry in the past decade. Recently, diversity combining and blind adaptive beamforming have been proposed to combat multipath fading and cochannel interference. However blind adaptive beamformer has a low convergence rate, and is only applied to signals with constant modulation. Both techniques have limitations on tracking complicated channels while cochannel interference and multipath effects coincide [10].

In this work, we develop a general spatial smoothing method for arrays of arbitrary geometry to make MUSIC, ESPRIT method and optimum adaptive beamforming algorithms operative in a coherent interference environment and meanwhile success robustness in performance. We compare general methods with proposal methods for arrays of arbitrary geometry.

## 2. Weight Adaptive Algorithm

Most adaptive beamforing algorithms may be categorized in two classes according to whether a training signal is used or not. One class of these algorithms is non-blind adaptive algorithm in which a training signal is used to adjust the array weight vector. On the other hand, blind adaptive algorithms do not require a training signal. In non-blind adaptive algorithm, a training signal, which is known to both the transmitter and receiver, is sent from the transmitter to the receiver during the training period [11]. The beamformer in the receiver uses the information of the training signal to compute the optimal weight vector. After the training period, data is sent and the beamformer uses the weight vector computed previously to process the received signal. If the channel and the interference characteristics remain constant from one training period to the next, the weight vector will remain unchanged. The training signal therefore samples the channel and must obey the well know Nyquist sampling theorem [12]. The remainder of this section reviews the development of well-known adaptive algorithms used in adaptive antennas. Initially the wiener solution is developed, which subsequent adaptive algorithms are used to compute. The application of adaptive algorithms avoids direct computation of the wiener solution which would otherwise require matrix inversion which is computationally intensive and can to an unstable [13].

**Table 1. Comparison of DOA Estimation algorithms**

| Algorithm | Advantages | Disadvantage |
|---|---|---|
| MUSIC | High resolution | Lower performance than ESPRIT<br>Sensitive to gain and phase errors<br>Sensitive to coherent multipath |
| ESPRIT | High resolution | Computationally complex<br>Limited by array geometries<br>Requires multiple snapshots |
| ML | Statistically optimum results | Computationally complex<br>Requires many snapshots |

Most non-blind algorithms are designed to minimize the mean-squared error between the desired signal d(t) and the array output y(t). This is termed the wiener solution and the following shows how it is developed for the signals relating to an adaptive beamformer. Then the error signal is given by [14-15]

$$\mathrm{e(t) = d(t) - y(t)} \quad (1)$$

$$y(\mathrm{t}) = \mathrm{W}^{H}\ \mathrm{r(t)} \quad (2)$$

The mean squared error is defined by the cost function as follow

$$\mathrm{J = E}[|e(t)|^2] \quad (3)$$

Were E[ ] denotes the expectation value. It should be noted that the cost function J attains its minimum when all the elements of its gradient vector are simultaneously zero. Substituting equation (1) and into equation(4.10) can be written

$$\mathrm{J = E[|d(k) - y(k)|^2} \quad (4)$$

$$= \mathrm{E[\{d(k) - y(k)\}}\,\{d(k) - y(k)\}^{H}] \quad (5)$$

$$= \mathrm{E[\{d(t) - W^H\ r(t)\}\{d(t) - W^H\ r(t)\}^*]} \quad (7)$$

$$= \mathrm{E}[|d(t)|^2] - \mathrm{P}^{H}\,\mathrm{W} - \mathrm{P\,W}^{H} + \mathrm{W}^{H}\,\mathrm{R\,W} \quad (8)$$

Where

$$\mathrm{R = E[r(t)r^H(t)]} \quad (9)$$

$$\mathrm{P = E[x(t)d^*(t)]} \quad (10)$$

In Eq(8), R is auto correlation matrix of the input data vector, and P is the cross correlation vector between the input data vector and the desired signal. The gradient vector of J can be written

$$\nabla(\mathrm{J}) = 2\frac{\partial \mathrm{J}}{\partial \mathrm{W}^*} \tag{11}$$

Where $\frac{\partial}{\partial \mathrm{W}^*}$ denotes the conjugate derivative with respect to the complex vector. When the mean squared error J is minimized, the gradient vector will be equal to a null vector.

$$\nabla(\mathrm{J})|_{W_{OPT}} = 0 \tag{12}$$

Substituting Eq(8) into Eq(11) gives

$$-2\mathrm{P} + 2\mathrm{R}\,\mathrm{W}_{OPT} = 0 \tag{13}$$

$$\mathrm{R}\,\mathrm{W}_{OPT} = \mathrm{P} \tag{14}$$

Eq(14) represents the matrix form of the so called winer hopf equation. Pre multiplying both sides of Eq(14) by the inverse of the correlation matrix, we obtain the optimum weight vector

$$\mathrm{W}_{OPT} = \mathrm{R}^{-1}\,\mathrm{P} \tag{15}$$

Eq(15) is also called the winer solution. From Eq(15), it is obvious that the computation of the optimum weight vector, $\mathrm{W}_{\mathrm{OPT}}$, requires knowledge of two quantities: the correation matrix R of the input data vector, and the corss correlation vector P between the input data vector and the desired signal.

## 3. Proposed Estimation signal

Signal processing techniques have been introduced with a wide range of functionalities, such as finding of the DOA of a desired signal or cancellation of interfering signals. The performance and applicability of these algorithms have been gauged assuming provision of error free input parameters, uncorrelated signals and distortion free analogue components. Algorithms have to be analytically, numerically or practically tested to see how sensitive they are to input errors or as often referred to, to perturbations of the input parameters, one of the most important of which are the array weighting coefficients. As such, beamforming schemes, DOA algorithms and other adaptive algorithms rely in one form or another on a perfect estimate of these.

We consider array consisting a uniform linear array $M$ narrow band signals with additive white Gaussian noise, element spacing $d$ and consisting of $p$ identical elements. The array output covariance matrix can be written as follow [16-17]

$$\mathrm{R} = \mathrm{E}[r(\mathrm{t})r^H(\mathrm{t})] = \mathrm{A}\,R_S\,A^H + \sigma^2 I \tag{16}$$

Where $r(\mathrm{t})$ is the received signal, $(\,)^H$ is hermite matrix $A$ is steering matrix and $\sigma^2$ is the variance of the white Gaussian noise. In the case of uniformly spaced linear array, with a sensor spacing, the spatial smoothing method can be applied to achieve the nonsingularity of the modified covariance matrix of the signals. This method begins by dividing a uniformly spaced linear array of $p$ sensors in $K$ overlapping subarrays of size $[1, \cdots, p]$ , with sensors forming$[2, \cdots, p]$ the first subarray, and sensors forming the second subarray [18]

$$A_k = A_1 E^{(k-1)} \quad \textbf{(17)}$$

Where $A_k$ is steering matrix consisting of steering vectors associated with the $k^{\text{th}}$ subarray and $E^k$ is the $k^{\text{th}}$ power of a $M \times M$ diagonal matrix. The spatially smoothed covariance matrix is defined as the average of the subarray covariance.

$\overline{\mathrm{R}}$ can be written as follow[16]

$$\overline{\mathrm{R}} = \mathrm{A}_1 \bar{R}_s \mathrm{A}_1^H + \sigma^2 I \quad \textbf{(18)}$$

Where $\mathrm{R}_k$ is the covariance matrix associated with the $k^{\text{th}}$ subarray, $\bar{R}_s$ is the modified covariance matrix of the signals, and has been proved to be full rank. The signals are thus progressively decorrelated. However linear arrays has limitations in the domain of estimable direction of arrivals. $\bar{R}_s$ can be written as follow

$$\bar{R}_s = \frac{1}{K} B B^H \quad \textbf{(19)}$$

Where $\mathrm{B} = \mathrm{P}\,\mathrm{A}^T$ with $\mathrm{P} = \mathrm{diag}(p_1, p_2, \cdots, p_M)$

$$\mathrm{P} = \begin{bmatrix} p_1 & 0 & \cdots & 0 \\ 0 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_M \end{bmatrix} \quad \textbf{(20)}$$

Where $p_1$ is $\exp(-\mathrm{j}2\frac{\pi}{\lambda}\mathrm{d}\,\sin\theta_1)$.

$$\mathrm{A} = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ e^{-j2K\pi d/\lambda \sin(\theta_1)} & \cdots & e^{-j2K\pi d/\lambda \sin(\theta_M)} \end{bmatrix} \quad \textbf{(21)}$$

Where $\lambda$ denotes wave length, $\lambda = \frac{c}{f}$. Correlation matrix of $N$ subarry matrix can be as follow

$$\overline{\mathrm{R}} = \frac{1}{N} \sum_{n=1}^{N} \bar{R}_n \quad \textbf{(22)}$$

$$= \mathrm{A}\left[\frac{1}{N} \sum_{n=1}^{N} P^{n-1} \ \bar{R}_s\right] \mathrm{A}^H + \sigma^2 \mathrm{I} \quad \textbf{(23)}$$

## 4. Simulation

In this section, we present simulation results on MUSIC method to show the effectiveness and applications of spatial smooth subarray. We choose a linear array, which has an orientational invariance structure, central symmetric, and a sensor spacing less than half wavelength. We use a 12 sensor linear array as spacing of half wavelength. We divide the both arrays into four overlapped subarrays. We get 6 sensors in each subarray of the linear array. We consider two narrow band coherent signals with DOA at $10^{o}$ and $-10^{o}$. The SNR is 20dB. A toalt of 500 snapshots are taken from the array. We use spatial smooth subarray method as a pre-processing scheme for MUSIC. In Figure 1, we illustrate the spatial spectrum obtained by different methods. Figure 1 a shows the result when there are two closely placed signals at $-10^{o}$ and $10^{o}$. Figure 1 shows a graph to estimate arrival direction by MUSIC method and two arrival directions. It showed DOA estimation an error about $1.3^{o}$ in Figure 1. Figure 2 shows a graph to estimate arrival direction by the proposal method. It showed correctly DOA estimation by proposal method in Figure 2. It denotes DOA estimation angle in Table 1.

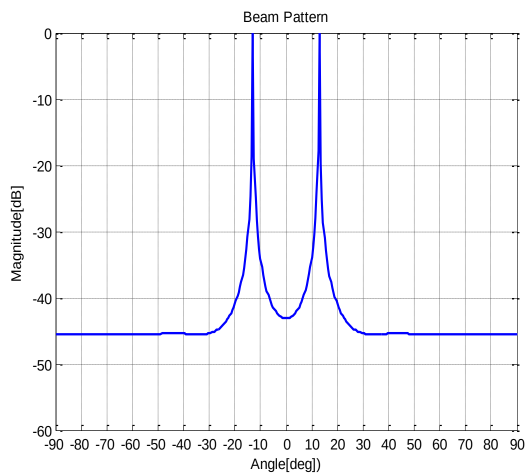


**Figure 2. DOA of Existing Method**

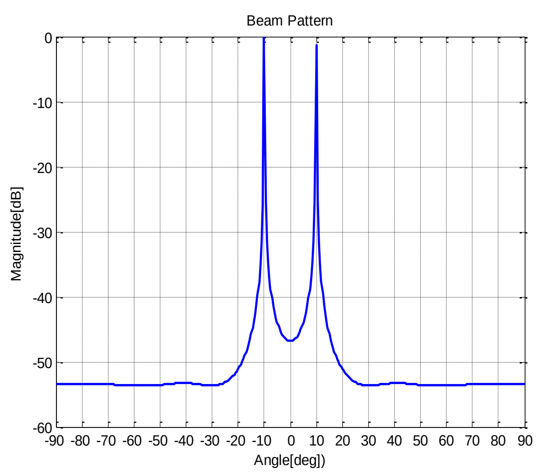


**Figure 2. DOA of proposed Method**

**Table 2. DOA Estimation Angle**

| Method | Estimation angle [-10$^o$ 10$^o$] |
|---|---|
| MUSIC | [-11.3789$^o$ 11.5987$^o$] |
| Proposed | [-10.1245$^o$ 10.0658$^o$] |

## 5. Conclusion

In this paper, we developed DOA methods for applying spatial smoothing using subarray method, thus making MUSIC and adaptive beamformers operative in a coherent interference environment to significantly improve performance robustness in DOA estimation and in adaptive beamforming. In order to apply spatial smooth subarray method, this array must have an orientational invariance structure and its center array must be ambiguity free. Also the number of subarrays must be greater than or equal to the largest number of mutually coherent signals. In order to apply spatial smooth subarray method in mix with MUSIC, all the subarrays must also be ambiguity free, and the number of sensors in each subarrays must be larger than the number of incoming signals.

We find that we can choose a square array with a sensor spacing less than half wavelength to meet all the conditions required for applying spatial smooth subarry. Simulation results show that for DOA estimation of coherent signals using spatial smooth subarry methods, a s square array has a preferred geometry in terms of the DOA estimation resolution and performance robustness.

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## Author

**Kwan Hyeong Lee**

Received ph.D degree in Electronics Engineering from Cheongju University, cheongju, korea, in 2004 respectively. From 1998 to 2004, He was work an associate professor, Dep. Information & Communication Gangneung Yeongdong College. From 2005 to 2007, He was work at full time lecture in Cheongju University. From 2007 to 2010, He was work at Agency for defense development. Currently, He is working an associate professor in Daejin University. He is interested the field that like to wireless communication and sensor network.