Active Tip Pointing Control of a Lightly Damped Beam using Integral-Derivative Feedback Control

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Abstract

This paper presents analysis on the performance and stability of an integral-derivative feedback system for active tip pointing control of a very flexible and slender cantilever beam. Since a piezoceramic PZT actuator at the fixed-end and a non-contacting displacement sensor at free-end of the beam are used for the control, the non-collocation of the sensor and the actuator makes that the plant is a non-minimum phase system. So this tip pointing control for a lightly damped beam requires a careful consideration on robust stability. Although the integral-derivative controller is simple to implement and its performance is modest, it can provide very robust stability. This is really important in some critical control systems. After computer simulation, it is observed that the integral-derivative controller can reduce the settling time to 50 % with robust stability and control gains. The stability and performance of the control systems in terms of the time domain and the frequency domain are discussed in detail. Thus the integral-derivative controller could be very effective to a tip pointing control of a flexible cantilever beam.

Keywords: *Integral-derivative feedback control, Lightly damped system, Non-minimum phase*

1. Introduction

A lightly damped beam has a long transient motion after a sudden input and this makes control of beam tip pointing at a certain position very difficult. Flexible structures also exhibit characteristics such as low-frequency, densely spaced and lightly damped modes, which make their control problem different from other control problems [1-3]. So far, the main emphasis in active vibration control field has been put to the suppression of unwanted vibration, for example vibration suppression in large space structures [4-7].

The object of this study is to investigate a classical PID method of actively controlling the tip pointing at a certain position of a flexible cantilever beam with piezoceramic actuators. The strategy is to actively control the position of the beam so that it follows a desired trajectory (set-point tracking), as required in robotics or positioning mechanism. In general, active control systems which can be defined as electrical or mechanical systems using a technique of controlling the outcome of the systems by the employment of actuators into the systems, whose output is dependent on the response of the systems. Thus, they are distinct from passive control systems which can alter the outcome of the systems without the introduction of actuators.

Active control techniques have been widely used in various systems especially in the area of sound and vibration control [7-9]. A system used to actively control the behavior of a structure must be comprised of both electrical and mechanical components: sensor, controller, plant and actuator. The role of sensors in active control systems is to measure the response of the system and to provide the information to the controller. In the study, the position control system for a single input single output (SISO) system requires one position sensor which measures the motion of the flexible beam.

A systematic approach for the integral-derivative feedback controller will be presented. The stability of active control system should be investigated carefully in the early stage of controller design [3, 9]. However, the performance of controllers will be degraded as a consequence of increasing of stability margins. Hence the trade-off between performance and stability is the key to controller design. This paper explains the beam dynamics in terms of frequency response function first and the design of the integral-derivative feedback controller is discussed. After that, computer simulation on the open-loop and the closed-loop response are described in detail.

2. Flexible Beam and Piezocermic Actuator Dynamics

Consider a very flexible cantilever beam (damping ratio ζ=0.01) with the length L_b = 1200 mm, the width B_b = 30 mm and the thickness t_b = 2 mm which has the fixed-free boundary condition and it is subjected to a harmonic bending moment $M(x,t)$ at $x = L_a$ as shown in Figure 1(a) [4, 8]. The tip motion $w(L_b,t)$ of the beam is expressed as [5],

$$w(L_b,t)=\sum_{n=1}^{\infty}B_n(t)\phi_n(L_b), \tag{1}$$

where $B_n(t)$ is the nth flexural modal amplitude and $\phi_n(L)$ is the nth flexural mode shape at beam tip. As shown in Figure 1(a), a bending moment is generated by a piezoceramic PZT actuator which is attached near the clamped end on the beam. The PZT actuator is assumed 100 mm long (L_a), 30 mm wide, and 1 mm thick.

By considering the boundary conditions of the beam, the theoretical plant model for the deflection response at $x = L_b$ to the moment at $x = L_a$ due can be derived as [5]

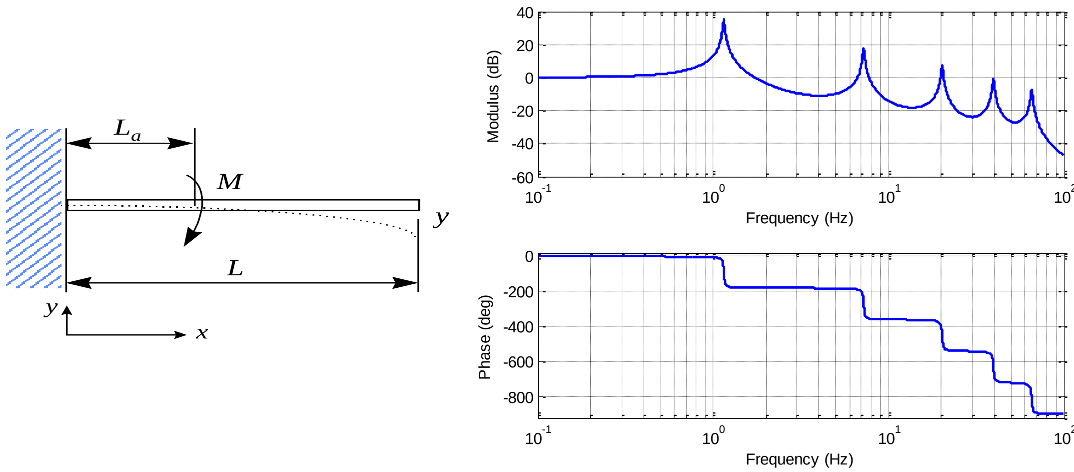
$$G(j\omega)=\frac{w(L_b)}{M(L_a)}=\sum_{n=1}^{\infty}\frac{\phi_n(L_b)\phi_n'(L_a)}{M_n[(\omega_n^2-\omega^2)+j2\zeta_n\omega_n\omega]}, \tag{2}$$

where ζ_n is the damping ratio of the each mode of the beam and $\phi_n'(L_a)$ is the spatial derivative of $\phi_n'(x)$ at $x = L_a$. As an actuator, the piezoceramic PZT patch induces bending moment at the end of the patch [4, 8].

The relationship between the bending moment M induced by the actuator when the applied input voltage $V(t)$ to the actuator can be written by [3, 4]

$$M=\frac{{t_b}^2E_bB_b(1+T)}{6+\varphi+12T+8T^2}\cdot\frac{d_{31}}{t_a}\cdot V(t) \tag{3}$$

where $T=\frac{t_a}{t_b}$, $\varphi=\frac{E_bB_bt_b}{E_aB_at_a}$, d_{31} is a piezoelectric constant (mV^{-1}) and B and t are the breadth and thickness respectively, and the subscripts a and b indicate the actuator and the beam. The calculated FRF (frequency response function) of the plant is illustrated in Figure 1(b), and it shows five flexural resonances at 1.15, 7.21, 20.19, 39.56 and 65.40 Hz below 100 Hz. The phase response indicates that a steep phase drop of 180 ^{o} at each mode due to the non-minimum phase property of the plant model as illustrated in Figure 1(b) [8, 9].



(a) (b)

Figure 1. (a) A flexible cantilever beam with a piezoceramic PZT actuator by the fixed end. The actuator generates a bending moment; (b) The calculated FRF of the plant model

3. Analogue Feedback Controller Design

A single input single output classical analogue feedback system is investigated in this study. Although the PID feedback controller has modest performance, it offers a robust stability to the control system without complicated electronics even if the plant is very lightly damped [6-9]. This makes the PID controller reviewed for better stability of the flexible system. Also it is assumed that the locations of the sensor and the actuator are apart as the actuator is on the fixed end and the sensor is at the free end. Thus the sensor-actuator response is non-minimum phase and this makes the control even more difficult.

The piezoceramic actuator is used to drive the beam motion so that its tip response is close to a given required command signal $r(j\omega)$. The position sensor detects the tip position of the flexible beam, that is the system output $y(j\omega)$. The positioning error signal $e(j\omega)$ is obtained from the difference between $r(j\omega)$ and $y(j\omega)$. This positioning error signal $e(j\omega)$ is fed through the feedback controller $H_I(j\omega)$ and the output of the controller is used to, then, drive the plant $G(j\omega)$ to track the command. As an analogue feedback controller, a combined form (integrator H_I in Figure 2(a) + differentiator H_D in Figure 2(b)) feedback controller H_{ID} is introduced for the position control system as plotted in Figure 2(c). This controller can provide better steady-state and transient response as normal PID controllers can provide an acceptable degree of error reduction simultaneously with acceptable stability and damping. As shown from Figure 2(c), this controller consists of two independent controllers H_I for reducing system errors and H_D for improving stability with two control loops (inner loop and outer loop).

The H_D feedback controller and the H_I feedback controller are defined by [6]

$$H_I(j\omega) = \frac{1}{j\omega RC} = \frac{K_I}{j\omega} \qquad H_D(j\omega) = RCj\omega = K_D j\omega \tag{4,5}$$

where R is resistance, C is capacitance, K_I and K_D are the integration and derivative feedback controller gains. The control signal is the integral of error signal at first, then it is the time rate of changes of the error. The inner loop can be regarded as $G'(j\omega)$ which has the H_D feedback controller, whereas the outer loop has the H_I feedback controller.

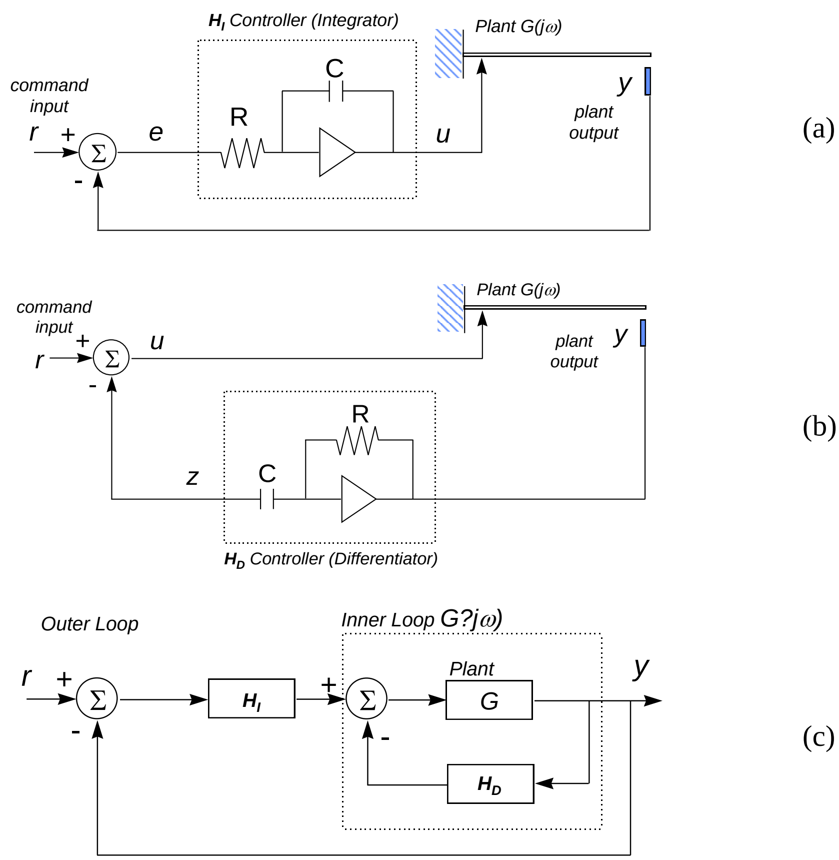


Figure 2. The control block diagrams of integral (*I*) feedback control system (a) H_I controller is an integrator; (b) The block diagram of the *I* feedback control system; (c) The block diagram of integral-derivative (*ID*) feedback control system in which the controller is a combined form of the integral controller and the derivative controller

As the frequency response of the inner loop $G'(j\omega)$ can be expressed as [6]

$$G'(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)H_D(j\omega)} \tag{6}$$

The frequency response of the open-loop with the integral-derivative feedback controller is expressed with

$$\left(\frac{y(j\omega)}{r(j\omega)}\right)_{OL} = G'(j\omega)H_I(j\omega) \tag{7}$$

Thus, the closed-loop response with the integral-derivative feedback controller H_{ID} is defined as [6]

$$\left(\frac{y(j\omega)}{r(j\omega)}\right)_{CL} = \frac{G'(j\omega)H_I(j\omega)}{1+G'(j\omega)H_I(j\omega)} = \frac{G(j\omega)H_I(j\omega)}{1+G(j\omega)(H_I(j\omega)+H_D(j\omega))}. \tag{8}$$

Therefore, the two control parameters (K_I and K_D) must be determined at the same time with a proper gain margin in the open-loop response.

4. Control Simulation, Results and Discussions

4.1 Integral-derivative feedback controller

A flexible cantilever beam for tip pointing control is considered and the beam length L_b = 1200 mm, the width B_b = 30 mm and the thickness t_b = 2 mm. The damping ratio of the beam is assumed =0.01. Computer simulations for two cases are accomplished.

At first case simulation, a constant derivative gain K_D = 0.001 is used whereas three different integral gains K_I = 0.055, 0.115 and 0.170 are applied for the integral-derivative controller. The frequency response functions of each gain combination are plotted in Figure 3(a), where the dotted line, the thick line and the thin line represent K_I = 0.055, 0.115 and 0.170 respectively. From Figure 3(a), the ID controllers introduce notches around 1 - 2 Hz due to the double actions by the differentiator and the integrator. The phase responses begin from -90^o and change to +90^o after the notch frequencies.

For second case simulation, a constant integral gain K_I = 0.115 is used whereas three different derivative gains $K_D = 10^{-2.915}$, 10^{-3} and $10^{-4.5}$ are applied for the integral-derivative controller. The frequency response functions of each gain combination are plotted in Figure 3(b), where the dotted line, the thick line and the thin line represent $K_D = 10^{-2.915}$, 10^{-3} and $10^{-4.5}$ respectively. From Figure 3(b), the ID controllers introduce notches around 1 - 10 Hz which are much larger than the first case.

4.2 Nyquist plots of open-loop responses

The stability of the control system must be examined carefully to design a stable controller. In order to assess the robustness of the controller, the Nyquist criterion is widely used which can be formulated in the frequency domain. This involves an investigation of the polar plot of the open-loop frequency response $G(j\omega)\ H_{ID}(j\omega)$ as shown in Figure 4. A close-loop system is stable only if the Nyquist plot of the stable open-loop frequency response does not enclose the Nyquist point (-1, j*0) in the s-domain [6].

The open-loop Nyquist plots for the previous two cases are illustrated in Figure 4. In first case simulation, the gains set the gain margins (GM) to be 9 dB (K_D = 0.001, K_I = 0.055, dotted line), 6 dB (K_D = 0.001, K_I = 0. 115, thick line) and 0.9 dB (K_D = 0.001, K_I = 0. 170, thin line) in the open-loop responses as shown in Figure 4(a).

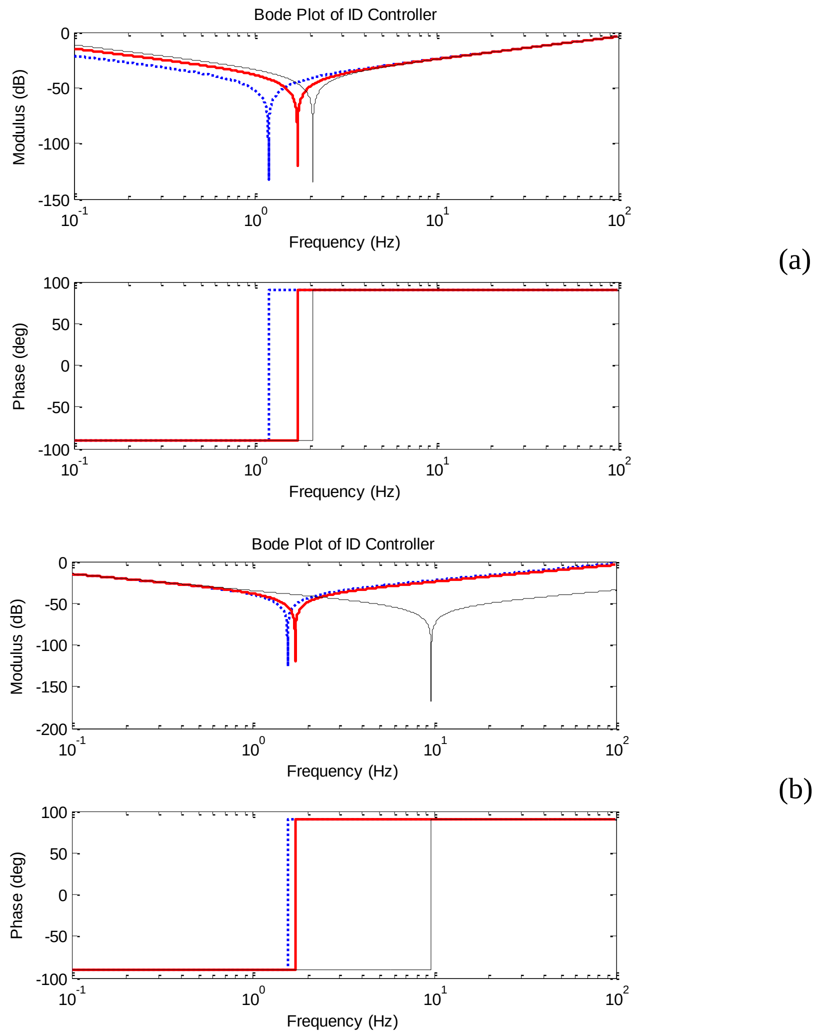


Figure 3. Comparison of the integral-derivative controller responses against frequency: (a) Integral gains of K_I = 0.055 (dotted line), 0.115 (thick line) and 0.170 (thin line) when the fixed derivative gain of K_D = 0.001 is used; (b) Derivative gains of $K_D = 10^{-2.915}$ (dotted line), 10^{-3} (thick line) and $10^{-4.5}$ (thin line) when the fixed integral gain of K_I = 0.115 is used

In the second case simulation, the gains set the GMs to be 8 dB (K_I = 0.115, $K_D = 10^{-2.915}$, dotted line), 6 dB (K_I = 0.115, $K_D = 10^{-3}$, thick line) and 0.9 dB (K_I = 0.115, $K_D = 10^{-4.5}$, thin line) in the open-loop responses as shown in Figure 4(b). It is noted that the largest response in the left half of the *s*-plane is the first bending mode of the plant and this threatens the control system stability in both cases.

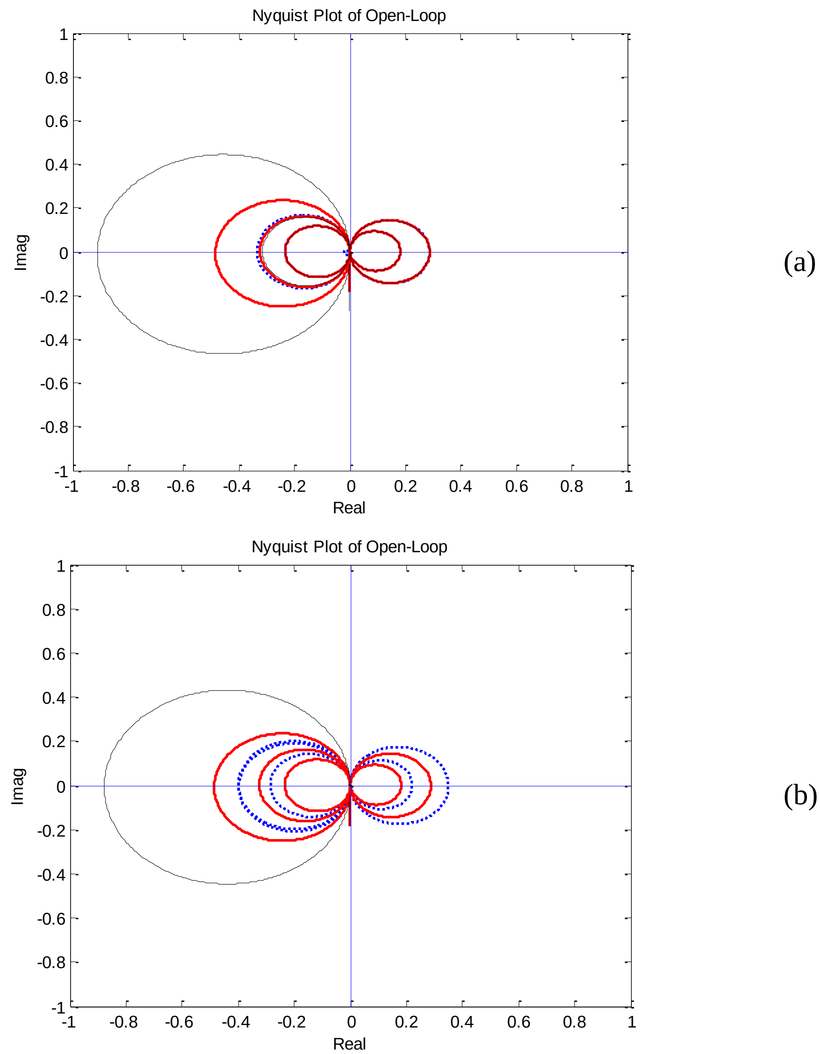
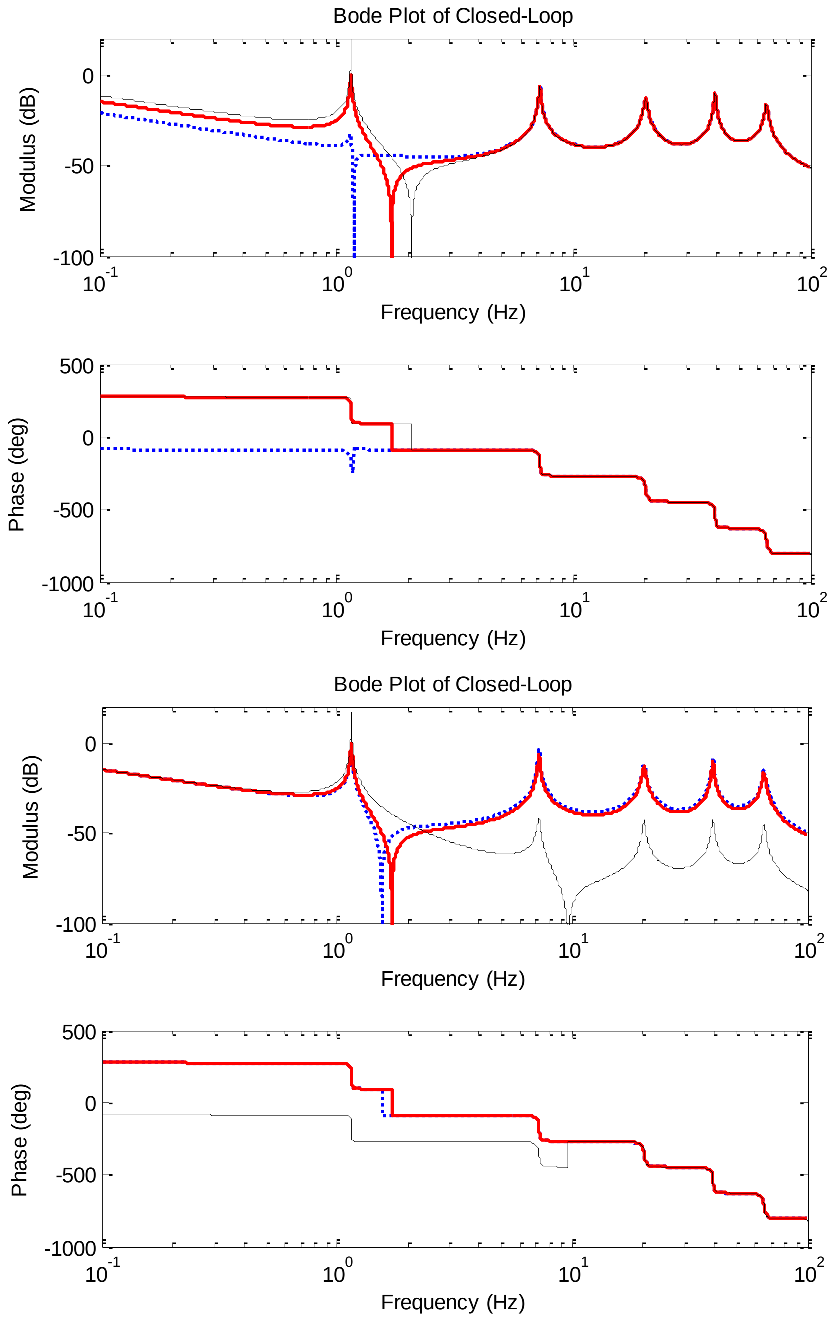


Figure 4. Comparison of the Nyquist plots of the integral-derivative controller responses: (a) First case: integral gains of K_I = 0.055 (9 dB GM, dotted line), 0.115 (6 dB GM, thick line) and 0.170 (0.9 dB GM, thin line) when the fixed K_D = 0.001 is used; (b) Second case: derivative gains of K_D = $10^{-2.915}$ (8 dB GM, dotted line), 10^{-3} (6 dB GM, thick line) and $10^{-4.5}$ (0.9 dB GM, thin line) when the K_I = 0.115 is used

4.3 Closed-loop frequency response functions

The frequency response functions of the closed-loop systems are plotted in Figure 5. In the first case simulation, the dotted line, the thick line and the thin line are the responses with GM = 9 dB (K_D = 0.001, K_I = 0.055), GM = 6 dB (K_D = 0.001, K_I = 0. 115) and GM = 0.9 dB (K_D = 0.001, K_I = 0. 170) respectively as shown in Figure 5(a).



(a)

(b)

Figure 5. Comparison of the closed-loop frequency response functions with the integral-derivative feedback controller: (a) First case: integral gains of K_I = 0.055 (9 dB GM, dotted line), 0.115 (6 dB GM, thick line) and 0.170 (0.9 dB GM, thin line) when the fixed K_D = 0.001 is used; (b) Second case: derivative gains of $K_D = 10^{-2.915}$ (8 dB GM, dotted line), 10^{-3} (6 dB GM, thick line) and $10^{-4.5}$ (0.9 dB GM, thin line) when the fixed K_I = 0.115 is used

In the second case simulation, the dotted line, the thick line and the thin line are the responses with GM = 8 dB (K_I = 0.115, $K_D = 10^{-2.915}$), GM = 6 dB (K_I = 0.115, $K_D = 10^{-3}$) and GM = 0.9 dB (K_I = 0.115, $K_D = 10^{-4.5}$) respectively as can be seen from Figure 5(b).

In the first case, the important variations are observed around the first bending modes due to the changes of the integral gains when the derivative gains are fiexed in the controllers. However, in second case, the responses of all five bending modes below 100 Hz can be affected by the changes of the derivative gains when the integral gains are fixed in the controllers.

4.4 Closed-loop step responses

It is known that an integral controller improves steady-state response in low frequency by feeding back the integrated signal of the tip position. However, the integral control gain must be decided carefully so as not to cause an unwanted transient response. The derivative feedback gain improves damping in the first resonance mode by feeding back the derivative signal of the tip position. Thus the integral-derivative controller can offer better performance than integral and derivative feedback controllers alone with proper gains.

In Figure 6, the step reponses of the closed-loop system are plotted to compare the performances of each integral-derivative controller. Figure 6(a) is the step response of the plant model with any control action and its settling time is about 80 seconds.

The first case is shown in Figure 6(b) as the step responses of the controllers are represented with the thin line (GM = 0.9 dB, K_D = 0.001, K_I = 0. 170), the thick line (GM = 6 dB, K_D = 0.001, K_I = 0. 115) and the dotted line (GM = 9 dB, K_D = 0.001, K_I = 0.055) and their settling times are about 30, 40 and 75 senonds respectively.

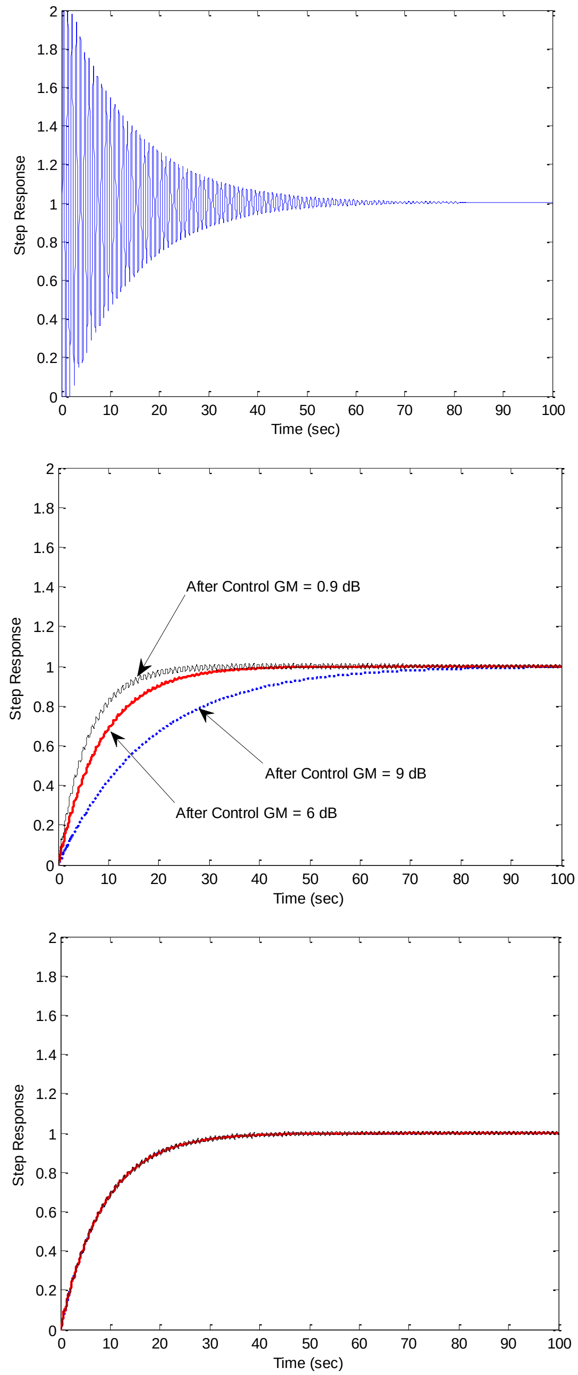
The second case is shown in Figure 6(c) as the step responses of the controllers are represented with the thin line (GM = 0.9 dB, K_I = 0.115, $K_D = 10^{-4.5}$), the thick line (GM = 6 dB, K_I = 0.115, $K_D = 10^{-3}$) and the dotted line (GM = 8 dB, K_I = 0.115, $K_D = 10^{-2.915}$) and their settling times are all about 40 seconds.

It is especially noted that the step response of the plant *before control* shows a very large and long ringing motion as plotted in Figure 6(a). So the introduction of the integral-derivative feedback control provides to minimize the ringing motion at the beam tip.

4.5 Power spectrum before and after control

The power spectra before and control are plotted in Figure 7(a) for the first case and Figure 7(b) for the second case. From Figure 7(a) as the result of the first case, the power spectrum after control at the first bending mode is dramatically reduced compared to that before control by the increase of the integral gain while the derivative gain is fixed.

However the power spectrum at the second and the above doesn’t changed that much. Also the integraton action at very low frequency range below 1 Hz gives a large effect to the closed-loop response which reduces the steady-state error.



(a)

(b)

(c)

Figure 6. Step responses before and after control. (a) Before control. (b) First case: After control with K_I = 0.055 (9 dB GM, dotted line), 0.115 (6 dB GM, thick line) and 0.170 (0.9 dB GM, thin line) when the fixed K_D = 0.001 is used. (c) Second case: After control with $K_D = 10^{-2.915}$ (8 dB GM, dotted line), 10^{-3} (6 dB GM, thick line) and $10^{-4.5}$ (0.9 dB GM, thin line) when the fixed K_I = 0.115 is used.

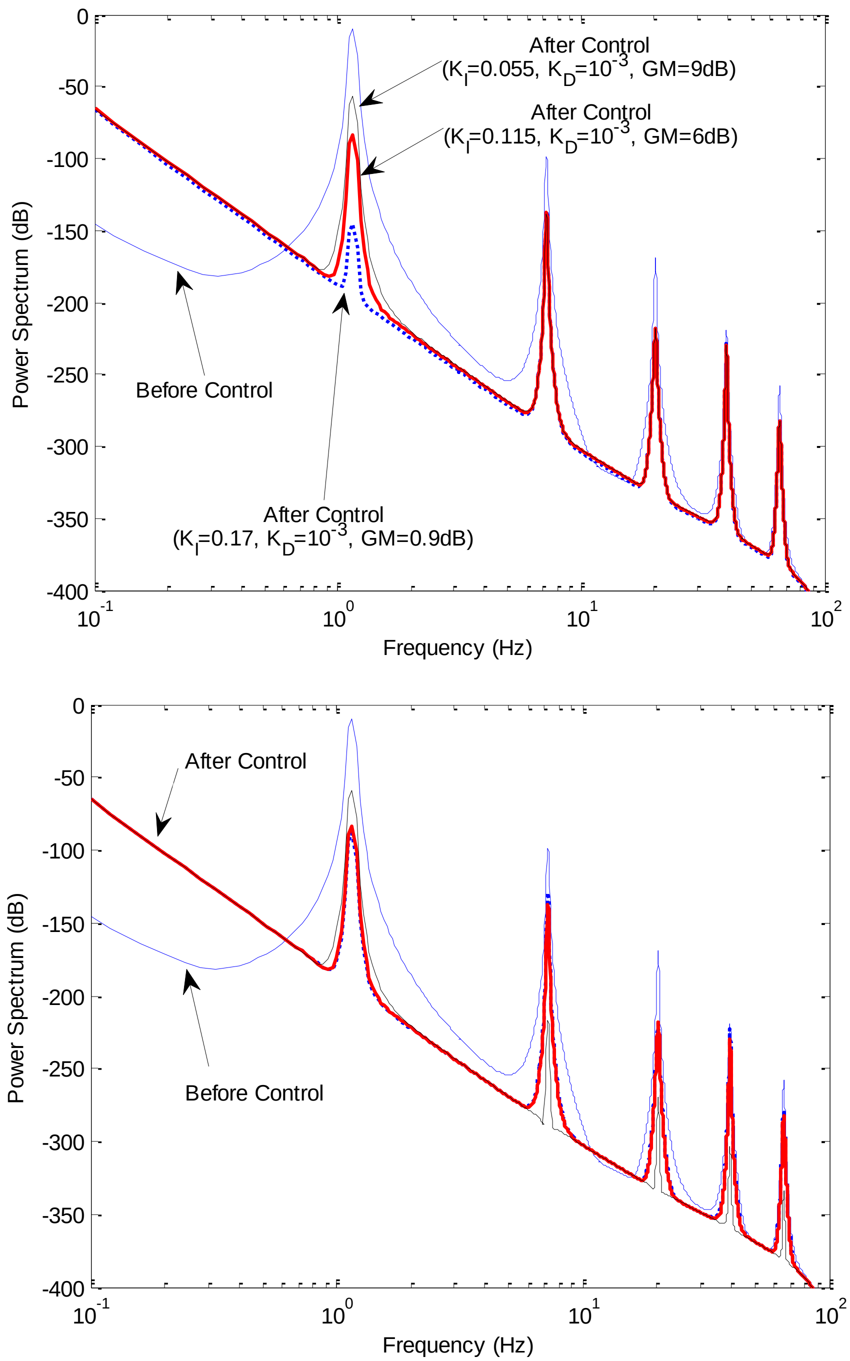


Figure 7. Comparison of the power spectra before and after control: (a) First case: K_I = 0.055 (9 dB GM, dotted line), 0.115 (6 dB GM, thick line) and 0.170 (0.9 dB GM, thin line) while the fixed K_D = 0.001 is used; (b) Second case: $K_D = 10^{-2.915}$ (8 dB GM, dotted line), 10^{-3} (6 dB GM, thick line) and $10^{-4.5}$ (0.9 dB GM, thin line) while the fixed K_I = 0.115 is used

The power spectrum after control plotted in Figure 7(b), as the result of the second case, is very similar to the first case in Figure 7(a).

It should be noted that the integral action enhances the low frequency response and the derivative action introduces the damping to the closed-loop system by the integral-derivative controller. Especially this controller is very effective to a tip pointing control of a flexible cantilever beam.

5. Conclusions

A flexible cantilever beam with a piezoceramic actuator near the fixed end and a non-contacting position sensor located at the tip is investigated in this paper. The very low damping and non-minimum phase make the tip pointing control difficult. For this problem, an analogue PID feedback controller using integration and derivation is considered. The integral-derivative feedback control shows a good stability and modest performance for the tip pointing control of the cantilever beam, although the beam is very lightly damped and slender. The integral action enhances the low frequency response and the derivative action introduces the damping to the closed-loop system by the integral-derivative controller. Thus the integral-derivative controller is very effective to a tip pointing control of a flexible cantilever beam.

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