Transient Response Improvement for Uncertain Time-varying Nonlinear Systems via Switching Control

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Abstract

Because nonlinear systems have more complex behaviors than linear systems, it is hard to apply classical linear system theories and techniques to analyze and control nonlinear systems. So, to apply these theories and techniques, the feedback linearization method is used when the nonlinear system satisfies certain conditions. In the linearizing procedure, there exist infinite pairs of coordinates transformations and feedback controllers. The freedom in the exact linearization technique is used to improve the feedback system in some sense. So, by the switching control with the proper switching rule, we can obtain the better transient behavior than that of using one diffeomorphism. For many real processes, nonlinear systems have uncertainties. Because the linearizing method requires the accurate mathematical models of the system for the exact linearization, the uncertainties have an effect on stability of the nonlinear system. But, if we know the bounds of uncertainties, we can compensate the uncertainties based on the Lyapunov redesign technique. In this paper, we propose a switching rule for taking the transient behavior improvements in the case of uncertain time-varying nonlinear system.

Keywords: *Multi-diffeomorphism, Feedback Linearization, Uncertainty, Switching Rule*

1. Introduction

In real life, most of phenomena and systems can be modeled in the form of nonlinear systems which involve linear systems. It is a well-known fact that because nonlinear systems have more complex behaviors than linear systems, it is hard to apply classical linear system theories and techniques to analyze and control nonlinear systems. So, in recent years, many scientist and engineers have conducted serious and meaningful researches to develop new theories to deal with nonlinear systems. For example, there are Lyapunov function theory to analyze stability of nonlinear systems, feedback linearization method to transform the original nonlinear system dynamics into equivalent linear systems, and many control techniques (backstepping control, gain scheduling control, sliding mode control, Lyapunov redesign control, *etc.*) to control nonlinear systems.

As mentioned above, feedback linearization method transforms the nonlinear systems into the equivalent linear forms. Thus, we can apply linear system theories and techniques to control nonlinear systems. In general, the feedback linearzation method requires the accurate mathematical models of the systems for the exact linearization of the closed-system. However, for many real processes, there exist inevitable uncertainties in their constructed models. The uncertainties may include parameter uncertainties, modeling error, and input disturbances. Since the linearization procedure relies on an exact cancelation of nonlinear terms, the existence of uncertainties degrades the control performances and sometimes influences the stability of the systems. Therefore, there have been many researches to get over the uncertainty problems. Lyapunov redesign is one of the typical examples. [13] and [14] compensates the model uncertainties that do not satisfy the conventional matching condition but states and tracking error remain to be bounded. [15] Presents a result of the robust stabilization of a class of nonlinear systems exhibiting parametric uncertainties. In [5], non-high gain feedback controller and high gain controller with a parameter diffeomorphism are employed to achieve robust stability with matched uncertainties and unmatched uncertainties respectively.

Many researches about feedback linearization method with uncertainties are limited to time-invariant nonlinear systems and uncertainties as in [5, 14 and 15]. But, in real cases, many systems can be modeled by time-varying nonlinear systems. Thus, the problem of applying feedback linearization method for time-varying system and compensating time-varying uncertainty has been studied. Feedback linearization of time-varying nonlinear systems was dealt with in [2] and time-varying uncertainty was dealt with in [17 and 16].

Many systems encountered in practice exhibit switching between several subsystems that is depend on various environmental factors. Some examples of such systems are discussed in [18] and [20]. Control techniques based on switching between different controllers have been applied extensively in recent years, particularly in the adaptive context, where they have been shown to achieve stability and improve transient behavior. Switched systems have numerous applications in control of mechanical systems, the automotive industry, aircraft and air traffic control, switching power converters, and many other fields. The stability of the closed-loop controlled system depends on the switching strategy.

Switched control between stable systems not necessarily implies a stable transient behavior [9]. On the other hand, by a proper switching strategy, switching between unstable systems can make the system stable [10]. In [11], min-projection strategy that the vector field associated with the smallest projection is selected for each state is proposed. [7] Addresses a new stability analysis of switched systems by introducing the concepts of minimum/ maximum holding time and redundancy as a tool for Lyapunov stability. Switching control also can be applied in the discrete system. [12] Addresses the problem of stability analysis and control synthesis of switched systems in the discrete-time domain.

Now, returning to the feedback linearizarion, feedback linearization technique is one of the most effective methods in the field of nonlinear control. In order to use feedback linearzation technique, we should solve a series of P.D.E's(partial derivative equation). The freedom in exact linearization technique make it possible to improve the transient behavior of the nonlinear systems [3]. That is, feedback linearizable system has various diffeomorphisms that the transient behavior improvement exist each other and by using a proper switching strategy, the controlled system is improved more than that using one diffeomorphism [4].

In this paper, we propose a switching rule to take the transient behavior improvements for uncertain time-varying nonlinear system. First, we design the controllers to compensate the uncertainties. After design the controllers, the switching rule is proposed.

2. Problem Formulation

Consider the following single-input uncertain time-varying system

$$\dot{x} = f(x,\theta(t)) + \Delta f(x,\theta(t)) + (g(x,\theta(t)) + \Delta g(x,\theta(t)))u \tag{1}$$

where $\theta(t) \in D_\theta \subset R^P$ is a vector of time-varying parameters, $f(x,\theta(t))$, $\Delta f(x,\theta(t))$, $g(x,\theta(t))$, $\Delta g(x,\theta(t))$ are smooth on R^n. $\Delta f(x,\theta(t))$, $\Delta g(x,\theta(t))$ are uncertainties or unmodeled parts of the system.

Assumption 1. The following single-input time-varying system without uncertainties

$$\dot{x} = f(x,\theta(t)) + g(x,\theta(t))u \tag{2}$$

is input-state feedback LTIsable [3].

Let $z = \hat{T}_1(x)$ is a time-varying diffeomorphism such that it transforms (2) into a linear system. Then, by $z = \hat{T}_1(x)$ and a feedback controller $u = \alpha(x,\theta(t),\cdots,\theta^n(t)) + \gamma^{-1}(x,\theta(t),\cdots,\theta^{n-1}(t))v$ $= -\dfrac{\hat{L}_f^n h(x,\theta(t))}{\bar{L}_g \hat{L}_f^{n-1} h(x,\theta(t))} + \dfrac{1}{\bar{L}_g \hat{L}_f^{n-1} h(x,\theta(t))} v,$ where $h(x,\theta(t))$ is satisfied with following partial derivative equations :

$$\bar{L}_{\overline{ad}_f^i g} h(x,\theta(t)) = 0, \quad i = 0,1,\cdots,n-2 \tag{3}$$

$$\bar{L}_g \hat{L}_f^{n-1} h(x,\theta(t)) \neq 0 \tag{4}$$

(2) becomes the following form :

$$\dot{z} = A_c z + B_c v \tag{5}$$

where $\{A_c, B_c\}$ is Brunovsky canonical pair. And then, by setting $v = Kz = k_1 z_1 + \cdots + k_n z_n$, we get

$$\dot{z} = Az \tag{6}$$

and K should be chosen that A is Hurwitz. With this procedure, we can get a pair of exactly linearizing diffeomorphism and feedback controller. However, there exist infinite pairs of diffeomorphisms and feedback controllers since (3) and (4) have infinitely many solutions. If we get one solution of the PDEs, then all the solutions of the PDEs are given by the following lemma.

Lemma 1. If a function $\varphi(x,\theta(t))$ solve the PDEs (3) and (4) by substituting $\varphi(x,\theta(t))$ for $h(x,\theta(t))$ near x^0, then the function $\lambda(x,\theta(t))$ (locally) satisfies the PDEs if

$$\lambda(x,\theta(t)) = \psi(\varphi(x,\theta(t))) \tag{7}$$

$$\frac{\partial \psi}{\partial \varphi} \neq 0 \tag{8}$$

$$\bar{L}_g \hat{L}_f^i \left(\frac{\partial \psi}{\partial \varphi} \right) = 0 \ (i = 0,1,\cdots,n-2) \tag{9}$$

hold for some smooth function ψ in a neighborhood of x^0.
Proof. $\lambda(x,\theta(t))$ satisfies the PDEs (3) and (4).
By using $z = \hat{T}_i(x,\theta(t),\cdots,\theta^{n-1}(t))$, the system (1) is transformed into the the following form :

$$\begin{aligned}
\dot{z} &= A_c z + B_c \gamma_i (x,\theta(t),\cdots,\theta^{n-1}(t))(u_i - \alpha_i(x,\theta(t),\cdots,\theta^n(t))) \\
&\quad + \frac{\partial \hat{T}_i}{\partial x} \Delta f(x,\theta(t)) + \frac{\partial \hat{T}_i}{\partial x} \Delta g(x,\theta(t)) u_i \\
&= A_c z + B_c \gamma_i (x,\theta(t),\cdots,\theta^{n-1}(t))(u_i - \alpha_i(x,\theta(t),\cdots,\theta^n(t))) \\
&\quad + \Delta \bar{f}_i (z,\theta(t),\cdots,\theta^n(t)) + \Delta \bar{g}_i (z,\theta(t),\cdots,\theta^n(t)) u_i
\end{aligned} \tag{10}$$

for $x \in D_x$, $z \in D_z = \hat{T}_i(D_x,\theta(t),\cdots,\theta^{n-1}(t))$ where $\{A_c, B_c\}$ is Brunovsky canonical pair and $\gamma_i(x,\theta(t),\cdots,\theta^{n-1}(t)) = \bar{L}_g \hat{L}_f^{n-1} h_i(x,\theta(t))$, $\alpha_i(x,\theta(t),\cdots,\theta^n(t)) = -\dfrac{\hat{L}_f^n h_i(x,\theta(t))}{\bar{L}_g \hat{L}_f^{n-1} h_i(x,\theta(t))}$ are satisfied.

Assumption 2. There exist some constants $c_{\bar{f}i\_1}$, $c_{\bar{f}i\_2}$, $c_{\bar{g}i\_1}$, and $c_{\bar{g}i\_2}$ that satisfy following inequalities :

$$\left\|\Delta\bar{f}_i(z,\theta(t),\cdots,\theta^n(t))\right\| \le c_{\bar{f}i\_1}\|z\| + c_{\bar{f}i\_2} \tag{11}$$

$$\left\|\Delta\bar{g}_i(z,\theta(t),\cdots,\theta^n(t))\right\| \le c_{\bar{g}i\_1}\|z\| + c_{\bar{g}i\_2} \tag{12}$$

for $z \in D_z$.

Assumption 3. The uncertainties satisfy following conditions.

$$\Delta f(x,\theta(t)) = span\{g(x,\theta(t))\} = g(x,\theta(t))\, f^*(x,\theta(t)) \tag{13}$$

$$\Delta g(x,\theta(t)) = span\{g(x,\theta(t))\} = g(x,\theta(t)) g^*(x,\theta(t)) \tag{14}$$

Lemma 2. If the assumption 2 and 3 are satisfied, the following inequalities are satisfied.

$$\left\|z^T P\Delta\bar{f}_i(z,t)\right\| \le (c_{\bar{f}i\_1}\|z\| + c_{\bar{f}i\_2})\left\|z^T PB_c\right\| \tag{15}$$

$$\left\|z^T P\Delta\bar{g}_i(z,t)\right\| \le (c_{\bar{g}i\_1}\|z\| + c_{\bar{g}i\_2})\left\|z^T PB_c\right\| \tag{16}$$

Proof. From (13), we have

$$\begin{aligned}
\Delta\bar{f}_i(z,\theta(t),\cdots,\theta^n(t)) &= \left[\frac{\partial\hat{T}_i}{\partial x}\Delta f(x,\theta(t))\right]_{x=\hat{T}_i^{-1}(z,\theta(t),\cdots,\theta^{(n-1)}(t))} \\
&= \left[\frac{\partial\hat{T}_i}{\partial x}g(x,\theta(t))\, f^*(x,\theta(t))\right]_{x=\hat{T}_i^{-1}(z,t)} \\
&= \left[B_c\gamma_i(x,\theta(t),\cdots,\theta^{(n-1)}(t))\, f^*(x,\theta(t))\right]_{x=\hat{T}_i^{-1}(z,t)} \\
&= B_c\Delta\hat{f}_i(z,t)
\end{aligned} \tag{17}$$

From (17), we have

$$\begin{aligned}
\left\|z^T P\Delta\bar{f}_i(z,t)\right\| &= \left\|z^T PB_c\Delta\hat{f}_i(z,t)\right\| \\
&\le \left\|z^T PB_c\right\|\left\|\Delta\hat{f}_i(z,t)\right\| \\
&= \left\|z^T PB_c\right\|\left\|B_c\Delta\hat{f}_i(z,t)\right\| \\
&= \left\|z^T PB_c\right\|\left\|\Delta\bar{f}_i(z,t)\right\| \\
&\le (c_{\bar{f}i_1}\|z\| + c_{\bar{f}i_2})\left\|z^T PB_c\right\|
\end{aligned} \tag{18}$$

Similarly, from (14), we have

$$\left\|z^T P\Delta\bar{g}_i(z,t)\right\| \le (c_{\bar{g}i\_1}\|z\| + c_{\bar{g}i\_2})\left\|z^T PB_c\right\| \tag{19}$$

3. Controller Design

In this subsection, we propose a controller that compensates the uncertainties based on the Lyapunov redesign. We consider the following control structure of the form:

$$\begin{aligned}
u_i &= \alpha_i(x,\theta(t),\cdots,\theta^n(t)) + \gamma_i^{-1}(x,\theta(t),\cdots,\theta^{n-1}(t))v_i \\
v_i &= v_{i_1} + v_{i_2}
\end{aligned}$$

where $v_{i\_2}(t)$ is a controller to overcome the uncertainties of the system. Our control law is as follows.

Theorem 1. The uncertain nonlinear system (10) is asymptotically stable by using the control law given by

$$u_i = \alpha_i(x,\theta(t),\cdots,\theta^n(t)) + \gamma_i^{-1}(x,\theta(t),\cdots,\theta^{(n-1)}(t))(v_{i\_1}+v_{i\_2}) \quad (20)$$

$$v_{i\_1} = Kz = k_1 z_1 + \cdots + k_n z_n \quad (21)$$

$$v_{i\_2} = -\eta_i \frac{z^T PB_c}{\left\| z^T PB_c \right\|} \quad (22)$$

$$\eta_i = \frac{c_{\bar{f}i\_1}\|z\| + c_{\bar{f}i\_2} + \left(c_{\bar{g}i\_1}\|z\| + c_{\bar{g}i\_2}\right)\left(\|\alpha_i(x,t)\| + \left\|\gamma_i^{-1}(x,t)\right\|\|Kz\|\right)}{1-\left(c_{\bar{g}i\_1}\|z\| + c_{\bar{g}i\_2}\right)\left\|\gamma_i^{-1}(x,t)\right\|} \quad (23)$$

for $z \in D_z{}' = \{z \mid z \in D_z \ and \left(c_{\bar{g}i\_1}\|z\| + c_{\bar{g}i\_2}\right)\left\|\gamma_i^{-1}(x,t)\right\| < 1\}$ where $z = \hat{T}_i(x,t)$ is satisfied.

Proof. Let the Lyapunov function be $V = z^T Pz$ where $P > 0$, $Q > 0$, $A = A_c + B_c K$, and $A^T P + PA = -Q$. By taking the derivative of V, we have

$$\begin{aligned}\dot{V} &= \dot{z}^T Pz + z^T P\dot{z} \\ &= [A_c z + B_c \gamma_i(x,t)(u_i - \alpha_i(x,t)) + \Delta\bar{f}_i(z,t) + \Delta\bar{g}_i(z,t)u_i]^T Pz \\ &\quad + z^T P[A_c z + B_c \gamma_i(x,t)(u_i - \alpha_i(x,t)) + \Delta\bar{f}_i(z,t) + \Delta\bar{g}_i(z,t)u_i]\end{aligned}$$

By our control law (20)-(23), we have

$$\begin{aligned}\dot{V} \le& -\lambda_{\min}(Q)\|z\|^2 + 2z^T PB_c v_{i\_2} + 2\left\|z^T P\Delta\bar{f}_i(z,t)\right\| + 2\left\|z^T P\Delta\bar{g}_i(z,\cdot)\right\|\|\alpha_i(x,t)\| \\ &+2\left\|z^T P\Delta\bar{g}_i(z,t)\right\|\left\|\gamma_i^{-1}(x,t)\right\|\|Kz\| + 2\left\|z^T P\Delta\bar{g}_i(z,t)\right\|\left\|\gamma_i^{-1}(x,t)\right\|\left\|v_{i\_2}\right\|\end{aligned}$$

By Lemma 2, we have

$$\begin{aligned}\dot{V} \le& -\lambda_{\min}(Q)\|z\|^2 - 2\left\{1-\left(c_{\bar{g}i\_1}\|z\| + c_{\bar{g}i\_2}\right)\left\|\gamma_i^{-1}(x,t)\right\|\right\}\left\|z^T PB_c\right\|\eta_i \\ &+2\left(c_{\bar{f}i\_1}\|z\| + c_{\bar{f}i\_2}\right)\left\|z^T PB_c\right\| \\ &+2\left(c_{\bar{g}i\_1}\|z\| + c_{\bar{g}i\_2}\right)\left(\|\alpha_i(x,t)\| + \left\|\gamma_i^{-1}(x,t)\right\|\|Kz\|\right)\left\|z^T PB_c\right\| \\ =& -\lambda_{\min}(Q)\|z\|^2\end{aligned}$$

Thus, the system (10) is asymptotically stable. Moreover, it is exponentially stable.

4. Switching Rule for Multi-diffeomorphism

The system given by (1) is transformed to (10) by $z = \hat{T}_i(x,t)$ and feedback controller u_i in Theorem 1, where $i = 1,2,\cdots,m.$ By the feedback controller u_i, the closed system is presented as

$$\dot{x} = f_i(x,t), i = 1,2,\cdots,m \quad (24)$$

Let $\dot{x} = f_{k_j}(x,t)$, $k_j \in [1,m]$ and $j \in [0,\infty)$, denote the system that is active for $t_j \le t < t_{j+1}$. Then, in order to establish the switching rule, following assumption is needed.

Assumption 4. There exists a nonzero limit of $\alpha_{k_j}(x,t)$ as $x \to 0$ where $\alpha_{k_j}(x,t) \triangleq \frac{\|\hat{T}_{k_j}(x,t)\|}{\|x\|}$, $k_j \in [1,m]$ and $j \in [0,\infty)$ for $x \in D_x$.

Let $V(z) = z^T P z$, where $P > 0,\ Q > 0,$ and $A^T P + PA = -Q,$ be a Lyapunov function of (10). Then (24) has a Lyapunov function as

$$V_i(x,t) = \hat{T}_i(x,t)^T P \hat{T}_i(x,t) \tag{25}$$

Lemma 3. The solution of (24) is exponentially decayed as

$$\|x(t)\| \le \left(\frac{M}{m}\right)^{\frac{1}{2}} \frac{\alpha_i(x(t_0),t_0)}{\alpha_i(x(t),t)} \|x(t_0)\| e^{-\frac{\gamma}{2M}(t-t_0)} \tag{26}$$

where $\lambda_{\min}(P) = m,\ \lambda_{\max}(P) = M,\ \lambda_{\min}(Q) = \gamma,$ and $\|\hat{T}_i(x,t)\| = \alpha_i(x,t)\|x(t)\|.$

Proof. The proof is obtained by using the equation (25).

Theorem 2. The origin of switched system (24) with the following switching rule is exponentially stable. Furthermore, the exponential upper bound of the switched system is smaller than that of the one system given by $\dot{x} = f_i(x,t),\ t \in [t_0,\infty).$

-Switching rule:

Step 1 : Set $i = 0$.

Step 2 : If

$$\min_{k_{i+1}\in\{2,\cdots,m\}} \frac{\alpha_{k_{i+1}}(x(t_i),t_i)}{\alpha_{k_{i+1}}(x(t),t)} < \left(\frac{M}{m}\right)^{-1} \frac{\alpha_1(x(t_i),t_i)}{\alpha_1(x(t),t)} \tag{27}$$

at $t = t_{i+1} > t_i,$ then $\dot{x} = f_{k_{i+1}}(x,t)$ is active for $t \ge t_{i+1}.$ Otherwise, $\dot{x} = f_1(x,t)$ is active for $t \ge t_{i+1}.$

Step 3 : Let $i = i+1,$ go to Step 2.

Proof. For $t_1 < t \le t_2,\ \dot{x} = f_{k_1}(x)$ is active.

$$\begin{aligned}\|x(t)\| &\le \left(\frac{M}{m}\right)^{\frac{1}{2}} \frac{\alpha_{k_1}(x(t_1),t_1)}{\alpha_{k_1}(x(t),t)} \left(\frac{M}{m}\right)^{\frac{1}{2}} \frac{\alpha_1(x(t_0),t_0)}{\alpha_1(x(t_1),t_1)} \|x(t_0)\| e^{-\frac{\gamma}{2M}(t_1-t_0)} e^{-\frac{\gamma}{2M}(t-t_1)} \\ &< \frac{\alpha_1(x(t_1),t_1)}{\alpha_1(x(t),t)} \frac{\alpha_1(x(t_0),t_0)}{\alpha_1(x(t_1),t_1)} \|x(t_0)\| e^{-\frac{\gamma}{2M}(t-t_0)} \\ &\le \left(\frac{M}{m}\right)^{\frac{1}{2}} \frac{\alpha_1(x(t_0),t_0)}{\alpha_1(x(t),t)} \|x(t_0)\| e^{-\frac{\gamma}{2M}(t-t_0)}\end{aligned}$$

Similarly, for $t_j \to \infty,$ that is, for all t,

$$\|x(t)\| < \left(\frac{M}{m}\right)^{\frac{1}{2}} \frac{\alpha_1(x(t_0))}{\alpha_1(x(t))} \|x(t_0)\| e^{-\frac{\gamma}{2M}(t-t_0)}$$

5. Simulation Result

Consider the following single-input time-varying uncertain nonlinear system

$$\dot{x}_1 = x_1^2 + x_2$$

$$\dot{x}_2 = x_1 \cos^2 t + x_2 + \Delta f(x, \theta(t)) + (1 + \Delta g(x, \theta(t)))u$$

where $\Delta f(x,\theta(t)) = 0.5 \sin x_1 \cos^2 t$ and $\Delta g(x,\theta(t)) = -0.5$ are satisfied.
Let $h_1(x,\theta(t)) = x_1$, then we can design the stabilizing feedback controller as follows.

$$u_1 = \alpha_1(x,t) + \gamma_1^{-1}(x,t)(v_{1\_1} + v_{1\_2})$$

$$v_{1\_1} = Kz = -z_1 - 2z_2$$

$$v_{1\_2} = -\eta_1 \frac{z_1 + z_2}{\|z_1 + z_2\|}$$

$$\eta_1 = \frac{0.5\|z\| + 0.5\left(\|\alpha_1(x,t)\| + \|-z_1 - 2z_2\|\right)}{0.5}$$

$$\alpha_1(x,t) = -\{2x_1(x_1^2 + x_2) + x_1 \cos^2 t + x_2\}$$

$$\gamma_1(x,t) = 1$$

Also, from Lemma 1, we can set $z_1 = \psi(x_1) = \sinh x_1$ because $\frac{\partial \psi(x_1)}{\partial x_1} \neq 0$ and $\bar{L}_g \hat{L}_f^i \left(\frac{\partial \psi(x_1)}{\partial x_1} \right) = 0$ for $i = 0, \cdots, n-2$. Thus, we can design the stabilizing feedback controller as follows.

$$u_2 = \alpha_2(x,t) + \gamma_2^{-1}(x,t)(v_{2\_1} + v_{2\_2})$$

$$v_{2\_1} = Kz = -z_1 - 2z_2$$

$$v_{2\_2} = -\eta_2 \frac{z_1 + z_2}{\|z_1 + z_2\|}$$

$$\eta_2 = \frac{0.6 + 0.6\left(\|\alpha_2(x,t)\| + \|-z_1 - 2z_2\| \|\gamma_2^{-1}(x,t)\|\right)}{1 - 0.6\|\gamma_2^{-1}(x,t)\|}$$

$$\alpha_2(x,t) = -\frac{\sinh x_1 \{x_1^2 + x_2\}^2 + \cosh x_1 \{2x_1(x_1^2 + x_2) + x_1 \cos^2 t + x_2\}}{\cosh x_1}$$

$$\gamma_2(x,t) = \cosh x_1$$

The results are depicted in Figure 1 and Figure 2.

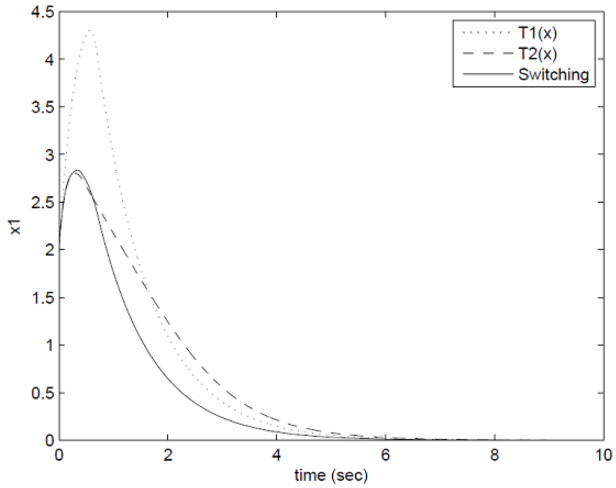

Figure 1. Comparison of three cases for x_1

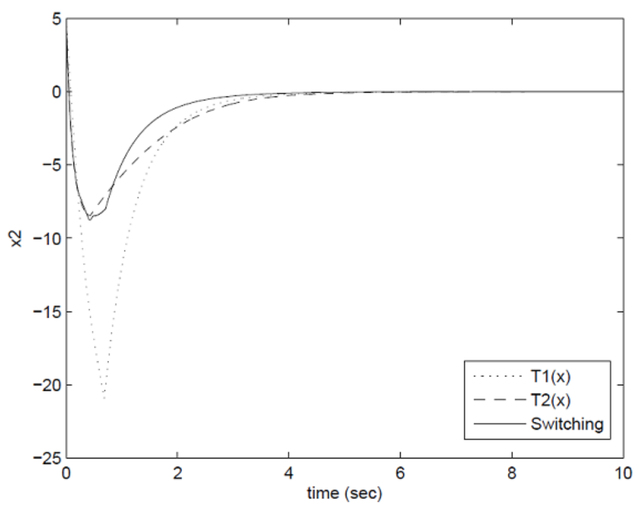

Figure 2. Comparison of three cases for x_2

The trajectory that generated by the switching control has better transient behavior than that of using one control input.

6. Conclusion

The aim of this paper is to propose a switching rule for uncertain time-varying nonlinear system by using multi-diffeomorphism. We formulate a set of problem mathematically in case of the existence of uncertainties. And then, the controllers and the switching rule for the switching control to improve transient behavior are proposed, analyzed, and simulated in case of time-varying system. The simulation results show that the trajectory of the switched system has improved transient behavior than that of each system.

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