Multi-segment Identification Method for Multi-sensor Multi-channel ARMA Signal with White Measurement Noise

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Abstract

For the multi-sensor multi-channel ARMA(the autoregressive and moving average model) signal with white measurement noise, unknown model parameters and noise variance, a multi-segment identification method is proposed. The simulation example shows the effectiveness of this method. These identification methods can be used to design self-turning information filters.

Keywords: *noise variance; information fusion; RIV*

1. Problem Formulation

Consider the multi-sensor system with the white observation noises

$$A(q^{-1})s(t) = C(q^{-1})w(t) \tag{1}$$

$$y_i(t) = s(t) + \xi(t) + v_i(t), \quad i = 1, \cdots, L \tag{2}$$

where $y_i(t) \in R^m$, $\xi(t) \in R^m$, $v_i(t) \in R^m$, $s(t) \in R^m$, $w(t) \in R^m$ is the measurement of the ith sensor, public interference noise, measurement noise, signal, and the input noise. $A(q^{-1})$, $C(q^{-1})$ are the polynomial matrices with the form

$$A(q^{-1}) = I_m + A_1 q^{-1} + \cdots + A_{n_a} q^{-n_a}, \quad C(q^{-1}) = C_0 + C_1 q^{-1} + \cdots + C_{n_c} q^{-n_c} \tag{3}$$

Where $C_0 = I_m$, $n_a > n_c$.

The objectives are found the Model parameters $A(q^{-1})$, $C(q^{-1})$ and the covariance matrix of the noise statistics Q_w, Q_{v_i} and Q_ξ based on the measurement $y_i(t)$ where $i = 1, \cdots, L$.

Assumption 1. The observational data $y_i(t)$ $(i = 1, \cdots, L)$ is bounded.

Assumption 2. The matrix polynomial $A(q^{-1})$ and $C(q^{-1})$ are stable, and they are relatively prime numbers.

Assumption 3. The matrix A_{n_a} is non-singular.

With the assumption 1-3, the objective is how to get the estimated value of $A(q^{-1})$, $C(q^{-1})$, Q_w, Q_{v_i} and Q_ξ. We can get the model below from(1) , (2).

$$A(q^{-1})y_i(t) = C(q^{-1})w(t) + A(q^{-1})\xi(t) + A(q^{-1})v_i(t), \ i = 1, \cdots, L \tag{4}$$

If $$e_i(t) = C(q^{-1})w(t) + A(q^{-1})\xi(t) + A(q^{-1})v_i(t)\ ,$$

the formula (4) an be changed into

$$A(q^{-1})y_i(t) = e_i(t)\ ,\ \ i = 1,\cdots,L \tag{5}$$

1.1. The First Segment

Utilize recursive instrumental variable method (RIV), using the method of average local model parameter estimators to get the unknown model parameters online information fusion estimators.

We have the model parameters of locally estimated $\hat{A}_i(q^{-1})$ by using MRIV algorithm

$$\hat{A}_f(q^{-1}) = \frac{1}{L}\sum_{i=1}^{L}\hat{A}_i(q^{-1})$$

Where $$A_f(q^{-1}) = I_m + A_{1f}q^{-1} + \cdots + A_{n_a f}q^{-n_a}$$

1.2. The Second Segment

Utilize related methods, using the method of average local noise variance estimates to get the unknown noise variance online information fusion estimators.

Estimating Q_ξ and Q_{v_i}

If $$r_i(t) = A(q^{-1})y_i(t) \tag{6}$$

then $$r_i(t) = C(q^{-1})w(t) + A(q^{-1})\xi(t) + A(q^{-1})v_i(t) \tag{7}$$

Substitute the fusion simulation $\hat{A}_f(q^{-1})$ into the formula (6), we can have

$$\hat{r}_i(t) = \hat{A}_f(q^{-1})y_i(t)$$

The recursion formula of the sample correlation function valuation is

$$\hat{R}_{rij}^{(t)}(k) = \hat{R}_{rij}^{(t-1)} + \frac{1}{t}[r_i(t)r_j^{\mathrm{T}}(t-k) - \hat{R}_{rij}^{(t-1)}]$$

When $k = n_a$, the correlation function of the formula (7) is

$$R_{rij}(n_a) = A_{n_a}Q_\xi\ ,\ \ i \neq j$$

Then $$Q_\xi = A_{n_a}^{-1}R_{rij}(n_a)$$

Substitute the fusion valuation $\hat{A}_{n_a f}$ and $\hat{R}_{rij}^{(t)}(n_a)$ into the local estimation

$$Q_{\xi ij} = \hat{A}_{n_a f}^{-1}\hat{R}_{rij}^{(t)}(n_a)\ ,\ \ (i, j = 1,\cdots,L; i \neq j)$$

We can get the fusion valuation by averaging

$$\hat{Q}_{\xi f} = \frac{2}{L(L-1)}\sum_{i=1}^{L-1}\sum_{j=i+1}^{L}\hat{Q}_{\xi ij}$$

where we use the fact $R_{ij}(k) = R_{ji}(k)$

When $i = j, k = 0,1,\cdots,n_a$, the correlation function of the formula (7) is

$$R_{rii}(k)=\sum_{\alpha=0}^{n_c}C_\alpha Q_w C_{\alpha-k}^{\mathrm{T}}+\sum_{\alpha=0}^{n_a}A_\alpha Q_\xi A_{\alpha-k}^{\mathrm{T}}+\sum_{\alpha=0}^{n_a}A_\alpha Q_{v_i} A_{\alpha-k}^{\mathrm{T}} \tag{8}$$

When $u\neq v, k=0,1,\cdots,n_a$, the correlation function of the formula (7) is

$$R_{ruv}(k)=\sum_{\alpha=0}^{n_c}C_\alpha Q_w C_{\alpha-k}^{T}+\sum_{\alpha=0}^{n_a}A_\alpha Q_\xi A_{\alpha-k}^{\mathrm{T}} \tag{9}$$

Use formula (8) to minus formula (9), we can get

$$R_{rii}(k)-R_{ruv}(k)=\sum_{\alpha=0}^{n_a}A_\alpha Q_{v_i} A_{\alpha-k}^{\mathrm{T}}$$

Substitute the fusion valuation $\hat{A}_f(q^{-1})$ and $\hat{R}_{rij}^{(t)}(k)$ into the formula above, we can get the local estimation

$$\hat{R}_{rii}^{(t)}(k)-\hat{R}_{ruv}^{(t)}(k)=\sum_{\alpha=0}^{n_a}\hat{A}_{\alpha f} Q_{viuvk}\hat{A}_{(\alpha-k)f}^{\mathrm{T}}\ ,(u,v=1,\cdots,L;u\neq v;k=0,1,\cdots,n_a)$$

Solve the correlation function matrix equations in order to get the local estimation $\hat{Q}_{v_{ijk}}$, we can get the fusion valuation by averaging.

$$\hat{Q}_{v_{if}}=\frac{2}{L(L-1)(n_a+1)}\sum_{k=0}^{n_a}\sum_{u=1}^{L-1}\sum_{v=u+1}^{L}\hat{Q}_{viuvk}$$

1.3. The Third Segment

Utilize related methods and Gevers-Wouters algorithm with dead zones to get the MA model information fusion parameter estimators.

Identify $C(q^{-1})$ and Q_w

If $$z(t)=C(q^{-1})w(t)$$

then $$R_z(k)=\mathrm{E}[z(t)z^{\mathrm{T}}(t-k)]$$

$$R_z(k)=\sum_{\alpha=k}^{n_c}C_\alpha Q_w C_{\alpha-k}^{\mathrm{T}}\ ,\ k=0,\cdots n_c$$

From formula (7), we can get

$$R_{ri}(k)=R_z(k)+\sum_{\alpha=k}^{n_a}A_\alpha Q_\xi A_{\alpha-k}^{\mathrm{T}}+\sum_{\alpha=k}^{n_a}A_\alpha Q_{v_i} A_{\alpha-k}^{\mathrm{T}}$$

Then $$R_z(k)=R_{ri}(k)-\sum_{\alpha=k}^{n_a}A_\alpha Q_\xi A_{\alpha-k}^{\mathrm{T}}-\sum_{\alpha=k}^{n_a}A_\alpha Q_{v_i} A_{\alpha-k}^{\mathrm{T}}$$

Substitute $\hat{R}_{ri}^{(t)}(k)$ and the fusion valuation $\hat{Q}_{\xi f}$, $\hat{Q}_{v_{if}}$ and $\hat{A}_f(q^{-1})$, we can get

$$\hat{R}_{zi}(k)=\hat{R}_{ri}^{(t)}(k)-\sum_{\alpha=k}^{n_a}\hat{A}_{\alpha f}\hat{Q}_{\xi f}\hat{A}_{(\alpha-k)f}^{\mathrm{T}}-\sum_{\alpha=k}^{n_a}\hat{A}_{\alpha f}\hat{Q}_{v_{if}}\hat{A}_{(\alpha-k)f}^{\mathrm{T}}\ ,\ i=1,\cdots,L$$

Using $\hat{R}_{zi}(k)$ to replace $R_z(k)$, we can get the local estimation $\hat{C}_i(q^{-1})$ and $\hat{Q}_{wi}$ with the G-W algorithm.

Define

$$\hat{C}_f(q^{-1}) = \frac{1}{L}\sum_{i=1}^{L}\hat{C}_i(q^{-1})$$

$$\hat{Q}_{wf} = \frac{1}{L}\sum_{i=1}^{L}\hat{Q}_{wi}$$

Theorem For the Constant linear discrete random system (1-4) with assumption, the parameter fusion estimation $\hat{A}_f(q^{-1})$, $\hat{C}_f(q^{-1})$, $\hat{Q}_{wf}$, $\hat{Q}_{\xi f}$ and $\hat{Q}_{v_{if}}$

$$\hat{A}_f(q^{-1}) \to A(q^{-1}), \quad \hat{C}_f(q^{-1}) \to C(q^{-1}) \; \hat{Q}_{wf} \to Q_w,$$

$$\hat{Q}_{\xi f} \to Q_\xi, \quad \hat{Q}_{v_{if}} \to Q_{v_i}, \quad t \to \infty, \quad \text{w.p.1}$$

2. Simulation Example

Consider the multi-sensor system with the white observation noise

$$A(q^{-1})s(t) = C(q^{-1})w(t) \tag{10}$$

$$y_i(t) = s(t) + \xi(t) + v_i(t), \quad i = 1,2,3 \tag{11}$$

where 1 $y_i(t) \in R^m$, $\xi(t) \in R^m$, $v_i(t) \in R^m$, $s(t) \in R^m$, $w(t) \in R^m$ $(m = 2)$ is the measurement of the ith sensor, public interference noise, measurement noise, signal, and the input noise. $A(q^{-1}), C(q^{-1})$ are the polynomial matrices with the form

$$A(q^{-1}) = I_2 + A_1 q^{-1} + A_2 q^{-2}, \quad C(q^{-1}) = I_2 + C_1 q^{-1}$$

The objectives are found the model parameters $A(q^{-1}), C(q^{-1})$ and the covariance matrix of the noise statistics Q_w, Q_{v_i} and Q_ξ based on the measurement $y_i(t)$ where $i = 1,2,3$

In the simulation,

$$A_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{13} & a_{14} \end{bmatrix} = \begin{bmatrix} -0.1 & -0.5 \\ -0.7 & 1.1 \end{bmatrix}, A_2 = \begin{bmatrix} a_{21} & a_{22} \\ a_{23} & a_{24} \end{bmatrix} = \begin{bmatrix} -0.3 & -0.36 \\ -0.35 & 0.3 \end{bmatrix}, C_1 = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$Q_w = \begin{bmatrix} 0.85 & 0 \\ 0 & 0.95 \end{bmatrix}, Q_\xi = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, Q_{v1} = \begin{bmatrix} \sigma_{v11}^2 & 0 \\ 0 & \sigma_{v12}^2 \end{bmatrix} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.16 \end{bmatrix},$$

$$Q_{v2} = \begin{bmatrix} \sigma_{v21}^2 & 0 \\ 0 & \sigma_{v22}^2 \end{bmatrix} = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.2 \end{bmatrix}, Q_{v3} = \begin{bmatrix} \sigma_{v31}^2 & 0 \\ 0 & \sigma_{v32}^2 \end{bmatrix} = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.25 \end{bmatrix}$$

From the formula (10),(11), we can get the AR models

$$A(q^{-1})y_i(t) = e_i(t), i = 1,2,3 \tag{12}$$

where $$e_i(t) = C(q^{-1})w(t) + A(q^{-1})\xi(t) + A(q^{-1})v_i(t)$$

First, we can use the MRIV identification Formula (12) to get the local estimation of the model parameter $\hat{A}_i(q^{-1}) = I_2 + \hat{A}_{i1}q^{-1} + \hat{A}_{i2}q^{-2}$, $i = 1,2,3$

The simulation valuation is $\hat{A}_f(q^{-1}) = \frac{1}{3}\sum_{i=1}^{3}\hat{A}_i(q^{-1})$

The formula of the model parameter fusion valuation is as below

$$\hat{A}_f(q^{-1}) = I_2 + \hat{A}_{1f}q^{-1} + \hat{A}_{2f}q^{-2}$$

Next, estimate the noise variances Q_ξ and Q_{v_i} ,

If
$$r_i(t) = A(q^{-1})y_i(t) \tag{13}$$
then
$$r_i(t) = C(q^{-1})w(t) + A(q^{-1})\xi(t) + A(q^{-1})v_i(t) \tag{14}$$

If the correlation function of $r_i(t)$ is $R_{rij}(k) = E[r_i(t)r_j^{\mathrm{T}}(t-k)]$

Substitute the fusion valuation $\hat{A}_f(q^{-1})$ into the formula (13), we can get

$$\hat{r}_i(t) = \hat{A}_f(q^{-1})y_i(t)$$

Define the sample function estimation of $\hat{r}_i(t)$ as

$$\hat{R}_{rij}^{(t)}(k) = \hat{R}_{rij}^{(t-1)}(k) + \frac{1}{t}[\hat{r}_i(t)\hat{r}_j^{\mathrm{T}}(t-k) - \hat{R}_{rij}^{(t-1)}(k)] \tag{15}$$

When $i \neq j, k = 2$, the correlation function of formula (7) is

$$\begin{aligned} R_{rij}(2) &= E[r_i(t)r_j^{\mathrm{T}}(t-2)] \\ &= E\{[w(t)+C_1w(t-1)+\xi(t)+A_1\xi(t-1)+A_2\xi(t-2)+v_i(t)+A_1v_i(t-1)+A_2v_i(t-2)]\times \\ &\quad [w(t-2)+C_1w(t-3)+\xi(t-2)+A_1\xi(t-3)+A_2\xi(t-4)+v_j(t-2)+A_1v_j(t-3)+A_2v_j(t-4)]^{\mathrm{T}}\} \\ &= A_2Q_\xi \end{aligned}$$

then
$$Q_\xi = A_2^{-1}R_{rij}(2)\,,\ (i, j = 1,2,3; i \neq j)$$

From formula(15), we can get

$$\hat{R}_{rij}^{(t)}(2) = \hat{R}_{rij}^{(t-1)}(2) + \frac{1}{t}[\hat{r}_i(t)\hat{r}_j^{\mathrm{T}}(t-2) - \hat{R}_{rij}^{(t-1)}(2)]$$

Substitute $\hat{R}_{rij}^{(t)}(2)$ and the fusion valuation $\hat{A}_{2f}$ into the formula above

$$\hat{Q}_{\xi 12} = \hat{A}_{2f}^{-1}\hat{R}_{r12}^{(t)}(2)\,,\ \hat{Q}_{\xi 13} = \hat{A}_{2f}^{-1}\hat{R}_{r13}^{(t)}(2)\,,\ \hat{Q}_{\xi 23} = \hat{A}_{2f}^{-1}\hat{R}_{r23}^{(t)}(2)$$

Solve the correlation function matrix equations above in order to get the local estimation.

The fusion valuation is $\hat{Q}_{\xi f} = \frac{1}{3}(\hat{Q}_{\xi 12} + \hat{Q}_{\xi 13} + \hat{Q}_{\xi 23})$

When $i = j, k = 0$, the correlation function of formula (7) is

$$R_{rii}(0) = E[r_i(t)r_i^{\mathrm{T}}(t)]$$

$$=E\{[w(t)+C_1w(t-1)+\xi(t)+A_1\xi(t-1)+A_2\xi(t-2)+v_i(t)+A_1v_i(t-1)+A_2v_i(t-2)]\times$$
$$[w(t)+C_1w(t-1)+\xi(t)+A_1\xi(t-1)+A_2\xi(t-2)+v_i(t)+A_1v_i(t-1)+A_2v_i(t-2)]^{\mathrm{T}}\}$$
$$=Q_w+C_1Q_wC_1^{\mathrm{T}}+Q_\xi+A_1Q_\xi A_1^{\mathrm{T}}+A_2Q_\xi A_2^{\mathrm{T}}+Q_{v_i}+A_1Q_{v_i}A_1^{\mathrm{T}}+A_2Q_{v_i}A_2^{\mathrm{T}} \tag{16}$$

When $u\neq v, k=0$, the correlation function of formula (7) is

$$R_{ruv}(0)==Q_w+C_1Q_wC_1^{\mathrm{T}}+Q_\xi+A_1Q_\xi A_1^{\mathrm{T}}+A_2Q_\xi A_2^{\mathrm{T}} \tag{17}$$

Use formula (16) to minus formula (17), we can get

$$R_{rii}(0)-R_{ruv}(0)=Q_{v_i}+A_1Q_{v_i}A_1^{\mathrm{T}}+A_2Q_{v_i}A_2^{\mathrm{T}}$$

Then we can get

$$\begin{cases}(1+a_{11}^2+a_{21}^2)\sigma_{vi1}^2+(a_{12}^2+a_{22}^2)\sigma_{vi2}^2=R_{rii}^{(1,1)}(0)-R_{ruv}^{(1,1)}(0)\\(a_{13}a_{11}+a_{23}a_{21})\sigma_{vi1}^2+(a_{14}a_{12}+a_{24}a_{22})\sigma_{vi2}^2=R_{rii}^{(2,1)}(0)-R_{ruv}^{(2,1)}(0)\end{cases}$$

and

$$\begin{cases}(a_{11}a_{13}+a_{21}a_{23})\sigma_{vi1}^2+(a_{12}a_{14}+a_{22}a_{24})\sigma_{vi2}^2=R_{rii}^{(1,2)}(0)-R_{ruv}^{(1,2)}(0)\\(1+a_{13}^2+a_{23}^2)\sigma_{vi1}^2+(a_{14}^2+a_{24}^2)\sigma_{vi2}^2=R_{rii}^{(2,2)}(0)-R_{ruv}^{(2,2)}(0)\end{cases}$$

Solve the equations, we can get

$$\begin{cases}\sigma_{vi1}^2=\dfrac{[(a_{14}a_{12}+a_{24}a_{22})(R_{rii}^{(1,1)}(0)-R_{ruv}^{(1,1)}(0))-(a_{12}^2+a_{22}^2)(R_{rii}^{(2,1)}(0)-R_{ruv}^{(2,1)}(0))]}{(a_{14}a_{12}+a_{24}a_{22})(1+a_{11}^2+a_{21}^2)-(a_{12}^2+a_{22}^2)(a_{13}a_{11}+a_{23}a_{21})}\\ \sigma_{vi2}^2=\dfrac{[(1+a_{13}^2+a_{23}^2)(R_{rii}^{(1,2)}(0)-R_{ruv}^{(1,2)}(0))-(a_{11}a_{13}+a_{21}a_{23})(R_{rii}^{(2,2)}(0)-R_{ruv}^{(2,2)}(0))]}{(1+a_{13}^2+a_{23}^2)(a_{12}a_{14}+a_{22}a_{24})-(a_{11}a_{13}+a_{21}a_{23})(a_{14}^2+a_{24}^2)}\end{cases}$$

Substitute $\hat{R}_{rii}^{(t)}(0)(i=1,2,3)$, $\hat{R}_{rij}^{(t)}(0)(i,j=1,2,3;i\neq j)$, the fusion valuation $\hat{A}_{1f}$ and $\hat{A}_{2f}$ in, we can get

$$\begin{cases}\hat{\sigma}_{vi10}^2=\dfrac{[(\hat{A}_{1f}^{(2,2)}\hat{A}_{1f}^{(1,2)}+\hat{A}_{2f}^{(2,2)}\hat{A}_{2f}^{(1,2)})(\hat{R}_{rii}^{(1,1)}(0)-\hat{R}_{ruv}^{(1,1)}(0))-((\hat{A}_{1f}^{(1,2)})^2+(\hat{A}_{2f}^{(1,2)})^2)(\hat{R}_{rii}^{(2,1)}(0)-\hat{R}_{ruv}^{(2,1)}(0))]}{(\hat{A}_{1f}^{(2,2)}\hat{A}_{1f}^{(1,2)}+\hat{A}_{2f}^{(2,2)}\hat{A}_{2f}^{(1,2)})(1+(\hat{A}_{1f}^{(1,1)})^2+(\hat{A}_{2f}^{(1,1)})^2)-((\hat{A}_{1f}^{(1,2)})^2+(\hat{A}_{2f}^{(1,2)})^2)(\hat{A}_{1f}^{(2,1)}\hat{A}_{1f}^{(1,1)}+\hat{A}_{2f}^{(2,1)}\hat{A}_{2f}^{(1,1)})}\\ \hat{\sigma}_{vi20}^2=\dfrac{[(1+(\hat{A}_{1f}^{(2,1)})^2+(\hat{A}_{2f}^{(2,1)})^2)(\hat{R}_{rii}^{(1,2)}(0)-\hat{R}_{ruv}^{(1,2)}(0))-(\hat{A}_{1f}^{(1,1)}\hat{A}_{1f}^{(2,1)}+\hat{A}_{2f}^{(1,1)}\hat{A}_{2f}^{(2,1)})(\hat{R}_{rii}^{(2,2)}(0)-\hat{R}_{ruv}^{(2,2)}(0))]}{(1+(\hat{A}_{1f}^{(2,1)})^2+(\hat{A}_{2f}^{(2,1)})^2)(\hat{A}_{1f}^{(1,2)}\hat{A}_{1f}^{(2,2)}+\hat{A}_{2f}^{(1,2)}\hat{A}_{2f}^{(2,2)})-(\hat{A}_{1f}^{(1,1)}\hat{A}_{1f}^{(2,1)}+\hat{A}_{2f}^{(1,1)}\hat{A}_{2f}^{(2,1)})((\hat{A}_{1f}^{(2,2)})^2+(\hat{A}_{2f}^{(2,2)})^2)}\end{cases}$$

Therefore

$$\hat{Q}_{v_{i0}}=\begin{bmatrix}\hat{\sigma}_{vi10}^2 & 0\\ 0 & \hat{\sigma}_{vi20}^2\end{bmatrix}, i=1,2,3$$

When $i=j, k=1$, the correlation function of formula (7) is

$$R_{rii}(1)=E[r_i(t)r_i^{\mathrm{T}}(t-1)]$$
$$=E\{[w(t)+C_1w(t-1)+\xi(t)+A_1\xi(t-1)+A_2\xi(t-2)+v_i(t)+A_1v_i(t-1)+A_2v_i(t-2)]\times$$
$$[w(t-1)+C_1w(t-2)+\xi(t-1)+A_1\xi(t-2)+A_2\xi(t-3)+v_i(t-1)+A_1v_i(t-2)+A_2v_i(t-3)]^{\mathrm{T}}\}$$

$$=C_1Q_w+A_1Q_\xi+A_2Q_\xi A_1^{\mathrm{T}}+A_1Q_{v_i}+A_2Q_{v_i}A_1^{\mathrm{T}} \tag{18}$$

When $u\neq v,k=1$, the correlation function of formula (7) is

$$R_{ruv}(1)=C_1Q_w+A_1Q_\xi+A_2Q_\xi A_1^{\mathrm{T}} \tag{19}$$

Use formula (18) to minus formula (19), we can get

$$R_{rii}(1)-R_{ruv}(1)=A_1Q_{v_i}+A_2Q_{v_i}A_1^{\mathrm{T}}$$

Then we can get

$$\begin{cases}(a_{11}+a_{21}a_{11})\sigma_{vi1}^2+a_{22}a_{12}\sigma_{vi2}^2=R_{rii}^{(1,1)}(1)-R_{ruv}^{(1,1)}(1)\\(a_{13}+a_{23}a_{11})\sigma_{vi1}^2+a_{24}a_{12})\sigma_{vi2}^2=R_{rii}^{(2,1)}(1)-R_{ruv}^{(2,1)}(1)\end{cases}$$

and

$$\begin{cases}a_{21}a_{13}\sigma_{vi1}^2+(a_{12}+a_{22}a_{14})\sigma_{vi2}^2=R_{rii}^{(1,2)}(1)-R_{ruv}^{(1,2)}(1)\\a_{23}a_{13}\sigma_{vi1}^2+(a_{14}+a_{24}a_{14})\sigma_{vi2}^2=R_{rii}^{(2,2)}(1)-R_{ruv}^{(2,2)}(1)\end{cases}$$

Solve the equations, we can get

$$\begin{cases}\sigma_{vi1}^2=\dfrac{[a_{24}(R_{rii}^{(1,1)}(1)-R_{ruv}^{(1,1)}(1))-a_{22}(R_{rii}^{(2,1)}(1)-R_{ruv}^{(2,1)}(1))]}{a_{24}(a_{11}+a_{21}a_{11})-a_{22}(a_{13}+a_{23}a_{11})}\\\sigma_{vi2}^2=\dfrac{[a_{23}(R_{rii}^{(1,2)}(1)-R_{ruv}^{(1,2)}(1))-a_{21}(R_{rii}^{(2,2)}(1)-R_{ruv}^{(2,2)}(1))]}{a_{23}(a_{12}+a_{22}a_{14})-a_{21}(a_{14}+a_{24}a_{14})}\end{cases}$$

Substitute $\hat{R}_{rii}^{(t)}(1)(i=1,2,3)$, $\hat{R}_{rij}^{(t)}(1)(i,j=1,2,3;i\neq j)$, the fusion valuations $\hat{A}_{1f}$ and $\hat{A}_{2f}$, we can get

$$\begin{cases}\hat{\sigma}_{vi11}^2=\dfrac{[\hat{A}_{2f}^{(2,2)}(\hat{R}_{rii}^{(1,1)}(1)-\hat{R}_{ruv}^{(1,1)}(1))-\hat{A}_{2f}^{(1,2)}(\hat{R}_{rii}^{(2,1)}(1)-\hat{R}_{ruv}^{(2,1)}(1))]}{\hat{A}_{2f}^{(2,2)}(\hat{A}_{1f}^{(1,1)}+\hat{A}_{2f}^{(1,1)}\hat{A}_{1f}^{(1,1)})-\hat{A}_{2f}^{(1,2)}(\hat{A}_{1f}^{(2,1)}+\hat{A}_{2f}^{(2,1)}\hat{A}_{1f}^{(1,1)})}\\\hat{\sigma}_{vi21}^2=\dfrac{[\hat{A}_{2f}^{(2,1)}(\hat{R}_{rii}^{(1,2)}(1)-\hat{R}_{ruv}^{(1,2)}(1))-\hat{A}_{2f}^{(1,1)}(\hat{R}_{rii}^{(2,2)}(1)-\hat{R}_{ruv}^{(2,2)}(1))]}{\hat{A}_{2f}^{(2,1)}(\hat{A}_{1f}^{(1,2)}+\hat{A}_{2f}^{(1,2)}\hat{A}_{1f}^{(2,2)})-\hat{A}_{1f}^{(2,1)}(\hat{A}_{1f}^{(2,2)}+\hat{A}_{2f}^{(2,2)}\hat{A}_{1f}^{(2,2)})}\end{cases}$$

Therefore

$$\hat{Q}_{v_{i1}}=\begin{bmatrix}\hat{\sigma}_{vi11}^2&0\\0&\hat{\sigma}_{vi21}^2\end{bmatrix},\ i=1,2,3$$

When $i=j,k=2$, the correlation function of formula (7) is

$$R_{rii}(2)=E[r_i(t)r_i^{\mathrm{T}}(t-2)]$$

$$=E\{[w(t)+C_1w(t-1)+\xi(t)+A_1\xi(t-1)+A_2\xi(t-2)+v_i(t)+A_1v_i(t-1)+A_2v_i(t-2)]\times$$
$$[w(t-2)+C_1w(t-3)+\xi(t-2)+A_1\xi(t-3)+A_2\xi(t-4)+v_i(t-2)+A_1v_i(t-3)+A_2v_i(t-4)]^{\mathrm{T}}\}$$

$$=A_2Q_\xi+A_2Q_{v_i} \tag{20}$$

When $u \neq v, k=2$, the correlation function of formula (7) is

$$R_{ruv}(2) = A_1 Q_{\xi} \tag{21}$$

Use formula (20) to minus formula (21), we can get

$$R_{rii}(2) - R_{ruv}(2) = A_2 Q_{v_i}$$

Then we can get $$Q_{v_i} = A_2^{-1}(R_{rii}(2) - R_{ruv}(2))$$

Substitute $\hat{R}_{rii}^{(t)}(2)(i=1,2,3)$, $\hat{R}_{rij}^{(t)}(2)(i,j=1,2,3; i \neq j)$ into the fusion valuation $\hat{A}_{2f}$, we can get

$$\hat{Q}_{v_{i2}} = \hat{A}_{2f}^{-1}(\hat{R}_{rii}^{(t)}(2) - \hat{R}_{ruv}^{(t)}(2))$$

Then the fusion valuation is

$$\hat{Q}_{v_{if}} = \frac{1}{3}(\hat{Q}_{v_{i0}} + \hat{Q}_{v_{i1}} + \hat{Q}_{v_{i2}}), \; i = 1,2,3$$

If $$z(t) = C(q^{-1})w(t)$$

then $$R_z(k) = \mathrm{E}[z(t)z^{\mathrm{T}}(t-k)]$$

From the formulas (16) and(18), we can get

$$R_{zi}(0) = R_{rii}(0) - Q_{\xi} - A_1 Q_{\xi} A_1^{\mathrm{T}} - A_2 Q_{\xi} A_2^{\mathrm{T}} - Q_{v_i} - A_1 Q_{v_i} A_1^{\mathrm{T}} - A_2 Q_{v_i} A_2^{\mathrm{T}}$$

$$R_{zi}(1) = R_{rii}(1) - A_1 Q_{\xi} - A_2 Q_{\xi} A_1^{\mathrm{T}} - A_1 Q_{v_i} - A_2 Q_{v_i} A_1^{\mathrm{T}}$$

Substitute $\hat{R}_{rii}^{(t)}(k)(i=1,2,3; k=0,1)$, fusion valuation $\hat{Q}_{\xi f}$, $\hat{Q}_{v_{if}}$, $\hat{A}_{1f}$ and $\hat{A}_{2f}$ in

$$\hat{R}_{zi}(0) = \hat{R}_{rii}^{(t)}(0) - \hat{Q}_{\xi f} - \hat{A}_{1f}\hat{Q}_{\xi f}\hat{A}_{1f}^{\mathrm{T}} - \hat{A}_{2f}\hat{Q}_{\xi f}\hat{A}_{2f}^{\mathrm{T}} - \hat{Q}_{v_{if}} - \hat{A}_{1f}\hat{Q}_{v_{if}}\hat{A}_{1f}^{\mathrm{T}} - \hat{A}_{2f}\hat{Q}_{v_{if}}\hat{A}_{2f}^{\mathrm{T}} \tag{22}$$

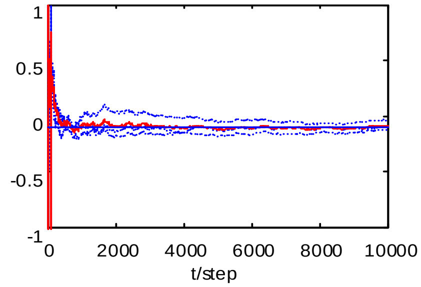
$$\hat{R}_{zi}(1) = \hat{R}_{rii}^{(t)}(1) - \hat{A}_{1f}\hat{Q}_{\xi f} - \hat{A}_{2f}\hat{Q}_{\xi f}\hat{A}_{1f}^{\mathrm{T}} - \hat{A}_{1f}\hat{Q}_{v_{if}} - \hat{A}_{2f}\hat{Q}_{v_{if}}\hat{A}_{1f}^{\mathrm{T}} \tag{23}$$

From the formulas (22) and (23), we can get the local estimation $\hat{C}_i(q^{-1})$, $\hat{Q}_{wi}$ $(i=1,2,3)$, by using the G-W algorithm with dead zone the fusion valuation is

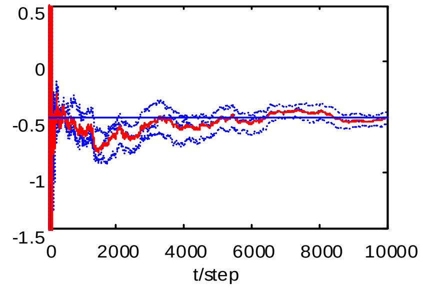
$$\hat{C}_{1f} = \frac{1}{3}(\hat{C}_{11} + \hat{C}_{21} + \hat{C}_{31}), \; \hat{Q}_{wf} = \frac{1}{3}(\hat{Q}_{w1} + \hat{Q}_{w2} + \hat{Q}_{w3})$$

The part of simulation results are shown in Figures 1 and 2]

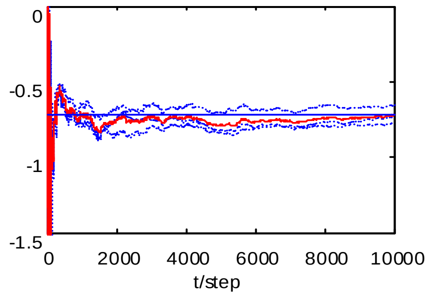
$\hat{A}_f(q^{-1})$:



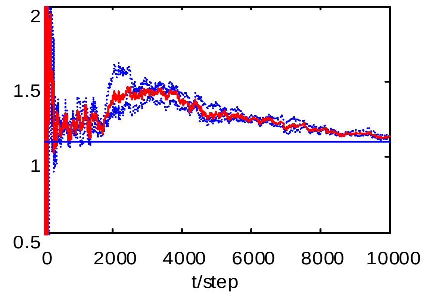
a) $a_{11}, \hat{A}_{1i}^{(1,1)} (i = 1,2,3, f)$



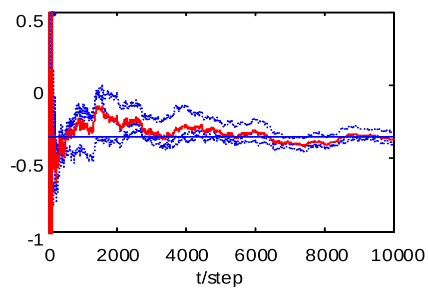
b) $a_{12}, \hat{A}_{1i}^{(1,2)} (i = 1,2,3, f)$



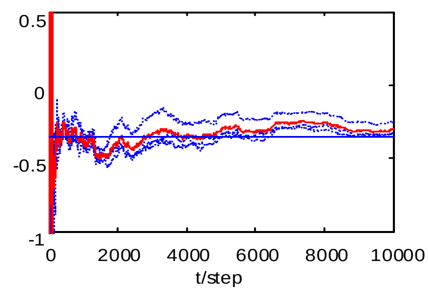
c) $a_{13}, \hat{A}_{1i}^{(2,1)} (i = 1,2,3, f)$



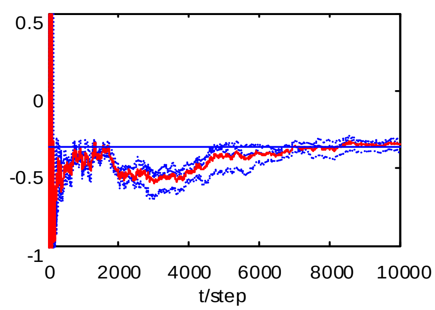
d) $a_{14}, \hat{A}_{1i}^{(2,2)} (i = 1,2,3, f)$



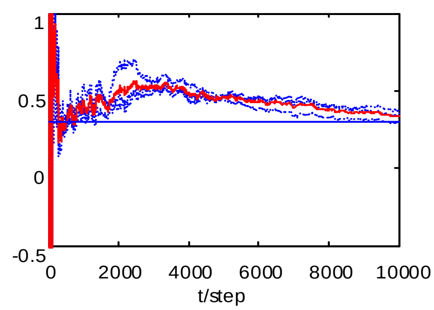
e) $a_{21}, \hat{A}_{2i}^{(1,1)} (i = 1,2,3, f)$



f) $a_{22}, \hat{A}_{2i}^{(1,2)} (i = 1,2,3, f)$

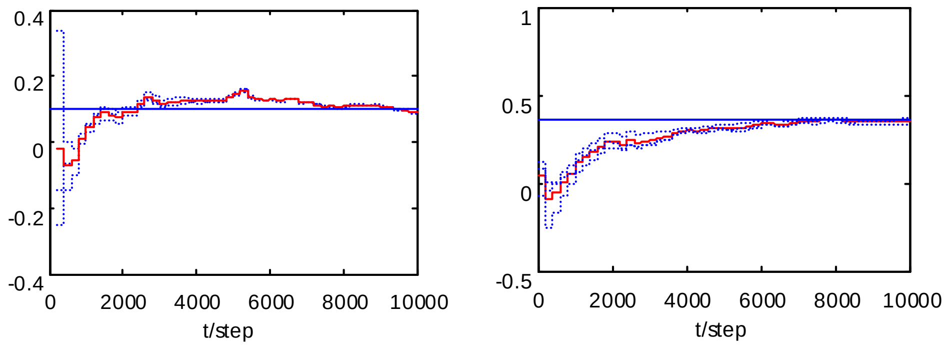


g) $a_{23}, \hat{A}_{2i}^{(2,1)} (i = 1,2,3, f)$



h) $a_{24}, \hat{A}_{2i}^{(2,2)} (i = 1,2,3, f)$

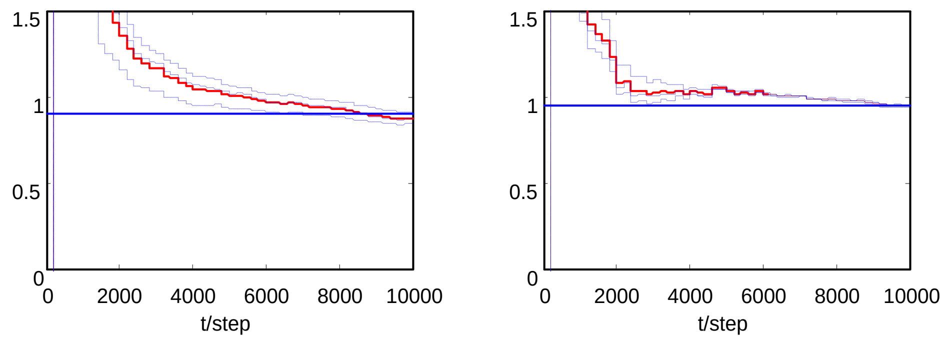
$C(q^{-1})$:



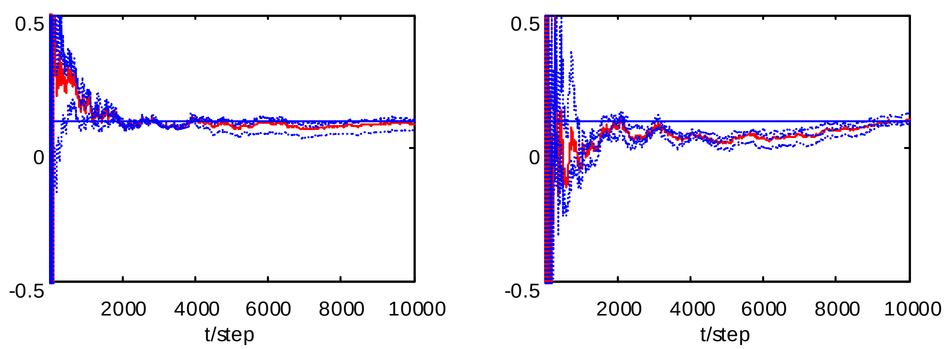
i) $C_1^{(1,1)}, \hat{C}_{1i}^{(1,1)} (i = 1, 2, 3, f)$ j) $C_1^{(2,2)}, \hat{C}_{1i}^{(2,2)} (i = 1, 2, 3, f)$

Figure 1. The convergence of the model parameter estimators

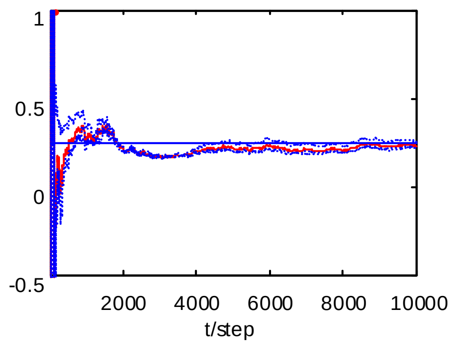
Q_ξ and Q_{v_i} :



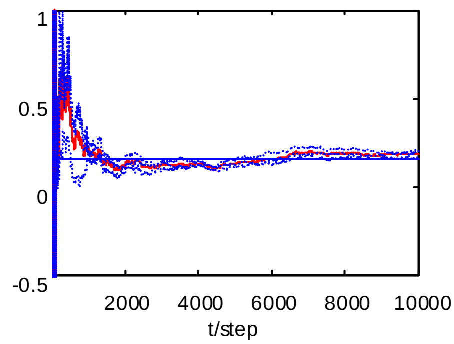
a) $Q_w^{(1,1)}, \hat{Q}_{wi}^{(1,1)} (i = 1, 2, 3, f)$ b) $Q_w^{(2,2)}, \hat{Q}_{wi}^{(2,2)} (i = 1, 2, 3, f)$



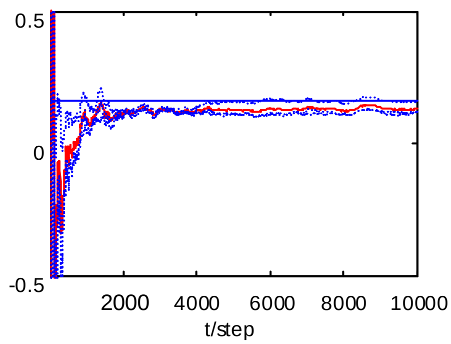
c) $Q_\xi^{(1,1)}, \hat{Q}_{\xi ij}^{(1,1)} (i, j = 1, 2, 3; i \neq j), \hat{Q}_{\xi f}^{(1,1)}$ d) $Q_\xi^{(2,2)}, \hat{Q}_{\xi ij}^{(2,2)} (i, j = 1, 2, 3; i \neq j), \hat{Q}_{\xi f}^{(2,2)}$



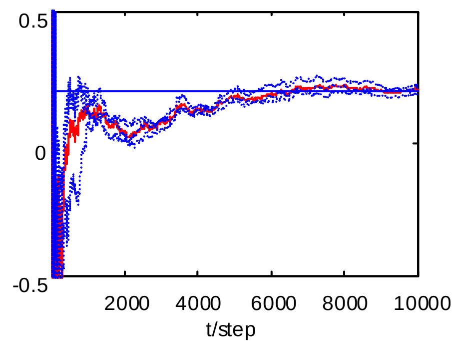
e) $\sigma_{v11}^2, \hat{Q}_{v_{1i}}^{(1,1)} (i = 0,1, f)$



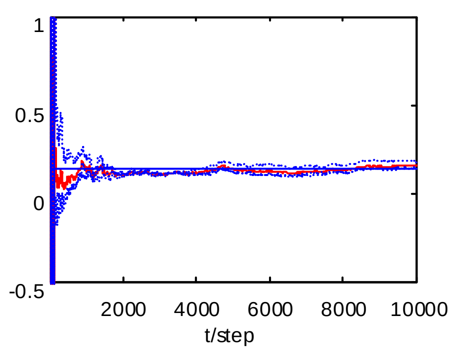
f) $\sigma_{v12}^2, \hat{Q}_{v_{1i}}^{(2,2)} (i = 0,1, f)$



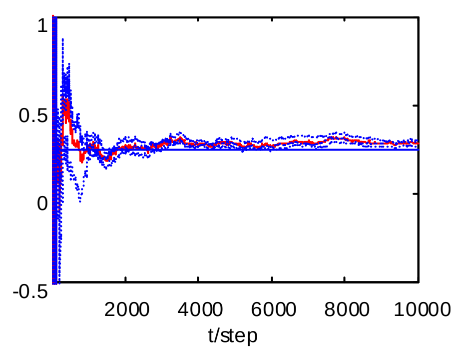
g) $\sigma_{v21}^2, \hat{Q}_{v_{2i}}^{(1,1)} (i = 0,1, f)$



h) $\sigma_{v22}^2, \hat{Q}_{v_{2i}}^{(2,2)} (i = 0,1, f)$



i) $\sigma_{v31}^2, \hat{Q}_{v_{3i}}^{(1,1)} (i = 0,1, f)$



j) $\sigma_{v32}^2, \hat{Q}_{v_{3i}}^{(2,2)} (i = 0,1, f)$

Figure 2. The convergence of the noise variance estimators

In the simulation figures, the straight lines say true value, and the solid-line curves say the fusion valuation, the imaginary curves say local valuation. The simulation example shows the effectiveness of this method. These identification methods can be used to design self-turning information fusion filters

References

[1] A. J. Roedker and C. D. Gillem, "Comparison of two-sensor tracking methods based on state vector fusion and measurement fusion", IEEE Transaction on Aerospace and Electronic Systems, vol. 24, (**1998**), pp. 447-449.

[2] D. Kim, J. Ha and K. You, "Adaptive Extended Kalman Filter Based Geolocation Using TDOA/FDOA", IJCA, vol. 4, no. 2, (**2010**), pp. 49-58.

[3] Y. Manai and M. Benrejeb, "New Condition of Stabilisation for Continuous Takagi-Sugeno Fuzzy System based on Fuzzy Lyapunov Function", IJCA, vol. 4, no. 4, **(2010)**, pp. 51-64.

[4] S. Bourennane and C. Fossati, "Multiway Filtering based on Multilinear Algebra Tools", IJSIP, vol. 3, no.1, **(2010)**, pp. 51-64.

[5] Z. L. Deng, Y. Gao, L. Mao, Y. Li and G. Hao, "New Approach to Information Fusion Steady-State Kalman Filtering", Automatica, vol. 41, no. 10, (**2005)**, pp. 1695-1707**.**