Trajectory Optimization for Large Scale Systems via Block Pulse Functions and Transformations

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Abstract

In this paper an optimal tracking algorithm for large scale systems has been suggested using orthogonal functions and their operational transform matrices. Orthogonal functions such Walsh and block pulse functions (bpf) belong to the class of piece wise constant basis functions that have been developed in twentieth century and have played an important role in control engineering applications. And Walsh and block pulse functions have been proposed to solve the problems related to systems identification, analysis and optimal control. In addition, orthogonal functions and transforms can be applied to develop signal transduction and communication theory by allowing the application scope has widened its coverage. The applied method is very useful to solve the two point boundary vale problem for optimal tracking of large scale system and it is superior to conventional numerical methods.

Keywords: *Walsh and bpf, operational matrix and transforms, optimal trajectory problem*

1. Introduction

In control engineering, the optimal tracking problem is to determine control inputs so that the system states track the desired state trajectory. Typically optimal tracking problem of large scale systems is to determine control inputs. In this case, optimal trajectory problems will lead to a very high order system, thus it cause any difficulties to solve the optimal tracking problem. In addition, due to the high order system controller, computational burden can be increased. Thus, by applying the hierarchical system theory and orthogonal functions, such as Walsh functions, block pulse functions and Haar functions, we can solve these problems. In the 1960s, after Harmuth applied orthogonal functions in communication problems, in the computerized data processing and analysis of linear and nonlinear systems, real-time signal data processing, non-sinusoidal communication fields, the control theory of orthogonal functions has been established.

2. Walsh Operational Matrices and Transforms

The incomplete set of Rademacher functions was completed by Walsh in 1923, to form the complete orthogonal set of rectangular functions, now known as Walsh functions [1]. The set of Walsh functions $\Phi(t)$ is closed. Thus, every signal $f(t)$ which is absolutely integral in $t \in [0, 1)$ can be expanded in series of the form.

We write:

$$f(t) = \sum_{n=0}^{N} C_n \, \phi_n(t) \tag{2.1}$$

where $$C_n = \int_0^1 \emptyset(t) f(t) dt \quad (2.2)$$

C_n is ith coefficient of Walsh functions $\Phi(t)$. It can be determined such that the following integral square error ε is minimized [2],

$$\varepsilon = \int_0^1 [f(t) - \sum_{n=0}^{N} C_n \emptyset_n(t)]^{\,2} dt \quad (2.3)$$

For example, if we expand $f(t)=t$, the result is

$$f(t) = \frac{1}{2}\emptyset_0(t) - \frac{1}{4}\emptyset_1(t) - \frac{1}{8}\emptyset_2(t) - \frac{1}{16}\emptyset_3(t) + \;\ldots.. \quad (2.4)$$

Let us take $\Phi_0(t)$, $\Phi_1(t)$, $\Phi_2(t)$,…, $\Phi_7(t)$ and integrate them: we will have very various triangular waves. If we evaluate the Walsh coefficients for these triangular waves, we will easily get the following formula for approximation [3]:

$$\int_0^t \emptyset_{(8)}(t) dt = P_{(8x8)} \emptyset_{(8)}(t) \quad (2.5)$$

A general formula $P_{(mxm)}$ can be written as follows:

$$P_{(mxm)} = \begin{bmatrix} P_{(\frac{m}{2}x\frac{m}{2})} & -\frac{1}{2m} I_{(\frac{m}{2}x\frac{m}{2})} \\ \frac{1}{2m} I_{(\frac{m}{2}x\frac{m}{2})} & 0_{(\frac{m}{2}x\frac{m}{2})} \end{bmatrix} \quad (2.6)$$

Since Walsh functions operational matrices and transform are naturally more suited for digital computation, an effort has been made to gradually replace the Fourier transform by Walsh type transforms. The operational matrices (2.6) perform integration for integer calculus nicely. Therefore, it can be used to solve optimal trajectory problems. Figure 1 is signal flow graph of fast Walsh transforms.

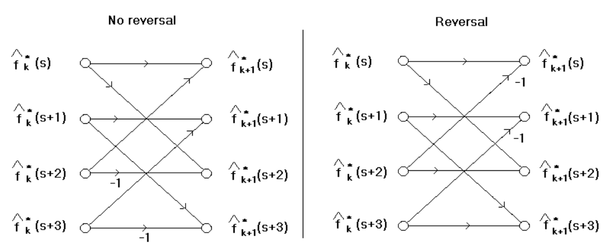

Figure 1. Fast Walsh transforms

3. BPF Operational Matrices and Transforms

Block pulse functions(bpf) are a set of orthogonal functions with piecewise conatant value and are usually applied as a useful tool in the analysis, synthesis, identification and other problems of control and systems science [4]. This set of functions was first introduced to electrical engineer in 1969, but for about seven years it has not received any attention with regard to pratical applications. Until the middle of the seventies, several reasearchers discussed the block pulse functions and their operational matrix for integration in order to reduce the complexity of expressions in solving certain control problems via Walsh functions. Since then, the block pulse functions have been extensively applied due to their simple and easy operaion [5]. As proposed by many authors, a set of block pulse functions for block pulse functions for block pulse transform defined in the unit interval *(0, 1]* as follows, where $i=0, 1, \ldots, m-1$.

$$Bpf_i(t) = \begin{cases} 1, & \frac{i}{m} \le t < \frac{i+1}{m} \\ 0, & otherwise \end{cases} \quad (3.1)$$

If problems are studied in an interval *τ∈[a, b)*, we can always transform them into the equivalent problems in the interval *t∈[0, T)*through:

$$t = \frac{\tau - a}{b - a} t_f \quad (3.2)$$

After the interval length *T* is determined, the accuracy of the results obtained from block pulse functions technique can be adjusted properly by the integer *m*, or expressing more clearly, by *T/m*. For the convenience of expressions, we usually denote the equidistant subinterval or the width of block pulse as *h=T/m*. In solving certain problems of control and systems science, the advantages of using the block pulse function technique are their easy operations and satisfactory approximations [6, 7]. These advantages are due to the distinct properties of block pulse functions. The elementary properties are as follows.

The block pulse functions are disjointed with each other in the interval *t∈[0, T):*

$$Bpf_i(t) Bpf_i(t) = \begin{cases} Bpf_i(t), & i = j \\ 0, & i \ne j \end{cases} \quad (3.3)$$

And the block pulse functions are orthogonal with each other in the interval *t∈[0, T):*

$$\int_0^{t_f} Bpf_i(t)\, dt = \begin{cases} h, & i = j \\ 0, & i \ne j \end{cases} \quad (3.4)$$

where *i, j = 1, 2, ...m.* The orthogonal property of block pulse functions is the basis of expanding functions into their block pulse series [8]. An arbitrary real bounded function *f(t),* which is square integral in the interval *t∈[0, T),* can be expanded into a block pulse series in the sense of minimizing the mean square error between *f(t)* and its approximation:

$$f(t) \cong \hat{f}(t) = \sum_{i=0}^{m-1} F_i\, Bpf_i(t) \quad (3.5)$$

Where ˆ*f(t)* is the block pulse series of the original *f(t)*, and F_i is the block pulse coefficient with respect to the i*th* block pulse function $Bpf_i(t)$. As an example, we can expand $f(t)=t^2$ into its block pulse series in the interval *t∈[0, 1)* with *m=8.*

$$F_i = \frac{1}{h} \int_0^1 f(t)\, Bpf_i(t) dt = \frac{m}{t_f} \int_{i\frac{t_f}{m}}^{(i=1)\frac{t_f}{m}} f(t) dt$$

$$= \frac{1}{2} \left[f\left(i \frac{t_f}{m} \right) + f((i+1) \frac{t_f}{m}) \right] \quad (3.6)$$

In order to expand the integral of a function into its block pulse series, we first consider the integral of each single block pulse function $Bpf_i(t)$. For the case of *t∈[0, i*h*)*, we have:

$$\int_0^t Bpf_i(t)\, dt = 0 \quad (3.7)$$

For the case of *t∈[i*h, *(i +1)*h*)*, we have:

$$\int_0^t Bpf_i(t)\, dt = \int_0^{ih} Bpf_i(t)\, dt + \int_{ih}^t Bpf_i(t)\, dt = t - ih \quad (3.8)$$

And in the case of *t∈[(i +1)*h, t_f*), we have:*

$$\int_0^t Bpf_i(t)\, dt = \int_0^{ih} Bpf_i(t)\, dt + \int_{ih}^{(i+1)h} Bpf_i(t)\, dt + \int_{(i+1)h}^t Bpf_i(t)\, dt = h \quad (3.9)$$

From the above discussion, the block pulse series of the integral of all the *m* block pulse functions can be written in a compact form.

$$\int_0^t Bpf_i(t)\,dt \cong PBpf_i(t) \tag{3.10}$$

where the operational matrix *P*:

$$P = \frac{h}{2}\begin{bmatrix} 1 & 2 & 2\cdots 2 \\ 0 & 1 & 2\cdots 2 \\ 0 & 0 & 1\cdots 2 \\ \vdots & \vdots & \vdots\ \ddots \vdots \\ 0 & 0 & 0\cdots 1 \end{bmatrix} \tag{3.11}$$

4. Optimal Trajectory Problem Solutions by Orthogonal Functions

4.1. Optimal Trajectory and Control

Now consider the following time invariant large scale system:

$$\frac{d}{dt}x(t) = Ax(t) + Bu(t),\ x(t_0) = x_0 \tag{4.1}$$

where $x(t) \in R^{n\times 1}$, $u(t) \in R^{m\times 1}$, $A \in R^{n\times n}$, and $B \in R^{n\times m}$.

Then the quadratic cost function is defined as equation (4.2).

$$\hat{J} = \frac{1}{2}\left\|x(t_f) - r(t_f)\right\|_F^2 + \frac{1}{2}\int_{t_0}^{t_f}(\|x(t) - r(t)\|_Q^2 + \|u(t)\|_R^2)dt \tag{4.2}$$

where F and Q is positive semi-definite matrix, R is positive definite matrix. And $r(t) \in R^{n\times 1}$ is desired state trajectory. Thus we can get the Hamiltonian H from equation (4.1) and (4.2).

$$\mathrm{H} = \frac{1}{2}\|x(t) - r(t)\|_Q^2 + \frac{1}{2}\|u(t)\|_R^2 + \lambda^T(t)[Ax(t) + Bu(t)] \tag{4.3}$$

where $\lambda(t) \in R^{n\times 1}$ is co-state vector.

The two point boundary problem can be obtained from equation (4.3) using Pontryagin's maximum principle.

$$\begin{bmatrix} x(\dot{t}) \\ \lambda(\dot{t}) \end{bmatrix} = \begin{bmatrix} A - B & R^{-1}B^T \\ -Q & -A^T \end{bmatrix}\begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} + \begin{bmatrix} 0 \\ Qr(t) \end{bmatrix} \tag{4.4}$$

Thus optimal control vector u(t) is given as equation (4.5).

$$\mathrm{u(t)} = -R^{-1}B^T\lambda(t) \tag{4.5}$$

In this case, however, such derived two point boundary value problem will has fairly high order number, which resulted in much difficulty in having the solution. Thus hierarchical system control theory approach and iterative algorithm using block pulse functions are applied to solve above problems. We can divide the large scale system into interrelated N sub systems. Thus *i*th sub system and its quadratic cost function can be expressed as follows:

$$\mathrm{x_i}(\dot{t}) = A_i x_i(t) + B_i u_i(t) + C_i \pi_i(t) \tag{4.6}$$

where $\pi_i(t) = \sum_{j \neq i}^{N} D_{ij} x_j(t)$, πi(t) is model coordination variable.

$$J = \sum_{i=1}^{N} J_i = \sum_{i=1}^{N} \{ \frac{1}{2} \|x(t_f) - r(t_f)\|_{H_i}^2 + \frac{1}{2} \int_{t_0}^{t_f} (\|x_i(t) - r_i(t)\|_Q^2 + \frac{1}{2} \|u_i(t)\|_{R_i}^2$$
$$+ \beta_i^T(t)[\pi_i(t) - \sum_{j \neq 1}^{N} D_{ij} x_j(t)]) dt\} \quad (4.7)$$

where $\beta_i(t)$ is Lagrange multipliers or goal coordination variable. From equation (4.6) and (4.7), we can define divided Hamiltonian

$$H_i = \sum_{i=1}^{N} \{ \frac{1}{2} \|x_i(t) - r_i(t)\|_{Q_i}^2 + \frac{1}{2} \|u_i(t)\|_{R_i}^2 + \beta_i^T(t)[\pi_i(t) - \sum_{j \neq 1}^{N} D_{ij} x_j(t)]$$
$$+ \lambda_i{}^T(t)[A_i x_i(t) + B_i u_i(t) + C_i \pi_i(t)]\} \quad (4.8)$$

The two point boundary problem of *i*th subsystem becomes

$$\begin{bmatrix} \dot{x(t)} \\ \dot{\lambda(t)} \end{bmatrix} = \begin{bmatrix} A_i & -B_i R_i^{-1} B_i^T \\ -Q_i & -A_i{}^T \end{bmatrix} \begin{bmatrix} x_i(t) \\ \lambda_i(t) \end{bmatrix} + \begin{bmatrix} C_i \sum_{j \neq i}^{N} D_{ij} x_j(t) \\ Q_i r_i(t) + \sum_{j \neq i}^{N} D_{ij} C_j^T \lambda_j(t) \end{bmatrix} \quad (4.9)$$

To get the optimal solution for divided sub systems, coordinator variables can be adjusted using interaction prediction principle. And *i*th two point boundary value problems can be evaluated from the Hamiltonian of unadjusted *i*th subsystem.

4.2. Solution using Walsh Functions

The Walsh operational matrices and transform for solving optimal trajectory problem is much simpler than general calculus for investing large scale systems. Because Walsh series is defined in the 0 to 1 interval we normalize the problem first by using $\lambda = {}^{\tau}/_{t_f}$ then equation (4.4) becomes,

$$\begin{bmatrix} \dot{x(\lambda)} \\ \dot{p(\lambda)} \end{bmatrix} = -t_f \begin{bmatrix} x(\lambda) \\ p(\lambda) \end{bmatrix}, \ 0 \leq \lambda < 1 \quad (4.10)$$

Next assume $\dot{x(\lambda)}$ and $\dot{p(\lambda)}$ to be expanded into Walsh series whose coefficients are to be determined.

$$\begin{bmatrix} \dot{x(\lambda)} \\ \dot{p(\lambda)} \end{bmatrix} = C\emptyset(\lambda) \quad (4.11)$$

where *C* is an *2n x m* matrix, and *Ø(λ)*, an m vector.

Then Walsh integration is applied to perform integration on (4.11).

$$\begin{bmatrix} x(\lambda) \\ p(\lambda) \end{bmatrix} = CP\emptyset(\lambda) + \begin{bmatrix} x(\lambda = 0) \\ 0_n \end{bmatrix} \quad (4.12)$$

Substituting (4.11) and (4.12) into (4.10) gives

$$C\emptyset(\lambda) = -t_f \left\{ CP + \begin{bmatrix} x(\lambda = 0) \\ 0_n \end{bmatrix}, \ 0_{2n} \dots 0_{2n} \right] \} \emptyset(\lambda) \quad (4.13)$$

Defining *k* as equation (4.14), then (4.13) is simplified into (4.15).

$$k = \begin{bmatrix} -t_f Ax(\lambda = 0) \\ -t_f Qx(\lambda = 0) \\ 0_{2n} \\ \vdots \\ 0_{2n} \end{bmatrix} \tag{4.14}$$

$$C = \left[I + t_f M \otimes P\cdot\right]^{-1} k \tag{4.15}$$

Solving equation (4.15) for C, we obtain the Walsh coefficients of the $x(\dot{\lambda})$ and $p(\dot{\lambda})$. Therefore we can get the optimal control vector potentially.

4.3. Solution using BPF

Block pulse functions have been applied to get optimal trajectory for a large scale system. The iterative algebraic equation is obtained through the reverse integral operator of block pulse functions.

$$\Phi_m = [I - -\left(\frac{t_f}{2m}\right)L]^{-1}\ ,\ \Phi_k = \Phi_{k+1}[I + \left(\frac{t_f}{2m}\right)L][I - \left(\frac{t_f}{2m}\right)L]^{-1} \tag{4.16}$$

where Φ_m is the coefficient of state transition matrix of block pulse functions expanded and k=0, 1, ….m-1. And the state equation (4.17) can be obtained by iterative operation that can be arranged and written as following equation (4.18).

$$x(\dot{t}) = [A - BR^{-1}B^T K(t)]x(t) + [M - BR^{-1}B^T S(t)] \tag{4.17}$$

$$x_1 = \eta_1[x_0 + \omega_1],$$

$$x_{k+1} = \eta_{k+1}[\rho_k x_k + \omega_{k+1} + \omega_k] \tag{4.18}$$

where k=0, 1, ….m-1 and η_k, ρ_k, ω_k are respectively as follows:

$$\eta_k = [\ I - \left(\frac{t_f}{2m}\right)(A - BR^{-1}B^T K_k)]^{-1} \tag{4.19}$$

$$\rho_k = [\ I + \left(\frac{t_f}{2m}\right)(A - BR^{-1}B^T K_k)] \tag{4.20}$$

$$\omega_k = \left(\frac{t_f}{2m}\right)(M_k - BR^{-1}B^T S_k) \tag{4.21}$$

Therefore we can get the optimal control vector potentially from equation (4.17). However we have to define and apply the error function in order to determine whether the optimal control input of each sub system is the same as the optimal control input of entire system.

$$\mathrm{E} = \frac{1}{m}\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{m}[(M_i)_j^{k+1} - (M_i)_j^k]}^T[(M_i)_j^{k+1} - (M_i)_j^k] + \frac{1}{m}\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{m}[(N_i)_j^{k+1} - (N_i)_j^k]}^T[(N_i)_j^{k+1} - (N_i)_j^k] \tag{4.22}$$

where *I* is the *i*th sub system, *j* is *j*th interval and (k+1) is next step of k.

5. Simulation

5.1. Example(1)

For computational examples are presented to illustrate the method proposed in chapter 4. Consider the following scaled system,

$$\dot{x}(t) = -x(t) + u(t) - Su(t-\tau), x(0) = 1.0, u(t) = 0, t \in [-\tau, 0] \quad (5.1)$$

$$\dot{x}(t) = Ax(t) + L\dot{x}(t-\tau) + bu(t) + Su(t-\tau) \quad (5.2)$$

$$J = 0.5x^T(t_f)G(t_f) + 0.5\int_0^{t_f}(x^TQx + u^TQx)dt \quad (5.3)$$

Now we can minimize J optimal control. Equation (5.1) is considered with values of S and τ;

$$S_1=0,\ S_2=\sqrt{0.5},\ \tau=2/3,\ S_3=0.5,\ \tau=5/12 \quad (5.4)$$

Comparing (5.1), we obtain A=-1.0, L=0.0, B=1.0, R=0.5, Q=1.0 G=0 and t_f=1.

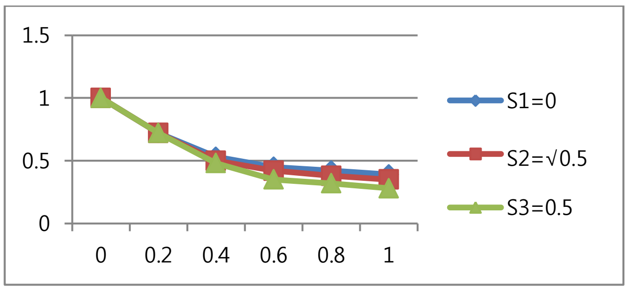

Figure 2. state trajectory of example(1)

Let's consider another scaled difference equation with λ=2;

$$x(k+2) - x(k) = -\frac{1}{3}h[x(\lambda k) + 4x(\lambda(k+1)) = x(k+2))] \quad (5.5)$$

and $x(0) = 1, x(1) = 0.9523$

Here l=2 and Walsh functions expansion coefficient vector for the solution (5.5) is x^T The approximate solutions obtained via different Walsh functions transform compared with the Chebyshev approximation method of the continuous scaled system are shown in Table 1.

Table 1. Results of example(1)

k	Laguerre	Chebyshev	Walsh functions transform
0.00	0.9146	1.0000	0.9999
0.05	0.8891	0.9523	0.9524
0.10	0.7645	0.9088	0.9088
0.15	0.6995	0.8689	0.8689
0.20	0.6784	0.8321	0.8321

5.2. Example(2)

Let's consider trajectory optimization solution for six order system plant with two inputs that have controllability using block pulse functions and hierarchical control method.

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \\ \dot{x_3}(t) \\ \dot{x_4}(t) \\ \dot{x_5}(t) \\ \dot{x_6}(t) \end{bmatrix} = \begin{bmatrix} -1.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2.5 \\ 0 & -3.5 & -3.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.0 & 2 & 0 \\ 0 & -0.1 & 0 & 0 & 0 & 1 \\ -0.2 & 0 & -0.6 & 0 & -3.25 & -2.25 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 1 \\ 0 & 0 \\ 0 & 0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \tag{5.6}$$

where initial conditions of state variable is x(0)=[-0.5 1.5 0 0.5 0.5 0]^T and desired state trajectory is r(t)=[0 0 1 0 0 1]^T. And the quadratic cost function is as follows:

$$\hat{J} = \frac{1}{2}[x(t_f) - r(t_f)]^T \mathrm{F}[x(t_f) - r(t_f)] + \frac{1}{2}\int_{t_0}^{t_f}\{[x(t) - r(t)]^T Q[x(t) - r(t)] + u(t)^T Ru(t)\}dt \tag{5.7}$$

where $Q = \begin{bmatrix} 550 & 0 & 0 & 0 & 0 & 0 \\ 0 & 150 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 550 & 0 & 0 \\ 0 & 0 & 0 & 0 & 150 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, F = Q, t_f = 10.0[sec], m = 200.$

The first sub system has the following expression.

$$\dot{x}_{sub1}(t) = A_{sub1}\dot{x}_{sub1}(t) + B_{sub1}u_{sub1}(t) + C_{sub1}\pi_{sub1}(t) \tag{5.8}$$

where $x_{sub1}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}, \pi_{sub1}(t) = x_{sub1}(t), x_{sub1}(0) = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}, r_{sub1}(t) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$

$$A_{sub1} = \begin{bmatrix} 0 & 2 & 0 \\ -1.0 & 0 & 1.5 \\ 0 & -4.5 & -4.0 \end{bmatrix}, B_{sub1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 1.0 \end{bmatrix}, C_{sub1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tag{5.9}$$

The quadratic cost function of first sub system is as follows:

$$\hat{J} = \frac{1}{2}[x_{\mathrm{sub1}}(t_f) - r_{\mathrm{sub1}}(t_f)]^T F_{\mathrm{sub1}}[x_{\mathrm{sub1}}(t_f) - r_{\mathrm{sub1}}(t_f)] + \frac{1}{2}\int_{t_0}^{t_f}\{[x_{\mathrm{sub1}}(t) - r_{\mathrm{sub1}}(t)]^T Q_{\mathrm{sub1}}[x_{\mathrm{sub1}}(t) - r_{\mathrm{sub1}}(t)] + \mathrm{u}_{\mathrm{sub1}}(t)^T R_{\mathrm{sub1}}\mathrm{u}_{\mathrm{sub1}}(t) \quad + \beta_{\mathrm{sub1}}^{(t)T}[\pi_{\mathrm{sub1}}(t) - \sum_{j\neq 1}^{N} D x_{sub2}(t)]\}dt \tag{5.10}$$

where $Q_{sub1} = \begin{bmatrix} 550 & 0 & 0 \\ 0 & 150 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_{sub1} = [1], F_{sub1} = Q_{sub1}.$

And the second sub system has the following expression. We can solve the second sub system as the same manner of the first sub system.

$$\dot{x}_{\mathrm{sub2}}(t) = A_{\mathrm{sub2}}\dot{x}_{\mathrm{sub2}}(t) + B_{\mathrm{sub2}}u_{\mathrm{sub2}}(t) + C_{\mathrm{sub2}}\pi_{\mathrm{sub2}}(t) \tag{5.11}$$

Therefore from divided two sub systems, we can decide optimal control inputs and state trajectory. The simulation results Figure 3~6 are shown as follow:

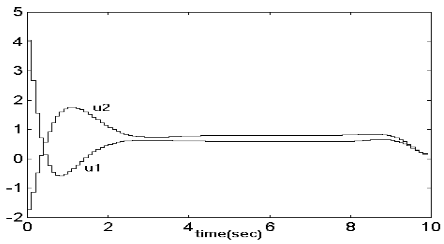

Figure 3. Control input u(t) before dividing

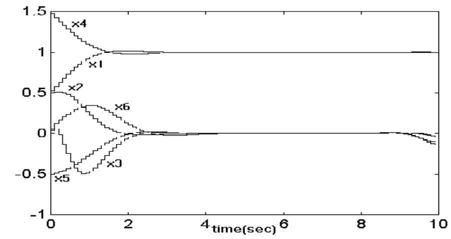

Figure 4. State trajectory T before dividing

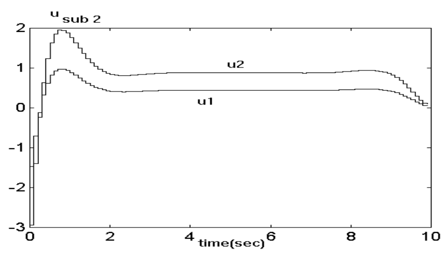

Figure 5. u(t) of 2nd sub system after dividing

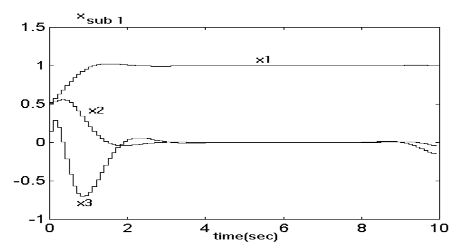

Figure 6. T of 2nd sub system after dividing

5. Conclusions

This paper presents an algorithm of optimal trajectory for large scale system using Walsh and block pulse functions. The optimal tracking problem is to decide the control input so that the system status would track the status orbit that we want. Generally, the problem in large scale system is to decide the control input by calculating the solution of the two point boundary value problem derived from the optimization control approach. If we want control to higher system, need high order controller, however increasing burden of computational complexity proportional to the order of the system, in some cases, the implementation of the controller may be impossible. The proposed method is quite simple and accurate to implement and has, therefore, obvious advantages in practical situations. The simulation results indicate that acceptable and optimal state trajectory can be obtained even in case where large scale system.

Acknowledgements

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