On the Optimal State Observer Synthesis for Discrete-time Linear Systems

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Abstract

This paper deals with the determination of the optimal state observer gain matrix for discrete-time linear systems. In this way, it has been shown that an improved optimal gain is reached by minimizing a quadratic criterion formulated as a quadratic output feedback control of the observation error system.

The gradient matrix operation is applied to the Lagrangian function in order to obtain necessary conditions, for minimizing the proposed criterion, to perform the optimal gain matrix. It has been shown, by Lyapunov stability theory, that this optimal gain ensures the asymptotic convergence of the observation error towards zero.

The necessary and sufficient conditions are presented by coupled discrete Lyapunov equations which resolution, by a proposed numerical algorithm, allows the calculus of the optimal observation gain.

The importance of the proposed criterion for the synthesis of the state observer has been illustrated through numerical simulation study of the state observation of a robot with flexible link which has highlighted the effectiveness of the developed method in relation to that optimizing the dual system.

Keywords: *State observer design; Optimal control; Output feedback control; Flexible link robot*

1. Introduction

In the literature, the continuous-time control and observation have a strong theoretical background. However, a large class of continuous-time systems is in fact computer controlled. In these cases information about the system (measurements) are only available at specific time instances and control law can only be changed at these time instances. Hence, it is important to synthesis these laws for the class of discrete-time systems [1, 2].

However, in many real world engineering applications, the knowledge of the system state is often required not only for control purpose but also for monitoring and fault diagnosis. In practice however, the measurements of the system state can be very difficult or even impossible, for example when an appropriate sensor is not available or economically viable [3, 4, 5, 6]. Model-based state estimation is a largely adopted strategy used in order to cope with this important problem. Typically a state estimation is provided by means of an observer whose inputs are the inputs and the outputs of the system and the outputs are the estimated states. Note that the structure of an observer is based on the mathematical model of the considered system. Therefore, the accuracy of the state estimation depends on the accuracy of the used mathematical model and the quality of the employed measurements.

Indeed, the theory of state observers for linear systems has been one of the most active research areas over the past decades and has become matured through extensive studies. Various approaches, such as transfer-function, geometric, algebraic, singular value decomposition, have been successfully proposed. The application of state observer theory can be found in a wide range of different fields. However, the observer theory ensured only the convergence of the observation error without limiting the gain matrix in order to make this problem practical [7, 8, 9].

Moreover, several methods are used for the determination of the observer gain matrix, such that the asymptotic stability of the error is ensured, as the poles placement technique [10, 7], the algebraic Lyapunov equation method [11, 12], the linear matrix inequality (LMI) approach [13, 14, 15, 16] and the optimization of a quadratic criterion [17, 18].

However, the quadratic criterion considered in the literature is constructed up on a dual system and so it does not express the real performances of the synthesized observer, even it is leading to satisfactory results.

In this paper we have also considered the optimal state observer gain determination to propose an optimization criterion which has a direct signification and interpretation regarding to the desired observer. Thus, this optimal gain is calculated from the gradient resolution of the designed Lagrangian function in order to obtain necessary conditions for minimizing such criterion.

A comparative study between the known criterion constructed on a dual system and the proposed one using the state observer is curried out on a physical system constituted by a robot with flexible link to show up the developed improvement.

Indeed, the robot with flexible link is a multivariable dynamical system where many variables are not measurable and its parameters are most often imprecisely known. For these reasons more often optimal state observers and estimation approaches are necessary for the control of such process [4, 19].

An outline of this paper is as follows: the optimal gain determination based on the dual system is presented in section 2. Section 3, is devoted for the proposed method for direct observer gain optimization. In section 4, an illustrative example of a robot with flexible link is presented to highlight the performances of proposed approach.

2. Optimal Gain Determination using Dual System

We consider the class of discrete-time linear dynamical systems described by the following state equations

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k \end{cases} \tag{1}$$

where $k \in \mathbb{N}$ is the discrete-time index, $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^p$ is the input vector, $y_k \in \mathbb{R}^m$ is the output vector and A, B and C are constant matrices of appropriate dimensions.

We assume that the pair (A, C) is observable. Then, the state observer for the discrete-time linear system (1) may be written as follows

$$\begin{cases} \hat{x}_{k+1} = A\hat{x}_k + Bu_k + L(y_k - \hat{y}_k) \\ \hat{y}_k = C\hat{x}_k \end{cases} \tag{2}$$

with $\hat{x}_k \in \mathbb{R}^n$ the observed state vector of x_k and $L \in \mathbb{R}^{n\times m}$ the observer gain matrix to be determined.

The state observer (2) is characterized by the gain matrix L such that the observation error given by

$$e_k = x_k - \hat{x}_k \tag{3}$$

converge exponentially towards zero.

The dynamics of the reconstruction error is expressed by the following equation

$$e_{k+1} = (A - LC)e_k \tag{4}$$

Among the technique of the matrix gain determination one can use the optimization of the quadratic criterion $\tilde{J}$

$$\tilde{J} = \sum_{k=0}^{\infty} \left(\tilde{e}_k^T Q \tilde{e}_k + \tilde{u}_k^T R \tilde{u}_k \right) \tag{5}$$

with $Q \in \mathbb{R}^{n\times n}$ the symmetric positive semi-definite matrix and $R \in \mathbb{R}^{m\times m}$ the symmetric positive definite matrix.

For the dual discrete-time linear system [18]

$$\tilde{e}_{k+1} = \tilde{A}\tilde{e}_k + \tilde{B}\tilde{u}_k \tag{6}$$

where $\tilde{A} = A^T$ and $\tilde{B} = C^T$.

The minimization of the quadratic criterion $\tilde{J}$ leads to:

- the predictor-observation gain matrix

$$L_p = -AP_pC^T(R - CP_pC^T)^{-1} \tag{7}$$

where $P_p \in \mathbb{R}^{n\times n}$ is the symmetric positive definite matrix solution of the following discrete Riccati equation

$$AP_pA^T + AP_pC^T(R - CP_pC^T)^{-1}CP_pA^T - P_p - Q = 0 \tag{8}$$

- the corrector-observation gain matrix

$$L_c = -AP_cA^TC^T(R - CAP_cA^TC^T)^{-1} \tag{9}$$

where $P_c \in \mathbb{R}^{n\times n}$ is the symmetric positive definite matrix solution of the following discrete Riccati equation

$$AP_cA^T + AP_cA^TC^T(R - CAP_cA^TC^T)^{-1}CAP_cA^T - P_c - Q = 0 \qquad (10)$$

The drawback of this technique is that the quadratic criterion $\tilde{J}$ which has been minimized has no direct physical interpretation regarding to the observation error.

In what follows we propose a new formulation of the optimization criterion which expresses the desired observer performances regarding to the error dynamical equation.

3. Optimal Gain Determination using Direct Method

The state observation error (4) can be described by the following dynamical system

$$\begin{cases} e_{k+1} = Ae_k + \eta_k \\ \eta_k = -Lv_k \\ v_k = Ce_k \end{cases} \qquad (11)$$

The system (11) expresses an output feedback control problem of the discrete-time linear system of order n with n dimensional input vector η_k and m dimensional output vector v_k.

The proposed output feedback control problem scheme can be optimized by minimizing the following quadratic criterion defined by

$$\begin{aligned} J &= \sum_{k=0}^{\infty}\left(e_k^TQe_k + \eta_k^TR\eta_k\right) \\ &= \sum_{k=0}^{\infty}e_k^T\left(Q + C^TL^TRLC\right)e_k \end{aligned} \qquad (12)$$

with $Q = Q^T \geq 0$ and $R = R^T > 0$.

For the state observer, the minimization of the quadratic criterion J can be interpreted as looking for a compromise between the minimization of observation error represented by the term $e_k^TQe_k$ and the minimization of the observer gain L written in the term $\eta_k^TR\eta_k$.

Using the solution of the equation (11), the equality (12) can be written as

$$J = \sum_{k=0}^{\infty}e_0^T\left(\left(A - LC\right)^k\right)^T\left(Q + C^TL^TRLC\right)\left(A - LC\right)^k e_0 \qquad (13)$$

where e_0 is the initial condition of the observation error.

The above expression can be put in the following form

$$J = e_0^TPe_0 = trace\left\{Pe_0e_0^T\right\} \qquad (14)$$

where $P = \sum_{k=0}^{\infty}\left(\left(A - LC\right)^k\right)^T\left(Q + C^TL^TRLC\right)\left(A - LC\right)^k$, symmetric positive definite matrix, satisfies the following discrete Lyapunov equation

$$\left(A-LC\right)^T P\left(A-LC\right)-P+Q+C^T L^T RLC=0 \tag{15}$$

The dependency of the optimal solution on the initial condition e_0 can be removed by considering that

$$E(e_0 e_0^T)=I_n \tag{16}$$

with $E(.)$ the average value.

Then, the expected value of the quadratic criterion $\bar{J}$ of the cost function (14) is simply evaluated as follows

$$\bar{J}=E\left\{J\right\}=trace\left\{P\right\} \tag{17}$$

Thus, that may have appeared to be a dynamical problem (11) is now formulated as a static quadratic criterion (17) which is minimized with respect to the observation gain matrix L and the symmetric positive definite matrix P subject to the constraint (15).

To obtain the necessary conditions for minimizing the quadratic criterion $\bar{J}$ with respect to matrices L and P with the constraint (15), we can apply the gradient matrix operations to the Lagrangian

$$\Im\left(L,P,\Gamma\right)=trace\left\{\Gamma^T\left[\left(A-LC\right)^T P\left(A-LC\right)-P+Q+C^T L^T RLC\right]\right\}+trace\left\{P\right\} \tag{18}$$

where $\Gamma \in \mathbb{R}^{n\times n}$ is a matrix of Lagrange multipliers may be selected symmetric positive definite.

By using gradient matrix operations [20], [21], the necessary conditions for L, P and Γ to be optimal are given by

$$\begin{cases} \dfrac{\partial \Im}{\partial L}\left(L,P,\Gamma\right)=2RLC\Gamma C^T-2P\left(A-LC\right)\Gamma C^T=0 \\ \dfrac{\partial \Im}{\partial P}\left(L,P,\Gamma\right)=\left(A-LC\right)\Gamma\left(A-LC\right)^T-\Gamma+I=0 \\ \dfrac{\partial \Im}{\partial \Gamma}\left(L,P,\Gamma\right)=\left(A-LC\right)^T P\left(A-LC\right)-P+Q+C^T L^T RLC=0 \end{cases} \tag{19}$$

From the first equation of system (19), we get the optimal observation gain matrix as follows

$$L=\left(R+P\right)^{-1} PA\Gamma C^T\left(C\Gamma C^T\right)^{-1} \tag{20}$$

In order to prove the asymptotic stability of the proposed form of the observation error (11), we look for a quadratic positive definite Lyapunov function defined by

$$V(e_k)=e_k^T Pe_k \tag{21}$$

with P the $(n\times n)$ symmetric positive definite matrix solution of the third equation of system (19).

Then, we have the difference between the Lyapunov function candidates (21) for two consecutive time instants along any trajectory of (11) as follows

$$\begin{aligned}\Delta V(e_k) &= V(e_{k+1}) - V(e_k) \\ &= e_{k+1}^T P e_{k+1} - e_k^T P e_k \\ &= e_k^T \Big[\big(A-LC\big)^T P\big(A-LC\big) - P\Big] e_k\end{aligned} \tag{22}$$

Using the third equation of the system (19), (22) becomes

$$\Delta V(e_k) = -e_k^T\big(Q + C^T L^T RLC\big)e_k \tag{23}$$

where L is the $(n\times m)$ optimal observation gain matrix given by equation (20), the matrices Q and R defined by (12) are such that $Q = Q^T \geq 0$ and $R = R^T > 0$. Then, the term $Q + C^T L^T RLC$ is positive definite.

Therefore, the difference of the quadratic Lyapunov function, given in (23), is characterized by

$$\Delta V(e_k) < 0 \tag{24}$$

for all $e_k \neq 0$.

From the above development, we can conclude that the observation error given by system (11) is asymptotically stable in the sense of Lyapunov stability theory.

Notice that to determine the optimal observation gain matrix L and the symmetric positive definite matrix P, it is clear that the three equations of the system (19) are coupled and can be written in the following form

$$\begin{cases} F_1(L,P,\Gamma): L = \big(R+P\big)^{-1} PA\Gamma C^T\left(C\Gamma C^T\right)^{-1} \\ F_2(L,P,\Gamma): \left(A-LC\right)\Gamma\left(A-LC\right)^T - \Gamma + I = 0 \\ F_3(L,P,\Gamma): \left(A-LC\right)^T P\left(A-LC\right) - P + Q + C^T L^T RLC = 0 \end{cases} \tag{25}$$

Then, to solve this system (25), it is important to use an iterative algorithm. The proposed iterative algorithm is developed as follows

Iterative Algorithm

1. Initialize : Set $n = 1$:

   Select $Q \geq 0$, $R > 0$ and L_1 such as $\rho(A - L_1 C) < 1$, where $\rho(A - L_1 C)$ is the spectral radius of $(A - L_1 C)$.

2. n[th] iterative:

- Using this value of L_n and the resolution of the discrete Lyapunov equation $F_3(L_n, P_n) = 0$, we obtain the value for P_n.

- With L_n and the resolution of the discrete Lyapunov equation $F_2(L_n, \Gamma_n) = 0$, we get Γ_n.
- Update L_{n+1}, for the obtained values P_n and Γ_n, with the relation $F_1(L_{n+1}, P_n, \Gamma_n)$.

3. $n = n+1$:

   Repeat the step 2 for $n = n+1$ to obtain the optimal values.

4. Terminate:

   Stop the algorithm if $\| P_n - P_{n-1} \| \leq \varepsilon$ (ε is a prescribed small number used to check the convergence of the algorithm).

   So, for $n = 1, 2, \ldots$, we have

   P_n is found from the discrete Lyapunov equation $F_3(L_n, P_n) = 0$,

   Γ_n is found from the discrete Lyapunov equation $F_2(L_n, \Gamma_n) = 0$,

   L_{n+1} is found from $F_1(L_{n+1}, P_n, \Gamma_n)$.

3. Numerical Example

To illustrate the availability and the efficiency of the proposed discrete-time linear optimal state observer design, we consider the system of a single link robot with a revolute elastic joint rotating in a vertical plane which is modelled by [4], [19], [22]:

$$\begin{cases} \dot{\theta}_m = \omega_m \\ \dot{\omega}_m = -\dfrac{F_m}{J_m}\omega_m + \dfrac{K}{J_m}(\theta_l - \theta_m) + \dfrac{K_\tau}{J_m}u \\ \dot{\theta}_l = \omega_l \\ \dot{\omega}_l = -\dfrac{F_l}{J_l}\omega_l - \dfrac{K}{J_l}(\theta_l - \theta_m) - \dfrac{Mgh}{J_l}\sin(\theta_l) \end{cases} \tag{26}$$

Where θ_m, ω_m, θ_l and ω_l are the motor angular displacement, the angular velocity of the motor, the link angular displacement and the angular velocity of the link respectively. J_m and J_l are the inertia of the motor and link respectively, $2h$ and M represent the length and mass of the link, F_m and F_l are the viscous friction coefficients, K is the elastic constant, g is the gravity constant and K_τ is the amplifier gain. The control u is the torque delivered by the motor.

The nonlinear state representation (26) is linearized around the operating point $\theta_{l0} = +\pi/3$, then we can express the linear time invariant model of the robot with flexible link as follows

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K}{J_m} & -\frac{F_m}{J_m} & \frac{K}{J_m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K}{J_l} & 0 & -\frac{Mgh}{J_l}cos(\theta_{l0}) - \frac{K}{J_l} & -\frac{F_l}{J_l} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_\tau}{J_m} \\ 0 \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \end{cases} \quad (27)$$

with $\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} \theta_m & \omega_m & \theta_l & \omega_l \end{bmatrix}^T$.

The performances of the proposed discrete-time linear optimal state observer with the optimal gain obtained by the proposed iterative algorithm, compared to the dual optimal one, was investigated by simulation for the discrete-time model of the flexible link robot discretized with the zero order hold at the sampling time $T_e = 0.05s$ and characterized by the following numerical parameters:

Table 1. Parameters of the Flexible Link Robot

Parameter	Numerical value
K	$1.8Nm/rad$
K_τ	$0.8Nm/V$
J_m	$37.9\times10^{-3}Kgm^2$
J_l	$94.6\times10^{-3}Kgm^2$
h	$0.15m$
M	$0.21Kg$
F_m	$47.3\times10^{-3}Nm/rad/s$
F_l	$0Nm/rad/s$

In the following, the procedure for the optimal state observer and the dual optimal one for discrete-time linear system are presented. For the computation of the observation gain matrix L, we select the same parameters, for the three design state observer approaches, $Q = 0.5*I_4$ and $R = 0.5*I_4$.

(i) Optimal gain matrix obtained by resolution of the dual system:

- the optimal predictor-observation gain matrix:

$$L_p = \begin{bmatrix} 0.1175 & 0.0364 \\ 1.6126 & -0.8185 \\ 0.5787 & -0.0215 \\ -0.6361 & 0.8276 \end{bmatrix}$$

- the symmetric positive definite matrix obtain with the optimal predictor-observation gain matrix:

$$P_p = \begin{bmatrix} 0.1793 & 0.1138 & 0.0622 & 0.1205 \\ 0.1138 & 6.2765 & 0.7617 & -1.3203 \\ 0.0622 & 0.7617 & 0.7818 & -0.2850 \\ 0.1205 & -1.3203 & -0.2850 & 1.3946 \end{bmatrix}$$

- the corresponding optimal quadratic criterion:

$$J_p = 8.6323$$

- the optimal corrector-observation gain matrix:

$$L_c = \begin{bmatrix} 0.1583 & 0.0602 \\ 1.1753 & -0.7452 \\ 0.4912 & -0.1550 \\ -0.1550 & 0.7432 \end{bmatrix}$$

- the symmetric positive definite matrix obtain with the optimal corrector-observation gain matrix:

$$P_c = \begin{bmatrix} 0.1753 & 0.2003 & 0.0792 & 0.0301 \\ 0.2003 & 5.5160 & 0.5877 & -0.3726 \\ 0.0792 & 0.5877 & 0.7456 & -0.0775 \\ 0.0301 & -0.3726 & -0.0775 & 0.8716 \end{bmatrix}$$

- the corresponding optimal quadratic criterion:

$$J_c = 7.3085$$

(ii) Gain matrix obtained by the proposed direct method:

In the objective to minimize moreover the quadratic criterion $\bar{J}$, one uses the observation gain matrix given by the dual system as initial stabilizable solution. We obtain with the iterative algorithm resolution, after $N = 10$ iterations:

- the optimal observation gain matrix:

$$L_{opt} = \begin{bmatrix} 0.0531 & 0.1981 \\ 0.0929 & -0.1131 \\ 0.8391 & -0.2027 \\ -0.8194 & 0.9492 \end{bmatrix}$$

- the symmetric positive definite matrix:

$$P_{opt} = \begin{bmatrix} 2.0084 & 0.0405 & -0.4364 & -0.4966 \\ 0.0405 & 0.0317 & 0.0693 & -0.0119 \\ -0.4364 & 0.0693 & 1.5426 & -0.3736 \\ -0.4966 & -0.0119 & -0.3736 & 1.2452 \end{bmatrix}$$

- the corresponding minimal quadratic criterion:

$$\bar{J}_{opt} = 4.8278$$

The performances of the proposed optimal state observers loaded by the optimization of the dual system and the proposed direct optimization, tested by numerical simulation, are shown in figures 1 to 4 which depict the evolution of the actual and the observed state variables of the studied flexible link robot: the motor angular position θ_m, the motor angular velocity ω_m, the link angular position θ_l and the link angular velocity ω_l.

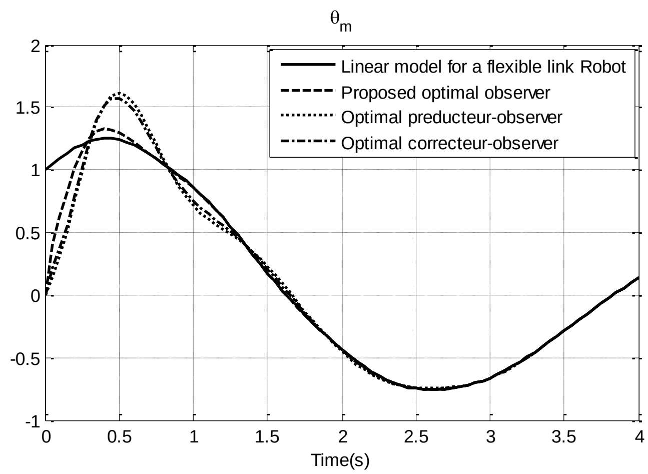

Figure 1. Actual and Observed Angular Position θ_m of the Motor

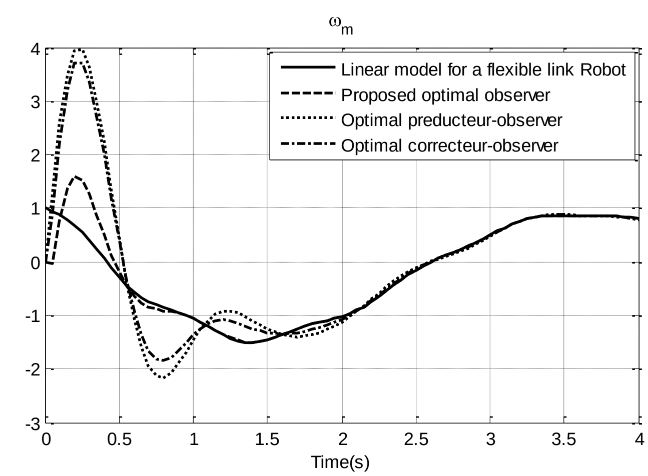

Figure 2. Actual and Observed Angular Velocity ω_m of the Motor

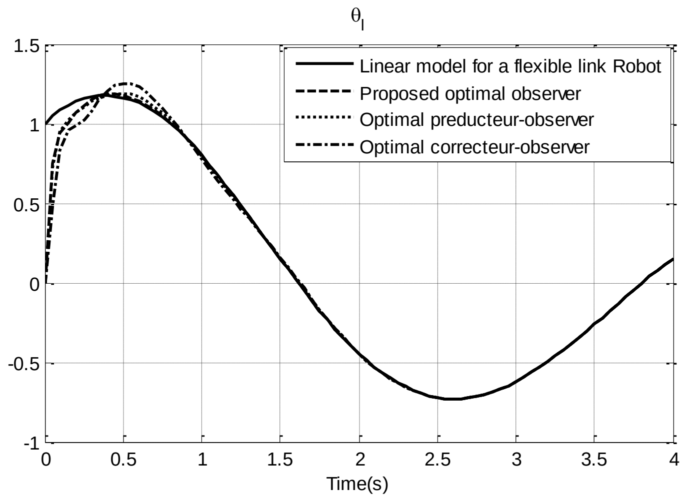

Figure 3. Actual and Observed Angular Position θ_l of the Link

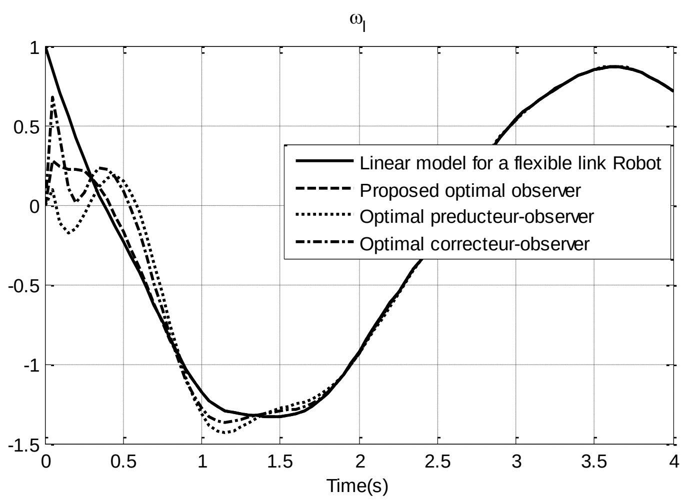

Figure 4. Actual and Observed Angular Velocity ω_l of the Link

It appears, from these simulations, that the optimal state observers for the dual discrete-time linear system allow a well reconstruction of the actual states. It can converge rapidly towards the state variable of the flexible link robot. The same graphics show the evolution of the observed state variables generated by the proposed direct optimization method. It's clear that this observer allows the reconstruction of the actual flexible link robot state variables with a remarkable superiority compared to the optimal state observers with dual system. Indeed, the high performances of the proposed optimal state observer with the direct optimization method show the improvement leaded by the use of the proposed iterative algorithm permitting the calculus of the optimal gain matrix.

4. Conclusion

Optimal state observer design for a class of discrete-time linear systems has been studied in this paper. The discrete-time linear optimal state observer is based on the determination of an optimal observation gain matrix derived by minimizing a proposed quadratic criterion characterized, in a new formulation, by a quadratic output feedback control problem of the observation error system. This criterion is expressed regarding to the performances of the state observer, and it bypasses the drawback of the actually used criterion which optimizes the dual system of the observation error.

It has been shown from the simulation results that the state observers allow the reconstruction of the unmeasurable state variables of the flexible link robot, but with a remarkable superiority of the optimal state observer derived by the proposed direct optimization method with respect to the optimal observers for the dual system.

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