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# Periodic Review Production Models with Variable Yield and Inventory Level Dependent Demand

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***Abstract***

*In this paper, we consider periodic review production or inventory models incorporating inventory level dependent demand and random yield models. Inventory level dependent demand models have been investigated by many marketing and production management researchers. Random yield inventory models have also been investigated by many production management researchers. There is, however, to our best knowledge, a few researches involving periodic review production model incorporating inventory level dependent demand and random yield. We, therefore, investigate single period and multiperiod inventory problems in which demand is inventory level dependent in a very general form and production yield is also random. For the single period problem, we obtain the complete analytical solutions and give the numerical example. For the multiperiod problem, we first investigate the two period models and extend to the n-period problem.*

***Keywords:*** *Production/Inventory Management, Periodic Review Production/Inventory Model, Inventory Level Dependent Demand, Random Yield*

## 1. Introduction

A subject in the area of inventory theory that has recently been receiving considerable attention is the class of inventory level dependent demand models [1]. In this model the presence of the inventory in some items is assumed to stimulate sales of the items. For the inventory control of these items, Baker and Urban [2, 3] investigated two basic models in which the demand rate of an item is a function of the instantaneous inventory level. After Baker and Urban's research, several other variations have been investigated such as Mandal Phaujdar [4], Datta and Pal [5], Goh [6], and so on [7]-[20]. Specially, Gerchak and Wang [9] investigated a very general form of inventory level dependent demand model. Their approach is to describe a demand as a general deterministic function of the starting inventory level, multiplied by a random variable. This makes both the modeling and analysis of the inventory model much easier, and permits the derivation of explicit, implementable solutions.

Another interesting area of inventory theory is random yield inventory models. When production activities are initiated or orders placed, the outputs or quantities received are often somewhat uncertain. The yield, or quantity received, might be itself uncertain, or possibly it is the usable portion of the yield which varies. These uncertainties could affect inventory stocking decision or production lot sizes [22]. There are several factors which might contribute to discrepancies between quantity ordered and quantity received. These might be clerical errors and damage in transit, inadequacies of raw materials, and rounding off by suppliers to achieve certain scale economies [23]. In Gerchak et al. [22], they investigated a periodic review production model with variable yield and uncertain demand. They provided a complete analysis of the single period model. Their results showed that the order point is not affected by the yield distribution and the expected yield is not order up to type. For other random yield inventory models refer to the papers [24]-[31].

In this research we consider single period and multi-period inventory models incorporating inventory level dependent demand and random yield models. For the previous researches of the incorporated inventory models, Lee and Chang [32] investigated the optimal order quantity in single period inventory level dependent demand and random yield models. This research is the extended version of the paper [32]. As another research, Lee et al. [33] are investigating periodic review production models with random yield, and stochastic endogenous demand. Moreover, Bar-Lev et al. [34] investigated the EOQ model with inventory-level-dependent demand rate and random yield. This paper, however, only dealt with the EOQ-type inventory problems. In this paper, we deals with the newsvendor-type inventory problems.

To develop the incorporated inventory model, we use Gerchak and Wang [9]'s demand model. Their approach is to describe a period's demand as a general deterministic function of the starting inventory level, multiplied by a random variable. This explicit and stochastic functional form is a major distinguishing feature of the paper [9]. It makes both the modeling and analysis of the periodic-review model much easier, and permits the derivation of explicit, implementable solutions. With these benefits, we decide to adopt Gerchak and Wang's demand model. For the random yield, we use Gerchak et al. [22]'s yield model. Since the multiplicative yield model in the paper [22] is more convenient than the additive model we adopt the Gerchak et al.'s yield model.

By incorporating these two models we provide complete analyses of the single period and two-period inventory problems in which demand is inventory level dependent in a very general form and production yield is also random. We also provide the solution structure of the n-period problem. In the incorporated inventory models, we showed that the objective function is concave in some conditions and that the optimal policy is of a "reorder-point" type. That is, there exists a unique and finite value such that if the initial stock is greater than this value the order quantity is zero, otherwise the order quantity is greater than zero. Expressions for determining the reorder point and the optimal order quantity are derived. When only inventory level dependent demand is considered [9], the reorder point policy reduces to an order-up-to type.

In the next section, we review the single period (news-vendor) problem. We, then, develop an optimal order quantity in single period inventory level dependent demand with random yield. After the analysis of the single period model, we analyze two-period inventory model and provide the solution structure of the n-period model. Finally we conclude this paper with concluding remarks and future research.

## 2. The Classical Single Period Problem

In this section, we review the classical single period problem. After this section, we extend the single period problem to the inventory level dependent demand model incorporating random yield. We will use the following notation in this section and throughout the paper.

r = unit revenue from any sold item.
c = unit production/purchase cost. We will assume that c is always less than r and the cost c is proportional to the realized yield.

h = unit holding cost associated with each unsold item at the end of each period in a multiperiod setting.
v = unit cost associated with each unsold item at the end of the period in single period setting. v may be negative, corresponding to a salvage value. Assume
–v<c.
p = unit shortage penalty cost per unit of unsatisfied demand at the end of a period.
Q = order quantity at the beginning of a period, which is the decision variable for the problem. We assume that the only chance to replenish the inventory is at the beginning of the period, and that the lead time is zero.
D = demand in a period. We assume that the demand D is a random variable and has a density function k(d) and a distribution function K(d).

Then, the profit per period is

$$\pi = \begin{cases} (r-c)Q - p(D-Q), & \text{if } D \geq Q, \\ rD - v(Q-D) - CQ, & \text{if } D < Q, \end{cases}$$

Simplifying and taking the expected value of $\pi$ gives the following expected profit.

$$E[\pi] = (r+p-c)\int_Q^\infty Qk(d)dd - p\int_Q^\infty dk(d)dd + (r+\mathrm{v})\int_0^Q dk(d)dd - (c+v)\int_0^Q Qk(d)dd$$

We can show that $E[\pi]$ is concave and the sufficient optimality condition has the following form.

$$F(Q^*) = \frac{r+p-c}{r+p+v}$$ , where Q* is the optimal order quantity.

## 3. A Single Period Model Incorporating the Inventory Level Dependent Demand with Random Yield

In this section, we develop a single period inventory models incorporating inventory level dependent demand with random yield. First consider notation and assumptions. In addition to the notation in the previous section, we use the following notations in and after this section.

β = discount factor per period. $0 < \beta < 1$
I = initial stock level of a period.
$Y_Q$= yield when the input level is Q.

In our inventory system, the demand D in any period is assumed to be randomly dependent on the starting inventory level x in the form

D=H(x)*W,

where H(x) is a general deterministic function of x and W is a nonnegative random variable with a known probability distribution G, density g and mean α. We assume that the random variable W is independent of x.

The specific form of H(x) may vary for different items or situations, but we will have the following general assumptions:

1. H(x) is positive and increasing in x; i.e. H(x)>0 and H'(x) ≥ 0.
2. H(x) is a concave function; i.e., H''(x)<0.
3. As x goes to infinity, the first-order derivative of H(x) goes to zero; i.e. H'(∞)=0.

All these assumptions are rather plausible, for the discussion of the plausibility of the assumptions refer to Gerchak and Wang [9].

The yield is assumed to be contingent on the order quantity Q in the following manner

$Y_Q$=Q*U

where U is a non-negative random variable which is not contingent on Q, and is independent of D and U has mean $\mu$ with density f( ).

Since demand, and hence revenue, are affected by our inventory decisions, the natural model will be a profit maximization.

In a single-period scenario, with an initial inventory I and an order quantity Q at the beginning of the period, the stating inventory level of the period will be x=I+ $Y_Q$ because $Y_Q$=Q*U is the yield at an order quantity Q. So, the resulting random demand will be given by

D=H(I+ $Y_Q$)*W=H(I+Q*U)*W

Now, let ∏ be the random profit. Suppose that its initial stock is I and an order quantity Q, the realized profit will be

$$\pi = \begin{cases} -cQu + rH(I+Qu)w - v[(I+Qu) - H(I+Qu)w], & \text{if } I+Qu \geq H(I+Q)w, \\ -cQu + r(I+Qu) - p[H(I+Qu)w - (I+Qu)], & \text{if } I+Qu < H(I+Q)w, \end{cases} \tag{1}$$

We wish to maximize E(∏), where the expectation is taken with respect to the joint distribution of W and U. That is, for any given initial inventory level I, we seek the order quantity Q which maximizes the expected profit. Let Ψ(Q|I)≡E(Π). Then from (1), we have

$$\begin{aligned} \psi(Q \mid I) = -cQ\mu + \int_{u=0}^{\infty} f(u) \int_{w=0}^{\frac{I+Qu}{H(I+Qu)}} g(w)[(r+v)H(I+Qu)w - v(I+Qu)]dwdu \\ + \int_{u=0}^{\infty} f(u) \int_{w=\frac{I+Qu}{H(I+Qu)}}^{\infty} g(w)[(r+p)(I+Qu) - pH(I+Qu)w]dwdu. \end{aligned} \tag{2}$$

Note that if H(x)=1 i.e., when demand is inventory-level independent, and the yield is deterministic, the model reduces to the classical news vendor problem.

In the following, the main task is to analyze the properties of the above model and its corresponding optimal order policy. These properties are obtained under the following two assumptions about the cost parameters, the demand function H(x) and the distributions of W and U.

ASSUMPTIONS:

The following inequalities (3) and (4) hold for all x ≥ 0;

1.

$$-(r+p+v)[\int_{u=0}^{\infty} g(\frac{x}{H(x)})\frac{H(x)-xH'(x)}{(H(x))^2}u^2 f(u)du-\int_{u=0}^{\infty} H'(x)g(\frac{x}{H(x)})\frac{xH(x)-x^2H'(x)}{(H(x))^3}u^2 f(u)du]$$

$$+(r+v)\int_{u=0}^{\infty}u^2 f(u)H''(x)\int_{w=0}^{\frac{x}{H(x)}}wg(w)dwdu-p\int_{u=0}^{\infty}u^2 f(u)H''(x)\int_{w=\frac{x}{H(x)}}^{\infty}wg(w)dwdu<0, \qquad (3)$$

where x is a function of u.

2.

$$-(r+p+v)\mu g(\frac{x}{H(x)})\frac{H(x)-xH'(x)}{(H(x))^2}+(r+v)[H''(x)\int_{u=0}^{\infty}uf(u)\int_{w=0}^{\frac{x}{H(x)}}wg(w)dwdu$$

$$+\mu H'(x)\frac{x}{H(x)}g(\frac{x}{H(x)})\frac{H(x)-xH'(x)}{(H(x))^2}]$$

$$-\text{p}[H''(x)\int_{u=0}^{\infty}uf(u)\int_{w=\frac{x}{H(x)}}^{\infty}wg(w)dwdu-\mu H'(x)\frac{x}{H(x)}g(\frac{x}{H(x)})\frac{H(x)-xH'(x)}{(H(x))^2}]<0. \qquad (4)$$

3. $$r+p-c-p\alpha H'(0)>0. \qquad (5)$$

Let

$$A(x)=[\int_{u=0}^{\infty} g(\frac{x}{H(x)})\frac{H(x)-xH'(x)}{(H(x))^2}u^2 f(u)du-\int_{u=0}^{\infty} H'(x)g(\frac{x}{H(x)})\frac{xH(x)-x^2H'(x)}{(H(x))^3}u^2 f(u)du]$$

$$+\int_{u=0}^{\infty}u^2 f(u)H''(x)\int_{w=\frac{x}{H(x)}}^{\infty}wg(w)dwdu,$$

where x is a function of u.

If A(x)>0, condition 1 holds for all values of the cost parameters; if A(x)<0, condition 1 can be written as

$$p<-(r+v)[A(x)+\alpha\int_{u=0}^{\infty}u^2 f(u)H''(x)du]/A(x).$$

Let

$$B(x)=\mu g(\frac{x}{H(x)})\frac{H(x)-xH'(x)}{(H(x))^2}+H''(x)\int_{u=0}^{\infty}uf(u)\int_{w=\frac{x}{H(x)}}^{\infty}wg(w)dwdu$$

$$-\mu H'(x)\frac{x}{H(x)}g(\frac{x}{H(x)})\frac{H(x)-xH'(x)}{(H(x))^2}]$$

If B(x)>0, condition 2 holds for all values of the cost parameters; if B(x)<0, condition 2 can be written as

$$p < -(r+v)[B(x) - H''(x)\mu\alpha]/B(x).$$

Now, condition 3 is guaranteed if $1-\alpha H'(0) > 0$; if $1-\alpha H'(0) < 0$, the condition translates to

$$p < (r-c)/[\alpha H'(0) - 1].$$

Interestingly, the condition 3 is the same as condition 2 in Gerchak and Wang [9]. As mentioned in Gerchak and Wang [9], since the explicit shortage penalty will tend to be relatively low, possibly zero, these two assumptions are thus plausible.

Now, let us prove the following:

PROPOSITION 1: $\Psi(Q\,|\,I)$ is strictly concave in *Q*.

PROOF: From (2) it follows, after some algebra, that

$$\frac{\partial\Psi(Q\,|\,I)}{\partial Q} = (r+p-c)\mu - (r+p+v)\int_{u=0}^{\infty} G\left(\frac{I+Qu}{H(I+Qu)}\right)uf(u)du$$
$$+(r+v)\int_{u=0}^{\infty} H'(I+Qu)uf(u)\int_{w=0}^{\frac{I+Qu}{H(I+Qu)}} wg(w)dwdu - p\int_{u=0}^{\infty} H'(I+Qu)uf(u)\int_{w=\frac{I+Qu}{H(I+Qu)}}^{\infty} wg(w)dwdu \quad (6)$$

and

$$\frac{\partial^2\Psi(Q\,|\,I)}{\partial Q^2} = -(r+p+v)\Big[\int_{u=0}^{\infty} g\left(\frac{I+Qu}{H(I+Qu)}\right)\frac{H(I+Qu)-(I+Qu)H'(I+Qu)}{(H(I+Qu))^2}u^2 f(u)du$$
$$-\int_{u=0}^{\infty} H'(I+Qu)g\left(\frac{I+Qu}{H(I+Qu)}\right)\frac{(I+Qu)H(I+Qu)-(I+Qu)^2 H'(I+Qu)}{(H(I+Qu))^3}u^2 f(u)du\Big]$$
$$+(r+v)\int_{u=0}^{\infty} u^2 f(u)H''(I+Qu)\int_{w=0}^{\frac{I+Qu}{H(I+Qu)}} wg(w)dwdu - p\int_{u=0}^{\infty} u^2 f(u)H''(I+Qu)\int_{w=\frac{x}{H(x)}}^{\infty} wg(w)dwdu < 0 \quad (7)$$

The inequality in (7) follows from assumption (3). Thus, Proposition 1 is proved.

Our second goal is to show that there exists a unique and finite critical level of initial stock above which no order will be placed. From (6), the first-order derivative of $\Psi(Q\,|\,I)$ with respect to Q at Q=0 is given by

$$\frac{\partial\Psi(0\,|\,I)}{\partial Q} = (r+p-c)\mu - (r+p+v)\int_{u=0}^{\infty} G\left(\frac{I}{H(I)}\right)uf(u)du$$
$$+(r+v)H'(I)\int_{u=0}^{\infty} uf(u)\int_{w=0}^{\frac{I}{H(I)}} wg(w)dwdu - pH'(I)\int_{u=0}^{\infty} uf(u)\int_{w=\frac{I}{H(I)}}^{\infty} wg(w)dwdu \quad (8)$$

Thus, it follows that

$$\frac{d}{dI}[\frac{\partial\Psi(0|I)}{\partial Q}] = -(r+p+v)\mu g(\frac{I}{H(I)})\frac{H(I)-IH'(I)}{(H(I))^2} + (r+v)[H''(I)\int_{u=0}^{\infty} uf(u)\int_{w=0}^{\frac{I}{H(I)}} wg(w)dwdu + \mu H'(I)\frac{I}{H(I)}g(\frac{I}{H(I)})\frac{H(I)-IH'(I)}{(H(I))^2}]$$
$$- p[H''(I)\int_{u=0}^{\infty} uf(u)\int_{w=\frac{I}{H(I)}}^{\infty} wg(w)dwdu - \mu H'(I)\frac{I}{H(I)}g(\frac{I}{H(I)})\frac{H(I)-IH'(I)}{(H(I))^2}] < 0. \quad (9)$$

where the inequality follows from assumption (4). Thus, we see that the first order derivative of $\Psi(Q|I)$ with respect to Q at Q=0 is decreasing in the initial stock level I.
Also from (8), since H(x)>0 by assumption, which implies $I/H(I)|_{I=0}=0$, we have

$$\frac{\partial\Psi(0|I)}{\partial Q}\Big|_{I=0} = r+p-c-p\alpha H'(0) > 0, \quad (10)$$

where the inequality follows from assumption (5). The assumption H'(∞) = 0 implies that $I/H(I)|_{I\to\infty}\to\infty$, so it follows that

$$\lim_{I\to\infty}\frac{\partial\Psi(0|I)}{\partial Q} = (r+p-c)\mu - (r+p+v)\mu = -(c+v)\mu < 0 \quad (11)$$

Now, since $\frac{\partial\Psi(0|I)}{\partial Q}$ is strictly decreasing in I, $\frac{\partial\Psi(0|I)}{\partial Q} > 0$ as I=0 and $\frac{\partial\Psi(0|I)}{\partial Q} < 0$ as I→∞; thus we have proved the following (8)—(10)

PROPOSITION 2: There exists a unique value $I_0$, $0< I_0<\infty$, of initial stock level, such that the optimal order quantity Q* satisfies

$$\begin{cases} Q^* = 0, & \text{if } I \ge I_0 \\ Q^* > 0, & \text{if } I < I_0 \end{cases}$$

where $I_0$ is given by equating (8) to zero, that is, by

$$(r+p+v)\int_{u=0}^{\infty} G(\frac{I_0}{H(I_0)})uf(u)du - (r+v)H'(I_0)\int_{u=0}^{\infty} uf(u)\int_{w=0}^{\frac{I_0}{H(I_0)}} wg(w)dwdu + pH'(I_0)\int_{u=0}^{\infty} uf(u)\int_{w=\frac{I_0}{H(I_0)}}^{\infty} wg(w)dwdu = (r+p-c)\mu. \quad (11)$$

Now, consider the optimal order quantity Q*=Q(I) for I< $I_0$, which we know by Proposition 2 to be strictly greater than zero. Since $\Psi(Q|I)$ is strictly concave in Q by Proposition 1, Q* will be given by equation (6) to zero; that is, Q*=Q(I) solves

$$(r+p+v)\int_{u=0}^{\infty} G(\frac{I+Qu}{H(I+Qu)})uf(u)du - (r+v)\int_{u=0}^{\infty} H'(I+Qu)uf(u)\int_{w=0}^{\frac{I+Qu}{H(I+Qu)}} wg(w)dwdu + p\int_{u=0}^{\infty} H'(I+Qu)uf(u)\int_{w=\frac{I+Qu}{H(I+Qu)}}^{\infty} wg(w)dwdu = (r+p-c)\mu. \quad (12)$$

By the result of Gerchak et al. (1988), for I< $I_0$, $E[Y_Q]=\mu Q^*(I) \neq I_0 - I$. Therefore, the optimal policy does not call for order quantity which is expected to be bring stock up to a constant level.

PROPOSITION 2: The optimal order quantity for our single period problem is given by

$$Q^* = Q(I) = \begin{cases} 0, & \text{if } I \ge I_0 \\ Q^{**}, & \text{if } I < I_0 \end{cases}$$

where Q** is given by (12).

EXAMPLE 1: Let H(x) be given by

$$H(x) = (x+1)^{1/2}.$$

All the assumptions we made about H(x) are satisfied by this example. That is, a=1, d=1, b=1/2. We also assume that the distribution of the random multiplier W in the demand function is uniform; i.e.,

$$g(w) = 1/2, \quad 0 \le w \le 2;\ \alpha$$

For the distribution of the random multiplier U in the yield, we assume

$$f(u) = 1, \quad 0 \le u \le 1, \mu = \text{so}$$

In addition we assume that unit revenue r=10, unit production/purchase cost c=2, unit cost associated with each unsold item at the end of the period in single period setting v=1, unit shortage penalty cost p=1

By the Proposition 2, $I_0$=4.16573. That is, if initial inventory I is greater than or equal to $I_0$=4.16573, we then do not order any item in this example. However, if initial inventory I is less than $I_0$=4.16573, we order some items. To determine order quantity, we use Proposition 3. In this example, let us assume that initial stock level of a period I=0. Since I=0 is less than I0=4.16573, by using Proposition 3 the order quantity Q**=0.6705.

## 4. Two Period Model

Consider now a two-period problem with random demands D1 and D2 and random multipliers U1 and U2. Then the decision problem in the first period is to find the value of Q which maximizes

$$\Psi_2(Q|I) \equiv \Psi(Q|I) + \beta E \phi_1[(I + QU_1) - H(I + QU_1)W_1] \quad (13)$$

where $\Psi(Q|I)$ is the expected profit in first period (with h as unit holding cost), and $\phi_1[(I + QU_1) - H(I + QU_1)W_1]$ is the final period value function [22].

PROPOSITION 3: Define

$$\Phi_1(I) = \max_{Q \ge 0} \Psi(Q|I) = \begin{cases} \Psi(Q(I)|I) & \text{if } I < I_0 \\ \Psi(0|I) & \text{if } I < I_0 \end{cases}.$$

$\phi_1$ is concave if

$$(r+h-\beta c)\int_{u=0}^{\infty} f(u)\int_{w=0}^{\frac{I+Qu}{H(I+Qu)}} H''(I+Qu)wg(w)dwdu-(r+h-\beta c)\int_{u=0}^{\infty} f(u)[H(I+Qu)-H'(I+Qu)]^2\frac{1}{H(I+Qu)}g(\frac{I+Qu}{H(I+Qu)})du$$
$$-p\int_{u=0}^{\infty} f(u)\int_{w=\frac{I+Qu}{H(I+Qu)}}^{\infty} H''(I+Qu)wg(w)dwdu+p\int_{u=0}^{\infty} f(u)H'(I+Qu)(I+Qu)g(\frac{I+Qu}{H(I+Qu)})\frac{H(I+Qu)-(I+Qu)H'(I+Qu)}{H^3(I+Qu)}du$$

< 0 for Q≥0

Proof:

(a) $I< I_0$

$$\Phi_1'(I)=\frac{d\Psi(Q|I)}{dI}=(r-c)u-(r+h-\beta c)\int_{u=0}^{\infty} f(u)\int_{w=0}^{\frac{I+Qu}{H(I+Qu)}}(1-H'(I+Qu)w)g(w)dwdu$$

.

$$-p\int_{u=0}^{\infty} f(u)\int_{w=\frac{I+Qu}{H(I+Qu)}}^{\infty} H'(I+Qu)wg(w)dwdu \qquad (14)$$

and

$$\Phi''(I)=$$
$$(r+h-\beta c)\int_{u=0}^{\infty} f(u)\int_{w=0}^{\frac{I+Qu}{H(I+Qu)}} H''(I+Qu)wg(w)dwdu-(r+h-\beta c)\int_{u=0}^{\infty} f(u)[H(I+Qu)-H'(I+Qu)]^2\frac{1}{H(I+Qu)}g(\frac{I+Qu}{H(I+Qu)})du$$
$$-p\int_{u=0}^{\infty} f(u)\int_{w=\frac{I+Qu}{H(I+Qu)}}^{\infty} H''(I+Qu)wg(w)dwdu+p\int_{u=0}^{\infty} f(u)H'(I+Qu)(I+Qu)g(\frac{I+Qu}{H(I+Qu)})\frac{H(I+Qu)-(I+Qu)H'(I+Qu)}{H^3(I+Qu)}du$$

(b) $I \geq I_0$

In this range Q( I )=0 for all I, so it follows that

$$\Phi_1'(I)=(r-c)u-(r+h-\beta c)\int_{u=0}^{\infty} f(u)\int_{w=0}^{\frac{I}{H(I)}}(1-H'(I)w)g(w)dwdu$$
$$-p\int_{u=0}^{\infty} f(u)\int_{w=\frac{I}{H(I)}}^{\infty} H'(I)wg(w)dwdu$$

and

$\Phi''(I) =$

$$(r+h-\beta c)\int_{u=0}^{\infty} f(u)\int_{w=0}^{\frac{I}{H(I)}} H''(I)wg(w)dwdu-(r+h-\beta c)\int_{u=0}^{\infty} f(u)[H(I)-H'(I)I]^2\frac{1}{H^3(I)}g(\frac{I}{H(I)})du$$

$$-p\int_{u=0}^{\infty} f(u)\int_{w=\frac{I}{H(I)}}^{\infty} H''(I)wg(w)dwdu+p\int_{u=0}^{\infty} f(u)H'(I)(I)g(\frac{I}{H(I)})\frac{H(I)-IH'(I)}{H^3(I)}du$$

Now, since both terms on the right in (13) are concave in Q, so is $\Psi_2$. Also, from (6), (13) and (14) we have

$$\lim_{Q\to\infty}\partial\Psi_2(Q\,|\,I)/\partial Q=(r+p-c)\mu-(r+p+v)\mu$$
$$+\beta E\{\lim_{Q\to\infty}(U_1-H'(I+QU_1)U_1W_1)\Phi_1'(I+QU_1-H(I+QU_1)W_1)\}$$
$$=-(c+v)\mu+\beta(-v)\mu=-(c+v+\beta v)\mu<0 \qquad (15)$$

In addition, by (8) we have

$$\partial\Psi_2(0\,|\,I)/\partial Q=(r+p-c)\mu-(r+p+v)\int_{u=0}^{\infty} G(\frac{I}{H(I)})uf(u)du$$
$$+(r+v)H'(I)\int_{u=0}^{\infty} uf(u)\int_{w=0}^{\frac{I}{H(I)}} wg(w)dwdu-pH'(I)\int_{u=0}^{\infty} uf(u)\int_{w=\frac{I}{H(I)}}^{\infty} wg(w)dwdu$$
$$+\beta E\{(U_1-H'(I)U_1W_1)\Phi_1'(I-H(I)W_1)\}$$

and thus

$$\lim_{I\to\infty}\partial\Psi_2(0\,|\,I)/\partial Q=(r+p-c)\mu-(r+p+v)\mu$$
$$+\beta(-v)\mu=-(c+v+\beta v)\mu<0. \qquad (16)$$

As well, we have

$$\partial\Psi_2(0\,|\,I)/\partial Q\,|_{I=0}=r+p-c-p\alpha H'(0)$$
$$+\beta E[(u_1-H'(0)u_1w_1)\Phi_1'(-H(0)W_1)]>0. \qquad (17)$$

The conclusion from (15), (16), (17) and the concavity of $\Psi_2$ is summarized in the following.

Proposition 4: For the first period in a two period problem, there exists $I_1$ such that the optimal order quantity Q* satisfies

$$\begin{cases} Q^* = 0, & \text{if } I \geq I_1 \\ Q^* > 0, & \text{if } I < I_1 \end{cases}$$

where $I_1$ is given by equating (8) to zero, that is, by

$$(r+p+v)\int_{u=0}^{\infty} G(\frac{I}{H(I)})uf(u)du$$

$$-(r+v)H'(I)\int_{u=0}^{\infty} uf(u)\int_{w=0}^{\frac{I}{H(I)}} wg(w)dwdu + pH'(I)\int_{u=0}^{\infty} uf(u)\int_{w=\frac{I}{H(I)}}^{\infty} wg(w)dwdu$$

$$-\beta E\{(U_1 - H'(I)U_1W_1)\Phi_1'(I - H(I)W_1)\} = (r+p-c)\mu$$

Now, consider the optimal order quantity Q*=Q(I) for I<$I_1$.

Proposition 5: For the first period in a two period problem, there exists $I_1$ such that the optimal order quantity Q* satisfies

$$Q^* = Q(I) = \begin{cases} 0, & \text{if } I \geq I_1 \\ Q^{**}, & \text{if } I < I_1 \end{cases}$$

where Q** is given by the following equation.

$$0 = \partial\Psi(Q\,|\,I)/\partial Q + \beta E\{(U_1 - H'(I+QU_1)U_1W_1)\Phi_1'(I + QU_1 - H(I+QU_1)W_1)\}$$

where $\partial\Psi(Q\,|\,I)/\partial Q$ is given by (6)

## 5. Multiperiod Model

For the n period problem, the decision in the first period is to find the value of Q which maximizes

$$\Psi_n(Q|I) \equiv \Psi(Q|I) + \beta E\phi_{n-1}[(I + QU_1) - H(I + QU_1)W_1]$$

Using a similar approach to the two period model, we can show:

1. $\Psi_n(Q|I)$ is concave in Q.
2. For any I, $\partial\Psi(0\,|\,I)/\partial Q < 0$ for Q sufficiently large.
3. For the first period in n period problem, there exists $I_n$ such that the optimal order quantity Q* satisfies

$$\begin{cases} Q^* = 0, & \text{if } I \geq I_{n-1} \\ Q^* > 0, & \text{if } I < I_{n-1} \end{cases}$$

4. For the first period in n period problem, there exists $I_n$ such that the optimal order quantity Q* satisfies

$$Q^* = Q(I) = \begin{cases} 0, & \text{if } I \geq I_{n-1} \\ Q^{**}, & \text{if } I < I_{n-1} \end{cases}$$

where Q** is given by the following equation,

$$0 = \partial\Psi(Q\,|\,I)/\partial Q + \beta E\{(U_1 - H'(I + QU_1)U_1W_1)\Phi_{n-1}'(I + QU_1 - H(I + QU_1)W_1)\}$$

where $\partial\Psi(Q\,|\,I)/\partial Q$ is given by (6).

## 5. Concluding Remarks and Future Research

In this research we consider single period and multi-period inventory models incorporating inventory level dependent demand with random yield. To develop the incorporated inventory model, we use Gerchak and Wang [9]'s demand model since it makes both the modeling and analysis of the periodic-review model much easier, and permits the derivation of explicit, implementable solutions. For the random yield, we use Gerchak et al. [22]'s yield model since the multiplicative yield model in the paper [22] is more convenient than the additive model. In the incorporated inventory models, we showed that the objective function is concave in some conditions and that the optimal policy is of a "reorder-point" type. That is, there exists a unique and finite value such that if the initial stock is greater than this value the order quantity is zero, otherwise the order quantity is greater than zero. Expressions for determining the reorder point and the optimal order quantity are derived. When only inventory level dependent demand is considered, the reorder point policy reduces to an order-up-to type.

For the future research, we are investigating multiperiod inventory model in which demand is inventory level dependent and yield is random and production capacity is variable.

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