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Kalman Filtering based Adaptive Frequency Domain Channel Estimation with Low Pilot Overhead for OFDM Systems

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Abstract

In this paper, we propose an adaptive frequency domain channel estimation method based on a modified Kalman filtering, which requires low pilot overhead, for estimating and tracking time-varying channel in orthogonal frequency division multiplexing (OFDM) systems. The proposed frequency domain Kalman filtering channel estimation achieves the minimum pilot overhead by exploiting periodically inserted pilot symbols and decision directed symbols between them. The pilot overhead efficiency and tracking performance of the proposed method is studied through simulation. Simulation results show that the proposed method has adequate tracking performance with a pilot overhead of below 5%.

Keywords: *Kalman filter, adaptive channel estimation, time-varying channel, orthogonal frequency division multiplexing (OFDM)*

1. Introduction

Orthogonal frequency division multiplexing (OFDM) has been widely applied in broadband wireless communication systems such as long term evolution (LTE), world interoperability for microwave access (WiMAX) and IEEE 802.20 etc. One of the most important properties of OFDM technique is its high level of robustness against multipath delay spread. If the channel delay spread is shorter than the guard interval, ISI can be perfectly equalized. Thus, OFDM system can achieve high data transmission rate [1]-[3].

Channel impulse response or accurate channel estimation is required at the receiver to equalize and detect transmitted data from received OFDM signals. However, the channel impulse responses at receiver are unknown a priori in most wireless communication systems. Hence, various channel estimation schemes have been studied during the last decade. A standard approach is pilot assisted channel estimation (PACE) [4], where a priori known pilot symbols are periodically sent through the channel. In most previous studies, channel impulse responses frequently have been assumed time invariant over a block or packet. Unfortunately, in fast time-varying channels, this approach not only requires increased number of pilot symbols in cost of overall data rate deduction, but also suffers from performance degradation in time-varying channel estimation. Therefore, it is designed for a channel estimation scheme to have ability to track the channel variations adaptively for wireless communications in high mobility environments. In [5], adaptive filtering schemes such as least mean squares (LMS) or recursive least squares (RLS) algorithm have been used for channel tracking. In [6], Kalman filtering (KF) also has been applied to channel estimation and tracking by modeling time-varying channels as a low order autoregressive (AR) process. Low order

AR models are enough to capture most of the channel dynamics and lead to effective symbol-wise channel tracking.

In this paper, we propose a modified KF based adaptive frequency domain channel estimation scheme which requires low pilot overhead for OFDM systems. The proposed channel estimation method is applied in frequency domain after DFT at OFDM receiver. KF based frequency domain channel estimation scheme had been studied in [7, 8]. In most previous works [6]-[8], KF based channel estimation requires frequent pilot symbols to track for channel variation. We apply periodical pilot symbols and decision directed blind KF based tracking between inserted pilot symbols. Furthermore, we apply frequency channel interpolation between pilot subcarriers by exploiting the correlation between adjacent subcarriers. Through these approaches, the proposed method shows satisfactory channel estimation and tracking with pilot overhead of below 5%.

The rest of this paper is organized as follows. The system model is presented in Section II. In Section III, we describe proposed KF based adaptive frequency domain channel estimation and pilot arrangement. Simulation results are shown in Section IV. Finally, Section V draws our conclusion.

2. System Model

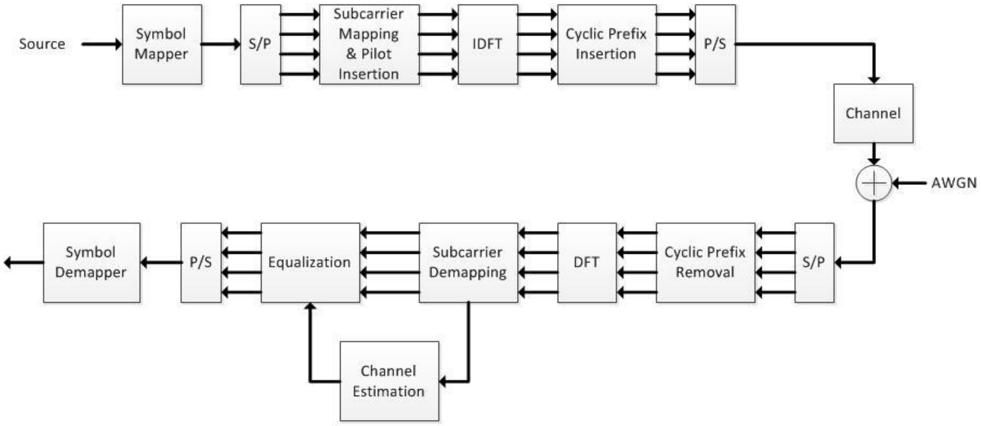

Figure 1. Block Diagram of an OFDM System

We consider a baseband OFDM system based on pilot assisted channel estimation which is given in Figure 1. The information bit sequences are mapped according to the modulation scheme in symbol mapper. The modulated symbol sequences of length N are then split into K subcarriers for OFDM modulation. The pilot symbols are inserted into all subcarriers with a specific period (Block type) or uniformly into subcarriers apart from each other (Comb type). Inverse discrete Fourier transform (IDFT) is used to transform q-th OFDM symbol on k-th subcarrier $X^{(q)}(k)$ into time domain signal $x^{(q)}(n)$.

$$\begin{aligned} x^{(q)}(n) &= IDFT\{X^{(q)}(k)\} \quad n = 0,1,\cdots,N-1 \\ &= \frac{1}{\sqrt{K}}\sum_{k=0}^{N-1} X^{(q)}(k)e^{j(2\pi kn/N)} \end{aligned} \tag{1}$$

Following IDFT, a cyclic prefix, which is chosen to be longer than expected channel delay spread, is added at the beginning of each OFDM symbol before transmission to avoid inter-symbol interference (ISI).

The transmitted signal will pass through time-varying channel with additive noise. The received signal is given by:

$$y^{(q)}(n)=\sum_{t=0}^{L_c-1} h^{(q)}(t)x^{(q)}(n-t)+w^{(q)}(n) \tag{2}$$

where $h^{(q)}(t)$ is the time domain channel impulse response, L_c denotes the maximum channel memory and $w^{(q)}(n)$ is additive white Gaussian noise (AWGN), which is independent identically distributed complex Gaussian random variables with zero mean and variance σ_w^2. The time-varying channel is assumed to be block fading, which means that the channel remains constant for an OFDM symbol period T.

To generate the time-varying frequency-selective fading channels, a complex autoregressive (AR) process of order p is employed [9]. The channel tap $h^{(q)}(t)$ can be represented by:

$$h^{(q)}(t)=\sum_{l=0}^{p} a_l h^{(q-l)}(t)+z(t) \tag{3}$$

where a_l ($1 \leq l \leq p$) are the AR coefficients, and $z(t)$ is a zero-mean i.i.d. complex Gaussian noise with uncorrelated real and imaginary components. According to the WSSUS model of Bello, all channel taps are independent. Thus, the time-varying part of each channel tap is a zero-mean, wide-sense-stationary complex Gaussian process. The AR coefficient a_l and the variance σ_z^2 of $z(t)$ are determined from the time-autocorrelation of the channel tap $h^{(q)}(t)$, which is given by

$$E[h^{(q_1)}(t)h^{(q_2)}(t)^*]=J_0\left(2\pi f_d T \mid q_1-q_2 \mid\right) \tag{4}$$

where J_0 is the zero-order Bessel function of the first kind, f_dT is the Doppler frequency normalized to the OFDM symbol period T.

The received signal in frequency domain is given by

$$Y^{(q)}(k)=H^{(q)}(k)X^{(q)}(k)+W^{(q)}(k) \tag{5}$$

where $Y^{(q)}(k)$ is obtained by discrete Fourier transform (DFT) of time domain received signal.

$$\begin{aligned} Y^{(q)}(k) &= DFT\left\{y^{(q)}(n)\right\} \quad k=0,1,\cdots,K-1 \\ &= \frac{1}{\sqrt{K}}\sum_{n=0}^{N-1} y^{(q)}(n)e^{-j(2\pi kn/N)} \end{aligned} \tag{6}$$

The frequency domain channel $H^{(q)}(k)$ and $W^{(q)}(k)$ are determined by DFT of $h^{(q)}(t)$ and $w^{(q)}(n)$, respectively.

$$H^{(q)}(k)=\sum_{t=0}^{L_c-1} h^{(q)}(t)e^{-j(2\pi kt/K)} \tag{7}$$

$$W^{(q)}(k)=\sum_{n=0}^{N-1} w^{(q)}(n)e^{-j(2\pi kt/K)} \tag{8}$$

3. Proposed KF Channel Estimation

In order to apply KF, which is a state-space based algorithm, to channel estimation, we first need the state-space model [10], which consists of a process equation and a measurement equation. The frequency domain channel $H^{(q)}(k)$ is used as the dynamic state variable. The measurement equation is the relation between the received signal $Y^{(q)}(k)$ and the channel $H^{(q)}(k)$, which is given by (5). The process equation, which is to represent the time-varying channel, can be obtained from (3).

$$H^{(q)}(k) = H^{(q-1)}(k)q + \Lambda^{(q)}(k) \tag{9}$$

where q denotes the state transition variable which is $J_0(2\pi f_d T)$, and $\Lambda^{(q)}(k)$ denotes complex zero mean white Gaussian noise with the variance of σ_z^2.

3.1. Proposed KF Algorithm

The time-varying fading channel can be modeled as an AR process of order p, and then the KF is the optimal minimum mean square error (MMSE) estimator. The proposed KF algorithm operates parallel on pilot inserted subcarriers, based on the MMSE criterion [10]. The proposed algorithm exploits decision directed approach for blind KF estimation when pilot symbols are not available. Our proposed algorithm can be summarized as in Table 1.

Table 1. The Proposed Algorithm Procedure

Initialization	q=0
Step 1:	If mod(q, N_t)=0 (N_t is pilot period in time) Using the transmitted pilot to estimate the channel $X^{(q)}(k)$ = pilot symbol else Using decision directed tentative symbol $X^{(q)}(k) = \hat{X}^{(q)}(k)$ $= \underset{\hat{X}^{(q)}(k)}{\arg\min} Y^{(q)}(k) - \tilde{H}^{(q\mid q-1)}(k)\hat{X}^{(q)}(k)$
Step 2:	Kalman channel estimation with $X^{(q)}(k)$
Step 3:	q=q+1, return to Step 1.

$$\tilde{H}^{(q\mid q-1)}(k) = \tilde{H}^{(q-1\mid q-1)}(k)q \tag{10}$$

$$\alpha^{(q)}(k) = Y^{(q)}(k) - \tilde{H}^{(q\mid q-1)}(k)X^{(q)}(k) \tag{11}$$

$$P^{(q\mid q-1)}(k) = qP^{(q-1\mid q-1)}(k)q^{*} + \sigma_z^2 \tag{12}$$

$$K^{(q)}(k) = \frac{X^{(q)^{*}}(k)P^{(q\mid q-1)}(k)}{X^{(q)^{*}}(k)P^{(q\mid q-1)}(k)X^{(q)}(k) + \sigma_z^2} \tag{13}$$

$$\tilde{H}^{(q\mid q)}(k) = \tilde{H}^{(q\mid q-1)}(k)q + \alpha^{(q)}(k)K^{(q)}(k) \tag{14}$$

$$P^{(q\,|\,q)}(k) = P^{(q\,|\,q-1)}(k)\left(1 - X^{(q)}(k)K^{(q)}(k)\right) \tag{15}$$

where $\tilde{H}^{(q\,|\,i)}(k)(i = q-1, q)$ denotes the linear MMSE estimate of $H^{(q)}(k)$ given $Y^{(0)}(k), \cdots, Y^{(i)}(k,$

$$P^{(q\,|\,i)}(k) = E\left[e^{(q\,|\,i)*}(k)e^{(q\,|\,i)}(k)\right] \tag{16}$$

$$e^{(q\,|\,i)}(k) = \tilde{H}^{(q\,|\,i)}(k) - H^{(q\,|\,i)}(k) \tag{17}$$

denoting the channel estimation error. $K^{(q)}(k)$ denotes the frequency domain Kalman gain. $P^{(q\,|\,q\text{-}1)}(k)$ and $P^{(q\,|\,q)}(k)$ are the prediction error covariance and the filtering error covariance, respectively.

3.2. Pilot Arrangement

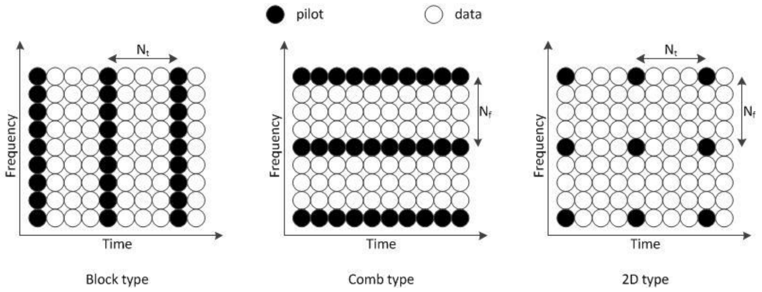

Figure 2. Pilot Arrangement for OFDM Channel Estimations

The three basic pilot arrangement methods are illustrated in Figure 2. The block type pilot arrangement is developed under the assumption of the relatively slow time-varying channel, and it is performed by inserting pilot symbols into all subcarriers of OFDM symbols within a specific period N_t. Secondly, the comb type pilot arrangement is introduced to satisfy the need for equalizing when the time-varying channel changes even from one OFDM block to the subsequent one. It is thus performed by inserting pilot symbols into every N_f th subcarriers of each OFDM symbol, where the interpolation is needed to estimate the channel of data subcarriers. The comb type pilot arrangement is suitable for the relatively fast time-varying channel. Finally, in two-dimensional (2D) type pilot arrangement, the pilot symbols are inserted on every N_f th subcarrier in every N_t th OFDM symbol.

3.3. Channel Interpolation

The idea of channel interpolation is to take advantage of the correlation between adjacent subcarriers. Since KF channel estimator operates parallel on pilot inserted subcarriers, in comb type or 2D type pilot assisted channel estimation, an efficient interpolation technique is necessary in order to estimate channel at data subcarriers by using the estimated channel at pilot inserted subcarriers.

In this paper, the low-pass interpolation (LPI) is used for frequency domain channel interpolation. The LPI method is performed by inserting zeros into the periodically estimated channel in frequency domain and then applying a low-pass finite-length impulse response (FIR) filter (the *interp* function in MATLAB), which allows the periodically estimated channel to pass through unchanged and interpolates such that the mean square error between the interpolated points and their ideal values are minimized.

4. Simulation Results

We consider an OFDM system with $K = 128$ subcarriers to demonstrate the performance of the proposed scheme. Each frame is made of 100 OFDM blocks with quadrature phase-shift keying modulation (QPSK) modulation. The pilot period in time domain, $N_t = 8$ is used for block type pilot, and the pilot period in frequency domain, $N_f = 8$ is used for comb type pilot. For 2D type pilot pattern, $N_t = 4$ and $N_f = 8$ are used. The time-varying channel is generated by an AR model of order 200, which provides a sufficiently accurate frequency selective Rayleigh fading channel. The channel memory length is $L_c = 5$, and a cyclic prefix of length $L_{cp} = 6$ is used.

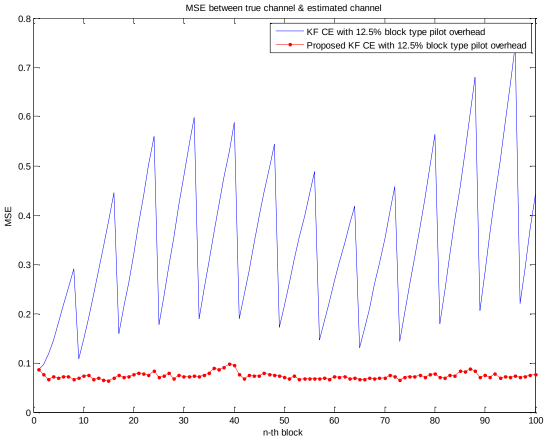

Figure 3. MSE between True and Estimated Channel over a Frame with Doppler shift f_d = 100Hz and SNR = 20dB

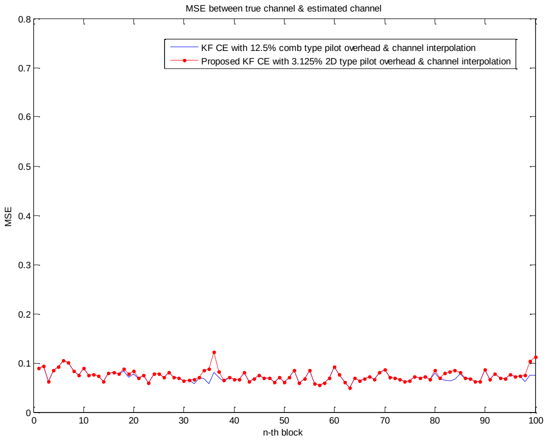

Figure 4. MSE between True and Estimated Channel over a Frame with Doppler shift f_d = 100Hz and SNR = 20dB

Figure 3 shows that mean square error (MSE) between true channel and estimated channel when block type pilot symbols are used. Since conventional block type channel estimation updates channel estimate when pilot symbols are available, it has bad tracking performance, especially in relatively fast time-varying channel. However, when the proposed algorithm is applied to block type pilot, channel estimation and tracking can be performed on data blocks. Thus, the proposed algorithm provides significant tracking performance improvement with 12.5% pilot overhead, which is the same as conventional block type pilot channel estimation.

Figure 4 shows that MSE between true and estimated channel when conventional comb type pilot channel estimation with 12.5% pilot overhead and the proposed algorithm with 2D type pilot pattern with 3.125% pilot overhead are used. Conventional comb type pilot channel estimation demonstrates consistent tracking performance in fast time-varying channel. Despite occasional tracking performance degradation, which is not significant, on blind KF estimation, the proposed algorithm shows good performance comparable to comb type pilot channel estimation with much lower pilot overhead.

5. Conclusions

In this paper, we proposed a modified KF based adaptive frequency domain channel estimation method with low pilot overhead for fast time-varying channels in OFDM systems. The proposed method minimizes pilot overhead by decision directed blind KF channel estimation between pilot symbols and channel interpolation. Despite occasional tracking performance degradation due to false decisions, the proposed algorithm exhibits satisfactory channel estimation performance for fast time-varying channels under moderate SNR.

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