Robust Adaptive Backstepping Control of Inverted Pendulum on Cart System

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Abstract

In this paper a design methodology for a novel robust adaptive backstepping controller for the stabilization control of an inverted pendulum on a movable cart, which is a benchmark control problem in the nonlinear control system paradigm, has been presented in a systematic manner. The proposed control law provides a systematic iterative formulation of a Lyapunov energy function for the inverted pendulum system to ensure its stabilization and convergence of the angle tracking error as well as the estimation error of its unknown parameters towards zero. For easier parameter adaptation, the model of the inverted pendulum has been transformed into a motion control model. The effectiveness of the proposed algorithm has been verified in simulation studies. The controller design has been evaluated not only for the tracking performance but also for the parameter convergence rate of the system. It is quite interesting to note that during the simulation it does not require any prior information about the parameters of the mathematical model of the inverted pendulum.

Keywords: *Robust Adaptive Backstepping, Inverted Pendulum, Nonlinear State Equation, Lyapunov Energy Function, Motion Control Model, Global stability*

1. Introduction

Adaptive control of inherently unstable nonlinear mechanical plants has been a major challenge in control system engineering. Inverted pendulum mounted on a movable cart is a common example of such system [8]. The inverted pendulum on cart has been an interesting and classical control problem for control system engineers since 1950s [1]. Balancing or stabilization control of such an inverted pendulum in the vertical upright position (which is its unstable equilibrium point) has become a very popular benchmark control problem for the derivation of advanced control algorithms for inherently unstable nonlinear systems [1, 4]. The present paper aims to elaborate the procedure of deriving a novel robust adaptive backstepping control law for the stabilization control problem of an inverted pendulum on a cart system at vertical upright position.

The dynamics of the inverted pendulum resembles the dynamics of numerous other systems of interest [8]. Therefore, inverted pendulum has been a popular candidate test rig for the research and illustration of various control methods like feedback stabilization, variable structure control, passivity based control, backstepping and forwarding control, nonlinear observer, friction compensation, task oriented control, hybrid system control, and chaotic system control [1]. It is quite interesting that biped walking robot control resembles the inverted pendulum control problem [13]. The main control objectives of the most of the research contributions were to control the inverted pendulum on the cart so that the unstable equilibrium point could be stabilized. However, when the point of interest is the lower equilibrium position, the same pendulum on a moving cart also resembles the problem of controlling a convey-crane carrying a heavy load from one starting point to another by keeping oscillations small [5]. Inverted pendulum also provides a simple model for stabilization control of rockets when it is being launched [8].

The single rod inverted pendulum is the most popular configuration for the control engineers and consists of a freely pivoted rod mounted on a motor driven cart as shown in the Figure 1. It has two equilibrium points: vertical upright equilibrium point and downward equilibrium point. When the pendulum is standing at the vertical upright position on the motionless cart and the resultant of the forces acting from all sides is zero it is said to be in the vertical upright equilibrium point. The vertical upright equilibrium point is inherently unstable, as any small disturbance may cause the pendulum to fall on the either side when the cart is at rest. However, this equilibrium can be maintained indefinitely by properly controlling the motion of the cart. When there is no external force acting on the pendulum it will come to the rest in the downward hanging position. This equilibrium position is stable. Apart from the control problem of the single rod inverted pendulum on a cart system, control aspects other types like double inverted pendulum on a cart, the rotational single-arm pendulum and the rotational two-link pendulum have also been reported in the literature [3, 6]. In this investigation, we have considered the model of a single rod inverted pendulum on a cart system for the development of the proposed control algorithm.

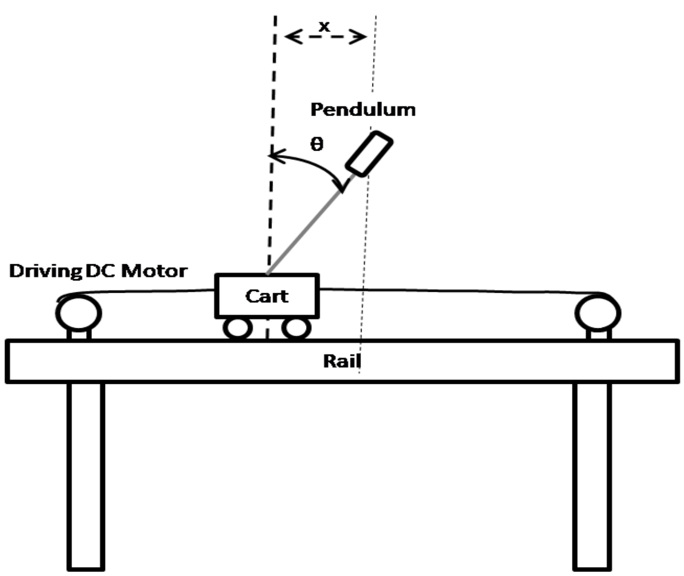

Figure 1. Inverted Pendulum Mounted on Cart

Numerous control techniques have been evolved for the stabilization of the inverted pendulum on the cart. It ranges from the simplest Proportional-Integral-Derivative (PID) type control to the modern nonlinear control techniques. Application of adaptive control for the stabilization of an inverted pendulum is a quite interesting proposition as the inverted pendulum on cart can be categorized as uncertain nonlinear dynamical systems [12]. However, when the knowledge of the parameters of the system is incomplete or approximate system data are available, approximation errors creep into the feedback loop which makes the system difficult to stabilize or keep an inherently unstable system in the stable equilibrium for a long time [2, 9]. Here, we have addressed a very challenging problem that of designing an adaptive controller for an inverted pendulum on a cart system without any prior knowledge of the parameters of the inverted pendulum model. This type of problems can be addressed with the help of robust adaptive controller [2, 9].

In this paper, we have proposed a robust adaptive nonlinear control scheme to stabilize an inverted pendulum on a cart system. The robust adaptive control law has been derived in two steps. First of all, the adaption law has been derived using the backstepping technique to keep the inverted pendulum in its vertical upright position. The main idea of the backstepping method is that the overall dynamic system is partitioned into two series cascaded subsystems, in which the states of the first subsystem are the control variables for the second. In this approach, The method computes the desired control input for the second subsystem, after which the control input for the first subsystem is computed so as to realize the desired state, which is the desired control input for the second subsystem [11]. Few methods for the design of adaptive backstepping controllers for inverted pendulums have been reported in the literature [3, 6]. However, they use simple adaptive backstepping techniques for parameter adaptation. The main limitation of these design methods is that there are many uncertain controller parameters which have to be estimated during the execution of the control algorithm. Moreover, when these control laws act on the system they suffer from a very high rate of adaptation which is not at all desirable from the perspective of robust performance of the system. This implies that some kind of robustification measures [2] in the adaptation laws (which have been designed by the backstepping method) may turn out to be useful to mitigate the problems that may arise due the high rate of adaptation, over adaption, etc. Therefore, a robustness scheme is then applied on the adaptation laws in the second step of the design of the control law with the help of continuous switching function.

The control algorithm proposed in this paper has been designed using a normalized adaptive backstepping control method. It reduces the number of uncertain controller parameters, which in turn offers a control algorithm with less computational cost. The proposed control algorithm also uses a continuous switching function in the adaptation to prevent the abrupt rate of change of parameter adaptation. To the best of the knowledge of the authors, design of robust adaptive control law to stabilize an inverted pendulum on a cart system using the backstepping technique has never been addressed in the literature.

The rest of the paper is organized as follows. The detail modeling of the inverted pendulum by means of Lagrangian mechanics has been presented in section II. In section III, we have designed the proposed control law using the adaptive backstepping control design method to keep the pendulum in its vertical upright position [10]. Simulation results of the robust adaptive backstepping controller have been presented in section IV. Various types of disturbance signals are also applied on the input force to the cart and also on the pendulum angular position directly (similar to that of a wind gust to a launching air vehicle). Finally we have presented the concluding remarks in section V.

2. Modeling of System Dynamics

A theoretical model of the pendulum on a cart system using Lagrangian dynamics [14] has been derived in this section. This model will be used for the design of the proposed control law and then in the simulation thereafter for the evaluation of the effectiveness of the proposed control law.

The total kinetic co-energy of the system is:

$$T^* = K_M + T_m^* = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(x^2 - 2xl\cos\theta + l^2\theta^2\right) + \frac{1}{2}I\dot{\theta}^2 \tag{1}$$

Since the cart moves only in the horizontal direction, the potential energy of the system is determined entirely by the pendulum angle θ from the vertical upright position

$$V = -mgl\cos\theta \tag{2}$$

Now we can calculate the Lagrangian from the kinetic co-energy and potential energy functions as:

$$L = T^{*} - V \tag{3}$$

After substitution of the T* and V from equations (1) and (2) respectively, in equation (3) we get:

$$L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(x^2 - 2xl\cos\theta + l^2\theta^2\right) + \frac{1}{2}I\dot{\theta}^2 + mgl\cos\theta \tag{4}$$

Now, in order to determine the Lagrangian dynamics of the system, we have to determine the following partial derivatives of L, one with respect to x and another with respect to θ:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F_x \tag{5}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \tag{6}$$

For the present system, equation (

$$\frac{d}{dt}\left(M\dot{x} + m\dot{x} - ml\cos\theta\dot{\theta}\right) - 0 = f(t) - c\dot{x} \tag{7}$$

This can be further reduced to:

$$(M + m)\ddot{x} + ml\sin\theta\dot{\theta}^2 - ml\cos\theta\ddot{\theta} = f(t) - c\dot{x} \tag{8}$$

Similarly, for the present system equation (6) becomes:

$$\frac{d}{dt}\left(-ml\dot{x}\cos\theta + ml^2\dot{\theta} + I\dot{\theta}\right) = \left(ml\dot{x}\sin\theta\dot{\theta} - mgl\sin\theta\right) = -b\dot{\theta} \tag{9}$$

This can be further reduced to:

$$\left(ml^2 + I\right)\ddot{\theta} - ml\ddot{x}\cos\theta = mgl\sin\theta - b\dot{\theta} \tag{10}$$

Simplifying and rearranging equations (8) and (10) we get the following equations of motion for the system:

$$(M + m)\ddot{x} + c\dot{x} - ml\cos\theta\ddot{\theta} + ml\sin\theta\dot{\theta}^2 = f(t) \tag{11}$$

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = ml\ddot{x}\cos\theta \tag{12}$$

3. Design of Robust Adaptive Backstepping Control Law

In this section, we will derive the robust adaptive backstepping control law in two steps. At first we will derive the adaptive backstepping control law by considering the Lyapunov stability criterion. Then we will formulate the robust adaptive backstepping control law by incorporating necessary robustification measures in the previously designed adaptive backstepping control law.

3.1. Adaptive Backstepping Control Law

The primary objective of control of the inverted pendulum on a cart system is to control the pendulum angle θ to zero degrees from the vertical upright position. Here we will derive an adaptive backstepping control law that will generate a control input to the system so that pendulum angle can be controlled very near to zero from the vertical upright position. We will use the mathematical model of the pendulum on a cart system that we have already derived in section II. From equation (11) & (12) we can find out a relationship between angular position and force applied on the cart of the inverted pendulum system as:

$$\Psi_1 \sec\theta\ddot{\theta} + \Psi_2 \tan\theta + \Psi_3\left(\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta\right) = f \tag{13}$$

where

$$\Psi_1 = \left(M + m\right)\left[\frac{\left(I + ml^2\right)}{ml}\right] \tag{14}$$

$$\Psi_2 = (M + m)g \tag{15}$$

$$\Psi_3 = ml \tag{16}$$

are the unknown parameters of the system.

Now, let us introduce the two state variables as, $z_1=\theta$ and $z_2=\dot{\theta}$. Then, equation (13) can be written in state space form as shown below:

$$\dot{z}_1 = z_2 \tag{17}$$

$$g(z_1)\dot{z}_2 = f - \Psi_2 \tan z_1 + \Psi_3 z_2^2 \sin z_1 \tag{18}$$

where,

$$g(z_1) = -\Psi_1 \sec z_1 + \Psi_3 \cos z_1 \tag{19}$$

We can also rewrite the equation (18) in the following form:

$$g(z_1)\dot{z}_2 = f - k(z_1) \tag{20}$$

Where

$$k(z_1) = \Psi_2 \tan z_1 - \Psi_3 z_2^2 \sin z_1 \tag{21}$$

For the sake of simplicity and ease of computation, let us introduce a normalized unknown function h, which is defined as $h = k(z_1)/g(z_1)$. With the introduction of this normalized function h, equation (20) now becomes:

$$f = g(z_1)(\dot{z}_2 + h) \tag{22}$$

Now, let us define an error variable e_1 as

$$e_1 = \theta_{rerf} - \theta \tag{23}$$

where θ_{ref} is the reference angle, it is a piecewise continuous and bounded function of time. For inverted pendulum θ_{ref} is zero.

Now our objective is to design a virtual control law z_{ref} which makes $e_1 \rightarrow 0$. Let us consider the following control Lyapunov function

$$V_1 = \frac{1}{2} e_1^2 \tag{24}$$

The first order time derivative of V_1 becomes

$$\dot{V}_1 = e_1 \dot{e}_1 = \dot{\theta}_{ref} - \dot{\theta} = \dot{\theta}_{ref} - z_2 \tag{25}$$

Now we can define the second error variable as

$$e_2 = z_{ref} - z_2 \tag{26}$$

where z_{ref} is a virtual control law for equation (25). In this step the objective of the design is to find out a suitable virtual control law for equation (25) which would make the above mentioned first order system stabilizable. A suitable choice of virtual control law is given below in equation (27)

$$z_{ref} = c_1 e_1 + \dot{\theta}_{ref} \tag{27}$$

where c_1 is a positive design constant, which guarantees the asymptotic stability of the system.

Then the time derivative of e_2 becomes

$$\begin{aligned} \frac{de_2}{dt} = \dot{z}_{ref} - z_2 \quad &= \left(\mathrm{c}_1 \dot{e}_1 + \ddot{\theta}_{ref}\right) - \frac{f}{g} + h \\ &= \left(\mathrm{c}_1 \left(-c_1 e_1 + e_2\right) + \ddot{\theta}_{ref}\right) - \frac{f}{g} + h \\ &= \left(-c_1 e_1^2 + c_1 e_2 + \ddot{\theta}_{ref}\right) - \frac{f}{g} + h \end{aligned} \tag{28}$$

In the above equation c_2 is a positive design constant.

Now, from equation (27) we can find out the expression for force f, which is actually the control input in our case, which gives us the desired dynamics for the system expressed by the equation (17) and (18). Thus,

$$f = \hat{g}(z_1)\left(\left(1-c_1^2\right)e_1 + \left(c_1+c_2\right)e_2 + \ddot{\theta}_{ref} + \hat{h}\right) \tag{29}$$

In equation (28) the estimated functions $\hat{g}$ and $\hat{h}$ represent the functions g and h of equation (27). Although the expression of the force is determined by equation (28), the control design is still incomplete, because the parameter adaptation law is not yet determined. Now, let us define the following parameter errors:

$$\left.\begin{aligned} \bar{g} &= g-\hat{g} \\ \bar{h} &= h-\hat{h} \end{aligned}\right\} \tag{30}$$

Substituting the expression of force f from equation (28) in the expression (27), we get the following error dynamics:

$$\frac{de_2}{dt} = -c_2e_2 - e_1 + \frac{\bar{g}}{g}\{(1-c_1^2)e_1 + (c_1+c_2)e_2 + \ddot{\theta}_{ref} + \hat{h}\} + \bar{h} \tag{31}$$

In order to find out the parameter update laws for $\hat{g}$ and $\hat{h}$ the following augmented Lyapunov function has been formulated for the closed loop system:

$$V_2 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2\gamma_1 g}\bar{g}^2 + \frac{1}{2\gamma_2}\bar{h}^2 \tag{32}$$

Here, the augmented Lyapunov function includes all error variables as well as the parameter errors. The time derivative of V_2 is given by:

$$\begin{aligned} \dot{V}_2 &= e_1\dot{e}_1 + e_2\dot{e}_2 + \frac{\bar{g}}{g\gamma_1}\left(-\frac{d\hat{g}}{dt}\right) + \frac{\bar{h}}{h\gamma_2}\left(-\frac{d\hat{h}}{dt}\right) \\ &= -c_1e_1^2 - c_2e_2^2 + \frac{\bar{g}}{g}\{e_2((1-c_1^2)e_1 + (c_1+c_2)e_2 + \ddot{\theta}_{\mathrm{ref}} + \hat{h}) - \frac{1}{\gamma_1}\frac{d\hat{g}}{dt}\} + \bar{h}(e_2 - \frac{1}{\gamma_2}\frac{dh}{dt}) \end{aligned} \tag{33}$$

From the above equation the following parameter updates law can be constructed for the parameters $\hat{g}$ and $\hat{h}$

$$\frac{d\hat{g}}{dt} = \gamma_1 e_2((1-c_1^2)e_1 + (c_1+c_2)e_2 + \ddot{\theta}_{\mathrm{ref}} + \hat{h}) \tag{34}$$

$$\frac{d\hat{h}}{dt} = \gamma_2 e_2 \tag{35}$$

The above parameter update laws make the derivative of the augmented Lyapunov function V_2 negative definite as give below:

$$\dot{V}_2 = -c_1 e_1^2 - c_2 e_2^2 \tag{36}$$

Now, it is evident from equation (36) that the error dynamics of the system is asymptotically stable. Therefore, the derived adaptive backstepping control law stabilizes system.

3.2. Robust Adaptive Backstepping Control Law

During the design of adaptive backstepping control law, first we have assumed that the model of the plant is free from the un-modeled dynamics, parameters drift, and noise. But in actual systems these assumptions may not be valid. When the knowledge of the parameters of the inverted pendulum on a cart system is incomplete or approximate system data are available, approximation errors creep into the feedback loop which makes the system difficult to stabilize (or keep an inherently unstable system) in the stable equilibrium for a long time [2, 9]. Moreover, the stability of the actual systems may be severely affected by some bounded disturbances and high rate of adaptation. Therefore, while using a high rate of parameter adaptation gain a modification of the above control law is necessary so that the stability of the overall system is not affected. Here, we have proposed introduction of a continuous switching function [9] in the parameter adaption law which aims to mitigate the above problem. Now, the modified adaptation law can be written as:

$$\dot{\hat{g}} = \gamma_1 e_2 \left(\left(1 - c_1^2\right) e_1 + \left(c_1 + c_2\right) e_2 + \ddot{\theta}_{ref} + \hat{h} \right) - \gamma_1 \sigma_{gs} \hat{g} \tag{37}$$

$$\dot{\hat{h}} = \gamma_2 e_2 - \gamma_2 \sigma_{sh} \hat{h} \tag{38}$$

Where σ_{gs} & σ_{hs} are called the continuous switching function and are represented as:

$$\sigma_{gs} = \begin{cases} 0 & \text{if } \left|\hat{g}\right| < g_0 \\ \sigma_{g0}\left(\dfrac{\left|\hat{g}\right| - g_0}{\left|\hat{g}\right|}\right) & \text{if } g_0 \le \left|\hat{g}\right| \le 2g_0 \\ \sigma_{g0} & \text{if } \left|\hat{g}\right| \ge 2g_0 \end{cases} \tag{39}$$

$$\sigma_{hs} = \begin{cases} 0 & \text{if } \left|\hat{h}\right| < h_0 \\ \sigma_{h0}\left(\dfrac{\left|\hat{h}\right| - h_0}{\left|\hat{h}\right|}\right) & \text{if } h_0 \le \left|\hat{h}\right| \le 2h_0 \\ \sigma_{h0} & \text{if } \left|\hat{h}\right| \ge 2h_0 \end{cases} \tag{40}$$

In equation sets (38) & (39) $\hat{g}_0$, $\hat{h}_0$, σ_{go} & σ_{ho} are the design constants. Now, the magnitude of the constants in equation (38) and (39) may be chosen in actual systems by trial and error. The total control scheme has been depicted in Figure 2. The controller structure has been elaborated in Figure 3.

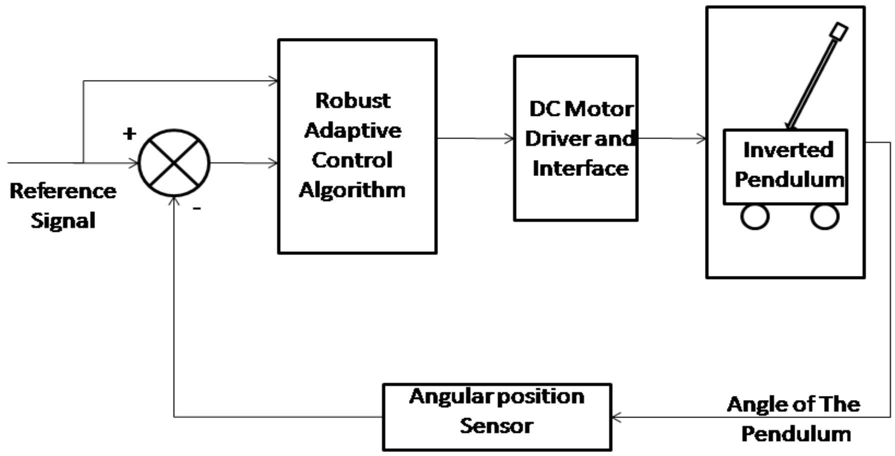

Figure 2. Total Control Scheme

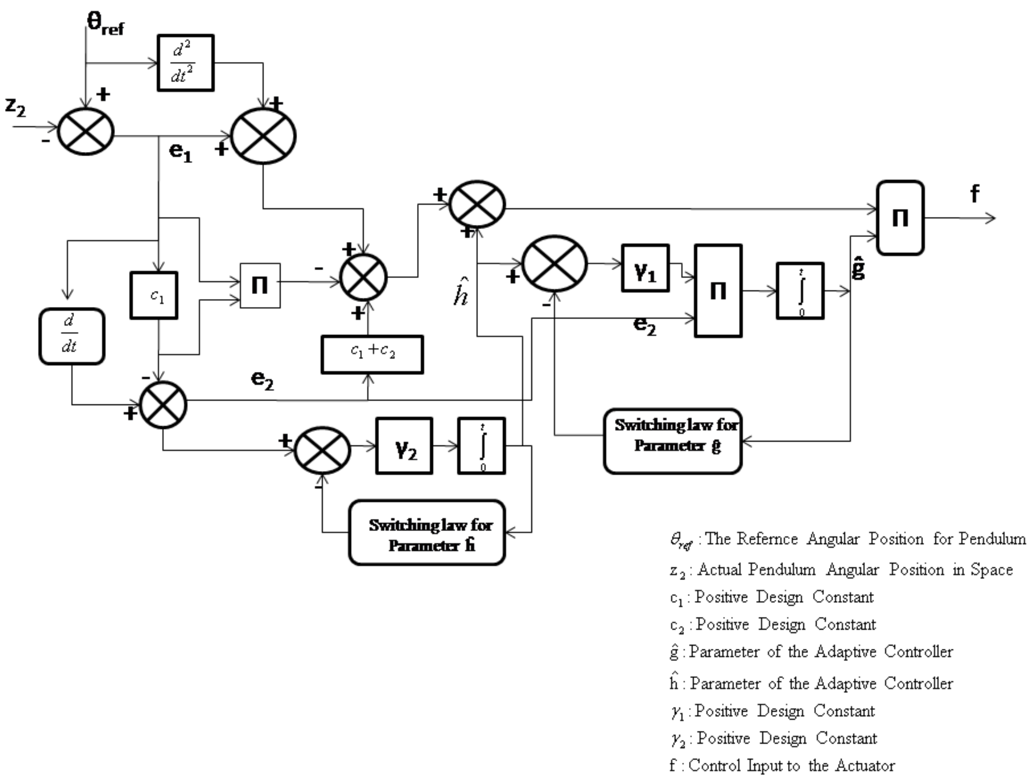

Figure 3. Controller Structure

4. Simulation Results

In this section the performance of the robust adaptive backstepping control law has been verified in simulation studies. We have considered the mathematical model of the inverted pendulum that we have derived in section II. The model of inverted pendulum and the model of the controller are simulated in MATLAB® and Simulink environment.

During simulation we have chosen M=1.25, m=0.1, l=0.1, g=0.1 in the inverted pendulum model. Controller parameters are selected as c_1=6, c_2=6, γ_1=9, γ_2=9, g_0=300, h_0=10 & σ_{g0}= σ_{h0}=1. Initial condition of the inverted pendulum rod angle is selected as 1 radian ≈57°.

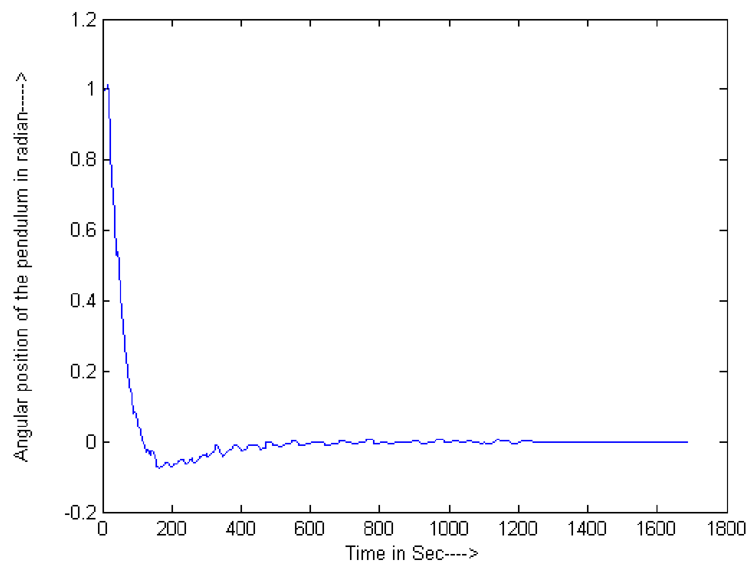

Figure 4. Angular Position of Pendulum in Space

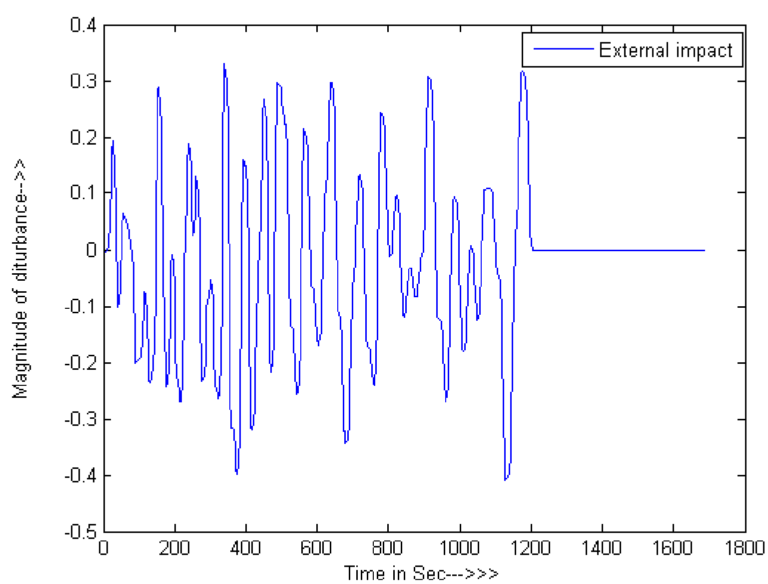

Figure 5. Disturbance Signal

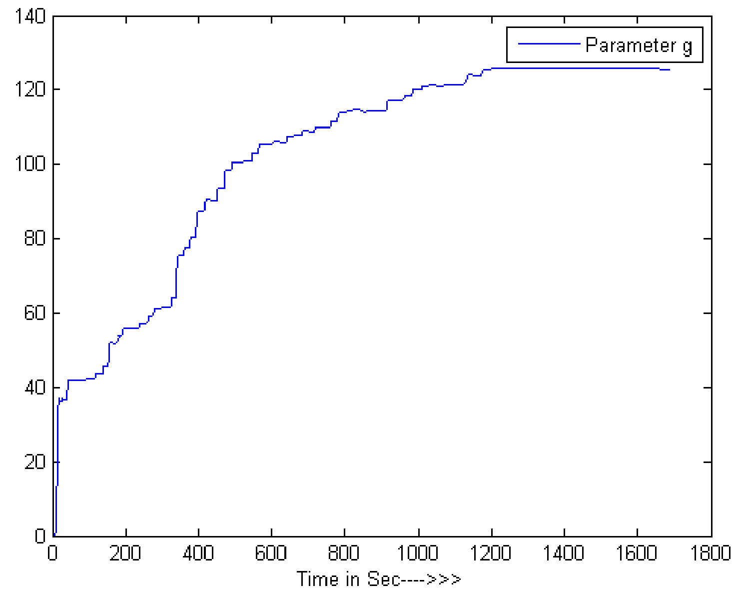

Figure 6. Variation of Controller Parameter g with Time

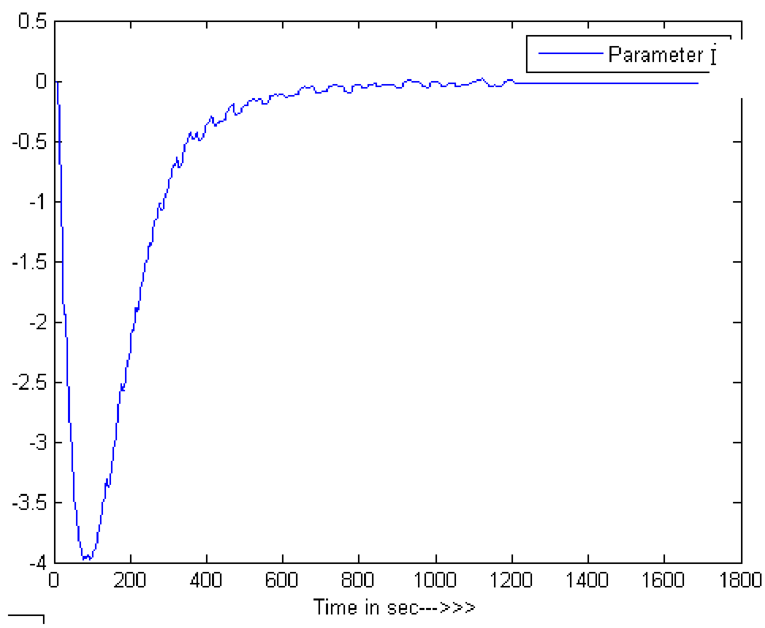

Figure 7. Variation of Controller Parameter h with Time

A band limited white noise signal and a Gaussian noise signal is also directly added with the angle of the pendulum as a disturbance at input signal to examine the stability of the controller against several types of disturbances. The controlled angular position of the pendulum as a function of time is plotted in Figure 4. The disturbance signal which has been added with the angular position is shown in Figure 5. The variation of parameter $\hat{g}$, and $\hat{h}$ are shown Figure 6 and Figure 7 respectively. It is clear from the above results that the controller is able to maintain the angle of the pendulum in its unstable equilibrium position (vertical upright) in spites of disturbance, noise, or parameter drift. The simulation results also reveal the fact that the parameter adaptation mechanism is robust. It is quite clear from Figure 6 and Figure 7 that the adaptive controller is able to execute a steady rate of parameter adaptation in spite of the presence of disturbance signal.

5. Conclusions

In this paper, a robust adaptive backstepping control law has been proposed to solve the stabilization control problem of an inverted pendulum. The control algorithm exhibits a stable control performance in the presence of unknown parameters of inverted pendulum on a cart system. We have used normalized adaptive backstepping control law which reduces the number of uncertain terms in control law. Therefore, the control algorithm is simple and considerably reduces the computational complexity. The control algorithm also uses continuous switching function to prevent the controller becoming unstable due to high rate of change of parameters during adaptation. The performance of the controller is tested against various types of noise and disturbance signals. The simulation results clearly reveal that the performance of the controller is very good in the presence of noise signals and also for the large initial deviation of pendulum from its desired position. The robust adaptive backstepping control method has the capability of quickly achieving the control objectives and exhibits excellent ability to stabilization when the pendulum is subjected to an external impact as disturbance.

List of Notations

M:	Mass of the cart
M:	Mass of the pendulum
I:	Moment of inertia of the pendulum
x:	Cart displacement
θ:	Pendulum angular displacement from the vertical upright position
l :	Rod length (pendulum)
T^* :	Total kinetic co-energy
V:	Potential energy
L:	Lagrangian
F_x :	Generic force in horizontal direction
f(t):	Force applied on the cart in horizontal direction
b:	Friction coefficient of cart with the rail
c:	Friction coefficient of pendulum
V1, V2:	Lyapunov functions

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