Robust H_∞ Controller Synthesis for Linear Uncertain Systems with Interval Time-delay: A Less Conservative Result

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Abstract

In this paper, we consider the problem of delay-dependent H_∞ control of a class of linear uncertain systems with interval time-varying delay and norm-bounded uncertainties using Lyapunov-Krasovskii approach. By exploiting a candidate Lyapunov-Krasovakii (LK) functional and using slack matrix variables in the delay-dependent stability analysis, a less conservative stability criterion is first derived in terms of linear matrix inequalities (LMIs). Subsequently, based on the criterion obtained, a delay-dependent condition for the existence of a static state feedback controller, which ensures asymptotic stability as well as a prescribed H_∞ performance of the closed loop system for all admissible uncertainties, is deduced. A numerical example is employed to demonstrate the effectiveness of the proposed controller.

Keywords: *H_∞ controller, Robust stability, Interval time-varying delay, Lyapunov-Krasovskii functional, Norm-bounded uncertainty, Linear Matrix Inequality.*

1 Introduction

The phenomena of time-delays are often encountered in various physical systems, like communication systems, air-craft stabilization, nuclear reactors, population dynamics, ship stabilization and electric power systems with lossless transmission lines, etc. [1–3]. These delays are time-varying in nature, and their presence in a system has an adverse impact not only on system performance, but also on its stability; therefore, neglecting the effects of delay in the analysis may lead to instability and incorrect design calculations. Hence, the problem of stability and H_∞ stabilization of systems with time-varying delay using static state feedback control law have received considerable attention in recent times [4–8]. Depending upon whether or not the existence conditions for H_∞ state feedback stabilization include the time-delay information, the stabilization criteria can be classified respectively into two categories: delay-dependent and delay-independent ones. While a delay-independent criterion guarantees asymptotic stability and a prescribed H_∞ performance level of closed loop system for any value of delay-size, the delay-dependent criterion ensures the same for a finite upper bound of the delay. In general, delay-dependent stability results are less conservative than the delay-independent ones if the delay size is very small. Hence, delay-dependent stabilization problem for the existence of a robust state feedback controller that guarantee asymptotic stability as well as prescribed H_∞ performance of the closed loop system has been receiving increasing attention of the control community in recent times [9–12].

In [9, 10], the range of the time-varying delay is assumed to vary from zero to an upper bound. Nevertheless, in certain time-delay systems, like networked control systems [13], the delay-range may have a non-zero lower bound, and such systems are referred to as interval time-varying delay systems. With rapid advancement in the networked control system technology, a number of significant results have been reported in the recent past for ascertaining delay-dependent stability of interval time-delay systems, notable among them being [14–16]. In [11,12], delay-dependent H_∞ stabilization problem for interval time-delay system is considered. In the delay-dependent stability analysis that leads to the stabilization criterion of [11], few useful terms are neglected while dealing with the time-derivative of the LK functional. This, in turn, leads to conservatism of the ensuing criterion. In [12], that is recently reported, a new H_∞ stabilization criterion is presented using a new LK functional candidate. However, in the delay-dependent stability analysis, the time-derivative of the LK functional is over-bounded paving way to conservatism. This offers motivation for deriving less conservative state feedback stabilization criterion that guarantees asymptotic stability of the closed loop system, ensuring, at the same time, a prescribed H_∞ performance level.

In this paper, a less conservative delay-dependent robust stability criterion is first derived for interval time-delay systems and norm-bounded uncertainties that guarantees a prescribed H_∞ performance. To reduce the conservatism, in the proposed robust stability analysis, a new LK functional candidate is employed, and the time-derivative of the functional is tightly bounded without neglecting any useful terms with minimal number of slack matrix variables. In the sequel, based on the criterion obtained, a delay-dependent condition for the existence of a static state feedback controller is derived, which ensures asymptotic stability as well as a prescribed H_∞ performance of the closed loop system for all admissible uncertainties. A numerical example is employed to illustrate the effectiveness of the proposed stabilization method.

Notations: Notations used in this paper are fairly standard: I and 0 represents the identity matrix and null matrix of appropriate dimensions; the superscript T stands for the matrix transposition; the notation $X = X^T > 0$ (respectively $X = X^T \geq 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix is real symmetric positive definite (respectively, positive semi definite); $\mathbb{R}$ ($\mathbb{Z}$) denotes the set of real numbers (integers); $diag(c_1,\ c_2,\ \ldots,\ c_m)$ denotes block diagonal matrix with diagonal elements $c_i,\ i = 1,\ 2,\ \ldots,\ m$. The symbol '$\star$' represents the symmetric elements in a symmetric matrix.

2 System Description and Problem Statement

Consider a class of linear uncertain systems with interval time-varying delay and norm-bounded uncertainties described by

$$\left.\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t-\tau(t)) + (B + \Delta B(t))u(t) + B_\omega\omega(t), \\ z(t) &= \begin{bmatrix} Cx(t) + D_\omega\omega(t) \\ C_d x(t-\tau(t)) \\ Du(t) \end{bmatrix} \\ x(t) &= \phi(t),\ \forall t \in [-\tau_M, 0], \end{aligned}\right\} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $\omega(t) \in \mathbb{R}^p$ is the disturbance input belonging to $\ell_2[0,\ \infty)$, $z(t) \in \mathbb{R}^l$ is the controlled output; $A,\ A_d,\ B,\ B_\omega,\ C,\ C_d,\ D_\omega$ and D are known, real matrices of appropriate dimensions; $\phi(t)$ is continuous-time initial function defined on $[-\tau_M, 0]$; $\Delta A(t)$, $\Delta A_d(t)$ and $\Delta B(t)$ are unknown, real, time-varying matrices of appropriate dimensions representing time-varying parametric perturbations; they are assumed to have the following form:

$$\begin{bmatrix} \Delta A(t) & \Delta A_d(t) & \Delta B(t) \end{bmatrix} = HF(t) \begin{bmatrix} E_a & E_d & E_b \end{bmatrix}, \tag{2}$$

where $H,\ E_a,\ E_d$ and E_b are known real constant matrices with appropriate dimensions, $F(t)$ is an unknown real time-varying matrix with Lesbesgue measurable elements satisfying $F^T(t)F(t) \leq I$. The time-varying delay $\tau(t)$ may be any one of the following:

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M;\ \dot{\tau}(t) \leq \mu, \tag{3}$$

or

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M; \tag{4}$$

where $\tau_m,\ \tau_M$ and μ are known constants. The main objective of this paper is to design a robust H_∞ state feedback controller for the delay system (1) subject to norm-bounded uncertainties (2) and interval time-delay (3) or (4) that guarantees asymptotic stability of the closed loop system as well as a prescribed H_∞ performance level. The H_∞ stabilization problem is stated below:

2.1 H_∞ Control Problem:

Given a scalar $\gamma > 0$, design a state feedback controller of the form:

$$u(t) = Kx(t) \tag{5}$$

where $K \in \mathbb{R}^{m\times n}$ is a constant matrix such that the closed loop system has a prescribed H_∞ performance for all admissible uncertainties (2) and time-delay satisfying (3) or (4) i.e.

1. The closed loop system is asymptotically stable for $\omega(t) = 0$,
2. The H_∞ performance $||z(t)||_2 < \gamma||\omega(t)||_2$ is guaranteed for all non-zero $\omega(t) \in \ell_2[0,\ \infty)$ and a prescribed scalar $\gamma > 0$ under the condition $x(t) = 0,\ \forall t \in [-\tau_M,\ 0]$.

Following lemmas are indispensable in deriving the proposed robust H_∞ controller:

Lemma 1. [17] For any constant matrix $W \in \mathbb{R}^{n\times n}$, a scalar $\gamma > 0$, and vector function $\dot{x} : [-\gamma, 0] \mapsto \mathbb{R}^n$ such that the integration $\int_{t-\gamma}^{t} \dot{x}^T(s)W\dot{x}(s)ds$ is well defined, then the following inequality holds:

$$-\gamma \int_{t-\gamma}^{t} \dot{x}^T(s)W\dot{x}(s)ds \leq \begin{bmatrix} x(t) \\ x(t-\gamma) \end{bmatrix}^T \begin{bmatrix} -W & W \\ \star & -W \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\gamma) \end{bmatrix}.$$

Lemma 2. [18] Suppose $r_1 \leq r(t) \leq r_2$, where $r(.) : \mathbb{R}_+$ (or $\mathbb{Z}_+$) $\to \mathbb{R}_+$ (or $\mathbb{Z}_+$). Then, for any $R = R^T > 0$, following integral inequality holds:

$$\begin{aligned} -\int_{t-r_2}^{t-r_1} \dot{x}^T(s)R\dot{x}(s)ds \ &\leq\ \delta^T(t)\ [(r_2 - r(t))TR^{-1}T^T + (r(t) - r_1)YR^{-1}Y^T \\ &+\ [Y \quad -Y+T \quad -T] + [Y \quad -Y+T \quad -T]^T]\ \delta(t), \end{aligned}$$

where $\delta(t) = [x^T(t-r_1)\ \ x^T(t-r(t))\ \ x^T(t-r_2)]^T$; $T = [T_1^T\ \ T_2^T\ \ T_3^T]^T$, and $Y = [Y_1^T\ \ Y_2^T\ \ Y_3^T]^T$ are free matrices of appropriate dimensions. For a proof, refer to the Appendix 1.

Lemma 3. [19] Suppose $\gamma_1 \leq \gamma(t) \leq \gamma_2$, where $\gamma(.) : \mathbb{R}_+$ (or $\mathbb{Z}_+$) $\to \mathbb{R}_+$ (or $\mathbb{Z}_+$). Then, for any constant matrices $\Xi_1,\ \Xi_2,$ and Ξ with proper dimensions, the following matrix inequality

$$\Xi + (\gamma(t) - \gamma_1)\Xi_1 + (\gamma_2 - \gamma(t))\Xi_2 < 0$$

holds, if and only if

$$\begin{aligned} \Xi \ &+\ (\gamma_2 - \gamma_1)\Xi_1 < 0, \\ \Xi \ &+\ (\gamma_2 - \gamma_1)\Xi_2 < 0. \end{aligned}$$

Lemma 4. [20] Given matrices $Q = Q^T,\ H,\ E$, and $R = R^T$ of appropriate dimensions, the inequality

$$Q + HFE + E^T F^T H^T < 0,$$

for all F satisfying $F^T F \leq R$ holds, if and only if there exists some scalar, $\epsilon > 0$ such that

$$Q + \epsilon HH^T + \epsilon^{-1} E^T RE < 0.$$

Fact 1. Schur complement: Given constant symmetric matrices $\Sigma_1,\ \Sigma_2$ and Σ_3 where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$, if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0,\ \text{or}\ \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

Fact 2. Definiteness of a matrix (Q) is invariant under congruent transformation by a full rank matrix (W). For example, if $Q = Q^T \in \mathbb{R}^{n\times n}$, and $W \in \mathbb{R}^{n\times n}$ is of full rank, then, $Q > 0 \Rightarrow W^T QW > 0$.

3 Robust H_∞ Performance Analysis

In this section, we propose a less conservative delay-range-dependent stability criterion for the following unforced system:

$$\left.\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t - \tau(t)) + B_\omega \omega(t), \\ z(t) &= \begin{bmatrix} Cx(t) + D_\omega \omega(t) \\ C_d x(t - \tau(t)) \\ 0 \end{bmatrix} \\ x(t) &= \phi(t),\ t \in [-\tau_M, 0]. \end{aligned}\right\} \tag{6}$$

Based on the criterion proposed, in the sequel, we design a robust H_∞ state feedback controller that guarantees asymptotic stability of the closed loop system as well as a prescribed H_∞ performance level. The proposed stability criterion is stated in the form of following theorem:

Theorem 1. Given scalars $\gamma > 0,\ 0 \leq \tau_m \leq \tau_M$, and μ, the system (6) with (3) is robustly asymptotically stable satisfying $||z(t)||_2 \leq \gamma||\omega(t)||_2$ for any nonzero $\omega(t) \in \ell_2[0,\ \infty)$ under $x(t) = 0,\ \forall t \in [-\tau_M,\ 0]$, if there exist real symmetric positive definite matrices $P,\ Q_1,\ Q_2,\ Q_3,\ R_1$ and R_2, scalars $\epsilon_i > 0,\ i = 1,\ 2$; free matrices $Y_j,\ T_j,\ S_j,\ j = 1,\ 2,\ 3$ of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} \Pi + \Upsilon + \Pi_1 + \Pi_1^T & (\tau_M - \tau_m)Y_a & \Upsilon_2 & \Upsilon_3 & \Phi & \epsilon_1 \Gamma^T \\ \star & -(\tau_M - \tau_m)R_2 & 0 & 0 & 0 & 0 \\ \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & I & 0 & 0 \\ \star & \star & \star & \star & -\epsilon_1 I & 0 \\ \star & \star & \star & \star & \star & -\epsilon_1 I \end{bmatrix} < 0, \tag{7}$$

$$\begin{bmatrix} \Pi + \Upsilon + \Pi_1 + \Pi_1^T & (\tau_M - \tau_m)T_a & \Upsilon_2 & \Upsilon_3 & \Phi & \epsilon_2 \Gamma^T \\ \star & -(\tau_M - \tau_m)R_2 & 0 & 0 & 0 & 0 \\ \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & I & 0 & 0 \\ \star & \star & \star & \star & -\epsilon_2 I & 0 \\ \star & \star & \star & \star & \star & -\epsilon_2 I \end{bmatrix} < 0, \tag{8}$$

where

$$\Pi = \begin{bmatrix} Q_1 - R_1 & R_1 & 0 & 0 & P & C^T D_\omega \\ \star & -Q_1 + Q_2 + Q_3 - R_1 & 0 & 0 & 0 & 0 \\ \star & \star & -(1-\mu)Q_3 & 0 & 0 & 0 \\ \star & \star & \star & -Q_2 & 0 & 0 \\ \star & \star & \star & \star & U & 0 \\ \star & \star & \star & \star & \star & -\gamma^2 I + D_\omega^T D_\omega \end{bmatrix},$$

$$\Upsilon = \begin{bmatrix} S_1 A + A^T S_1^T & 0 & S_1 A_d + A^T S_2^T & 0 & -S_1 + A^T S_3^T & S_1 B_\omega \\ \star & 0 & 0 & 0 & 0 & 0 \\ \star & \star & S_2 A_d + A_d^T S_2^T & 0 & -S_2 + A_d^T S_3^T & S_2 B_\omega \\ \star & \star & \star & 0 & 0 & 0 \\ \star & \star & \star & \star & -S_3 - S_3^T & S_3 B_\omega \\ \star & \star & \star & \star & \star & 0 \end{bmatrix},$$

$$\begin{aligned}
\Pi_1 &= \begin{bmatrix} 0 & Y_a & -Y_a + T_a & -T_a & 0 & 0 \end{bmatrix}, \\
Y_a &= \begin{bmatrix} 0 & Y_1^T & Y_2^T & Y_3^T & 0 & 0 \end{bmatrix}^T, \\
T_a &= \begin{bmatrix} 0 & T_1^T & T_2^T & T_3^T & 0 & 0 \end{bmatrix}^T, \\
\Phi &= \begin{bmatrix} H^T S_1^T & 0 & H^T S_2^T & 0 & H^T S_3^T & 0 \end{bmatrix}^T, \\
\Gamma &= \begin{bmatrix} E_a & 0 & E_d & 0 & 0 & 0 \end{bmatrix}, \\
\Upsilon_2 &= \begin{bmatrix} C & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
\Upsilon_3 &= \begin{bmatrix} 0 & 0 & C_d & 0 & 0 & 0 \end{bmatrix}^T,
\end{aligned}$$

with $U = \tau_m^2 R_1 + (\tau_M - \tau_m) R_2$.

Proof. Consider the following LK functional:

$$\begin{aligned}
V(x_t) &= x^T(t)Px(t) + \int_{t-\tau_m}^{t} x^T(s)Q_1 x(s)ds + \int_{t-\tau_M}^{t-\tau_m} x^T(s)Q_2 x(s)ds \\
&+ \int_{t-\tau(t)}^{t-\tau_m} x^T(s)Q_3 x(s)ds + \tau_m \int_{-\tau_m}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta \\
&+ \int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta, \qquad (9)
\end{aligned}$$

where P, Q_1, Q_2, Q_3, R_1 and R_2 are positive definite matrices. The time derivative of the LK functional along the trajectory of (6) is given by

$$\begin{aligned}
\dot{V}(x_t) &= 2x^T(t)P\dot{x}(t) + x^T(t)Q_1x(t) - x^T(t-\tau_m)(Q_1 - Q_2 - Q_3)x(t-\tau_m) \\
&- x^T(t-\tau_M)Q_2x(t-\tau_M) - (1-\dot{\tau}(t))x^T(t-\tau(t))Q_3x(t-\tau(t)) \\
&+ \dot{x}^T(t)U\dot{x}(t) - \tau_m\int_{t-\tau_m}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds - \int_{t-\tau_M}^{t-\tau_m} \dot{x}^T(s)R_2\dot{x}(s)ds. \qquad (10)
\end{aligned}$$

Now, for any $S = [S_1^T \;\; 0 \;\; S_2^T \;\; 0 \;\; S_3^T \;\; 0]^T$ and $\Sigma(t) = [x^T(t) \;\; x^T(t-\tau_m) \;\; x^T(t-\tau(t)) \;\; x^T(t-\tau_M) \;\; \dot{x}^T(t) \;\; \omega^T(t)]^T$, following holds good:

$$2\Sigma^T(t)S(-\dot{x}(t) + (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t-\tau(t)) + B_\omega \omega(t)) = 0. \qquad (11)$$

Using the upper bound of the delay-derivative and (11), we can rewrite (10) as follows:

$$\begin{aligned}\dot V(x_t) &\leq 2x^T(t)P\dot x(t) + x^T(t)Q_1x(t) - x^T(t-\tau_m)(Q_1 - Q_2 - Q_3)x(t-\tau_m)\\ &- x^T(t-\tau_M)Q_2x(t-\tau_M) - (1-\mu)x^T(t-\tau(t))Q_3x(t-\tau(t))\\ &+ \dot x^T(t)U\dot x(t) - \tau_m\int_{t-\tau_m}^{t}\dot x^T(s)R_1\dot x(s)ds - \int_{t-\tau_M}^{t-\tau_m}\dot x^T(s)R_2\dot x(s)ds\\ &+ 2\Sigma^T(t)S(-\dot x(t) + (A+\Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t-\tau(t)) + B_\omega\omega(t)). \quad (12)\end{aligned}$$

Now, for a prescribed scalar $\gamma > 0$, we define a performance index J as follows:

$$J = \int_0^\infty (z^T(s)z(s) - \gamma^2\omega^T(s)\omega(s))ds. \quad (13)$$

By adding $z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)$ to the both sides of (12), and using the decomposition of the parametric perturbation (2), and dealing the integral terms $-\tau_m\int_{t-\tau_m}^{t}\dot x^T(s)R_1\dot x(s)ds$ and $-\int_{t-\tau_M}^{t-\tau_m}\dot x^T(s)R_2\,\dot x(s)ds$ with Lemma 1 and Lemma 2 respectively, we get

$$\begin{aligned}\dot V(x_t) &+ z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \leq \Sigma^T(t)(\Pi + \Upsilon + \Pi_1 + \Pi_1^T + \Upsilon_2\Upsilon_2^T + \Upsilon_3\Upsilon_3^T\\ &+ \Phi F(t)\Gamma + (\Phi F(t)\Gamma)^T + (\tau(t)-\tau_m)Y_aR_2^{-1}Y_a^T + (\tau_M - \tau(t))T_aR_2^{-1}T_a^T)\Sigma(t). \quad (14)\end{aligned}$$

If

$$\begin{aligned}\Pi &+ \Upsilon + \Pi_1 + \Pi_1^T + \Upsilon_2\Upsilon_2^T + \Upsilon_3\Upsilon_3^T + \Phi F(t)\Gamma + (\Phi F(t)\Gamma)^T + (\tau(t)-\tau_m)Y_aR_2^{-1}Y_a^T\\ &+ (\tau_M - \tau(t))T_aR_2^{-1}T_a^T < 0, \quad (15)\end{aligned}$$

then

$$\dot V(x_t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) < 0. \quad (16)$$

Now, when $\omega(t) = 0$, $\dot V(x_t) < 0$; then, the system is asymptotically stable. For $\omega(t) \neq 0$, by integrating (16) from 0 to t, and letting $t \to \infty$ with zero initial condition, we get

$$\int_0^\infty z^T(s)z(s)ds \leq \int_0^\infty \gamma^2\omega^T(s)\omega(s)ds, \quad (17)$$

i.e. $||z(t)||_2 \leq \gamma||\omega(t)||_2$. Therefore, $J < 0$, and the proof is completed. By applying successively lemmas 3 and 4 and Schur complement to (15), we deduce the LMIs stated in Theorem 1. □

Remark 1. For systems without uncertainties i.e. $\Delta A(t) = \Delta A_d(t) = 0$, we have the following criterion for ascertaining delay-range-dependent stability:

Corollary 1. Given scalars $\gamma > 0$, $0 \leq \tau_m \leq \tau_M$, and μ, the system (6) without uncertainties is asymptotically stable satisfying $||z(t)||_2 \leq \gamma||\omega(t)||_2$ for any nonzero $\omega(t) \in \ell_2[0,\ \infty)$ under $x(t) = 0$, $\forall t \in [-\tau_M,\ 0]$, if there exist real symmetric positive definite matrices $P,\ Q_1,\ Q_2,\ Q_3,\ R_1$ and R_2; free matrices $Y_j,\ T_j,\ S_j,\ j = 1,\ 2,\ 3$ of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix}\Pi + \Upsilon + \Pi_1 + \Pi_1^T & (\tau_M - \tau_m)Y_a & \Upsilon_2 & \Upsilon_3\\ \star & -(\tau_M-\tau_m)R_2 & 0 & 0\\ \star & \star & -I & 0\\ \star & \star & \star & -I\end{bmatrix} < 0, \quad (18)$$

$$\begin{bmatrix}\Pi + \Upsilon + \Pi_1 + \Pi_1^T & (\tau_M - \tau_m)T_a & \Upsilon_2 & \Upsilon_3\\ \star & -(\tau_M-\tau_m)R_2 & 0 & 0\\ \star & \star & -I & 0\\ \star & \star & \star & -I\end{bmatrix} < 0. \quad (19)$$

Remark 2. In addition to the aforesaid condition, if there is no disturbance input i.e. $\omega(t) = 0$, then we have the following delay-range-dependent stability criterion:

Corollary 2. Given scalars $0 \leq \tau_m \leq \tau_M$, and μ, the system (6) without uncertainties and $\omega(t) = 0$ is asymptotically stable, if there exist real symmetric positive definite matrices $P,\ Q_1,\ Q_2,\ Q_3,\ R_1$ and R_2; free matrices $Y_j,\ T_j,\ S_j,\ j = 1,\ 2,\ 3$ of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} \Pi' + \Upsilon' + \Pi_2 + \Pi_2^T & (\tau_M - \tau_m)Y_b \\ \star & -(\tau_M - \tau_m)R_2 \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} \Pi' + \Upsilon' + \Pi_2 + \Pi_2^T & (\tau_M - \tau_m)T_b \\ \star & -(\tau_M - \tau_m)R_2 \end{bmatrix} < 0, \tag{21}$$

where

$$\begin{aligned}
\Pi' &= \begin{bmatrix} Q_1 - R_1 & R_1 & 0 & 0 & P \\ \star & -Q_1 + Q_2 + Q_3 - R_1 & 0 & 0 & 0 \\ \star & \star & -(1-\mu)Q_3 & 0 & 0 \\ \star & \star & \star & -Q_2 & 0 \\ \star & \star & \star & \star & U \end{bmatrix}, \\
\Upsilon' &= \begin{bmatrix} S_1A + A^TS_1^T & 0 & S_1A_d + A^TS_2^T & 0 & -S_1 + A^TS_3^T \\ \star & 0 & 0 & 0 & 0 \\ \star & \star & S_2A_d + A_d^TS_2^T & 0 & -S_2 + A_d^TS_3^T \\ \star & \star & \star & 0 & 0 \\ \star & \star & \star & \star & -S_3 - S_3^T \end{bmatrix}, \\
\Pi_2 &= \begin{bmatrix} 0 & Y_b & -Y_b + T_b & -T_b & 0 \end{bmatrix}, \\
Y_b &= \begin{bmatrix} 0 & Y_1^T & Y_2^T & Y_3^T & 0 \end{bmatrix}^T, \\
T_b &= \begin{bmatrix} 0 & T_1^T & T_2^T & T_3^T & 0 \end{bmatrix}^T.
\end{aligned}$$

Remark 3. It can be readily established (see, Appendix 2) that the delay-range-dependent stability criterion reported recently in [16] is a special case of Corollary 2, and can be obtained from the same by constraining the slack matrices $S_1 = P,\ S_2 = 0,\ S_3 = U,\ Y_1 = -Y_2 = T_2 = -T_3 = -\frac{R_2}{\tau_M - \tau_m}$, and $Y_3 = T_1 = 0$. Since slack matrices in the optimization problem are constrained, the search space of the solution is restricted, and as a consequence of this, the stability criterion of [16] is more conservative than Corollary 2.

4 Robust H_∞ Controller Synthesis

In this section, based on the result stated in the section 3, a state feedback H_∞ controller is designed to ensure asymptotic stability as well as a prescribed H_∞ performance of the closed loop system for all admissible uncertainties. The system (1) under the state feedback (5) is given by

$$\left.\begin{aligned}
\dot{x}(t) &= (A + BK + \Delta A(t) + \Delta B(t)K)x(t) + (A_d + \Delta A_d(t))x(t - \tau(t)) + B_\omega\omega(t), \\
z(t) &= \begin{bmatrix} Cx(t) + D_\omega\omega(t) \\ C_dx(t - \tau(t)) \\ DKx(t) \end{bmatrix} \\
x(t) &= \phi(t),\ t \in [-\tau_M, 0].
\end{aligned}\right\} \tag{22}$$

The proposed LMI criterion for H_∞ controller synthesis is stated below:

Theorem 2. Given scalars $\gamma > 0$, and $0 \le \tau_m \le \tau_M$, the uncertain system (1) with (3) is robustly asymptotically stabilizable satisfying $||z(t)||_2 \le \gamma||\omega(t)||_2$ for any nonzero $\omega(t) \in \ell_2[0,\ \infty)$ under $x(t) = 0,\ \forall t \in [-\tau_M,\ 0]$, if there exist real symmetric positive-definite matrices $\hat{P},\ \hat{Q}_1,\ \hat{Q}_2,\ \hat{Q}_3,\ \hat{R}_1$ and $\hat{R}_2$; scalars $\hat{\epsilon}_i > 0,\ i = 1,\ 2$ and free matrices $\hat{Y}_j,\ \hat{T}_j,\ j = 1,\ 2,\ 3;\ X,\ Y$ of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} \Pi'' + \Upsilon'' + \hat{\Pi}_1 + \hat{\Pi}_1^T & (\tau_M - \tau_m)\hat{Y}_a & \Upsilon_1' & \Upsilon_2' & \Upsilon_3' & \hat{\epsilon}_1\Phi' & \Gamma'^T \\ \star & -(\tau_M - \tau_m)\hat{R}_2 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -I & 0 & 0 & 0 & 0 \\ \star & \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -\hat{\epsilon}_1 I & 0 \\ \star & \star & \star & \star & \star & \star & -\hat{\epsilon}_1 I \end{bmatrix} < 0, \quad (23)$$

$$\begin{bmatrix} \Pi'' + \Upsilon'' + \hat{\Pi}_1 + \hat{\Pi}_1^T & (\tau_M - \tau_m)\hat{T}_a & \Upsilon_1' & \Upsilon_2' & \Upsilon_3' & \hat{\epsilon}_2\Phi' & \Gamma'^T \\ \star & -(\tau_M - \tau_m)\hat{R}_2 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -I & 0 & 0 & 0 & 0 \\ \star & \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -\hat{\epsilon}_2 I & 0 \\ \star & \star & \star & \star & \star & \star & -\hat{\epsilon}_2 I \end{bmatrix} < 0, \quad (24)$$

where

$$\Pi'' = \begin{bmatrix} \hat{Q}_1 - \hat{R}_1 & \hat{R}_1 & 0 & 0 & \hat{P} & B_\omega + XC^T D_\omega \\ \star & -\hat{Q}_1 + \hat{Q}_2 + \hat{Q}_3 - \hat{R}_1 & 0 & 0 & 0 & 0 \\ \star & \star & -(1-\mu)\hat{Q}_3 & 0 & 0 & B_\omega \\ \star & \star & \star & -\hat{Q}_2 & 0 & 0 \\ \star & \star & \star & \star & \hat{U} & B_\omega \\ \star & \star & \star & \star & \star & -\gamma^2 I + D_\omega^T D_\omega \end{bmatrix},$$

$$\Upsilon'' = \begin{bmatrix} (1,1) & 0 & A_d X^T + XA^T + Y^T B^T & 0 & -X^T + XA^T + Y^T B^T & 0 \\ \star & 0 & 0 & 0 & 0 & 0 \\ \star & \star & A_d X^T + XA_d^T & 0 & -X^T + XA_d^T & 0 \\ \star & \star & \star & 0 & 0 & 0 \\ \star & \star & \star & \star & -X - X^T & 0 \\ \star & \star & \star & \star & \star & 0 \end{bmatrix},$$

$$\Upsilon_1' = \begin{bmatrix} DY & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\Upsilon_2' = \begin{bmatrix} CX^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\Upsilon_3' = \begin{bmatrix} 0 & 0 & C_d X^T & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\Phi' = \begin{bmatrix} H^T & 0 & H^T & 0 & H^T & 0 \end{bmatrix}^T,$$

$$\Gamma' = \begin{bmatrix} E_a X^T + E_b Y & 0 & E_d^T X & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\hat{Y}_a = \begin{bmatrix} 0 & \hat{Y}_1^T & \hat{Y}_2^T & \hat{Y}_3^T & 0 & 0 \end{bmatrix}^T,$$

$$\hat{T}_a = \begin{bmatrix} 0 & \hat{T}_1^T & \hat{T}_2^T & \hat{T}_3^T & 0 & 0 \end{bmatrix}^T,$$

with $(1,1) = AX^T + XA^T + BY + Y^T B^T$ and $\hat{U} = \tau_m^2 \hat{R}_1 + (\tau_M - \tau_m)\hat{R}_2$. The robust H_∞ state feedback controller K is given by

$$K = YX^{-T}. \quad (25)$$

Proof. Under state feedback $u(t) = Kx(t)$, by Theorem 1, we can readily deduce the following LMIs for ascertaining delay-dependent stability of the closed loop system:

$$\begin{bmatrix} \Pi + \Upsilon|_{A=A+BK} + \Pi_1 + \Pi_1^T & (\tau_M - \tau_m)Y_a & \Upsilon_1 & \Upsilon_2 & \Upsilon_3 & \hat{\epsilon}_1\Phi & \Gamma^T|_{E_a=E_a+E_bK} \\ \star & -(\tau_M - \tau_m)R_2 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -I & 0 & 0 & 0 & 0 \\ \star & \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -\hat{\epsilon}_1 I & 0 \\ \star & \star & \star & \star & \star & \star & -\hat{\epsilon}_1 I \end{bmatrix} < 0,$$

$$\begin{bmatrix} \Pi + \Upsilon|_{A=A+BK} + \Pi_1 + \Pi_1^T & (\tau_M - \tau_m)T_a & \Upsilon_1 & \Upsilon_2 & \Upsilon_3 & \hat{\epsilon}_2\Phi & \Gamma^T|_{E_a=E_a+E_bK} \\ \star & -(\tau_M - \tau_m)R_2 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -I & 0 & 0 & 0 & 0 \\ \star & \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -\hat{\epsilon}_2 I & 0 \\ \star & \star & \star & \star & \star & \star & -\hat{\epsilon}_2 I \end{bmatrix} < 0,$$

where $\hat{\epsilon}_i = \epsilon_i^{-1}$, $i = 1, 2$ and $\Upsilon_1 = [KD \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T$. Now, we observe that $\Upsilon|_{A=A+BK}$ is nonlinear as it involves product of unknown matrices. Therefore, to obtain an LMI based result, replace $S_1 = S_2 = S_3 = X^{-1}$ where X is a nonsingular matrix. Then, by pre-multiplying with $diag(X, X, X, X, X, I, X, I, I, I, I, I)$ and post multiplying with its transpose, and defining $\hat{P} = XPX^T$, $\hat{Q}_i = XQ_iX^T$, $\hat{Y}_i = XY_iX^T$, $\hat{T}_i = XT_iX^T$, $i = 1, 2, 3$ and $Y = KX^T$, we deduce the LMIs stated in Theorem 2. This completes the proof. □

Remark 4. For systems with interval time-varying delay (4), wherein, no restriction is imposed on the upper bound of the delay-derivative, the results on robust stability and stabilization can be obtained readily from Theorem 1 and Theorem 2 by letting $Q_3 = 0$ and $\hat{Q}_3 = 0$ respectively.

Remark 5. The proposed stabilization approach can be readily extended to systems with $\omega(t) = 0$ and/or $\Delta A(t) = \Delta A_d(t) = \Delta B(t) = 0$. Though these results are not stated explicitly in this paper due to brevity, these cases are nevertheless considered while demonstrating the effectiveness of the proposed stabilization criterion using a numerical example in the next section.

5 Numerical Example

Consider the uncertain system in (1) with following parameters:

$$\begin{aligned} A &= \begin{bmatrix} 4 & 0.1 & -0.3 \\ -0.2 & 3 & -0.2 \\ 0.2 & -0.3 & 2 \end{bmatrix}, \ A_d = \begin{bmatrix} -0.4 & 0.1 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}, \ B = \begin{bmatrix} -4 & 0.2 & 0 \\ 0 & 3 & 1 \\ 0.1 & 0 & 3 \end{bmatrix}, \\ C &= C_d = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \ D_\omega = 0.1, \ B_\omega = \begin{bmatrix} 0.2 & 0 \\ 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \ D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \\ H &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \ E_a = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \ E_d = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.2 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \\ E_b &= \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}. \end{aligned}$$

By solving the LMI in Theorem 2, the closed loop system is found to be asymptotically stable for $1.2 \leq \tau(t) \leq 2.0$ with $\mu = 0.3$ and $\gamma = 0.6747$. The corresponding H_∞ state feedback controller is computed as

$$K = \begin{bmatrix} 1.4616 & -0.1108 & -0.0352 \\ 0.2768 & -1.6801 & 0.4802 \\ -0.0730 & 0.0385 & -1.1948 \end{bmatrix}.$$

On the other hand, for the same value of μ and γ, the recently reported result in [12] concluded that the closed loop system is stable for $1.2 \leq \tau(t) \leq 1.8$. Hence, the proposed controller synthesis approach is less conservative than that of [12].

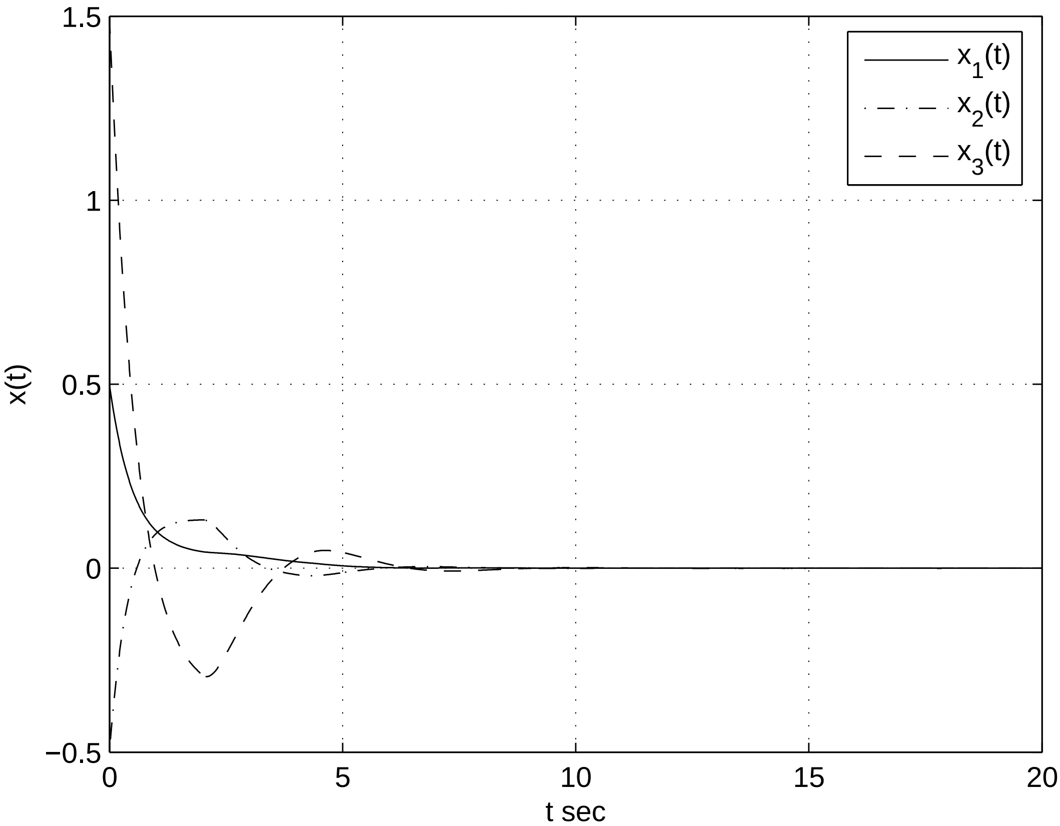

Fig. 1. State response of the closed-loop system

The state trajectories of the uncertain system under state feedback $u(t) = Kx(t)$ is shown in Fig. 1, where the disturbance signal is uniformly distributed random noise over $[-0.01 \;\; 0.01]$ and $x(0) = [0.5 \quad -0.5 \quad 1.5]^T$. The figure shows that under proposed state feedback controller, the closed loop system of the example is robustly asymptotically stable satisfying $||z(t)||_2 \leq \gamma||\omega(t)||_2$ for all values of non-zero $\omega(t) \in \ell_2[0, \;\infty)$ and a prescribed H_∞ performance level $\gamma = 0.6747$. The plot of $||u(t)||_2$ versus t for $1.2 \leq \tau(t) \leq 2.0$ is shown in Fig. 2. The number of decision variables involved in the proposed result is listed in Table 1 against the existing result of [12]. From the table, it is clear that the proposed stabilization criterion uses less number of decision variables; hence, it is computationally less expensive than the criterion of [12].

Case 1. When $\omega(t) = 0$, $\Delta A(t) \neq 0$, $\Delta B(t) \neq 0$ and $\Delta A_d(t) \neq 0$, the system (1) under (5) is asymptotically stable for $1.2 \leq \tau(t) \leq 3.1$, whereas, according to [12], the closed loop system is

Table 1: Number of decision variables

Method	No. of decision variables
[12]	$18.5n^2 + 4.5n + 1$
Theorem 2	$11n^2 + 3n + 2$

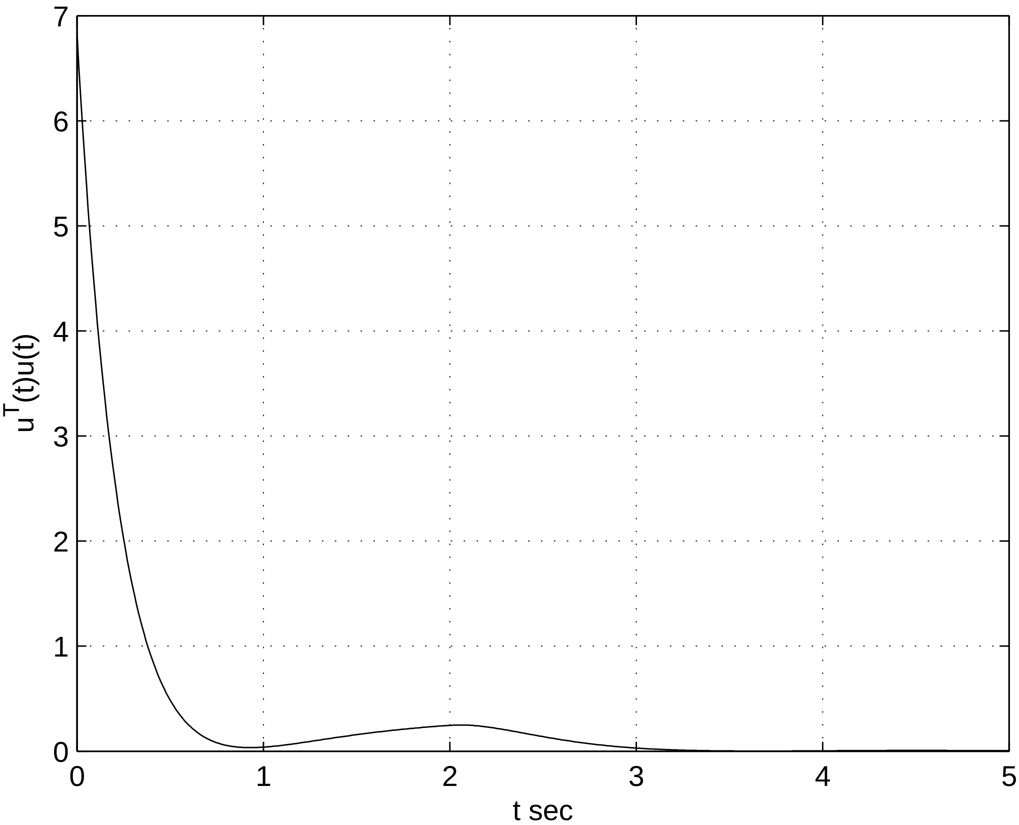

Fig. 2. Plot of $u^T(t)u(t)$ versus t

asymptotically stable for $1.2 \leq \tau(t) \leq 2.6$. The proposed state feedback controller K for this case is given by

$$K = \begin{bmatrix} 1.3491 & -0.0584 & -0.0611 \\ 0.1253 & -1.5446 & 0.4518 \\ -0.1038 & 0.1018 & -1.1178 \end{bmatrix}.$$

Case 2. When $\omega(t) = 0$, $\Delta A(t) = \Delta A_d(t) = 0$, and $\Delta B(t) = 0$ the nominal system under (5) is asymptotically stable for $1.2 \leq \tau(t) \leq 3.8$, and corresponding state feedback gain matrix is computed as

$$K = \begin{bmatrix} 1.3034 & -0.0387 & -0.0540 \\ 0.1135 & -1.4216 & 0.4204 \\ -0.1101 & 0.1013 & -1.0612 \end{bmatrix}.$$

The corresponding result, according to [12], reports that the closed loop system is asymptotically stable for $1.2 \leq \tau(t) \leq 3.2$.

6 Conclusion

In this paper, a less conservative approach is presented for the problem of delay-dependent H_∞ control of a class of linear uncertain systems with interval time-varying delay and norm-bounded uncertainties using Lyapunov-Krasovskii approach. By exploiting a candidate Lyapunov-Krasovakii (LK) functional and imposing tighter bounding on its time-derivative, a delay-dependent condition for the existence of a state feedback controller is derived in LMI framework that ensures asymptotic stability as well as a prescribed H_∞ performance of the closed loop system for all admissible uncertainties. A numerical example is employed to demonstrate the effectiveness of the proposed controller.

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Appendix

Appendix 1: Proof of Lemma 2

Proof. Newton-Leibniz formula and Jenson integral inequality are used to derive the Lemma; they are stated below:

Fact 3. Newton-Leibniz Formula: For a continuous function $f(x)$ and its derivative $f^{'}(x)$ defined on $[a,\ b]$, then

$$\int_a^b f^{'}(x)dx = f(b) - f(a).$$

Fact 4. Jenson Integral Inequality: For any symmetric positive definite matrix $M \in \mathbb{R}^{n\times n}$, scalars γ_1 and γ_2 satisfying $\gamma_1 < \gamma_2$, vector function $\omega : [\gamma_1,\ \gamma_2] \rightarrow \mathbb{R}^n$ such that following integrals are well defined, then

$$\int_{\gamma_1}^{\gamma_2} \omega^T(s)M\omega(s)ds \geq \frac{1}{\gamma_2-\gamma_1}\Bigg[\int_{\gamma_1}^{\gamma_2}\omega(s)ds\Bigg]^T M\Bigg[\int_{\gamma_1}^{\gamma_2}\omega(s)ds\Bigg]. \tag{26}$$

For any vectors $z,\ y$, and symmetric, positive definite matrix X, we know that the following inequality holds:

$$-2z^T y \leq z^T X^{-1} z + y^T X y.$$

Substituting $z = T^T\delta(t)$, $y = (x(t-r(t)) - x(t-r_2)) = \int_{t-r_2}^{t-r(t)} \dot{x}(s)ds$ (Fact 3), and $X = \frac{R}{(r_2-r(t))}$, we get:

$$\begin{aligned} -2\delta^T(t)T(x(t-r(t)) - x(t-r_2)) \quad &\leq \quad (r_2-r(t))\delta^T(t)TR^{-1}T^T\delta(t) \\ &+ \quad \frac{1}{(r_2-r(t))}\Bigg[\int_{t-r_2}^{t-r(t)}\dot{x}(s)ds\Bigg]^T R\Bigg[\int_{t-r_2}^{t-r(t)}\dot{x}(s)ds\Bigg]. \end{aligned}$$

Using Fact 4, we get

$$-2\delta^T(t)T(x(t-r(t)) - x(t-r_2)) \leq (r_2-r(t))\delta^T(t)TR^{-1}T^T\delta(t) + \int_{t-r_2}^{t-r(t)} \dot{x}^T(s)R\dot{x}(s)ds,$$

or in other words,

$$-\int_{t-r_2}^{t-r(t)} \dot{x}^T(s)R\dot{x}(s)ds \leq \delta^T(t)\ [(r_2-r(t))TR^{-1}T^T + [0 \quad T \quad -T] + [0 \quad T \quad -T]^T]\ \delta(t).$$

By analogy, we can deduce the following inequality as well:

$$-\int_{t-r(t)}^{t-r_1} \dot{x}^T(s)R\dot{x}(s)ds \leq \delta^T(t)\ [(r(t)-r_1)YR^{-1}Y^T + [Y \quad -Y \quad 0] + [Y \quad -Y \quad 0]^T]\ \delta(t).$$

Summation of the last two equations completes the proof of the Lemma. □

Appendix 2

In this section, we shall show that the delay-range-dependent stability criterion presented in [16] is a conservative version of the proposed stability criterion of Corollary 2. For this, let us first state the stability criterion of [16] in the following lemma:

Lemma 5. [16] Given scalars $0 \leq \tau_m \leq \tau_M$, and μ, the system (6) without uncertainties and $\omega(t) = 0$ is asymptotically stable, if there exist real symmetric positive definite matrices $P,\ Q_1,\ Q_2,\ Q_3,\ Z_1$ and Z_2 such that the following LMIs hold:

$$\begin{bmatrix} (1,1) & Z_1 & PA_d & 0 & A^TU' \\ \star & -Q_1+Q_2+Q_3-Z_1-Z_2 & Z_2 & 0 & 0 \\ \star & \star & (3,3) & 2Z_2 & A^TU' \\ \star & \star & \star & -Q_2-2Z_2 & 0 \\ \star & \star & \star & \star & -U' \end{bmatrix} < 0, \tag{27}$$

$$\begin{bmatrix} (1,1) & Z_1 & PA_d & 0 & A^TU' \\ \star & -Q_1+Q_2+Q_3-Z_1-2Z_2 & 2Z_2 & 0 & 0 \\ \star & \star & (3,3) & Z_2 & A^TU' \\ \star & \star & \star & -Q_2-Z_2 & 0 \\ \star & \star & \star & \star & -U' \end{bmatrix} < 0, \tag{28}$$

where $(1,1) = A^TP+PA+Q_1-Z_1$, $(3,3) = -(1-\mu)Q_3-2Z_2-Z_2$ and $U' = \tau_m^2 Z_1+(\tau_M-\tau_m)^2 Z_2$.

Now, define two non-singular transformation matrices:

$$\Theta_1 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & -\frac{I}{\tau_M-\tau_m} \\ 0 & 0 & I & 0 & 0 & \frac{I}{\tau_M-\tau_m} \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{I}{\tau_M-\tau_m} \end{bmatrix},$$

$$\Theta_2 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & -\frac{I}{\tau_M-\tau_m} \\ 0 & 0 & 0 & I & 0 & \frac{I}{\tau_M-\tau_m} \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{I}{\tau_M-\tau_m} \end{bmatrix}.$$

After substituting $S_j,\ j = 1,\ 2,\ 3$ as $S_1 = P,\ S_2 = 0$ and $S_3 = U$, congruence transformation of the LMIs (20) and (21) respectively with Θ_1 and Θ_2 yield the following equivalent inequalities:

$$\left[\begin{array}{ccccc|c} \Psi_{11} & R_1 & PA_d & 0 & A_d^TU & 0 \\ \star & \Psi_{22}-\frac{R_2}{\tau_M-\tau_m} & T_1+\frac{R_2}{\tau_M-\tau_m} & -T_1 & 0 & Y_1+\frac{R_2}{\tau_M-\tau_m} \\ \star & \star & \Psi_{33}+T_2+T_2^T-\frac{R_2}{\tau_M-\tau_m} & -T_2+T_3^T & A_d^TU & Y_2-\frac{R_2}{\tau_M-\tau_m} \\ \star & \star & \star & -Q_2-T_3-T_3^T & 0 & Y_3 \\ \star & \star & \star & \star & -U & 0 \\ \hline \star & \star & \star & \star & \star & -\frac{R_2}{\tau_M-\tau_m} \end{array}\right] < 0, \tag{29}$$

$$\left[\begin{array}{ccccc|c} \Psi_{11} & R_1 & PA_d & 0 & A_d^TU & 0 \\ \star & \Psi_{22}+Y_1+Y_1^T & -Y_1+Y_2^T & Y_3^T & 0 & T_1 \\ \star & \star & \Psi_{33}-Y_2-Y_2^T-\frac{R_2}{\tau_M-\tau_m} & -Y_3^T+\frac{R_2}{\tau_M-\tau_m} & A_d^TU & T_2+\frac{R_2}{\tau_M-\tau_m} \\ \star & \star & \star & -Q_2-\frac{R_2}{\tau_M-\tau_m} & 0 & T_3-\frac{R_2}{\tau_M-\tau_m} \\ \star & \star & \star & \star & -U & 0 \\ \hline \star & \star & \star & \star & \star & -\frac{R_2}{\tau_M-\tau_m} \end{array}\right] < 0, \tag{30}$$

where $\Psi_{11} = A^TP + PA + Q_1 - R_1$, $\Psi_{22} = -Q_1 + Q_2 + Q_3 - R_1$ and $\Psi_{33} = -(1-\mu)Q_3$. Since block $(2,2) < 0$ of the equivalent LMIs hold, it is clear that blocks $(1,1) < 0$ of (29) and (30) are exactly same as the LMIs (27) and (28) presented respectively in Lemma 5, if we constrain the slack matrices Y_i and $T_i,\ i = 1,\ 2,\ 3$ as

$$\begin{aligned} Y_1 &= -\frac{R_2}{\tau_M-\tau_m},\ Y_2 = \frac{R_2}{\tau_M-\tau_m},\ Y_3 = 0, \\ T_1 &= 0,\ T_2 = -\frac{R_2}{\tau_M-\tau_m},\ T_3 = \frac{R_2}{\tau_M-\tau_m}, \end{aligned}$$

and subsequently let $Z_1 = R_1$ and $Z_2 = \frac{R_2}{\tau_M-\tau_m}$. Hence, Corollary 2 of this paper, in addition to being less conservative than the delay-dependent criterion of [16], also includes the criterion as a special case. This is demonstrated using the following standard numerical example [14–16]:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

The maximum allowable delay-range bound provided by Corollary 2 of this paper and the stability criterion of [16] is listed in Table 1.

Table 2: Upper Delay bound τ_M for a given τ_m for different μ

μ	Method	$\tau_m = 0$	$\tau_m = 1$	$\tau_m = 2$	$\tau_m = 3$	$\tau_m = 4$
0.5	[16]	2.0723	2.2877	2.5048	3.2591	4.0744
	Corollary 2	2.3372	2.4147	2.6181	3.3173	4.0905
0.9	[16]	1.5304	1.8737	2.5048	3.2591	4.0744
	Corollary 2	1.8731	2.0665	2.6181	3.3173	4.0905
Any	[16]	1.5296	1.8737	2.5048	3.2591	4.0744
	Corollary 2	1.8680	2.0665	2.6181	3.3173	4.0905

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