

# Adaptive Extended Kalman Filter Based Geolocation Using TDOA/FDOA

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## Abstract

We propose a moving target tracking algorithm using the measurement signals of time difference of arrival (TDOA) and the frequency difference of arrival (FDOA) in this paper. The geolocation system using TDOA measurement does not have enough accuracy to estimate the position of target. We use both TDOA and FDOA measurement signals to estimate the target location and target's velocity at discrete times. The Kalman filter performs remarkably in calculation and location estimation. However, the estimation error can be large when the priori noise covariances are assumed with improper values. Therefore, we offer an adaptive extended Kalman filter (AEKF) to update the noise covariance at every measurement and estimation process to find proper noise covariance at each steps. The simulation results show our proposed algorithm reduces the position error effectively and improves the accuracy of target tracking greatly.

**Keywords:** Adaptive extended Kalman filter, time difference of arrival, frequency difference of arrival, target tracking, geolocation.

## 1. Introduction

Target tracking is an important technique in wide range of areas, such as wireless communications, search and rescue using satellites, locating and identifying a target in electronic warfare, etc. As main techniques for target tracking, there are several generally used methods: difference of arrival, time difference of arrival, frequency difference of arrival, etc. Many researches have been performed for position calculation. The Taylor-series expansion [1] method uses an iterative scheme to find an optimal solution of the position equation, and it needs an approximate initial position guess. Therefore, when the initial position is proper, it can get a high precision and the convergence is also good and fast. However, the problem is in that it is hard to get the initial value in the real environment which has much disturbance. Chan [2] proposed a two-stage weighted least square algorithm to linearize the TDOA and FDOA measurement. When the TDOA measurement is obtained by signal processing, the feature of TDOA measurement error can be indentified and the nonlinear equations of TDOA measurement can be transformed into linear equations by some mathematical operation. The analytic solution to the measurement with some constrains is introduced in [3]: DOA, TDOA, FDOA, the combination of TDOA and FDOA, etc.

The DOA is one of the passive conventional techniques with low location accuracy. The TDOA uses the hyperbola curves. The accuracy is higher than the DOA and it does not require any time reference [4]. The location of a target can be tracked by the signal intercepted at several sensors; one of the most effective methods is to use the measurement signals of time difference of arrival and/or frequency difference of arrival between pairs of signals received from the sensors. This method involves two problems concerning the

estimation of position: one is the signal processing to collect a set of TDOA and FDOA estimates, and the other is to process the results from the TDOA/FDOA data to estimate the target location. As a conventional way, target tracking uses only TDOA measurements which are not accurate to estimate the target location when the number of receivers is not enough [5]. To solve this problem, we use the TDOA and FDOA measurement signals together to estimate the location and the velocity of the target.

The Kalman filter is well-known for solving the problem of the target location. To overcome the nonlinear problems, the extended Kalman filter (EKF) estimates the state through a linearization process. The EKF uses priori guess to estimate the process and measurement noise covariance. As the circumstances change at different times, it's difficult to track the position precisely when the priori values are estimated with too much error from the real values. In this paper, we propose an adaptive extended Kalman filter for precise position tracking. Using the adaptive factor, the process and measurement error covariance can be modified to approach the real values. In section 2, we introduce the system modeling for target localization. Section 3 designs the adaptive extended Kalman filter algorithm for geolocation. The simulation results show the improved tracking accuracy in section 4. Conclusions are drawn in section 5.

## 2. System Modeling for Target Localization

The localization method is based on using the time difference of arrival (TDOA) and the frequency difference of arrival (FDOA) signals collected from receiver sensors or UGVs receiver equipped with sensors under the interference noise. To follow the fast moving target's location, it is required to reduce the computational process in real time system. In the real situations, the TDOA and FDOA signals are mostly affected by the external noises.

$$s = s_e + \Delta s \quad (1)$$

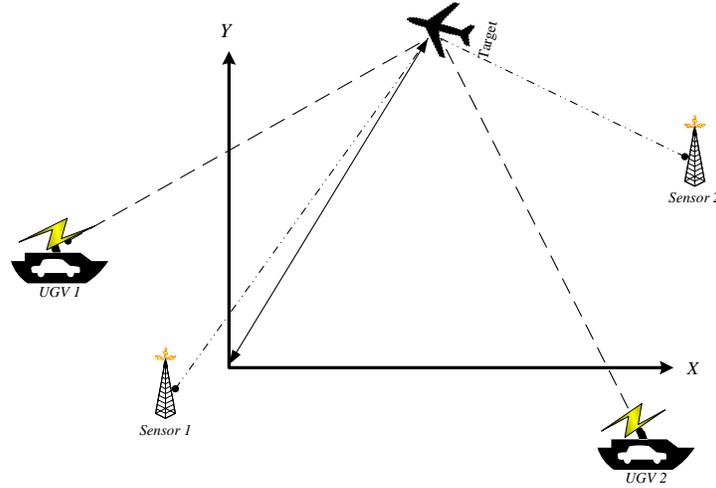
where  $s_e$  is the ideal measurement value without noises and  $\Delta s$  is an added noise value. As a robust estimation solution, Kalman filter algorithm is a classic and efficient method to estimate the position using TDOA and FDOA as measurement output.

To apply Kalman filter algorithm in the localization problem, the state-space equation needs to be formulated. Therefore the target tracking problem can be modeled as a discrete equation in two dimensions as following.

$$s(k+1) = As(k) + Bu(k) + w(k),$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ T_s & 0 \\ 0 & T_s \end{bmatrix} \quad (2)$$

where  $s(k) = [x \ y \ \Delta x \ \Delta y]^T$ ,  $(x, y)$  is the target position,  $(\Delta x, \Delta y)$  is the positional variation at each sampling time  $T_s$ ,  $u(k) = [v_x \ v_y]^T$  is the velocity of the moving target, and  $w(k)$  is the process noise modeled as a zero-mean white Gaussian noise. The output equation can be formulated using the measurements of TDOA and FDOA signals.



**Fig. 1. Concept of Target Location Using TDOA and FDOA Measurements**

$$z(k) = h(s(k), n(k)) \quad (3)$$

Where  $n(k)$  is the measurement noise also modeled as a white Gaussian noise. The output  $z(k)$  is the TDOA and FDOA measurements obtained from the definition in the following analytical methods. In this model, because of the nonlinear terms in the measurement output equation, we use the partial derivative approximate as measurement transition matrix.

$$H(k) \approx \frac{\partial h(s, n)}{\partial s} \quad (4)$$

The transmitted signal has a time difference and a frequency difference when it is sent to each receiver with known position and velocity by a moving target emitter with unknown position and velocity. To simplify the location process, we choose two pairs of receivers as shown in Fig. 1; one pair is just composed of receiver sensors, the other pair includes two UGVs equipped with sensors. The analytic method to find the target position can be summarized as follows.

Let the position and velocity of two receivers be denoted by  $r_1, r_2 \in \mathbb{R}^2$  and  $v_1, v_2 \in \mathbb{R}^2$ , respectively. The unit position vectors from the unknown target position  $r$  with velocity of  $v$  to the receivers are given by:

$$u_i = \frac{r_i - r}{\|r_i - r\|}, \quad i = 1, 2 \quad (5)$$

Although more receiver sensors can be used to identify for more accuracy, we just use two receivers for a simple model in this paper. Here, TDOA and FDOA measurements obtain signals from the two receiver sensors as following.

$$TDOA = \frac{1}{c} (\|r_2 - r\| - \|r_1 - r\|)$$

$$FDOA = -\frac{f_0}{c} \left[ (v_2 - v)^T u_2 - (v_1 - v)^T u_1 \right] \quad (6)$$

where  $c$  is the signal propagation velocity and  $f_0$  is the signal frequency. The TDOA and FDOA measurements are the nonlinear processes. To apply the EKF algorithm, we need to linearize the output measurement through the partial differentiation. The gradients are:

$$\begin{aligned} \nabla TDOA &= -\frac{1}{c} (u_2 - u_1) \\ \nabla FDOA &= \frac{f_0}{c} \left[ \frac{[I - u_2 u_2^T](v_2 - v)}{\|r_2 - r\|} - \frac{[I - u_1 u_1^T](v_1 - v)}{\|r_1 - r\|} \right] \end{aligned} \quad (7)$$

The above equation of (7) can be set as the measurement output matrix.

$$H(k) \approx \begin{bmatrix} \nabla TDOA \\ \nabla FDOA \end{bmatrix} = \frac{f_0}{c} \begin{bmatrix} -\frac{1}{f_0} (u_2 - u_1) \\ \frac{[I - u_2 u_2^T](v_2 - v)}{\|r_2 - r\|} - \frac{[I - u_1 u_1^T](v_1 - v)}{\|r_1 - r\|} \end{bmatrix} \quad (8)$$

### 3. Localization Using Adaptive Extended Kalman Filter

The extended Kalman filter (EKF) for target tracking is widely used in the position estimation of nonlinear system. However, the divergence of estimated results caused by a modeling error is considered to be a crucial weakness. Generally, the dynamic properties and errors are considered together in Kalman filter. The EKF algorithm uses a fixed priori estimates for the process and measurement noise covariances during the whole estimation process. However, the divergence of the nonlinear system model based on the TDOA and FDOA measurement values can happen in real environment if the real position values do not match with the values of estimated model system. It becomes difficult to apply the EKF method to track the accurate position with the past static priori values during the target's fast movement [6]. As a solution to prevent the divergence of extended Kalman filter, the adaptive extended Kalman filter (AEKF) is proposed to update the covariance of process noise and measurement noise in current states. As the AEKF algorithm uses the recent covariance values in the process and measurement state, it is effective to overcome the disturbance problem.

The summary for EKF algorithm consists of two parts and is given as [7]:

1) Time update (prediction part)

(a) The state projection:

$$\hat{s}^-(k+1) = f(\hat{s}(k), u(k), w(k)), \quad (9)$$

(b) The error covariance projection:

$$P^-(k) = A(k)P(k-1)A(k)^T + W(k)Q(k)W(k)^T, \quad (10)$$

2) Measurement update (correcting part)

(a) Kalman gain update:

$$K(k) = P^-(k)H(k)^T \left[ H(k)P^-(k)H(k)^T + V(k)R(k)V(k)^T \right]^{-1}, \quad (11)$$

(b) The error covariance update:

$$P(k) = [I - K(k)H(k)]P^-(k), \quad (12)$$

© The estimate update with measurement  $z(k)$ :

$$\hat{s}(k) = \hat{s}^-(k) + K(k)[z(k) - \hat{z}(k)]. \quad (13)$$

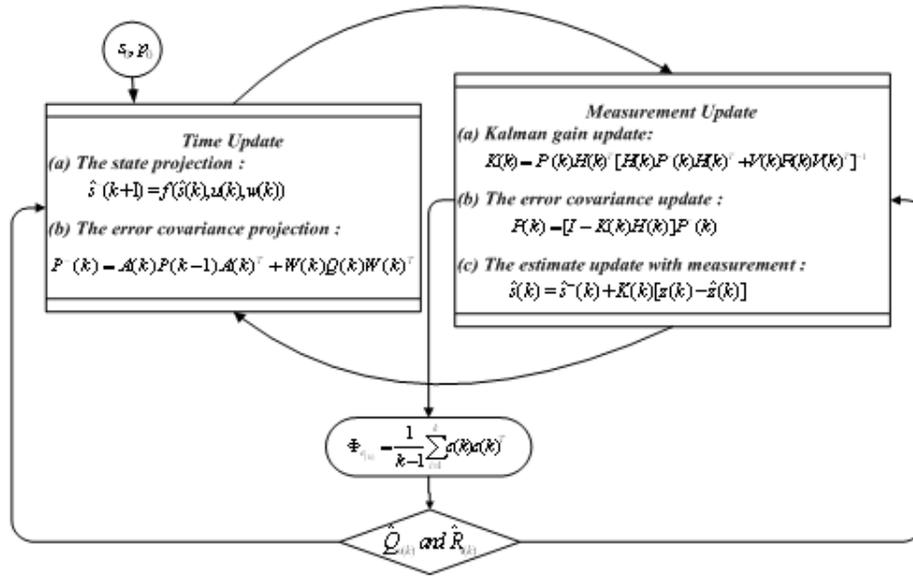
Here  $\hat{s}(k)$  is the predicted estimate of state at  $k$ -th step,  $\hat{s}^-(k)$  is a posteriori estimate of state at  $k$ -th step,  $u(k)$  is the control input with a velocity of a moving target,  $P^-(k)$  is the error covariance of  $\hat{s}^-(k)$ ,  $A(k)$  is the state transition matrix,  $K(k)$  is the gain matrix, and  $Q(k)$  and  $R(k)$  represent the process and measurement noise covariance, respectively.

The initial condition  $s_0$  is designed to be a zero-mean Gaussian random variable with a covariance  $p_0$  ( $p_0 > 0$ ), and  $w(k)$  and  $v(k)$  are independent zero-mean white Gaussian noises. We assume the process and measurement noise as,

$$\begin{aligned} E[w(i)w(k)^T] &= \begin{cases} Q(k), & i = k \\ 0, & i \neq k \end{cases} \\ E[v(i)v(k)^T] &= \begin{cases} R(k), & i = k \\ 0, & i \neq k \end{cases} \\ E[w(i)v(k)^T] &= 0, \text{ for all } k \text{ and } i. \end{aligned} \quad (14)$$

In Fig. 2, with initial values of  $s_0, p_0$  in time update process, we evaluate the estimate of system state by time flow and we calibrate the state estimation by comparing the differences between a real measurement and an estimate through a system modeling in measurement update process.

The implementation of Kalman filter requires a priori statistical knowledge of the process noise and measurement noise. Poor knowledge of the noise values may seriously degrade the function of Kalman filter and the divergence problem in the filtering process may happen. To



**Fig. 2. Operation of Adaptive Extended Kalman Filter**

fulfill the accuracy requirement, the adaptive Kalman filter can be utilized as the noise-adaptive filter to estimate the process and measurement noise covariance matrices  $Q(k)$ , and  $R(k)$ .

An innovation sequence utilizes the correlation and covariance matching techniques to estimate the noise covariance. Here, the maximum-likelihood estimation for the multivariate normal distribution approach is to make the actual value of the covariance consistent with its theoretical value. From the incoming measurement  $z(k)$  and the optimal prediction  $h(\hat{s}^-(k), u(k), v(k))$  obtained in the previous step, the innovation sequence is defined as  $e(k) = z(k) - \hat{z}(k)$ .

The innovation sequence can be written as:

$$e(k) \approx H(k)(s(k) - \hat{s}^-(k)) + v(k). \quad (15)$$

The covariance can be obtained by taking the variance on both sides of the Eq. (15).

$$\Phi_{e(k)} = H(k)P^-(k)H(k)^T + R_{v(k)}. \quad (16)$$

The covariance of  $e(k)$  is written as

$$\Phi_{e(k)} = E[e(k)e(k)^T]. \quad (17)$$

According the maximum-likelihood estimation for the multivariate normal distribution approach, the statistical sample variance of  $\Phi_{e(k)}$ :

$$\hat{\Phi}_{e(k)} = \frac{1}{k} \sum_{i=1}^k e(i)e(i)^T. \quad (18)$$

From Eq. (16), the estimate of the measurement noise covariance is as

$$\hat{R}_{v(k)} = \hat{\Phi}_{e(k)} - H(k)P^-(k)H(k)^T. \quad (19)$$

The estimate of the process noise covariance is

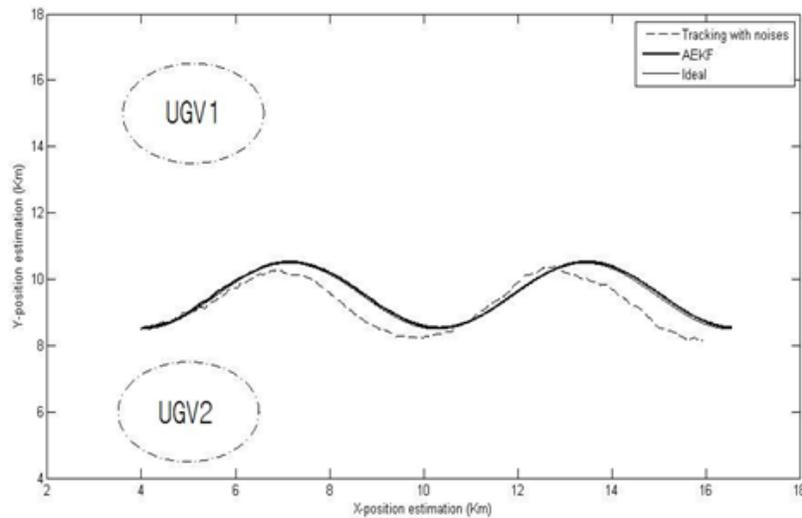
$$\begin{aligned} \hat{Q}_{w(k)} &= \frac{1}{k} \sum_{i=1}^k (s(k) - \hat{s}^-(k))(s(k) - \hat{s}^-(k))^T + P(k) - A(k)P(k-1)A(k)^T \\ &\approx K(k)\hat{\Phi}_{e(k)}K(k)^T. \end{aligned} \quad (20)$$

Two important values for the process and measurement noise covariance are modified adaptively by using the Eqs. (19) and (20)

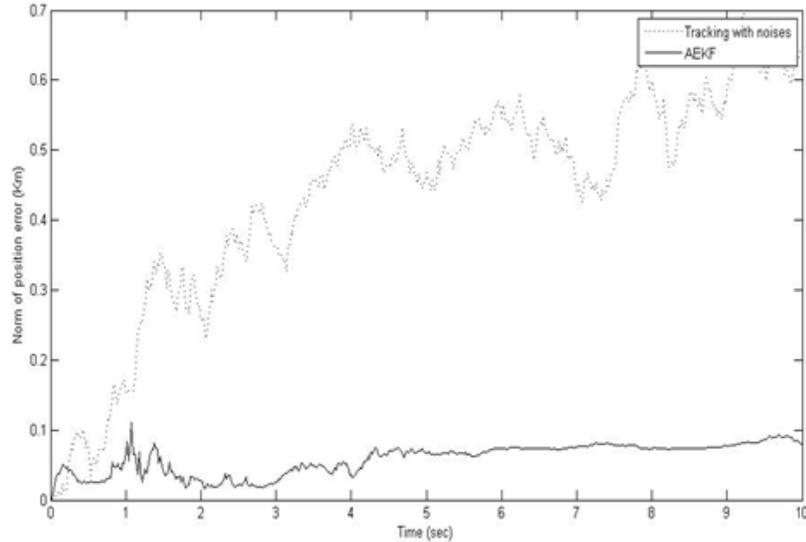
#### 4. Simulation results

In this section, through some simulation results, we demonstrate the effectiveness of the proposed tracking method using AEKF algorithm. The simulation conditions are as follows: the first receiver's sensor position is set as (8, 30) km, the initial velocity is set as (1, 0.9) km/s; the second receiver sensor is set as (70, 4) km, the velocity is set as (1, 0.5) km/s. The UGVs are also equipped with sensors as receivers. The positions of UGVs are set separately as (5, 6) km and (5, 15) km with a constant altitude and the UGV is moving following a circular orbit with a radius of 1 km.

Figure. 3 shows the position estimate of AEKF compared with the ideal position and the position tracking with noise. The ideal position without noise in Fig. 3 is indicated by the



**Fig. 3. Position Estimation of a Sinusoidal Movement**

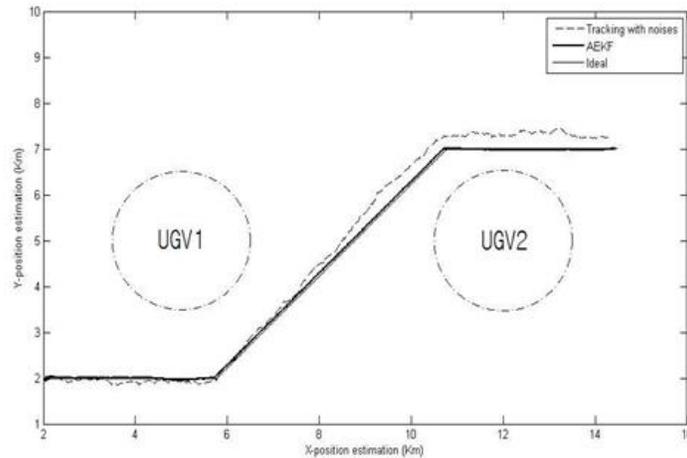


**Fig. 4. Norm of Position Error for Sinusoidal Movement**

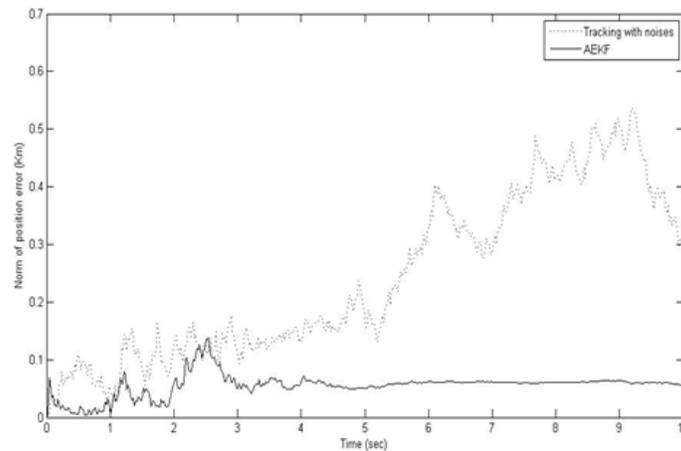
shallow solid line as a reference, the estimate of position tracking with noise is indicated by the long-dashed line. The solid bold line is the position estimate using the AEKF algorithm. It also confirms that the adaptive estimate algorithm updates the process and measurement noise covariance for each sampling time. The simulation proves that the AEKF method can track the target more accurately compared with the tracking result with noise. It also illustrates that the updated information on process and measurement noises plays an important role in locating the real position in localization problem.

Figure. 4 represents the norm of position error. The dotted line represents the norm of position error between the ideal position and the position tracking with error, the solid line means the norm of position error between the ideal value and the position estimate using the AEKF algorithm in Fig. 3. Fig. 4 shows that the position error of target is reduced when we use the AEKF algorithm and the proposed algorithm can estimate more closely to the real position with two moving UGVs' receivers.

Figure. 5 shows the simulation results with a different trajectory. The ideal position without noise is indicated by the solid line as reference for comparison and the position tracking with noise is indicated by the dashed line. The bold solid line is the AEKF algorithm estimate which updates the process and measurement noise covariance in the process. In the simulation, we use two UGV sensors as receivers with different location and constant velocity. Using the TDOA and FDOA measurement signals together to estimate the location and velocity of the target at different times, the simulation results prove that the AEKF method is effective to reduce the error and that the accuracy of the target tracking is increased. Figure. 6 represents the norm of position error. The norm of position error between the ideal position and position tracking with error is compared with that between the ideal position and position estimate using the AEKF algorithm. Fig. 6 shows that the position error with noise fluctuates widely more than that of AEKF. Through the AEKF algorithm, the position error is limited within the boundary of 0.15 km. The simulation result proves the effectiveness of the AEKF algorithm in target tracking.



**Fig. 5. Position Estimation of a Transitional Movement**



**Fig. 6. Norm of Position Error for a Transitional Movement**

## 5. Conclusions

This paper proposes an AEKF algorithm for tracking the target position using the TDOA and FDOA measurement signals together from the two UGVs' receivers and two general receivers in the modeling. The method that uses only TDOA measurement signals is not sufficient for estimating the velocity of the target. Therefore we use the TDOA and FDOA signals together to estimate the location and the velocity of the target at discrete times. Generally, EKF algorithm is easily and widely applied to the target tracking through linearization of the nonlinear process. However, the EKF algorithm uses a past static priori values as the process and measurement noise covariance during the update process. When the priori values are incorrectly estimated and when it deviates from the true values under the irregular environmental condition, it's difficult to track the accurate position of the target. We proposed the AEKF algorithm which updates the current process and the measurement noise covariance according to the changing environment at discrete times. Through the simulation results, the proposed estimate algorithm achieved more accurate target position and reduced the position error.

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