Design of Loop-Shaping and Internal Model Controller for Unstable and Communication Delay System

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Abstract

In this paper, we aim to stabilize the unstable system by loop shaping method with H∞ Controller. Moreover, after stabilizing the system, we also need to modify the system which is unstable due to time-delay elements by Internal Model Controller considering the uncertainty of control object and time-delay elements. Time-delay will happen during long distance communication. Therefore, when we have a control object in a long distance, the transmitted reference signal will certainly be delayed. For this reason, even though we stabilize the control object, it will be an unstable system by time-delay elements. In this paper, we consider an unstable control object, such as, ballistic missile. Generally, it is required to be controlled from a long distance. Therefore, we stabilize the control object with H∞ controller and modify the system later .It is unstable due to time-delay elements by Internal Model Controller.

Keywords: *H_∞ Controller, H_2 Controller, Time-Delay, Robust Control, Internal Model Controller*

1. Introduction

In this research, we propose control of the time-delay system by using H_∞ Controller and IMC (Internal Model Controller). Until now, there were many schemes of the designing controller for systems which include time-delay elements. For example, the classical way is PID [6] (Proportional-Integral-Derivative). However, this scheme does not guarantee the stability for large time-delay. On the other hand, LQI (Linear Quadratic Integration) [1], [2], [3], [4] method is a modern control scheme and it guarantees the stability even for a large time-delay. However, for MIMO system, it is very complicated to solve the Ricatti equation [5], [6], [7] and to design the optimum controller. Therefore, we consider IMC method [8], [9], [10] to control the MIMO system including time-delay elements. Time-delay will happen during utilization of long distance communication. The application of long distance communication is an important issue in aerospace engineering. When we have a control object in long distance, the transmitter's signal will be delayed. Therefore, the signal received at the control object will also be delayed. Moreover, the feedback signal to transmitter location will also be delayed.

So, in this case, we have a round trip delay. One delay is to reach the control object and another delay is to receive the feedback signal for comparison with the reference signal. Moreover, the control object of the system is usually an unstable system. Therefore, we have to consider the stabilization of the control object. In this paper, we propose the loop shaping method with H_∞ controller. By loop shaping method, we can stabilize the control object. Though, the control object will be unstable because of time-delay elements. Therefore, after stabilizing the control object, we use IMC method to improve tracking characteristics which were worsened because of time-delay elements. Also, we considered the uncertainty of the control object of time-delay elements which are estimated and approximated by using Pade approximation. Here, loop shaping method considers stability, robustness and performance. Therefore, the method forms a desirable loop shape of singular values of open loop. IMC method minimizes the amplitude of external disturbance of output signal by H_2 norm. Therefore, by this design, problems of the system instability and uncertainty have been overcome. Then, they can be solved by one of the robust controllers such as loop shape and IMC method.

2. The Theory of Time-Delay System and Background of Research

In this research, we consider a round trip time-delay system. One delay element which reaches the control object and another delay element which feedbacks the output signal in order to compare with reference signal. Figure 1 shows the block diagram of a round trip time-delay system without controller.

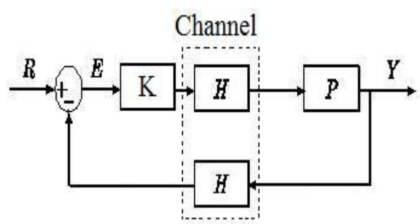

Fig. 1: Time-delay system

Here “Y”,”H”,”R”,”P” and “E” are the output signal of the system, time-delay element, reference signal, control object and error signal of the system, respectively. Through Figure 1, it is clear that the sensitivity function(S) and transfer function (T) are obtained as follows:

$$S = K^{-1}H^{-1}P^{-1}(I + KPH^2)^{-1}PHK \qquad (1)$$

$$T = (I + KPH^2)^{-1}PH \qquad (2)$$

Generally, in feedback control system, by adding controller “K” which is designed corresponding to control object, we can stabilize the system and decrease errors. In classical control, PID is used and in modern control, LQI method is used. In both of them, integrator gain is optimized. As a result, for PID controller, if time-delay is large, the system could not preserve stability. But for modern LQI method, we could stabilize the system without error. However, for high dimension and MIMO system, we could not design the optimum controller because of complexity of solving the Ricatti equation.

In this research, the system is already unstable and contains time-delay elements. Therefore, we propose to integrate the IMC method and loop shaping method with H_∞ controller into hybrid system controllers. At first, loop shaping method with H_∞ controller is designed corresponding to control object to stabilize it. Secondly, Internal Model Controller is designed corresponding to time-delay elements and uncertainty of control object. As a result, the estimated time-delay elements are determined after stabilization.

3. Loop Shaping Method with H∞ Controller

At first, it is required to stabilize the control object for the process of stabilizing the closed loop system. Therefore, in this case, we design H∞ controller by loop shaping method. For loop shaping method, we designed a stabilizing feedback controller to optimally shape the open loop frequency response of a MIMO feedback control system to match a desired loop shape. In this case, controller has the property to shape the open loop so that it matches the frequency of desired loop shape. Figure 2 shows the partition of control object which has state feedback control.

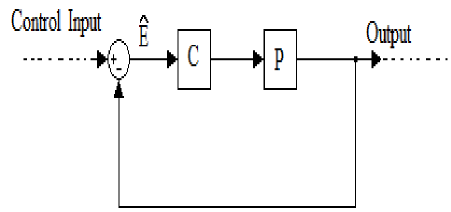

Fig.2: Block Diagram of State Feedback Control

Here "C" is H∞ Controller, "E" is error of system and "P" is control object. Figure 4 shows the singular value specifications on open loop ($\hat{L}$), sensitivity ($\hat{S}$), and closed loop ($\hat{T}$).

$$\hat{L} = PC \quad (3)$$

$$\hat{S} = (I + \hat{L})^{-1} \quad (4)$$

$$\hat{T} = (I + \hat{L})^{-1}\hat{L} \quad (5)$$

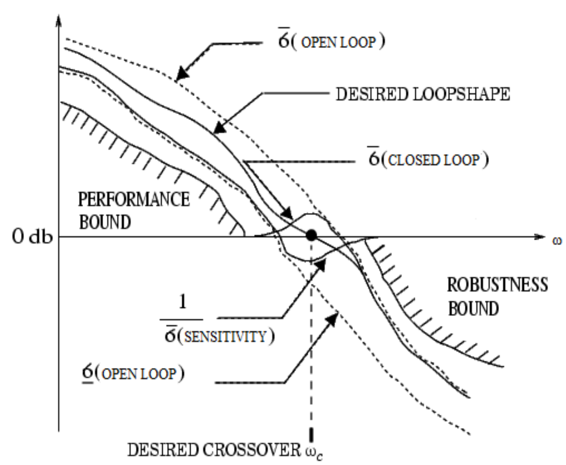

Fig.3: The Singular Value Specifications on Open Loop, Sensitivity, and Closed Loop

Figure 3 shows that the desired loop shape has high gain in low frequency and small gain in high frequency. Consequently, if open loop has high gain in low frequency, the output signal has better performance. If it has low gain in high frequency, it is robust to noise and uncertainty.

4. Internal Model Controller for Time-Delay System

Internal Model Controller is an optimum controller which minimizes the effect of disturbance to output signal and considers the uncertainty of control object. Also, in this research, we consider the existence of time-delay elements. Hence, most systems would be unstable due to time-delay elements. Consequently, we introduce the IMC method to modify the stability of the system and compensate the output signal. The main reason why we adopt the IMC method is due to consideration of the uncertainty of control object and time-delay elements. The effects of disturbance to output signal are minimized and it can correspond to MIMO (Multi Input and Multi Output) and high dimensional systems. In order to modify the unstable system due to time-delay elements, we have designed the internal model controller. Figure 4 shows the IMC system which "D" is external disturbance to output signal, "K" is Internal Model Controller. Furthermore, "P" and "H" are actual system and time-delay elements, "P ","H ", "C " are model system, approximated and predicted time-delay elements, model of loop shape controller, respectively.

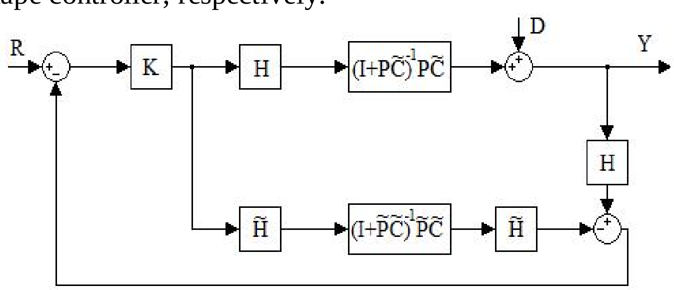

Fig.4: Internal Model Controller Including a Round Trip Time-Delay Elements and External Disturbance

Figure 4 shows the block diagram of IMC method. From this, we obtained relation between output and reference signal as follows:

$$Y = (I + P\tilde{C})^{-1} P\tilde{C}HM^{-1}KR + [I - (I + P\tilde{C})^{-1} P\tilde{C}HM^{-1}KH]D \qquad (6)$$

Where, $M = I + K\Delta P$ and $\Delta P = H(I + P\tilde{C})^{-1} P\tilde{C}H - \tilde{H}(I + P\tilde{C})^{-1}\tilde{P}\tilde{C}\tilde{H}$ which is uncertainty of system.

Since, in the above equation (6) "D" multiplied by $I - (I + P\tilde{C})^{-1} P\tilde{C}HM^{-1}KH$, IMC method minimizes the effect of disturbance. As a result we consider the minimization of H^2 norm of this coefficient. Equation (7), below, shows how to derive "K" which is Internal Model Controller:

$$\min_K \left\| I - (I + P\tilde{C})^{-1} P\tilde{C}HM^{-1}KH \right\|_2 \qquad (7)$$

that is:

$$\frac{\partial}{\partial K}\sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}[I - (I + P\tilde{C})^{-1} P\tilde{C}HM^{-1}KH]^{*}[I - (I + P\tilde{C})^{-1} P\tilde{C}HM^{-1}KH]d\omega}$$

Theoretically, the optimum Controller K in equation (8) is calculated as follows:

$$K = \tilde{C}^{-1}\tilde{P}^{-1}(I + \tilde{P}\tilde{C}) \tag{8}$$

where, "K" is the inverse system of model of control object. It is significant to note that to realize the controller, when K is an improper system, it is required to multiply a filter the same as K's dimension to make proper or strictly proper.

5. Uncertainty of Control Object and Time-Delay Elements

In IMC, we use the model of control object and the real control object which is an unknown system. As we obtained through equation (7) when K is the inverse system of model, it is the optimum case. But, if the model system does not completely match with the real system, errors will occur. Consequently, the ideal case is when M equals unit matrix, that is: when $\Delta P=0$ then, $M=I$.

In this case, the system is called nominal system which is ideal. Although, in the case of internal model controller, except the model system, we have to realize the predicted time-delay elements. Therefore, we used the Pade approximation for "L" approximated time-delay. Time-delay elements" $\tilde{H}$ " is shown as follows:

$$\tilde{H} = e^{-s\tilde{L}} \approx Pade(\tilde{L}, n) = \frac{\sum_{k=0}^{n}(-1)^k c_k \tilde{L}^k s^k}{c_k \tilde{L}^k s^k} \tag{9}$$

Where, $c_k = \dfrac{(2n-k)!}{2n!k!(n-k!)} \quad (k = 0,1,2,\ldots,n)$

Here we consider this approximated time-delay element as a system matrix.

$$\tilde{H} = \left[\begin{array}{c|c} A_{\tilde{H}} & B_{\tilde{H}} \\ \hline C_{\tilde{H}} & D_{\tilde{H}} \end{array}\right] \tag{10}$$

The key is that the dimension "n" of approximated matrix must be the same as control object. Here is the example of realization of Pade approximation for a 1 second time-delay with n=2.

$$\tilde{H} \approx Pade(1,2) = \frac{\sum_{k=0}^{2}(-1)^k c_k 1^k s^k}{c_k 1^k s^k} = \frac{s^2 - 6s + 12}{s^2 + 6s + 12}$$

$$\rightarrow \begin{cases} \dot{x}_{\tilde{H}} = A_{\tilde{H}} x_{\tilde{H}} + b_{\tilde{H}} u_{\tilde{H}} \\ y_{\tilde{H}} = c_{\tilde{H}} x_{\tilde{H}} + d_{\tilde{H}} u_{\tilde{H}} \end{cases}$$

where, $A_{\tilde{H}} = \begin{bmatrix} -6 & -3 \\ 4 & 0 \end{bmatrix}$, $b_{\tilde{H}} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$, $c_{\tilde{H}} = \begin{bmatrix} -3 & 0 \end{bmatrix}$ and $d_{\tilde{H}} = 1$

As it can be seen Pade approximated time-delay element is the same as a non-minimum phase filter.

6. Simulation and Results

For evaluating loop shape method, we will simulate an unstable system which is ballistic missile. Generally, for missiles, it is required to control from long distance. Therefore, it certainly has time-delay. The feedback signal will also be delayed. So, it can be applied to loop shape method. Figure 5 shows the control object attitude which is controlled by 2 inputs ε and μ.

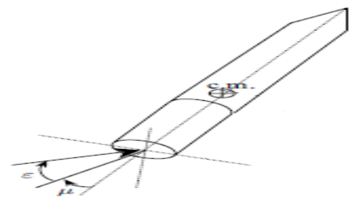

Fig. 5: Missile Attitude Control

Missile is an unstable system which has a round trip delay. Firstly, we will stabilize the control object by loop shaping method. Then, we will design a controller which is IMC method and corresponds to time-delay elements.

The dynamics of the missile is given by the following equations.

The real system:

$$\begin{cases} \dot{x} = Ax + Bu(t-L) \\ y = Cx \end{cases} \tag{11}$$

where L is time-delay and $u = \begin{bmatrix} \varepsilon \\ \mu \end{bmatrix}$

The actual system matrix is:

$$P = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

$$= \left[\begin{array}{cccccc|cc} -0.0675 & 0 & 0 & 0 & 2.3694 & 0 & 7.0084 & 0 \\ 0 & -0.0675 & 0 & 0 & 0 & -2.3694 & 0 & -7.0084 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.7285 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -0.0233 & 0 & -0.0353 & 0 & 0.0522 & 0 \\ 0 & -1 & 0 & 0.0233 & 0 & -0.0353 & 0 & 0.0522 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Actual time-delay: L= 5 seconds

The model system is:

$$\begin{cases} \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u(t-\tilde{L}) \\ \tilde{y} = \tilde{C}\tilde{x} \end{cases} \quad (12)$$

where $\tilde{A} = A + \Delta A$, $\tilde{B} = B + \Delta B$, $\tilde{C} = C + \Delta C$ and $\Delta L = |L - \tilde{L}|$

Predicted time-delay: $\tilde{L} = 1\text{second}$

$\Delta A, \Delta B, \Delta C$ are uncertainty matrices and $\tilde{L}$ is predicted time- delay.

The model system matrix and approximated time-delay are:

$\tilde{P} = \left[\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline \tilde{C} & \tilde{D} \end{array}\right]$ and $\tilde{H} = Pade(\tilde{L}, Dimesion(A))$.

Norm of uncertainty matrices: $\|\Delta A\| = 0.0033$, $\|\Delta B\| = \|\Delta C\| = 0.00$

Now we consider the process of controller designing with loop shape and then IMC. For loop shape first we choose the desired loop Gd which is:

$$G_d = \frac{100}{s} \quad (13)$$

Then design the H^∞ controller corresponding to model system. Finally designing the IMC "K" and obtain the equation (8). Here for making the "K" proper we have to add a filter which in this case we set to as follows:

$$F(s) = \frac{1}{s^4 + 4\Delta L s^3 + 6\Delta L s^2 + 4\Delta L s + 1} I_{2\times 2} \quad (14)$$

Results of Simulation

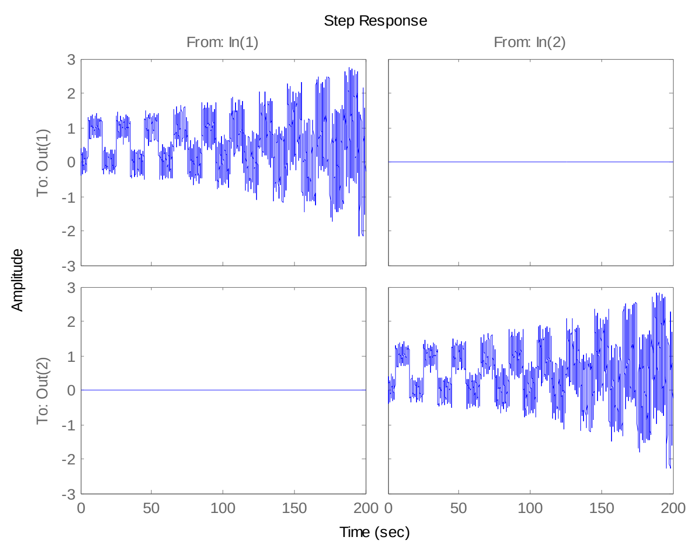

Fig.6: Step Response of Figure 1

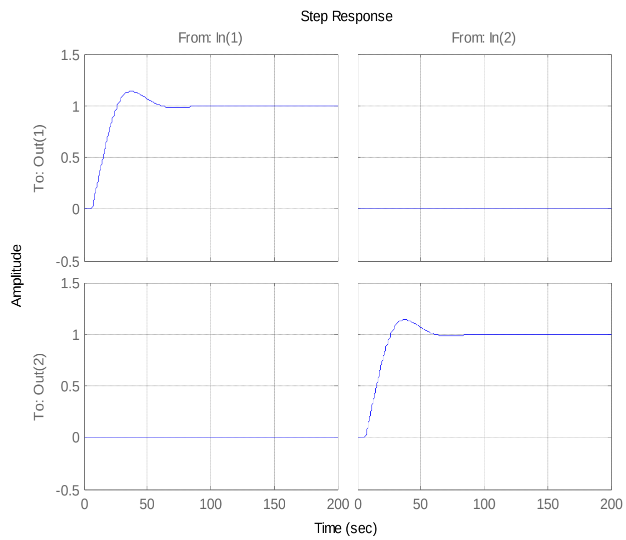

Fig.7: Step Response of Figure 4

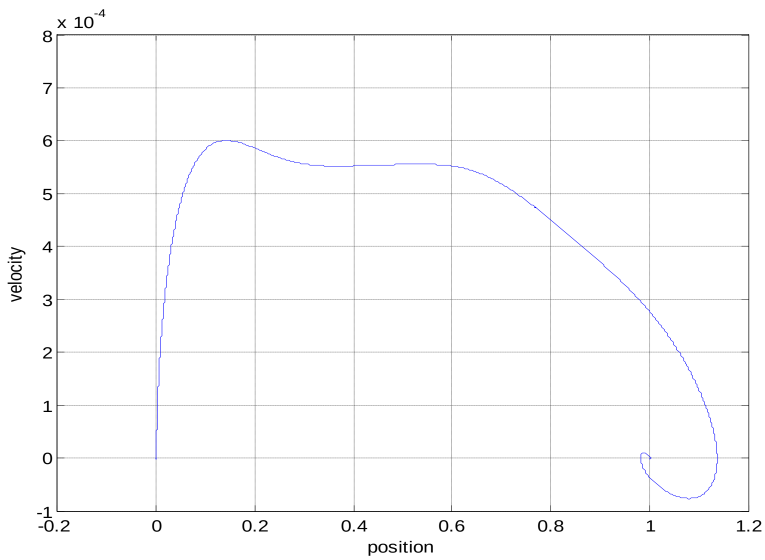

Fig. 8: Stability of System

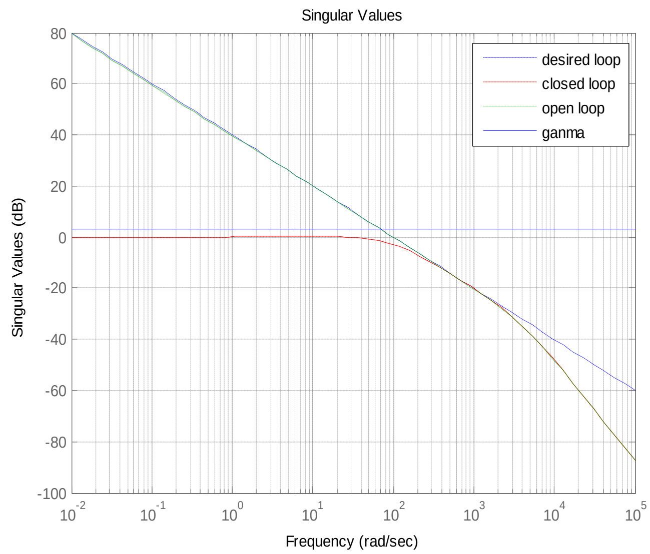

Fig.9: Singular Values of Actual Part of System with Controller and Desired Loop

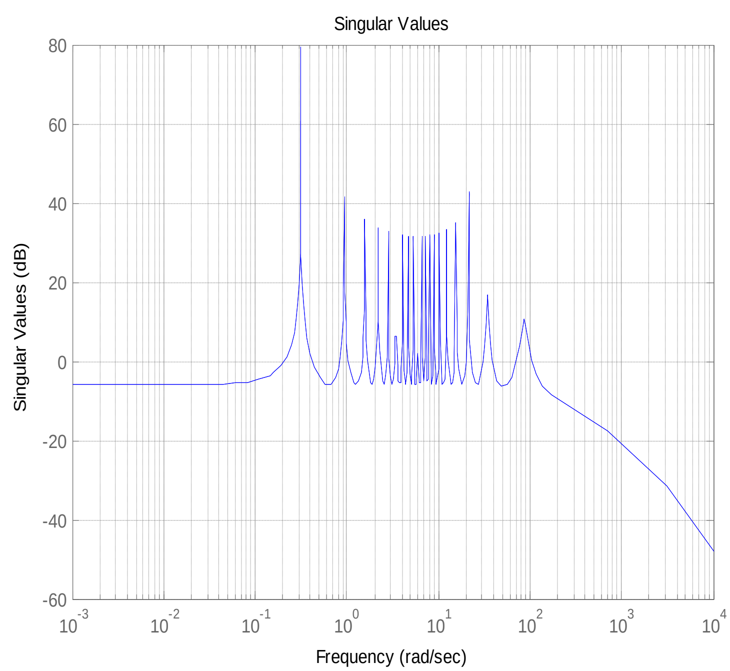

Fig.10: Singular Value of Closed Loop Figure 1

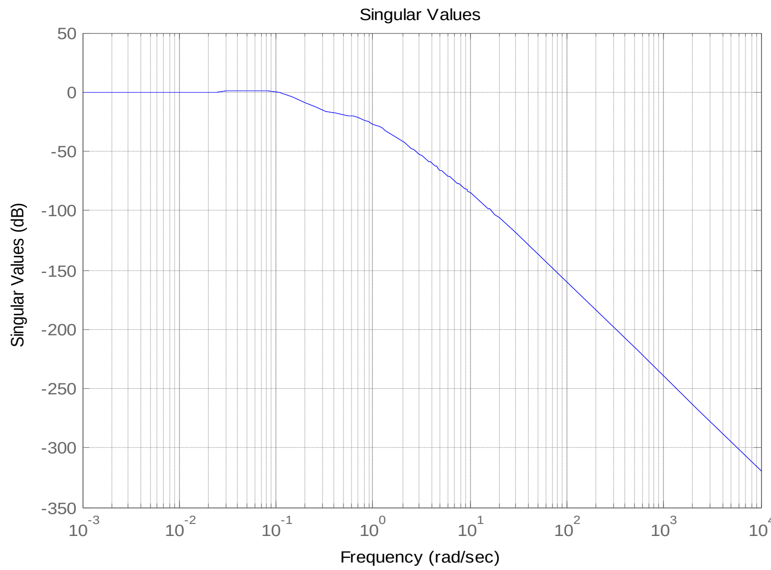

Fig. 11: Singular Value of Closed Loop Figure4

As we can see through figure 6 the step response of the system with time-delay is unstable and in figure 10 we can confirm that the singular value of closed loop in this system has many peak frequencies, so it is predictable that this system is unstable. By using the IMC-Loop shape method we could stabilize the step response as we can see in figure 7. Also in figure 8 we can confirm that the stability of the system, it converges to point (1, 0) means that the steady value of position converge to reference signal and velocity converge to zero. In Figure 9 the open loop shape and closed loop shape of actual part of control object, they mach to desired loop in high frequency so this mean the system is robust . Finally in figure 11 shows the singular value of IMC-loop shape closed loop, comparing to figure 10, it has no more peak and also it has low gain in high frequencies.

7. Conclusion

In this research we used loop shape controller to stabilize the control object and compensate the system with internal model controller which was unstable due to time-delay element. So the controller of this system is hybrid of loop-shape controller and IMC. The performance of output signal is depended on the filter which make IMC's controller proper. In this research we consider that the output signal reach to reference signal as soon as possible and little over shoot is permitted. Therefore we chose the filter equation. As a future work we are going to design others scheme designing of H^{∞} Controller and realize the hybrid controller such as IMC- H^{∞} controller in actual system.

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