

# Design of Fractional Order Controller Based on Evolutionary Algorithm for a Full Vehicle Nonlinear Active Suspension Systems

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## Abstract

*An optimal Fractional Order  $PI^\lambda D^\mu$  (FOPID) controller is designed for a full vehicle nonlinear active suspension system. The optimal values of FOPID controller parameters for minimizing the cost function are tuned using an Evolutionary Algorithm (EA), which offers an optimal solution to a multidimensional rough objective function. The fitness parameters of FOPID controller (proportional constant  $P$ , integral constant  $I$ , derivative constant  $D$ , integral order  $\lambda$  and derivative order  $\mu$ ) are selected from ranges of reliable values, depending on survival-to-the-fitness principle used in the biology science. A full vehicle nonlinear active suspension model including hydraulic actuators, nonlinear dampers and nonlinear springs has been proposed with structural and analytical details. The nonlinear frictional forces due to rubbing of piston seals with the cylinders wall inside the actuators are taken into account to find the real supply forces generated by the hydraulic actuator. The results of the full vehicle nonlinear suspension system using the FOPID controller are compared with the corresponding passive suspension system (system without controller). The controlled suspension system has been investigated under typical vehicle maneuvers: cruising on rough road surface, sharp braking and cornering. The results have clearly shown the effectiveness and robustness of the proposed controller.*

**Keywords:** Fractional Order  $PI^\lambda D^\mu$  Controller, Evolutionary Algorithm, Nonlinear Active Suspension Systems.

## 1. Introduction

In recent years, several models and controllers have been developed in attempts to enhance the riding and handling qualities in modern vehicles. Fuzzy logic control and adaptive fuzzy control were developed for the closed loop system of the quarter vehicle active suspension system [1]. A neuro-fuzzy controller was designed to improve damping and riding quality of a semi-active quarter vehicle model [2, 3]. A genetic algorithm based fuzzy PI/PD controller was designed for the quarter vehicle model in the Reference [4] to improve the riding comfort and holding ability under different road condition. In the Reference[5] a fuzzy control method based on adaptive technology was applied to active control of quarter automotive suspension system. Aiming at developing an active suspension for the quarter car model of a passenger car, a genetic algorithm based proportional derivative controller was developed to improve its performance [6]. A half vehicle model was used to design a fuzzy logic controller to isolate the vibration on rough road [7]. A combination of genetic algorithm based self-tuning PID controller and fuzzy logic controller were used in the Reference [8] to design a robust controller for a half car active suspension system. In all those publications, a

limitation is obvious that with a quarter or half vehicle linear model used to design the control system, it can only indicate the vehicle body vertical movement or/and the pitching movement, not include the rolling movement of the vehicle body. There are several researchers who have made progress by using a full vehicle model to design the control system to take into account the rolling movement. An active suspension control approach combining a filtered feedback controller scheme and input decoupling transformation was introduced to a full vehicle suspension system [9]. A feedback controller is designed to control separately the vehicle body motions of heave, pitch and roll taking into account the dynamics of the active and semi-active actuator [10]. Due to the complex mathematical relationships in nonlinear active suspension system, most of those researchers approximate the active suspension system as a linear system when designing the controller.

Since the nonlinearity and uncertainties inherently exist in suspension system, the nonlinear effect has to be taken into account and the nonlinear active suspension control law should be used. The PID (proportional integral derivative) controllers, which have been dominating industrial controllers, have been modified by the author using the notion of a fractional-order integrator and differentiator. It has been shown that two extra degrees of freedom from the use of a fractional-order integrator and differentiator make it possible to further improve the performance of traditional PID controllers. The ordinary PI, PD, and PID controllers are regarded as special cases of the fractional-order  $PI^\lambda D^\mu$  (FOPID) controller since the values of  $\lambda$  and  $\mu$  can be selected freely, which adds two more degree of freedom to the controller design. Classical PID type control design techniques (such as empirical Ziegler–Nichols, Chien–Hrones–Reswick formula, Cohen–Coon formula, refined Ziegler–Nichols tuning and Wang–Juang–Chan formula) cannot be used here because of the roughness of the objective function surface.

Several methods have been introduced to tune the parameters of FOPID controller. Reference [11] suggested a state-space design method based on feedback poles placement to design the FOPID controller. A frequency domain approach based on the expected crossover frequency and phase margin is proposed in Reference [12]. A frequency-band broken–line approximation method is suggested to tune the parameters of fractional-order  $PI^\lambda D^\mu$  controller in Reference [13]. The Particle Swarm Optimization Technique is suggested to design the FOPID controller in Reference [14].

In this paper, the parameters of optimal FOPID controller will be tuned by using EA to control of full vehicle nonlinear active suspension system. The novelty of the proposed method is to design FOPID controller by using EA for full vehicle nonlinear active suspension system and implementing this controller as robust controller even when significant disturbances occurred. An evolutionary algorithm (EA) is a modern technique for searching complex spaces for an optimum. This algorithm operates on a population of potential solutions applying the principle of survival of the fittest to produce better and better approximations to a solution. In this research EA has been used for tuning the optimal values of  $K_p$ ,  $K_i$ ,  $K_d$ ,  $\lambda$  and  $\mu$ . The proposed FOPID controller has been designed for full vehicle nonlinear active suspension model including hydraulic actuators in order to improve the ride comfort and road handling during various maneuvers: traveling, braking and cornering. Two types of disturbances have been applied to examine the robustness of the proposed controller: change of the amplitude of sine shaped road profile input and change of the amplitude of square shaped road profile input. The results will show how the robustness of proposed controller has been achieved.

## 2. Fractional order system

Fractional calculus is a field of mathematical analysis which studies the ability of taking real number power of the differential operator and integration operator. There are many definitions used to describe the fractional order function. The well established definitions include the Cauchy integral formula, the Grünwald–Letnikov definition, the Riemann–Liouville definition, and the Caputo definition. The Riemann–Liouville definition is the most frequently used definition in fractional-order calculus, in which the fractional order integration is defined as [13]

$${}_a D_t^{-\varphi} f(t) = \frac{1}{\Gamma(\varphi)} \int_a^t (t-\tau)^{\varphi-1} f(\tau) d\tau \quad (1)$$

where  $\varphi$  represents the real order of the differintegral ( $0 < \varphi < 1$ );  $a$  is the initial time instance, often assumed to be zero; and  $t$  is the parameter for which the differintegral is taken. The Laplace transform of the fractional derivative of  $f(t)$  is given by:

$$L[D_t^\varphi f(t)] = s^\varphi L[f(t)] - \sum_{k=1}^n s^k [D_t^{\varphi-k-1} f(t)]_{t=0} \quad (2)$$

If the derivatives of the function  $f(t)$  are all equal to 0 at  $t=0$ , the second part on right hand side of Equation 2 can be omitted. Therefore, Equation 2 can be rewritten as

$$L[D_t^\varphi f(t)] = s^\varphi F(s) \quad (3)$$

where  $F(s)$  is the Laplace transform of  $f(t)$ .

## 3. Fractional-order $PI^\lambda D^\mu$ controller

Fractional order differential equation is used to describe the fractional order  $PI^\lambda D^\mu$  controller. In PID controller case, three parameters  $K_p$ ,  $K_d$  and  $K_i$  should be tuned to design the controller. One of the possibilities to improve PID controllers is to use fractional-order controllers with real order of derivative and integral.

The differential equation of fractional order controller is described as [15]

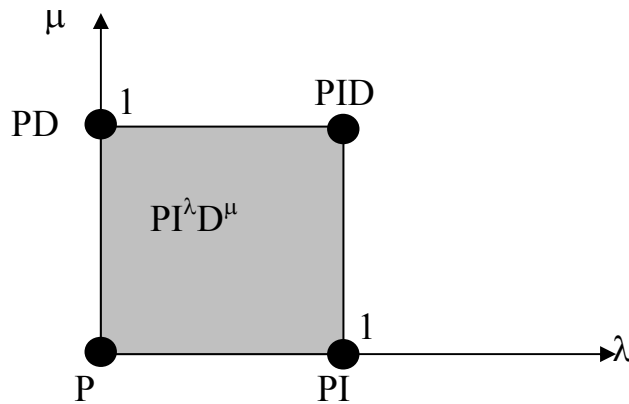
$$u(t) = K_p e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^\mu e(t) \quad (4)$$

where  $e(t)$  is the error between a measured process output variable and a desired set point and  $u(t)$  is the control output. Eq. (4) shows that the FOPID controller needs to tune five parameters  $K_p$ ,  $K_i$ ,  $K_d$ ,  $\lambda$  and  $\mu$ . Therefore, the integral order and derivative order add more flexibility to design an FOPID controller.

The continuous transfer function of FOPID is given by

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu \quad (5)$$

From Eq. 5 the PID controller can be obtained by setting  $\lambda=\mu=1$ . When  $\lambda=1$  (or 0) and  $\mu=0$  (or 1) a normal PI (or PD) controller can be obtained. Fig.1 shows that the fractional order  $PI^\lambda D^\mu$  controller generalizes the integer order PID controller and expands it from a point to a plane. This expansion gives the designer more flexibility to design an accurate controller.



**Figure 1. Generalized FOPID controller**

#### 4. Evolutionary algorithms

The Evolutionary Algorithm (EA) is an optimization algorithm used to search for optimal solutions to a problem [16]. It shares the same features with the genetic algorithm (GA) in terms of selection method, crossover strategy, mutation scheme and reinsertion policy. The main difference between evolutionary algorithm and genetic algorithm lies in the representation of the individuals solutions (population). In evolutionary algorithm, the individuals represent as real numbers, whereas genetic algorithm encodes the values of individuals to generate the new population.

Fig. 2 depicts the structure of a generic evolutionary algorithm. At the beginning of the computation a number of the individuals (each individual is D-dimension parameter vector) representing the candidate solutions are randomly initialized. Those candidate solutions represent the current population  $P(t)$  (first generation). The objective function is then evaluated for these individuals to obtain the fitness for each individual. If the optimization criteria are not met, the creation of new generation will start. The individuals that have high fitness are more likely to be selected as a parent to generate the offspring for a new generation. There are various selection methods used to select the beset parents from the population for crossing, such as Roulette wheel selection, Rank selection, Tournament selection and Stochastic universal sampling. Best parents are recombined to produce offspring. This process is called Crossover. Crossover operator is applied to the mating pool with the hope that it creates a better offspring. There are various crossover techniques, such as Single point crossover, Two points crossover, Multi-point crossover, Uniform crossover and three parent crossover. After a crossover, offspring will be subject to mutation. The mutation randomly reforms an offspring to generate new variants. The fitness of the offspring is then computed. By inserting the offspring into the population, a new generation  $P(t+1)$  is created. Different schemes of global reinsertion exist: a) replace all parents by the offspring (pure reinsertion); b) replace parents uniformly at random (uniform reinsertion) and c) replace the worst parents (elitist reinsertion). The cycle is performed until the maximum number of generations elapse or a desired level of fitness is reached.

#### 5. Nonlinear active suspension model

A framework is being suggested in which an active suspension controller generates the suitable command signals (inputs of the hydraulic actuators) to improve the vehicle

performance including riding comfort and road handling stability. The rigid comfort can be measured by evaluating the acceleration and displacement of the sprung mass. The handling stability can be obtained by minimizing the vertical motion of tires and the rotational motions of the vehicle body such as rolling and pitching movements during sharp manoeuvres of cornering and braking.

The full vehicle active suspension physical model is proposed and shown in Fig. 3. This model consists of five parts: the vehicle body mass ( $M$ ) and four unsprung masses  $m_i$  (where  $i \in [1,2,3,4]$ ). The vehicle body mass is assumed as rigid body and has freedom of motion in vertical, pitch and roll direction. The vertical displacements at each suspension point are denoted by  $z_1, z_2, z_3$  and  $z_4$ . The  $z_c, \alpha$  and  $\eta$  denote the displacement at the center of gravity of the vehicle, pitch angle and roll angle, respectively. The vertical displacements of unsprung masses are denoted by  $w_1, w_2, w_3$  and  $w_4$ .  $J_x$  and  $J_y$  are the moments of inertia about x-axis and y-axis, respectively. The cornering torque and braking torque are denoted by  $T_x$  and  $T_y$ , respectively. In the model, the disturbances  $u_1, u_2, u_3$  and  $u_4$  are caused by road roughness.

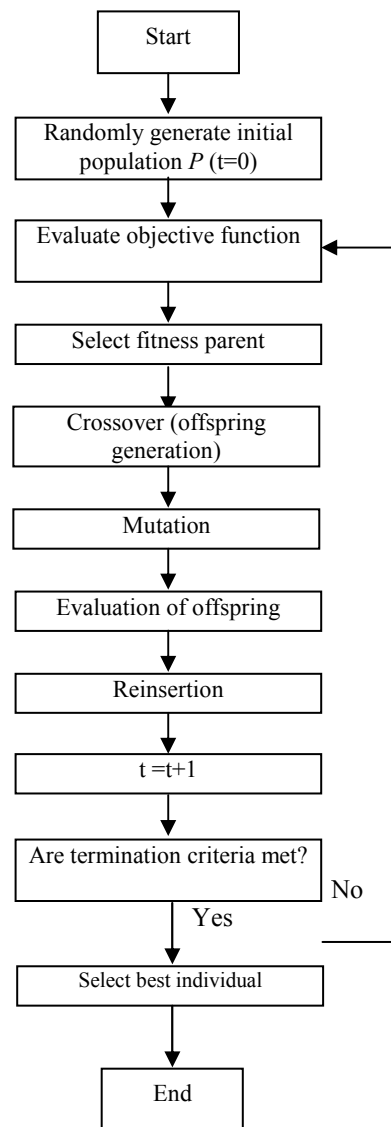
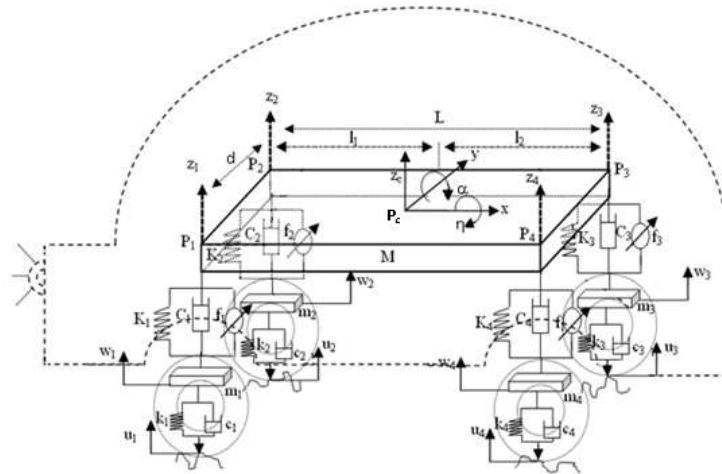


Figure 2. Structure of an evolutionary algorithm



**Figure 3. Full Vehicle Nonlinear Active Suspension System**

### 5.1. Nonlinear force characteristics

The suspension elements possess a nonlinear property. Therefore, each suspension will be assumed as specific nonlinear components placed in parallel (nonlinear spring model, nonlinear damper model and nonlinear hydraulic actuator). The main purpose of using the suspensions control is to generate a control force between the vehicle body mass and unsprung masses. The  $i^{th}$  nonlinear suspension has stiffness and damping coefficient denoted by  $K_i$  and  $C_i$ , respectively. Each tire will be simulated as linear oscillator with stiffness and damping coefficient denoted by  $k_i$  and  $c_i$ , respectively.

The motions of the vehicle body mass are governed by the following equations:

#### I. Vertical motion

Using Newton's Second law of motion

$$M\ddot{z}_c = -\sum_{i=1}^4 F_{Ki} - \sum_{i=1}^4 F_{Ci} + \sum_{i=1}^4 F_{Pi} \quad (6)$$

where  $F_{Ki}$  nonlinear suspension spring forces which can be written as [17]:

$$F_{Ki} = K_i(z_i - w_i) + \xi K_i(z_i - w_i)^3$$

$F_{Ci}$  nonlinear suspension damping force which can be written as [17]:

$$F_{ci} = C_i(\dot{z}_i - \dot{w}_i) + \xi C_i(\dot{z}_i - \dot{w}_i)^2 \text{sgn}(\dot{z}_i - \dot{w}_i)$$

$$F_{Pi} = F_{Ai} - F_{fi}$$

where  $F_{Ai}$  is the nonlinear hydraulic force provided by the  $i^{th}$  actuator and  $F_{fi}$  the nonlinear frictional force due to rubbing of piston seals with the cylinders wall inside the  $i^{th}$  actuator.

#### II. Pitching motion

Using Newton's law again

$$\begin{aligned}
 J_x \ddot{\alpha} = & (F_{K1} - F_{K2} - F_{K3} + F_{K4}) \frac{b}{2} + \\
 & (F_{C1} - F_{C2} - F_{C3} + F_{C4}) \frac{b}{2} + \\
 & (F_{P4} - F_{P1} + F_{P3} - F_{P2}) \frac{b}{2} + T_x
 \end{aligned} \quad (7)$$

where  $b$  is the distance between the front wheels (or rare wheels).

### III. Rolling motion

$$\begin{aligned}
 J_y \ddot{\eta} = & (F_{K3} + F_{K4})l_2 - (F_{K1} + F_{K2})l_1 + \\
 & (F_{C3} + F_{C4})l_2 - (F_{C1} + F_{C2})l_1 + \\
 & (F_{P1} + F_{P2})l_1 + (F_{P3} + F_{P4})l_2 + T_y
 \end{aligned} \quad (8)$$

where  $l_1$  is the distance between the centre of front wheel axle and centre of gravity of the vehicle.  $l_2$  is the distance between the centre of gravity of the vehicle and the centre of rare wheel axle.

The motion of the  $i^{th}$  unsprung mass is governed by the following equation:

$$\begin{aligned}
 m_i \ddot{w}_i = & -k_i(w_i - u_i) - c_i(\dot{w}_i - \dot{u}_i) + \\
 & F_{Ki} + F_{Ci} - F_{Pi}
 \end{aligned} \quad (9)$$

### 5.2 Hydraulic actuator dynamics

The nonlinear force produced by the active hydraulic actuator is applied between body and wheel axles. This force is governed by the following equation [18]

$$F_{Ai} = A_p P_{Li} \quad (10)$$

where  $A_p$  the cross section area of the piston inside the  $i^{th}$  actuator,  $P_{Li}$  the hydraulic pressure inside the  $i^{th}$  actuator.

The nonlinear pressure is given by:

$$\dot{P}_{Li} = -\beta P_{Li} - \sigma(A_p \dot{x}_{pi} - Q_i) \quad (11)$$

where  $\sigma = \frac{4\beta_e}{V_t}$ ,  $\beta = \sigma C_{tp}$  and  $x_{pi} = z_i - w_i$

$\beta_e$  the effective bulk modulus of hydraulic system.

$V_t$  the total volume of fluid under compression.

$C_{tp}$  leakage coefficient of piston.

$Q_i$  hydraulic flow through the piston inside the  $i^{th}$  actuator.

$Q_i$  is governed by the following equation:

$$Q_i = C_d \omega x_{vi} \sqrt{\frac{1}{\rho} (P_{si} - \text{sgn}(x_{vi}) P_{Li})} \quad (12)$$

where

$C_d$  is the discharge coefficient,

$\omega$  is the area gradient,

$x_v$  is the spool valve displacement,

$\rho$  is the fluid density,

$P_s$  is the supply piston pressure.

The spool valve displacement is controlled by an input voltage  $u_m$ . The corresponding dynamic relation can be simplified as a first order differential equation:

$$\dot{x}_{vi} = \frac{1}{\tau} (u_{mi} - x_{vi}) \quad (13)$$

### 5.3 Friction force

Due to rubbing of the piston with the inside actuator wall, heat will be generated. Therefore, the actual force generated by the  $i^{th}$  hydraulic actuator  $F_{Ai}$  is not equal to the force supply by  $i^{th}$  hydraulic actuator  $F_{Pi}$ . The difference between these two forces is named as frictional force  $F_{fi}$ . This force can not be neglected because the value of this force is big and can be greater than 200 Nm [19]. Frictional force is modelled with a smooth approximation of Signum function

$$F_{fi} = \begin{cases} \kappa \operatorname{sgn}(\dot{z}_i - \dot{w}_i) & \text{if } |\dot{z}_i - \dot{w}_i| > 0.01 \\ \kappa \sin\left(\frac{\dot{z}_i - \dot{w}_i}{0.01} \frac{\pi}{2}\right) & \text{if } |\dot{z}_i - \dot{w}_i| < 0.01 \end{cases} \quad (14)$$

### 5.4 Full state space model of vehicle with nonlinear suspension

The suspensions points  $P_1$ ,  $P_2$  and  $P_4$  satisfy a plane equation

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} = 0$$

Any points on the rigid plate satisfy

$$z = \frac{(z_4 - z_1)}{L} x + \frac{(z_2 - z_1)}{b} y + z_1. \quad (15)$$

The vertical displacement at centre of gravity  $z_c$  can be calculated from Equation 15 as following:

$$z_c = az_1 + 0.5z_2 + \frac{l_1}{L} z_4$$

where  $a = 0.5 - \frac{l_2}{L}$  and  $L = l_1 + l_2$

Applying Equation 15 at point  $P_3$ , the displacement can be written as

$$z_3 = -z_1 + z_2 + z_4.$$

The pitch angle and the roll angle can be calculated from the following equations:

$$\alpha = \frac{\partial z}{\partial y} = \frac{z_2 - z_1}{b}$$

$$\eta = \frac{\partial z}{\partial x} = \frac{z_4 - z_1}{L}$$

Let us assume that:



$$\begin{aligned}
 s_i &= (z_i - w_i)^3 \quad i \in \{1,2,3,4\} \\
 s_j &= (\dot{z}_i - \dot{w}_i)^2 \operatorname{sgn}(\dot{z}_i - \dot{w}_i) \quad j \in \{5,6,7,8\} \\
 s_r &= x_{vi} \sqrt{\frac{1}{\rho} (P_{si} - \operatorname{sgn}(x_{vi}) P_{Li})} \quad r \in \{9,10,11,12\} \\
 s_h &= \begin{cases} \operatorname{sgn}(\dot{z}_i - \dot{w}_i) & \text{if } |\dot{z}_i - \dot{w}_i| > 0.01 \\ \sin\left(\frac{\dot{z}_i - \dot{w}_i}{0.01} \frac{\pi}{2}\right) & \text{if } |\dot{z}_i - \dot{w}_i| < 0.01 \end{cases} \quad h \in \{13,14,15,16\}
 \end{aligned}$$

The input in the state space is governed by the following equation:

$$\dot{X} = B_1 X + B_2 S + B_3 U \quad (16)$$

The output equation can be written as:

$$Y = C_0 X + DU \quad (17)$$

where  $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}; U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$

$$X_1 = \begin{bmatrix} z_c \\ \alpha \\ \eta \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ P_{L1} \\ P_{L2} \\ P_{L3} \\ P_{L4} \\ x_{v1} \\ x_{v2} \\ x_{v3} \\ x_{v4} \end{bmatrix}; X_2 = \begin{bmatrix} \dot{z}_c \\ \dot{\alpha} \\ \dot{\eta} \\ \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \\ \dot{w}_4 \end{bmatrix}; S = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ s_9 \\ s_{10} \\ s_{11} \\ s_{12} \\ s_{13} \\ s_{14} \\ s_{15} \\ s_{16} \end{bmatrix}; U_1 = \begin{bmatrix} T_x \\ T_y \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_{m1} \\ u_{m2} \\ u_{m3} \\ u_{m4} \end{bmatrix}; U_2 = \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \\ \dot{u}_4 \end{bmatrix}$$

The output matrix can be given by

$$Y = [z_1 \ z_2 \ z_3 \ z_4 \ w_1 \ w_2 \ w_3 \ w_4 \ z_c \ \alpha \ \eta]^T$$

$B_1, B_2, B_3, C_0$  and  $D$  are coefficient matrices of the suspension system model.

## 6. Simulation and results

From Equation 6, five parameters  $K_p, K_d, K_i, \lambda$  and  $\mu$  are required to be designed for each suspension. Each individual vector has the FOPID parameters (five parameters). For reducing the time of optimization, the ranges of FOPID parameters are selected as  $K_p \in [0 \ 20000]$ ,  $K_d \in [0 \ 4000]$ ,  $K_i \in [0 \ 1500]$ ,  $\lambda \in [0 \ 1]$  and  $\mu \in [0 \ 1]$ . To evaluate the objective function, the following function will be used:

$$J_f = 0.5 \sum_{\varepsilon=1}^4 z_{\varepsilon}^2 \quad (10)$$

The stop criteria of the computation of evolutionary loop used was the one that defines the maximum number of generations being produced.

The nonlinear active suspension is presented in order to reduce the discomfort arising from road roughness and to increase the ride handling. The model parameters of this study are taken from Reference [20]. A MATLAB /SIMULINK program package has been used to simulate the full vehicle nonlinear active suspension model with the FOPID controller. Four FOPID controllers have been used (one controller for each suspension) and all controllers have the same parameter values. The initial values and optimal values of the FOPID controller parameters are shown in Table 1. Figures 4, 5, 6, 7 and 8 show the changing of the control parameters (values of  $K_p$ ,  $K_d$ ,  $K_i$ ,  $\lambda$  and  $\mu$ ) during the optimization process. After 225 optimization iteration steps, the objective function  $J_f$  has been minimized as shown in Fig. 9. During optimization phase, it is assumed that the inputs to the full vehicle model are just the road uneven excitation and control forces. The white noise input is supplied as rode profile.

**Table 1 Initial and Optimal Values of FOPID Controller**

Parameter	Initial value	Optimal value
$K_p$	100	12678.26
$K_d$	20	3253.92
$K_i$	1	768.1
$\lambda$	0.3	0.45
$\mu$	0.7	0.886

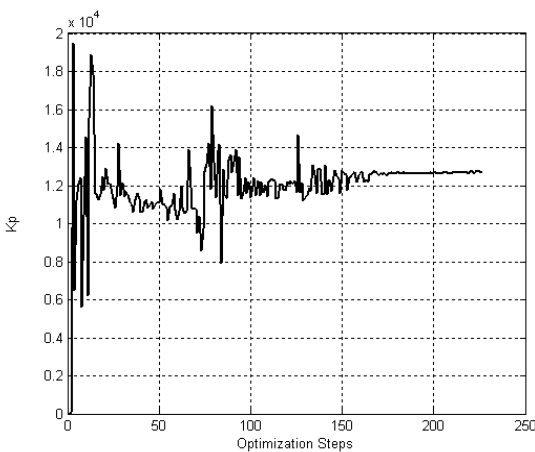


Figure 4. Changing Value of  $K_p$  During Optimization Steps

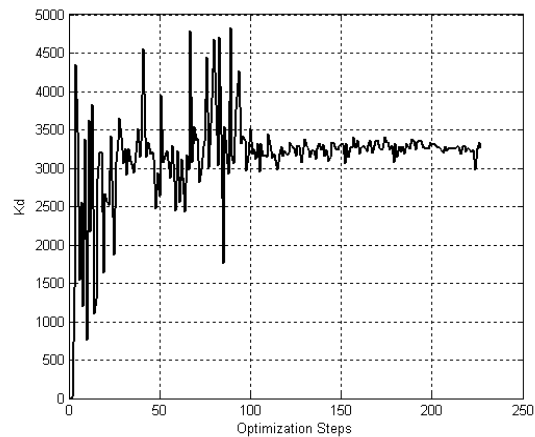


Figure 5. Changing Value of  $K_d$  During Optimization Steps

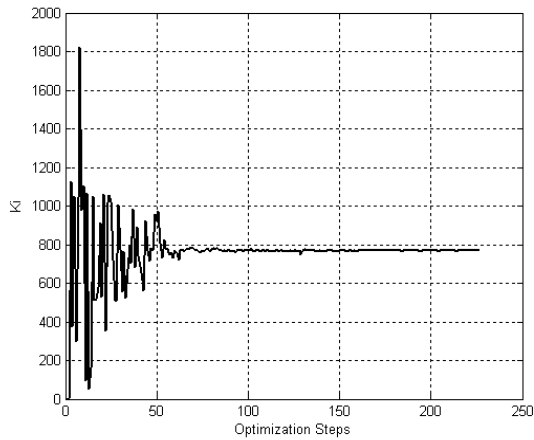


Figure 6. Changing Value of  $K_i$  During Optimization Steps

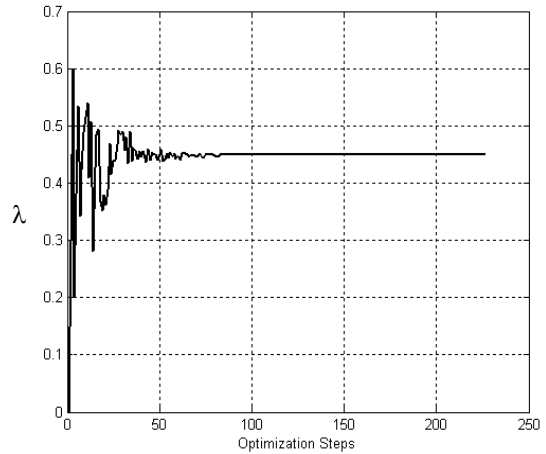


Figure 7. Changing Value of  $\lambda$  During Optimization Steps

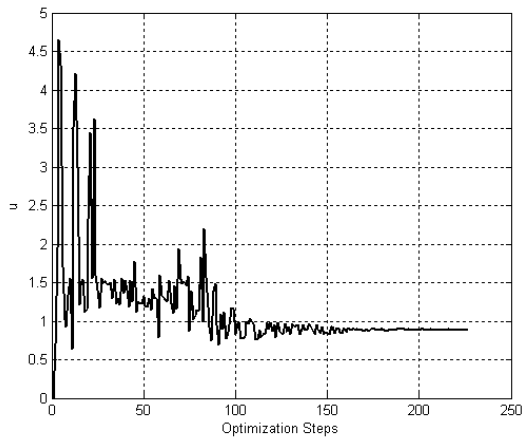


Figure 8. Changing Value of  $\mu$  During Optimization Steps

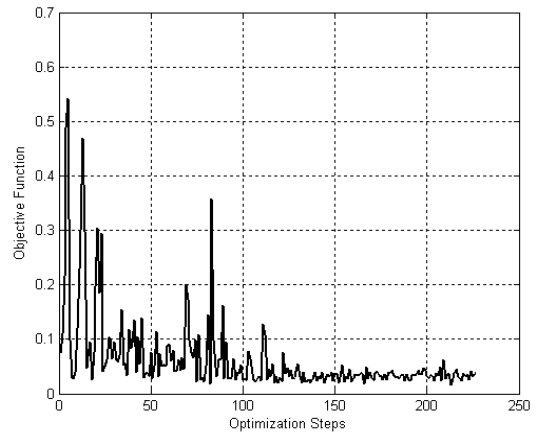


Figure 9. Objective Function

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## 7. Test of robustness of the proposed controller

An efficient controller is the one that is still stable even the disturbance signal is applied on the plant. Therefore, to establish the effectiveness for a controller the robustness should be examined. After the optimal values of the FOPID controller have been obtained, the robustness of the proposed FOPID controller with optimal values should be tested. Two types of disturbances are applied in turn to test the robustness of the FOPID controller.

### 7.1. Square input signal with varying amplitude applied as road input profile

The square input signal has been applied as road input profile. The amplitude of this signal has been changed from 0.01m to 0.08m. At each value the cost function (as described in Eq. 18) has been calculated:

$$\phi = 0.5 \sum_{h=1}^4 z_h^2 \quad (18)$$

Fig. 10 shows the time response of the cost function as function of amplitude of the square signal input.

### 7.2. Sine wave input signal with varying amplitude applied as road input profile

The different amplitude of sine wave input from 0.01m to 0.08m has been applied as road input profiles. The time response of the cost function for the passive system and the result of Fractional Order PID<sup>d</sup> controller are shown together in Fig. 11.

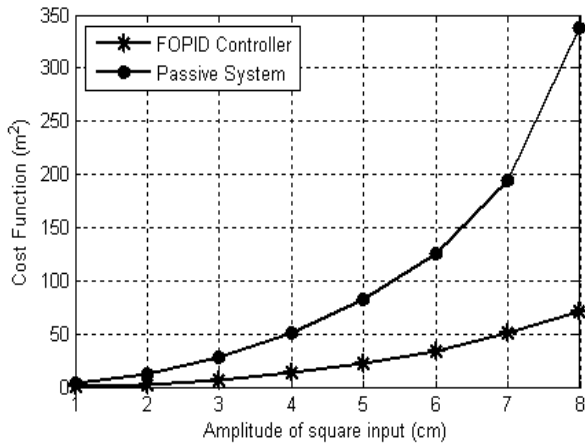


Figure 10. Time response of the cost functions against the different amplitudes of square input.

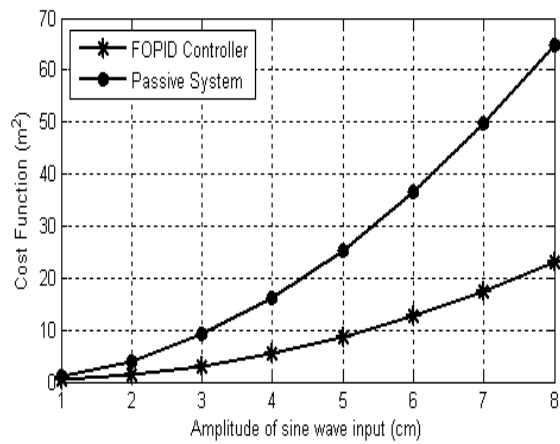


Figure 11. Time response of the cost functions against the different amplitudes of sine wave input.

## 8. Conclusion

Fractional Order PID<sup>d</sup> controller has been developed to optimize the vehicle motion control by minimizing the cost function. The Evolutionary Algorithm has been used to select the fitness values of the FOPID controller. The optimal FOPID controller has been designed to control the nonlinear active suspension system under typical disturbances. Two different road profiles as the system input have been applied with different amplitudes to assess the robustness of the proposed controller when the disturbances occur. The results have shown without doubt the effectiveness and robustness of the proposed FOPID controller.

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