Optimal Control Method for a Hydroelectric Power Development in Multi Level Dams

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Abstract
Generally, about 67% of hydropower generation is depending on the rainy season. In other word, the process of development of water resources is too limited to a particular season.
As a solution to this problem, this paper proposes an optimal control method of hydroelectric power development in multi level water dams.
The main advantage of the system is stabilization of the amount of power generate in dams. Simulation results show that the proposed optimal control method indicates development rate in Chungpyung Dam is about a 40%, 25% in Chungju Dam and 37.5% in Paldang.

Keywords: *Hydroelectric Power Plant, Multi Dam, Optimal Control Method, Power Generate.*

1 Introduction

Korea has four distinct seasons, but rainy season is restricted to July and August. Therefore, water resource for hydroelectric generation is too limited to particular season. Actually, about 67% of hydropower generation is depending on the rainy season. Since the operation ratio of power system for drainage system in Han River of Seoul is 30%, the cost for thermal power generation is saved when the hydroelectric power generation becomes higher. There is a problem that reserves the water below the current average water level to prevent from flood in July and August, flood season. February and March are dry season. Water for generation is insufficient in this season. In flood season, enough water should be reserved by means of the precise system that predicts, and prevents the flood.

In this paper, we propose the optimal control method of electric power generation in multilevel water dams for plummeting of water resources problem. In order to verify the development rate of the proposed optimal control method, and to compare it with the rainy season, we perform simulation on the total control system for hydroelectric power in Han River of Seoul.

2 Power system optimal control method by the regulation of dam water level -Simple Model

2.1 Optimal control of water level

Consider the following feedback problem.

$$J(x,\tau) = E\int_{\tau}^{T}\{h[x(t)-\bar{x}]^2 + c[u(t)-\bar{u}]^2\}e^{-\rho t}dt + \bar{s}\,h[x(T)-\bar{x}]^2e^{-\rho(T-\tau)} \quad (1)$$

$$\frac{dx(t)}{dt} = [-ax(t) - u(t) + s(t)] + \sigma\omega(t) \quad (2)$$

$$dx(t) = [-ax(t) - u(t) + s(t)]dt + \sigma d\omega(t) \quad (3)$$

Where, $0 \le \tau \le t \le T$, $x(0) = x_0$

Variables:
$0 \le \tau \le t \le T$: time
$x(t)$: dam water level at time t.
$u(t)$: generation speed(amount of release) at time t.
x_0: water level at $t = 0$, starting point.
$\bar{x}$: target water level of $x(t)$.
$\bar{u}$: target value for generation speed(amount of release), $u(t)$.
h: penalty for the deviation of water level at time t, $x(t)$, from target water level($\bar{x}$).
c: penalty for the deviation of generation speed, $u(t)$, from the target value, $\bar{u}$.
ρ: a constant for easy calculation by the computer.
Let $T = \infty$, $(0 \le \rho < 1)$; unnecessary to solve the differential equation,
a: natural extinction rate of water level(drying rate data is given)
s(t): water level change rate by external source (+inflow – drinking water – water for agriculture: data is given).
ω(t): water level change rate by random source; 0, average of ω(t), σ variance.
E: expected value of integral.

The model above is to minimize the cost for dam water level and for dam water release during the period [0,*T*] when there is relationship of differential equation (2) between water release, inflow, and water level. The determinant variable, u(t), should be defined for dam water releasing rate at each time to minimize the total cost. If u(t) is determined, then the optimized x(t) is also solved by equation (2), and the cost function (1) becomes minimized.

The optimization of a general model for the problem stated above is like shown below [1],[2],[3].

$$J(x,\tau) = E\{\int_{\tau}^{T} f^0\, e^{-\rho t}dt + S(T, x_T)e^{-\rho(T-\tau)}\} \quad (4)$$

$$\frac{dx(t)}{dt} = f + \sigma\omega\,, x_0 = x_0 \quad (5)$$

$$-J_t = \min_u\{f^0 + J_x f + \frac{1}{2}J_{xx}\sigma^2 - \rho J\} \quad (6)$$

$$f^0 = h[x(t) - \bar{x}]^2 + c[u(t) - \bar{u}]^2 \quad (7)$$

$$\mathrm{f} = -\mathrm{ax(t)} - \mathrm{u(t)} + \mathrm{s(t)} \tag{8}$$

Next, we apply the optimized general model to the problem of water level control

$$-\mathrm{J_t} = \min_u \{\, \mathrm{h[x-\bar{x}] + c[u-\bar{u}] + J_x[-ax-u+s]} + \frac{1}{2}\mathrm{J_{xx}}\sigma^2 - \rho\mathrm{J}\}\,\mathrm{u^*}$$
$$= \bar{\mathrm{u}} + \frac{1}{2\mathrm{c}}\mathrm{J_x} \Leftarrow \min_u \{-\mathrm{J_t}\}: \frac{\partial}{\partial u}\{-\mathrm{J_t}\} = 2\mathrm{c[u(t)-u]} - \mathrm{J} = 0 \tag{9}$$

$$-\mathrm{J_t} = \mathrm{h[x-\bar{x}]^2} + \mathrm{c}\left\{\left[\mathrm{u} + \frac{1}{2\mathrm{c}}\mathrm{J_x}\right] - \bar{\mathrm{u}}\right\}^2 + \mathrm{J_x}\left\{-\mathrm{ax} - \left[\bar{\mathrm{u}} + \frac{1}{2\mathrm{c}}\mathrm{J_x}\right] + \mathrm{s}\right\} + \frac{1}{2}\mathrm{J_{xx}}\sigma^2 - \rho\mathrm{J}$$
$$-\mathrm{J_t} = -\frac{1}{4\mathrm{c}}\mathrm{J_x}^2 + \mathrm{J_x}\,[-\mathrm{ax} - \bar{\mathrm{u}} + \mathrm{s}] + \mathrm{h[x-\bar{x}]} + \frac{1}{2}\mathrm{J_{xx}a^2} - \rho\mathrm{J} \tag{10}$$

The algorithm to find an optimized solution is derived as follows by letting the function like below.

$$\mathrm{J(t,x) \equiv xSx + qx + r \equiv V(t,x)}$$
$$\mathrm{J_t \equiv x\dot{S}x + \dot{q}x + \dot{r}}\ (*\ \mathrm{x\dot{S}x + \dot{q}x + \dot{r} = 0 \text{ for independant } t \text{ and } x \text{ in } V(t,x)})$$
$$\mathrm{J_x \equiv 2Sx + q}$$
$$\mathrm{J_{xx} \equiv 2S} \tag{11}$$

S,q,r are sub-variables. The optimization is shown below by using partial derivatives.

$$-[\mathrm{\dot{S}x + \dot{q}x + \dot{r}}] = -\frac{1}{4\mathrm{c}}[2\mathrm{x'S'} + \mathrm{q'}][2\mathrm{Sx+q}] + [-\mathrm{ax} - \bar{\mathrm{u}} + \mathrm{s}][2\mathrm{Sx+q}]$$
$$+ \mathrm{h[x-\bar{x}]^2} + \frac{1}{2}\sigma^2 2\mathrm{S} - \rho[\mathrm{Sx^2 + qx + r}]$$
$$-[\mathrm{\dot{S}x + \dot{q}x + \dot{r}}] = -\frac{1}{4\mathrm{c}}[4\mathrm{x'S'Sx} + 2\mathrm{q'Sx} + \mathrm{q'q}] + [-\mathrm{ax} - \bar{\mathrm{u}} + \mathrm{s}][2\mathrm{Sx+q}]$$
$$+ \mathrm{h[x^2 - 2\bar{x}x + \bar{x}^2]} + \sigma^2\mathrm{S} - \rho[\mathrm{Sx^2 + qx + r}]$$
$$0 = \mathrm{x^2}\left[\dot{\mathrm{S}} - \frac{1}{\mathrm{c}}\mathrm{S^2} - 2\mathrm{aS} + \mathrm{h} - \rho\mathrm{S}\right] + \mathrm{x}\left[\dot{\mathrm{q}} - \frac{1}{\mathrm{c}}\mathrm{qS} + (-\bar{\mathrm{u}} + \mathrm{s})2\mathrm{S} - \mathrm{aq} - 2\mathrm{h}\bar{\mathrm{x}} - \mathrm{pq}\right]$$
$$+ [\dot{\mathrm{r}} - \frac{1}{4\mathrm{c}}\mathrm{q} + (-\bar{\mathrm{u}} + \mathrm{s})\mathrm{q} + \mathrm{h}\bar{\mathrm{x}}^2 + \sigma^2\mathrm{S} - \rho\mathrm{r} \tag{12}$$

From last equation shown above, it should be 0 in order to satisfy x(t), (water level), at any time. That is,

$$\dot{\mathrm{S}}(\mathrm{t}) = \frac{1}{\mathrm{c}}\mathrm{S^2} + (2\mathrm{a} + \rho)\mathrm{S} - \mathrm{h},\ \mathrm{S(T)} = \bar{\mathrm{S}}\mathrm{he}^{-\rho\mathrm{T}}$$
$$\dot{\mathrm{q}}(\mathrm{t}) = \left[\frac{1}{\mathrm{c}}\mathrm{S} + \mathrm{q} + \mathrm{a}\right]\mathrm{q} + [2(\bar{\mathrm{u}} - \mathrm{s})\mathrm{S} + 2\mathrm{h}\bar{\mathrm{x}}],\ \mathrm{q(T)} = -2\bar{\mathrm{S}}\mathrm{h}\bar{\mathrm{x}}\mathrm{e}^{-\rho\mathrm{T}}$$
$$\mathrm{r(t)} = \rho\mathrm{r} + (\bar{\mathrm{u}} - \mathrm{s})\mathrm{q} + \frac{1}{4\mathrm{c}}\mathrm{q^2} - \mathrm{h}\bar{\mathrm{x}}^2 - \sigma^2\mathrm{S},\ \mathrm{r(T)} = 0 \tag{13}$$

When t=T, S(T), q(T), and r(T) are as follows.

$$\lambda\,(\mathrm{T}) = \frac{\partial\{\mathrm{Sh[x(T) - x]e}\}}{\partial\{\mathrm{x(T)}\}} = [2\mathrm{She}^{-\rho\mathrm{T}}]\mathrm{x(T)} + [-2\bar{\mathrm{S}}\mathrm{h}\bar{\mathrm{x}}\mathrm{e}^{-\rho\mathrm{T}}]$$
$$\equiv 2\mathrm{S(T)x(T)} + \mathrm{q(T)} = \mathrm{J_x(T)} \tag{14}$$

The algorithm to find an optimized solution is derived as follows by letting the function.

S,q,r are sub-variables. The optimization is shown below by using partial derivatives. From last equation shown above, it should be 0 in order to satisfy x(t), (water level), at any time. That is,

$$\begin{aligned} S(T) &= \bar{S}he^{-\rho T} \\ q(T) &= -2\bar{S}h\bar{x}e^{-\rho T} \\ r(T) &= 0 \end{aligned} \tag{15}$$

Let 0 < t < T, then the equations is as follows: $\dot{S}(t) \neq 0, \dot{q}(t) \neq 0, \dot{r}(t) \neq 0$
$S(t)$,q(t), and r(t) can be calculated with Euler method or Runge-kutta methods when 0 ≤ t ≤ T. Then the feedback control can be obtained with the following equation if x(t) is given.

$$u^*(t) = \bar{u}(t) + \frac{1}{2c}J_x = \frac{1}{c}S(t)x(T) + \left(\bar{u}(t) + \frac{1}{2c}q(t)\right) \tag{16}$$

$$* \; (* \; u_{stable}(t) = -a\bar{x} + s(t), 0 \le u(t) \le u_{max} \; , u_{max} \; : \text{Maximum Capacity})$$

2.2 Simulation of a model – Hwacheon Dam-

We can review Hwacheon dam, independently. In other words, the impact of change in the amount of release from Hwacheon dam can be relieved at dams in downstream such as Chuncheon dam, Uiam dam, and Chungpyung dam.

Table 1. Amount of inflow to Hwacheon dam per month (unit CMS=cubic meter per second = m3/sec: 1950-1982 average for 30 years

Month	**1**	**2**	**3**	**4**	**5**	**6**
Inflow	10.7	9.6	28.0	76.6	56.6	80.4
Cumulative	10.7	20.3	48.3	124.9	181.5	261.9 (m^3/sec)
Month	**7**	**8**	**9**	**10**	**11**	**12**
Inflow	248.6	322.2	253.1	26.3	23.8	21.3
Cumulative	510.5	832.7	1085.8	1112.1	1135.9	1157.2 (m^3/sec)

Average per month: 96.4 m3/sec =1157.2(m3/sec)/12;
Power plant statistics
Hwacheon dam
Capacity of Facility, MW: 108 MW=27MW*4;
Capacity of Facility, MW per year: 326,000 MWh;
Turbine: capacity: 30MW; numbers: 4, total capacity: 120MW;
Max usage: 185 m3/sec = 46.25 m3/sec* 4 turbines;

Simulation conditions,

$T = 24$; $dt = 0{:}1$;$KK = 240$; $x0 = x0 = 50$;$h = 1$; $c = 0{:}01$; $\bar{x} = xx = 60$; $\bar{u} = uu = 10$; $u_{max} = 11$; $\rho = 0{:}1$; $a = 0{:}05$;$s = 0$; a= 0;

The result is shown in the Figure.1.

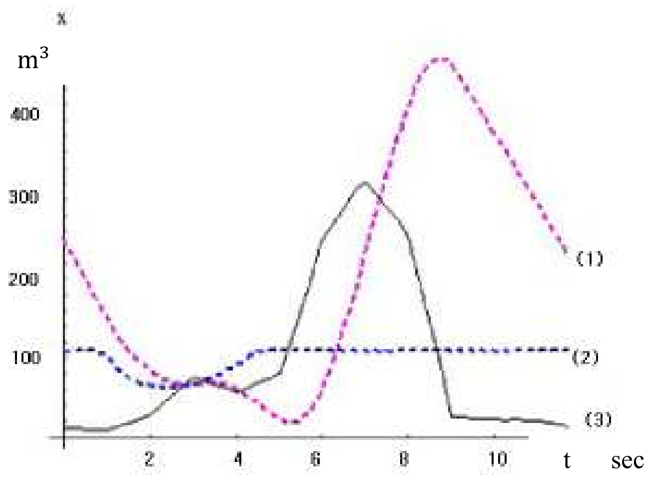

Figure. 1. Simulation of simple model in Hwacheon dam

In the Figureure, (1)is water level (m^3),(2) is water for generation in proposed algorithm (m^3/sec),and (3) is inflow (m^3 /sec).

3 Total Optimal Control Method for Hydroelectric Power Plant in Han River

3.1 Discrete optimization model

The previous researches have been on similar problems to next model with cost reduction model to generate the power.

$$\min F = \sum_{t=1}^{T} \left\{ \sum_{j} h_j\left(u_j(t) - Q_{j0}\right) + \left[P_R(t) - \sum_{j} h_j\left(u_j(t) - Q_{j0}\right) \right]^2 \right\} \tag{17}$$

$$x(t+1) = x_j(t) - u_j(t) + s_j(t) \tag{18}$$

$$0 \le t \le T, x_j(0) = x_0 \tag{19}$$

Variable definition

$s_j(t)$: [+inflow – drinking water – water for agriculture] at j reservoir at time t.
$x_i(t)$: amount of water reserved in j reservoir at time t.
$\underline{u}_j(t)$: output power(converted value that was used) of j hydroelectric power plant at time t.
$\overline{P}_R(t)$: expected power consumption of j at time t.
$\overline{R}_j(t), \overline{S}_j(t), \overline{Q}_j(t), \overline{P}_{Hj}(t), \overline{P}_{Si}(t)$: indicate the maximum of variables
$\underline{R}_j(t), \underline{S}_j(t), \underline{Q}_j(t), \underline{u}_{Hj}(t), \underline{v}_{Si}(t)$: indicate the minimum of variables.
$\overline{P}_R(t)$: power consumption at time t.
Explanation of cost:
$\sum_j h_j(u_j(t) - Q_{j0})$: cost for hydroelectric power generation.
$\left[P_R(t) - \sum_j h_j(u_j(t) - Q_{j0})\right]^2$: An approximate cost value for shortage of power supply or over power supply compared to power consumption.

$$P_R(t) \cong \sum_j \left[h_j(u_j(t) - Q_{j0})\right], P_{Hj}(t) \cong h_j(u_j(t) - Q_{j0}) \tag{20}$$

Each variable is located in the middle between maximum and minimum values.

$$\underline{u}_j(t) \le u_j(t) \le \overline{u}_j(t)$$

$$\underline{s}_j(t) \le s_j(t) \le \overline{s}_j(t)$$

$$\underline{P}_{Hj}(t) \le P_{Hj}(t) \le \overline{P}_{Hj}(t)$$

$$\underline{P}_{Si}(t) \le P_{Si}(t) \le \overline{P}_{Si}(t).$$

3.2 Total Optimized control model

The problem above can be converted to an optimized control problem.

$$\min J = \frac{1}{2}\int_0^T \{[x(t) - \tilde{x}(t)]'Q[x(t) - \tilde{x}(t)] + [u(t) - \tilde{u}(t)]'R[u(t) - \tilde{u}(t)] + [v(t) - \tilde{v}(t)]'P[v(t) - \tilde{v}(t)]\}\,dt + \frac{1}{2}[x(t) - \tilde{x}(t)]'Q[x(t) - \tilde{x}(t)] \tag{21}$$

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t) + Cv(t) + g(t),\ 0 \le t \le T,\ x(0) = x_0 \tag{22}$$

$$\underline{u}_j(t) \le u_j(t) \le \overline{u}_j(t), \underline{v}_j(t) \le v_j(t) \le \overline{v}_j(t), \underline{x}_j(t) \le x_j(t) \le \overline{x}_j(t), \qquad 0 \le t \le T$$

n: number of hydroelectric power plants; ′: transpose.
u(t): water release related to power generation.
v(t): drinking water + water release for flood prevention.
$u(t) = [u_1(t), \dots, u_n(t)];\ v(t) = [v_1(t), \dots, v_n(t)]; x(t) = [x_1(t), \dots x_n(t)]$
This can't be solved by feedback algorithm due to inequality sign. To solve the problem, apply Augmented Lagrangian Method [4],[5].

$$\max_{\lambda} \quad \min_{\underline{u} \le u \le \overline{u}, \overline{z} \ge 0, \underline{z} \ge 0; x, \lambda} \quad L(u, z, z; x, \lambda)$$

$$\begin{aligned} L(u, \overline{z}, \underline{z}) = \frac{1}{2}\int_0^T & \{[x(t) - \tilde{x}(t)]'Q[x(t) - \tilde{x}(t)] + [u(t) - \tilde{u}(t)]'R[u(t) - \tilde{u}(t)] \\ & + [v(t) - \tilde{v}(t)]'P[v(t) - \tilde{v}(t)]\}\,dt \\ & + \frac{1}{2}[x(t) - \tilde{x}(t)]'Q[x(t) - \tilde{x}(t)] \\ & + \int_0^T \left\{\lambda(t)\left[-\frac{dx(t)}{dt} + (Ax(t) + Bu(t) + Cv(t) + g(t))\right]\right\} dt \\ & + \int_0^T \{\eta_1(t)[x(t) - \overline{x}(t) + \overline{z}(t)] + \eta_2(t)[x(t) - \underline{x}(t) + \underline{z}(t)]\} dt \\ & + \int_0^T \left\{\frac{\gamma}{2}[x(t) - \overline{x}(t) + \overline{z}(t)]\left[x(t) - \underline{x}(t) + \underline{z}(t)\right] \right. \\ & \left. + \frac{\gamma}{2}[-x(t) + \overline{x}(t) + \overline{z}(t)][-x(t) + \underline{x}(t) + \underline{z}(t)]\right\} dt \end{aligned} \tag{23}$$

Lagrangian function L is defined above, λ(t) and sub variables like $\overline{z}(t)$, and z(t) are inserted. In order to solve in numeral method, effectively, multiply γ/2 to inequality sign, and relative term, and square them.

The converted problem can be solved by numerical method, Gradient method.
Euler method is used to solve the differential equations, and I can simplify if there is no restriction.
The algorithm for optimal numerical method is used for this system. Superscript i means an ith approximate value improved, calculate $x^{i+1}(t)$ value improved in left side by using the value in the right side. The calculation period and boundary condition are shown next.

$$0 \le t \le T,\ \ x^{i}(0) = x_0\,, \lambda^{'}(t) = Q\left[x^{i}(t) - \tilde{x}(t)\right]$$

From this, $x^{i}(t)$ is obtained when t=0, and $x^{i}(0) = x_0$ is known, and divide the period $0 \le t \le T$, into Δ_t. Calculate the value, $x^{i+1}(t + \Delta_t)$ by changing the period from t = 0, Δ_t, $2\Delta_t$, …, T. Variable, $\lambda(t)$, is the last value at t = T, $\lambda^{'}(t) = Q\left[x^{i}(t) - \tilde{x}(t)\right]$ divide the period, $0 \le t \le T$, into Δ_t. Calculate the value, $\lambda^{i+1}(t - \Delta_t)$ by changing the period from t = T, T-Δ_t, T-$2\Delta_t$, …, 0.

Hydroelectric power dams in Han River are shown in Figure. 2.

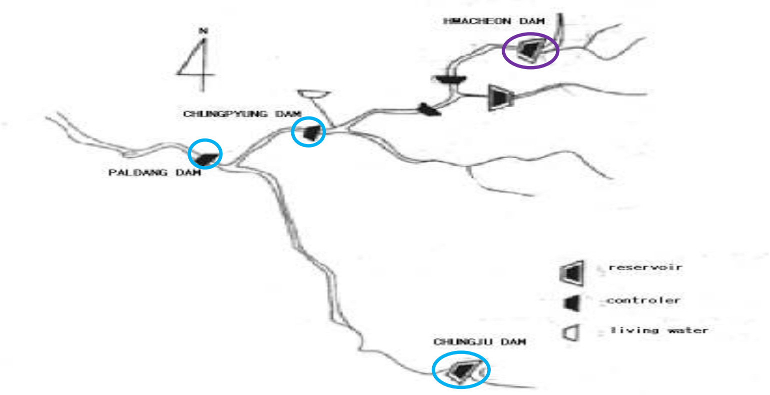

Figure. 2. Dams in Han River

Let's review to optimize all three dams in Han River: Paldang dam which is important for hydroelectric power, supplying water to Seoul, and flood control, and Chungpyung dam, and Chungjoo dam which are connected to Paldang dam. The general model of vector matrix above can be rewritten like the following:

$$\begin{aligned}\min J = \frac{1}{2}\int_0^T &\{q_1[x_1(t) - \tilde{x}_1(t)]^2 + q_2[x_2(t) - \tilde{x}_2(t)]^2 + q_3[x_3(t) - \tilde{x}_3(t)]^2 \\ &+ r_1[u_1(t) - \tilde{u}_1(t)]^2 + r_2[u_2(t) - \tilde{u}_2(t)]^2 \\ &+ r_3[u_3(t) - \tilde{u}_3(t)]^2 + p_1[v_1(t) - \tilde{v}_1(t)]^2 \\ &+ p_2[v_2(t) - \tilde{v}_2(t)]^2 + p_3[v_3(t) - \tilde{v}_3(t)]^2\}\,dt \\ &+ \frac{1}{2}\bar{q}_1[x_1(T) - \tilde{x}_1(T)]^2 + \frac{1}{2}\bar{q}_2[x_2(T) - \tilde{x}_2(T)]^2 \\ &+ \frac{1}{2}\bar{q}_3[x_3(T) - \tilde{x}_3(T)]^2\end{aligned} \tag{24}$$

$$\frac{dx_1(t)}{dt} = -a_1x_1(t) - [b_1u_1(t) + c_1v_1(t)] + g_1(t) \tag{25}$$

$$\frac{dx_2(t)}{dt} = -a_2x_2(t) - [b_2u_2(t) + c_2v_2(t)] + g_2(t) \tag{26}$$

$$\frac{dx_1(t)}{dt} = -a_3x_3(t) - [b_3u_3(t) + c_3v_3(t)] + [b_2u_2(t) + c_2v_2(t)] + [b_1u_1(t) + c_1v_1(t)] + g_3(t) \tag{27}$$

$$\underline{u}_1(t) \le u_1(t) \le \bar{u}_1(t), \underline{u}_2(t) \le u_2(t) \le \bar{u}_2(t), \underline{u}_3(t) \le u_3(t) \le \bar{u}_3(t)$$
$$\underline{v}_1(t) \le v_1(t) \le \bar{v}_1(t), \underline{v}_2(t) \le v_2(t) \le \bar{v}_2(t), \underline{v}_3(t) \le v_3(t) \le \bar{v}_3(t)$$
$$\underline{x}_1(t) \le x_1(t) \le \bar{x}_1(t), \underline{x}_2(t) \le x_2(t) \le \bar{x}_2(t), \underline{x}_3(t) \le x_3(t) \le \bar{x}_3(t)$$

Subscript 1 in all variables indicates the Chungpyung dam, subscript 2 is Chungjoo dam, and subscript 3 is Paldang dam. a1, a2, and a3 indicate natural evaporation from the amount of water reserved in dams. g1(1), g2(t), and g3(t) are the amount of inflow from raining, and other region. u1(1), u2(t), and u3(t) are the amount of released, and v1(t), v2(t), and v3(t) are drinking water and the amount of water released for flood control. The review period: $0 \le t \le T$, initial water reserved in each dam:

$x_1(0) = x_{10}, x_2(0) = x_{20}, x_3(0) = x_{30},$ Variables and constants are defined as below

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} \quad v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} \quad g(t) = \begin{bmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \end{bmatrix}$$

$$Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix} R = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix} P = \begin{bmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{bmatrix} \overline{Q} = \begin{bmatrix} \bar{q}_1 & 0 & 0 \\ 0 & \bar{q}_2 & 0 \\ 0 & 0 & \bar{q}_3 \end{bmatrix}$$

$$A = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \quad B = \begin{bmatrix} -b_1 & 0 & 0 \\ 0 & -b_2 & 0 \\ b_1 & b_2 & -b_3 \end{bmatrix} \quad Q = \begin{bmatrix} -c_1 & 0 & 0 \\ 0 & -c_2 & 0 \\ c_1 & c_2 & -c_3 \end{bmatrix}$$

$$\bar{x}(t) = \begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \\ \bar{x}_3(t) \end{bmatrix} \quad \bar{u}(t) = \begin{bmatrix} \bar{u}_1(t) \\ \bar{u}_2(t) \\ \bar{u}_3(t) \end{bmatrix} \quad \bar{v}(t) = \begin{bmatrix} \bar{v}_1(t) \\ \bar{v}_2(t) \\ \bar{v}_3(t) \end{bmatrix}$$

$$\underline{x}(t) = \begin{bmatrix} \underline{x}_1(t) \\ \underline{x}_2(t) \\ \underline{x}_3(t) \end{bmatrix} \quad \underline{u}(t) = \begin{bmatrix} \underline{u}_1(t) \\ \underline{u}_2(t) \\ \underline{u}_3(t) \end{bmatrix} \quad \underline{v}(t) = \begin{bmatrix} \underline{v}_1(t) \\ \underline{v}_2(t) \\ \underline{v}_3(t) \end{bmatrix}$$

$$\lambda(t) = \begin{bmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \lambda_3(t) \end{bmatrix} \quad \eta(t) = \begin{bmatrix} \eta_1(t) \\ \eta_2(t) \\ \eta_3(t) \end{bmatrix} \quad \bar{z}(t) = \begin{bmatrix} \bar{z}_1(t) \\ \bar{z}_2(t) \\ \bar{z}_3(t) \end{bmatrix} \quad \underline{z}(t) = \begin{bmatrix} \underline{z}_1(t) \\ \underline{z}_2(t) \\ \underline{z}_3(t) \end{bmatrix} \tag{28}$$

To find out the best optimum solution for 3 dam models with the algorithm to calculate the general optimization solution above, see below.

Chungpyung Dam

Table 2. Chungpyung Dam.

month	Water level (EL.m)	Rainfall (mm)	Inflow (m^3/sec)	Used for Generation (m^3/sec)	Released for prevention (m^3/sec)	Fluctuation Pondage (m^3/sec)
1	50.271	1.000	2335.800	2210.300	0.000	125.500
2	50.505	35.000	2270.500	2232.900	0.000	37.600
3	50.556	61.000	2967.400	3026.300	0.000	-58.900
4	50.407	75.000	3167.500	3146.300	0.000	21.200
5	50.426	110.500	4594.500	4645.200	0.000	-50.700
6	49.937	174.500	6675.100	6813.599	0.000	-138.500
7	49.645	836.500	29398.201	9631.800	19735.301	31.101
8	49.705	283.500	15410.399	10533.599	4884.300	-7.500
9	49.945	22.500	4464.000	4268.100	0.000	195.900
10	50.571	53.500	2025.400	2045.600	0.000	-20.200
11	50.592	53.000	2198.600	2198.400	0.000	0.200
12	50.270	19.000	2914.200	2931.800	0.000	-17.600
Aver/sum	50.236	1725.000	78421.602	53683.895	24619.602	118.104

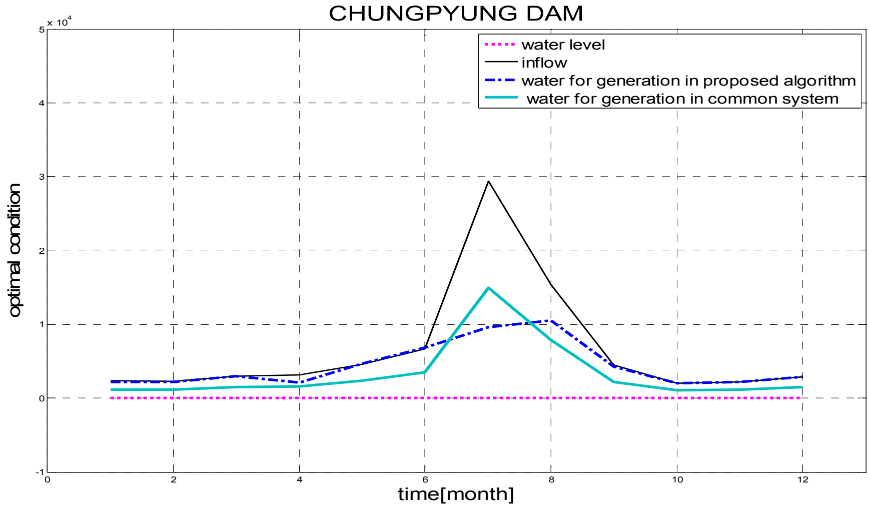

Figure. 3. Optimal solution of Chungpyung dam

Chungjoo dam

Table 3.. Chungjoo dam.

month	Water level (EL.m)	Rainfall (mm)	Inflow (m^3/sec)	Used for Generatio n (m^3/sec)	Released for prevention (m^3/sec)	Fluctuatio n Pondage (m^3/sec)
1	64.642	16.300	2347.803	2353.512	0.000	-5.709
2	64.537	27.000	1896.696	1876.879	40.616	-20.799
3	64.395	43.900	2035.505	2023.125	0.000	12.380

4	64.535	26.500	1991.507	1862.260	106.519	22.728
5	64.666	97.100	2463.776	2482.174	0.000	-18.398
6	64.631	96.700	2569.069	2456.870	84.315	27.884
7	64.617	356.700	25599.471	3399.189	22205.396	-5.115
8	64.754	144.000	6241.611	3937.936	2320.850	-17.175
9	64.745	56.700	2294.223	2282.063	0.765	11.395
10	64.724	38.900	2193.484	2183.365	8.683	1.436
11	64.671	41.700	1879.880	1874.649	11.674	-6.443
12	64.650	35.400	2452.162	2461.088	0.956	-9.882
Aver/sum	64.631	980.900	53965.188	29193.107	24779.773	-7.698

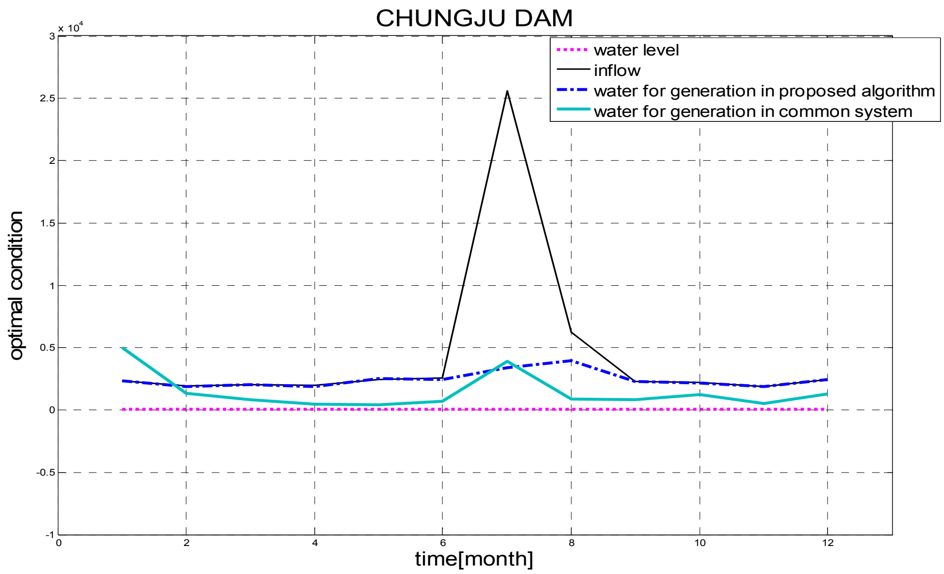

Figure. 4. . Optimal solution of Chungju dam

Paldang dam

Table 4. optimal solution Paldang dam

month	Water level (EL.m)	Rainfall (mm)	Inflow (m^3/sec)	Used for Generation (m^3/sec)	Released for prevention (m^3/sec)	Fluctuation Pondage (m^3/sec)
1	25.165	0.500	4000.300	3912.601	0.000	87.700
2	25.216	35.500	4056.600	4085.700	0.000	-29.100
3	25.176	65.000	4976.100	4960.801	2.900	12.399
4	25.101	53.000	4865.801	4881.700	0.000	-15.899
5	25.095	110.000	7630.000	7642.401	0.000	-12.400
6	25.050	131.000	9445.700	9483.700	0.000	-38.000
7	24.887	782.000	79447.516	15363.700	64054.902	28.913
8	24.910	248.000	26799.604	17914.201	8915.000	-29.598
9	25.120	48.500	6692.400	6592.400	0.000	100.000
10	25.125	53.000	4096.600	4167.500	0.000	-70.900
11	25.170	49.500	4313.600	4288.800	0.000	24.800
12	25.114	17.500	5101.800	5068.400	0.000	33.399

Aver/sum	25.096	1593.500	161426.016	88361.898	72972.797	91.315

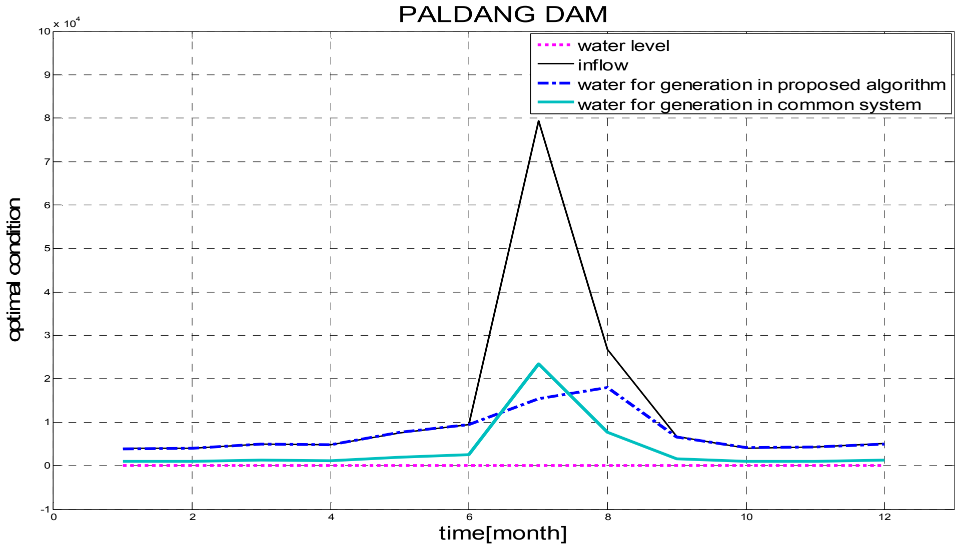

Figure. 5. Optimal solution of Paldang dam

4 Conclusion

Generally, about $\frac{2}{3}$ (67%) of hydropower generation is depend on the rainy season. In other word the process of development of water resources is too limited to a particular season. In order to develop this problem, we proposed an optimal control method of hydroelectric power development in multi level water dams. To verify the development rate of the proposed optimal control method, we simulated on the total control system for hydroelectric power in Han River of Seoul.

Simulation results show that the proposed optimal control method indicates development rate in Chungpyung Dam is about a 40%, 25% in Chungju Dam and 37.5% in Paldang. The main advantage of the proposed optimal control method is stabilization of the amount of power generate in dams.

Through the system for simple model, Hwacheon Dam, the development rate of hydropower is stabilized without big impact from inflow even in the rainy season or dry season. It would be helpful to maintain the water level in each dam not exceeding the limits, and the generate rate would be kept the constant power generation. However, this system has some negative side because of labile factor. It has not been performed that accurate measurement of each element in Han-River. The data of each element in this paper is approximate number derived by a formula. When the data of each factors correctly attained, this system can be valuable method.

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