Systematic Method for Kinematics Modeling of Legged Robots on Uneven Terrain

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Abstract

The paper develops a systematic method for kinematics modeling of multi-legged robots for walking on rough terrain. An extended D-H table is proposed for characterizing the robot joints and linkages parameters that capture the motion. The equations of motions are set up using this table for different frames starting from the robot reference frame, going through individual legs and finally reaching feet-terrain contact frames. The composite equation of motion is formed from those of individual legs. The formulation allows determining actuations to various joints for achieving a desired robot motion, while optimizing a performance index such as a stability measure. For illustration, the method is applied to SILO4, an articulated quadruped.

Keywords: *Legged Robots, Kinematics Modeling.*

1. Introduction

Two main tasks are involved in the design of locomotion strategies for multi-legged robots walking over rough terrain. One is the high level leg sequencing and foothold planning. The other is lower level actuation and coordinated control of robot joints to achieve a desired motion of the rover body. While the high level task is crucial and a large volume of research has been devoted to it (see e.g. [1]-[7]), the lower level task is also very important for the success of walking. This lower level task requires developing kinematics models for the multi-legged robot but has received little attention. In [8] the kinematics modeling is divided into an open chain for the swing leg and a closed chain for the body. A Jacobian based approach is proposed in [9] for a point foot quadruped where the swing leg inverse kinematics solution is projected into the null space of whole body Jacobian to improve the motion.

This paper extends and generalizes our previous work on wheeled mobile robots [10]-[11], and proposes a systematic approach to full kinematics modeling of a general multi-legged walking robot. The full kinematics model relates the motion of the body to motions of the legs. The kinematics model can be used for various applications, e.g. determining actuation to leg joints to achieve a desired robot trajectory, studying various parameters affecting motions of the body and legs, and interaction between the robot feet and the terrain. The systematic kinematics modeling can also remove many heuristics when dealing with multi-legged robot motion.

2. Kinematics modeling of multi-legged robots

The goal of kinematics modeling is to relate the motion of the body to those of the swing legs and legs in contact with the terrain. In order to achieve this, we attach a sequence frames starting at the body reference (e.g. body center of gravity) then going through a chain of frames (e.g.. shoulder, hip, knee, ankle) and ending at a foot's sole. A foot is either in contact with the terrain, or is swinging. Let $u_{i-1} = [\, x_{i-1} \quad y_{i-1} \quad z_{i-1} \,]^T$ and $u_i = [\, x_i \quad y_i \quad z_i \,]^T$ denote the position of the current and next frames, respectively. Similarly, let $\varphi_{i-1} = [\, \alpha_{i-1} \quad \beta_{i-1} \quad \gamma_{i-1} \,]^T$ and $\varphi_i = [\, \alpha_i \quad \beta_i \quad \gamma_i \,]^T$ be the orientation of the previous and current frames, respectively, where α_i, β_i and γ_i are the rotation around *x*, *y* and *z* axes, or roll, pitch and yaw, respectively. Using the D-H notation, four parameters describe the transformation from the frame F_{i-1} to the frame F_i given by rotation θ_i about *z*-axis, translation d_i along *z*-axis, translation a_i along *x*-axis, and rotation ε_i about x-axis. Let the D-H parameter vector be denoted by $\eta_i = [\, \theta_i \quad d_i \quad a_i \quad \varepsilon \,]$. It is shown in our previous paper that the velocity propagation from the frame F_{i-1} to the frame F_i is [11]

$$\begin{bmatrix} \dot{u}_i \\ \dot{\varphi}_i \end{bmatrix} = \begin{bmatrix} R_i & R_i S_{.i} \\ 0 & R_i \end{bmatrix} \begin{bmatrix} \dot{u}_{i-1} \\ \dot{\varphi}_{i-1} \end{bmatrix} + \begin{bmatrix} A_i \\ B_i \end{bmatrix} \dot{\eta}_i \qquad i = 1, \cdots, n \tag{1}$$

where R_i is a rotation matrix and S_i is a skew symmetric matrix given by

$$R_i = \begin{bmatrix} c\theta_i & s\theta_i & 0 \\ -c\varepsilon_i s\theta_i & c\varepsilon_i c\theta_i & s\varepsilon_i \\ s\varepsilon_i s\theta_i & -s\varepsilon_i c\theta_i & c\varepsilon_i \end{bmatrix}; \qquad S_i = \begin{bmatrix} 0 & d_i & -a_i s\theta_i \\ -d_i & 0 & a_i c\theta_i \\ a_i s\theta_i & -a_i c\theta_i & 0 \end{bmatrix} \tag{2}$$

The coefficient matrices in (1) are

$$A_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ a_i c\varepsilon_i & s\varepsilon_i & 0 & 0 \\ -a_i s\varepsilon_i & c\varepsilon_i & 0 & 0 \end{bmatrix} \qquad B_i = \begin{bmatrix} 0 & 0 & 0 & 1 \\ s\varepsilon_i & 0 & 0 & 0 \\ c\varepsilon_i & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

It is noted that η_i contains the robot joints and linkages values some of which may be zero or constant, depending on the geometric arrangement. Such joints and links do not contribute to the D-H parameter rate vector $\dot{\eta}_i$. Furthermore, some of the joints and linkages may be actuated and thus adjustable while others may be variable but unactuated. An example of the latter joints is the ankle joints that are generally used to conform to the topology of the terrain.

Equations (1) can be written in a more compact form as:

$$\begin{bmatrix} \dot{u}_i \\ \dot{\varphi}_i \end{bmatrix} = D_i \begin{bmatrix} \dot{u}_{i-1} \\ \dot{\varphi}_{i-1} \end{bmatrix} + E_i \, \dot{\eta}_i \;; \quad i = 1,2,\cdots,n \tag{4}$$

where D_i and E_i are, respectively, 6×6 and 6×4 matrices given in (2)-(3).

It is to be noted that (4) describes the kinematics of particular chain starting from the robot body reference and ending at a foot's sole where contact with the terrain takes place. In general the number of frames in each of the *m* chains can be different, where *m* is the number of feet. To take these aspects into consideration, it would be more precise to write (4) as

$$\begin{bmatrix} \dot{u}_{i_j} \\ \dot{\varphi}_{i_j} \end{bmatrix} = D_{i_j} \begin{bmatrix} \dot{u}_{i_{j-1}} \\ \dot{\varphi}_{i_{j-1}} \end{bmatrix} + E_{i_j} \, \dot{\eta}_{i_j} \;; \quad \begin{matrix} i = 1,\cdots,n \\ j = 1,2,\cdots,m \end{matrix} \tag{5}$$

where D_{i_j} and E_{i_j} are, respectively, 6×6 and 6×4 the transformation matrices relating velocities of the frame *i-1* to frame *i* in the *j*-th chain. There can be in general n_j frames in the *j*-th chain, though usually the chains are similar and have the same number of frames. It is noted that some of the four elements of $\dot{\eta}_j$ may be zero if the corresponding D-H parameter is constant, as discussed above.

Cascading the transformations in (5), we obtain an aggregate transformation for a kinematics chain leading to the sole of a foot. Since there are *m* feet (soles), there will be *m* such aggregate transformations which can be written

$$\begin{bmatrix} \dot{u}_{SE_j} \\ \dot{\varphi}_{SE_j} \end{bmatrix} = P_j \begin{bmatrix} \dot{u}_{BY} \\ \dot{\varphi}_{BY} \end{bmatrix} + Q_j \dot{\Gamma}_j \;; \quad j = 1,2,\cdots,m \tag{6}$$

where $\dot{u}_{SE\,j}$ and $\dot{\varphi}_{SE\,j}$, $j = 1,2,\cdots,m$; are the translational and rotational velocities at *j*-th foot sole, respectively, and $\dot{u}_{BY}$ and $\dot{\varphi}_{BY}$ are the corresponding robot body velocities. The vector

$$\dot{\Gamma}_j = \begin{bmatrix} \dot{\eta}_{1_j}^T & \cdots & \dot{\eta}_{n_j}^T \end{bmatrix}^T \tag{7}$$

is the combined vector of the D-H parameter rates of all frames in the j-th chain. When all four D-H parameters for all frames in the chain *j* are variable, which is a rare case in practice, the dimension of the vector in (7) is $4n_j \times 1$. The matrix $P_j = D_{0_j} D_{1_j} \cdots D_{(n-1)_j}$ is a 6×6 matrix and Q_j is a matrix similarly defined in terms of both $E_{0_j}, E_{1_j}, \cdots, E_{(n-1)_j}$ and $D_{0_j}, D_{1_j}, \cdots, D_{(n-1)_j}$, and its maximum dimension is $6 \times 4n_j$. We can now combine (6) into a single equation of the following form for the whole robot:

$$\begin{bmatrix} \dot{u}_{SE_1} \\ \dot{\varphi}_{SE_1} \\ \vdots \\ \dot{u}_{SE_m} \\ \dot{\varphi}_{SE_m} \end{bmatrix} = \begin{bmatrix} P_1 \\ \vdots \\ P_m \end{bmatrix} \begin{bmatrix} \dot{u}_{BY} \\ \dot{\varphi}_{BY} \end{bmatrix} + \begin{bmatrix} Q_1 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & Q_m \end{bmatrix} \begin{bmatrix} \dot{\Gamma}_1 \\ \vdots \\ \dot{\Gamma}_m \end{bmatrix} \quad (8)$$

The above equation relates the motions of the feet's soles to the motion of the body and joint angle rates. The fixed (non swing) feet have constant positions but their orientations can vary. Equation (8) can be written in a compact form as

$$\dot{V}_{SE} = P\dot{V}_{BY} + Q\dot{\Gamma} \quad (9)$$

where $\dot{V}_{SE}$ is the $6m \times 1$ vector of composite foot position/orientation rate, $\dot{V}_{BY}$ is the 6×1 robot body position/orientation rate, and $\dot{\Gamma}$ is the $mr \times 1$ composite joint rate vector; and r is the number of variable joints in each leg. Note that some of the variable joints are actuated and some are unctauted and are used for conforming to the terrain topology, as mentioned before. The $6m \times 6$ matrix P and $6m \times mr$ coefficient matrix Q are defined in (8). The following example illustrates the above kinematics modeling procedure.

3. Example

The SIL04 [4], shown in Fig. 1, is a versatile quadruped walking robot that has four identical legs, each with a shoulder joint, a hip joint and a knee joint that are actuated, and three passive joints, i.e. ankle, heel and sole that conform to the terrain. For each leg, we start from the body frame BY and assign the following consecutive frames: shoulder SR, hip HP, knee KE, ankle AE, heel HL, sole SE, and contact CT where the sole contacts with the terrain. The corresponding extended D-H table parameters are given in Table 1. In this table d_0, d_6 are fixed distances along the z axis of the frames, and $a_0, \cdots, a_3$ are fixed distances along x axis. This table shows the conventional D-H parameters as well as rates for variable parameters. It is seen that the only variable parameters are θ_i, which is the rotation about the *z*-axis of the frames, and therefore, derivatives of the other three D-H parameters are zero and not included in the table.

Figure 1. The SILO4 walking robot [4]

Table 1. D-H Parameters for one leg of the quadruped

From F_{i-1}	To F_i	θ_i	d_i	a_i	ε_i	$\dot{\theta}_i$
F0=*BY*	*F1= SR*	θ_0	d_0	a_0	*0*	*0*
F1= SR	*F2= HP*	θ_1	*0*	a_1	*90*	$\dot{\theta}_1$
F2=HP	*F3=KE*	θ_2	*0*	a_2	*0*	$\dot{\theta}_2$
F3=KE	*F4=AE*	θ_3	*0*	a_3	*0*	$\dot{\theta}_3$
F4=AE	*F5=HL*	θ_4	*0*	*0*	*90*	$\dot{\theta}_4$
F5=HL	*F6=SE*	θ_5+90	*0*	*0*	*-90*	$\dot{\theta}_5$
F6=SE	*F7=CT*	θ_6+90	d_6	*0*	*0*	$\dot{\theta}_6$

The variable joints includes the actuated shoulder, hip and knee joint angles, respectively, θ_1, θ_2 and θ_3 as well as three unctauted joints at the foot, i.e. ankle θ_4, heel θ_5 and sole θ_6. Since there are four legs, i.e. $m=4$, there are a total of 12 actuated and another 12 unactuated joints for this quadruped. It is to be noted that for simplifying notations, we have dropped the leg index *j*, in Table 1. To be precise, a parameter such as joint angle θ_2 must be written as θ_{2_1} to indicate the second joint angle in the first chain (leg).

Substituting the values in the first row if Table 1 into (1) - (4), we obtain the transformation from the quadruped base frame to the shoulder frame as

$$\begin{bmatrix}\dot{u}_{SR}\\ \dot{\varphi}_{SR}\end{bmatrix}=\begin{bmatrix} c\theta_0 & s\theta_0 & 0 & -d_0 s\theta_0 & d_0 c\theta_0 & 0\\ -s\theta_0 & c\theta_0 & 0 & -d_0 c\theta_0 & -d_0 s\theta_0 & a_0\\ 0 & 0 & 1 & a_0 s\theta_0 & -a_0 c\theta_0 & 0\\ 0 & 0 & 0 & c\theta_0 & s\theta_0 & 0\\ 0 & 0 & 0 & -s\theta_0 & c\theta_0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{u}_{BY}\\ \dot{\varphi}_{BY}\end{bmatrix} \tag{10}$$

Note that the relationship between the above two frames is fixed since all D-H parameter rates for this transformation are zero. The transformation between shoulder and hip frames are obtained similarly using (1)-(4) and the data in the second row of Table 1, as

$$\begin{bmatrix}\dot{u}_{HP}\\ \dot{\varphi}_{HP}\end{bmatrix}=\begin{bmatrix} c\bar{\theta}_1 & s\bar{\theta}_1 & 0 & 0 & 0 & 0\\ -s\bar{\theta}_1 & c\bar{\theta}_1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & c\bar{\theta}_1 & s\bar{\theta}_1 & 0\\ 0 & 0 & 0 & -s\bar{\theta}_1 & c\bar{\theta}_1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{u}_{SR}\\ \dot{\varphi}_{SR}\end{bmatrix}+\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\dot{\theta}_1 \tag{11}$$

where $\overline{\theta}_1 = \theta_1 - \theta_0$. It is noted that the shoulder and hip joint are in fact a compound joint with no offset. The two transformations from the hip *HP* to the knee *KE* is obtained as

$$\begin{bmatrix} \dot{u}_{KE} \\ \dot{\varphi}_{KE} \end{bmatrix} = \begin{bmatrix} s\theta_2 & -c\theta_2 & 0 & d_2 c\theta_2 & d_2 s\theta_2 & 0 \\ c\theta_2 & s\theta_2 & 0 & -d_2 s\theta_2 & d_2 c\theta_2 & a_2 \\ 0 & 0 & 1 & -a_2 c\theta_2 & -a_2 s\theta_2 & 0 \\ 0 & 0 & 0 & s\theta_2 & -c\theta_2 & 0 \\ 0 & 0 & 0 & c\theta_2 & s\theta_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{u}_{HP} \\ \dot{\varphi}_{HP} \end{bmatrix} + \begin{bmatrix} 0 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}_2 \tag{12}$$

The last transformation involving actuated joints is from the hip to the knee and is found to be

$$\begin{bmatrix} \dot{u}_{AE} \\ \dot{\varphi}_{AE} \end{bmatrix} = \begin{bmatrix} c\theta_3 & s\theta_3 & 0 & 0 & 0 & 0 \\ -s\theta_3 & c\theta_3 & 0 & 0 & 0 & a_3 \\ 0 & 0 & 1 & a_3 c\theta_3 & -a_3 c\theta_3 & 0 \\ 0 & 0 & 0 & c\theta_3 & s\theta_3 & 0 \\ 0 & 0 & 0 & -s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{u}_{KE} \\ \dot{\varphi}_{KE} \end{bmatrix} + \begin{bmatrix} 0 \\ a_3 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}_3 \tag{13}$$

We continue with these transformations using the extended D-H table 1 to go to the ankle frame AE, heel HL, sole SL and finally to the contact frame CT. Substituting these transformations, recursively, we obtain an equation of the form (6) for the first leg, i.e. for $j = 1$. The same procedure is applied to other legs, and finally the composite equations (8)-(9) are formed.

The hand derivations described above are done for illustrative purpose. However, we have developed a Matlab program that takes the extended D-H table and produces the kinematics model of a multi-legged robot either symbolically or numerically.

4. Forms and applications of the kinematics model

The kinematics equation developed in Section 3 can be used for various applications. Consider (10), and suppose the actuated joint angles are to be determined to achieve some desired trajectories for a swing foot and the body. The desired trajectories are generated by a higher level planner. Note that the trajectory for the fixed feet will have zero position values. Depending on the rank of Q, (10) may or may not have a unique solution for $\dot{\Gamma}$. There exists a unique solution if $rank(Q \vdots I \vdots -P) = rank(Q) = mr$ where *I* is the $6m \times 6m$ identity matrix. In this case we can solve the (9) as

$$\dot{\Gamma} = Q^{\#}(\dot{V}_{SE} - P\dot{V}_{BY}) \tag{14}$$

where $Q^{\#}$ is the $mr \times 6m$ pseudo-inverse of *Q*, i.e. $Q^{\#} = (Q^T Q)^{-1} Q^T$. The rank condition implies that the number of joints in each foot must be $r > 6$. However, in practice it is not necessary to specify position and orientation all feet and the rover reference frame. For example it may be only necessary to specify feet positions, and (x, y) and the heading (yaw) trajectories of the rover reference frame. In this case we must partition the quantities in (9) into known (specified) and unknown, as follows

$$\begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}\begin{bmatrix} \dot{\bar{V}}_{SE} \\ \dot{\tilde{V}}_{SE} \end{bmatrix} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}\begin{bmatrix} \dot{\bar{V}}_{BY} \\ \dot{\tilde{V}}_{BY} \end{bmatrix} + \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}\begin{bmatrix} \dot{\Gamma}_1 \\ \dot{\Gamma}_2 \end{bmatrix} \tag{15}$$

where the quantities with the bar represent those specified, and quantities with the tilde denote the unknown quantities that are to be determined. The matrices I_1, I_2 are appropriate identity matrices, and P_1, P_2 and Q_1, Q_2 are the partition matrices of P and Q, respectively. Equation (15) can be rearranged into known and unknown quantities as

$$\begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & -I_2 & P_2 \end{bmatrix}\begin{bmatrix} \dot{\Gamma} \\ \dot{\tilde{V}}_{SE} \\ \dot{\tilde{V}}_{BY} \end{bmatrix} = \begin{bmatrix} I_1 & -P_1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{\bar{V}}_{SE} \\ \dot{\bar{V}}_{BY} \end{bmatrix} \tag{16}$$

or

$$G\,X = H\,Y \tag{17}$$

where $X = \begin{bmatrix} \dot{\Gamma} \\ \dot{\tilde{V}}_{SE} \\ \dot{\tilde{V}}_{BY} \end{bmatrix}$ and $Y = \begin{bmatrix} \dot{\bar{V}}_{SE} \\ \dot{\bar{V}}_{BY} \end{bmatrix}$ are the unknown and known vectors, respectively, and G and H given in (16) are coefficient matrices. Since only foot positions, and robot (x, y) position and heading rates are specified, there are generally more unknown variables and the solution to (18) can be written as

$$X = G^{\#}\,Y + d\,(I_G - G^{\#}G)\,\rho \tag{18}$$

where I_G is an identity matrix with the same dimension as $G^{\#}G$, d is an arbitrary scalar and ρ is a free vector. The free vector can be used for optimization of a performance index ψ provided that $\rho = \partial\Psi / \partial X$ [12]. The performance index can be, for example, a measure for stability of the robot which will be discussed in the following paragraph.

Let w_j, $j = 1,\cdots,m-1$ represent vectors drawn from the body reference point to those feet soles that are in contact with the terrain. Each consecutive pair of these vectors (i.e., w_j and w_{j+1}) forms a plane denoted by π_j. The unit vector perpendicular to this plane is given by

$$\bar{s}_j = \frac{w_j \times w_{j+1}}{\left\| w_j \times w_{j+1} \right\|}\,;\;\; j = 1,\cdots,m-1;\;\; w_m = w_1 \tag{19}$$

For the quadruped with one swing leg, there will be three such planes. Assuming that the body frame BY is set at the center of robot mass, the unit gravity vector $\bar{g}$ can be expressed in terms of pitch and roll angles of the body as

$$\bar{g} = (s\beta_{BY} \quad -s\alpha_{BY}\, c\beta_{BY} \quad -c\alpha_{BY}\, c\beta_{BY})^T \tag{20}$$

Now we define the stability measure as the dot product between unit vectors $\bar{s}_j$ and $\bar{g}$, i.e.

$$\mu_j = \bar{g}^T \bar{s}_j \tag{21}$$

Higher value of μ_j represents a more stable robot. When the gravity vector $\bar{g}$ lies in any of the planes π_j , the vectors $\bar{g}$ and $\bar{s}_j$ become orthogonal, resulting in $\mu_j = 0$. Tipover occurs when $\mu_j < 0$. We must now define an objective function whose optimization results in a stable configuration. Consider maximization of an objective function of the form

$$\psi = k_1 \prod_{j=1}^{m-1} (\mu_j - k_2\alpha_{BY}^2 - k_3\beta_{BY}^2) \tag{22}$$

where k_1, k_2 and k_3 are weighting factors. The first term in (23) represents the stability defined above. In addition to stability, we would also like to keep the robot leveled and thus attempt to minimize the pitch and roll which are represented as the second and third terms in (23). The weighting factors k_1, k_2 and k_3 place relative emphasis between achieving robot stability margin and a balanced configuration.

5. Conclusions

A methodology has been proposed for kinematics modeling of multi-legged robots. The kinematics model utilizes an extended D-H table that includes a listing of variable joints and linkages to capture the motions of the robot moving parts. The equations of the motion can be used for actuation and control to achieve a desired robot motion, after foot and body trajectories have been specified by a higher level foot planner. The method allows optimization of a performance index such as maximization of a stability index. Based on the kinematics developed in this paper, we are currently working on a simulation and animation environment that will show the motion of a quadruped walking along a desired path over a rough terrain.

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