Adaptive TLS Approach for Nonlinearity Compensation in Laser Interferometer

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Abstract

The heterodyne laser interferometer has been widely used in linear displacement and precise measurement field. However the periodic nonlinearity that arises from incomplete laser sources and non-ideal optical components restricts the precise measurement at the nanometer level. In this paper, the total least squares (TLS) algorithm which can obtain optimal compensation parameters of nonlinearity is introduced. Using the TLS algorithm, the nonlinearity error is reduced and the measurement data can be more stabilized. The effectiveness of TLS approach is verified through the comparison of the experimental results with those obtained by a capacitance displacement sensor.

Keywords*: Laser Interferometer, TLS approach, Nonlinearity Compensation*

1. Introduction

Recently the laser interferometry has been widely used in the length-related measurement. The application includes photo lithography in semiconductor manufacture, metrology, and some velocity sensors as well. There are two main kinds of interferometry. The first one is a heterodyne laser interferometer which uses two orthogonal frequencies in a laser head. The other is a homodyne interferometer which uses a single-frequency laser [1]. Between two kinds of interferometry especially the heterodyne laser interferometer is well known for its easy alignment with the optical device, fast response, and high signal-to-noise ratio, etc. However, the measurement accuracy of heterodyne interferometry is restricted by a periodic nonlinearity due to the nonideal laser sources and imperfect optical devices [2-3]. The principal nonlinearity errors happen from frequency mixing, polarization mixing, polarization-frequency mixing, and ghost reflections.

Many researches have been carried out for nonlinearity compensation of a laser interferometer. Theoretical analysis studied by Guo [4] shows that the system nonlinearity arisen from the nonorthogonality and the ellipticity of two-frequency input lights can be compensated using a single phase compensator. Eom [5] proposed a simple digital control system for the frequency stabilization of an internal mirror He-Ne laser. Freitas [6] shows that the second harmonic errors are caused by the two orthogonal linearly polarized inputs of a heterodyne interferometer.

In this paper, we compensate the nonlinearity errors using total least squares (TLS) algorithm with the analytical nonlinearity modeling in [7]. Under the ideal environment the typical configuration for heterodyne interferometry is shown in figure 1. The two frequencies (ω_1, ω_2) are separated through PBS. One of them ($A\,\omega_1$) proceeds to a fixed-mirror while the other signal ($B\ \omega_2$) proceeds to a moving mirror. The reflected two frequencies are recombined again through PBS and are detected by a photo detector (B).

In Fig. 1 if there does not exist nonlinearity each electric field can be expressed as follows. (See [8] for detail.)

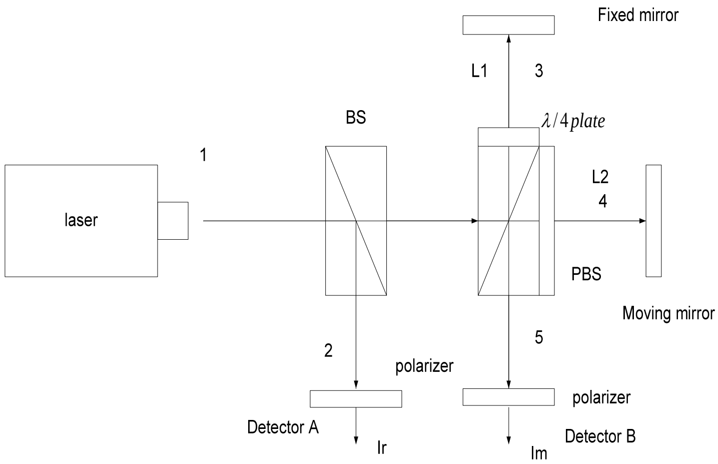

Figure 1. General configuration of heterodyne interferometry: (a) BS (beam splitter),
(b) PBS (polarization beam splitter). (1) $2\pi f_1 t$, $2\pi f_2 t$,
(2) $A2\pi f_1 t + B2\pi f_2 t$, (3) $A2\pi f_1 t$, (4) $B2\pi f_2' t$, (5) $A2\pi f_1 t + B2\pi f_2' t$.

$$
\begin{aligned}
EA1 &= \frac{1}{\sqrt{2}} Ae^{j(2\pi f_1 t)} + \frac{1}{\sqrt{2}} Ae^{j(\phi_A)} \\
EA2 &= \frac{1}{\sqrt{2}} Be^{j(2\pi f_2 t)} + \frac{1}{\sqrt{2}} Be^{j(\phi_B)} \\
EB1 &= \frac{1}{\sqrt{2}} Ae^{j(2\pi f_1 t)} + \frac{1}{\sqrt{2}} Ae^{j(\phi_A)} \\
EB2 &= \frac{1}{\sqrt{2}} Be^{j(2\pi f_2 t)} + \frac{1}{\sqrt{2}} Be^{j(\phi_B + \Delta\phi)}
\end{aligned} \tag{1}
$$

Each intensity of I_r and I_m detected from photo detector A and B is expressed respectively as follows.

$$
\begin{aligned}
I_r &\propto (EA1 + EA2)(EA1 + EA2)^* \\
&= \frac{1}{2}(A^2 + B^2) + AB\cos[2\pi\Delta f t + (\phi_B - \phi_A)] \\
I_m &\propto (EB1 + EB2)(EB1 + EB2)^* \\
&= \frac{1}{2}(A^2 + B^2) + AB\cos[2\pi\Delta f t + (\phi_B - \phi_A) + \Delta\phi]
\end{aligned} \tag{2}
$$

Here A and B are the amplitudes, ϕ_A and ϕ_B are the initial phase values, and Δf is the frequency difference of $f_2 - f_1$. The phase difference between the measurement signal and the reference signal is $\Delta\phi$ which can be transformed to a displacement as

$$\Delta\phi \approx \frac{4\pi n(L_2 - L_1)}{\lambda_m} = \frac{4\pi n \Delta L}{\lambda_m}$$
$$\Delta L = \frac{\Delta\phi\lambda_m}{4\pi n} \qquad (3)$$

where λ_m is a mean wavelength, n is a refractive index, and L stands for the displacement measurement.

2. Nonlinearity analysis in heterodyne laser interferometer

The measurement resolution of heterodyne laser interferometer is limited by the unwanted nonlinearity caused by frequency mixing, polarization mixing, frequency-polarization mixing, and ghost reflections. From the detector B, the intensities of the electric fields which include every error component can be expressed as following forms [7].

$$\begin{aligned} E_{B1} &= \bar{A}e^{j(2\pi f_1 t)} + \beta_f e^{j(2\pi f_2 t)} + \alpha_p e^{j(2\pi f_1' t)} \\ &\quad + \beta_{pf} e^{j(2\pi f_2 t + \pi/2)} \\ E_{B2} &= \bar{B}e^{j(2\pi f_2' t)} + \alpha_f e^{j(2\pi f_1' t)} + \beta_p e^{j(2\pi f_2 t)} \\ &\quad + \alpha_{pf} e^{j(2\pi f_1' t + \pi/2)} \end{aligned} \qquad (4)$$

where $\bar{A}$ and $\bar{B}$ represent $A\cos\phi_1$ and $B\cos\phi_2$, respectively, ϕ_1 and ϕ_2 are defined as the angles between the real polarization and the axes indicated by the PBS. α and β are the cross-reflected beams of Af_1 and Bf_2, respectively. The symbol of a prime means a Doppler-shifted frequency and the small subscripts of *f, p, pf* indicate the nonlinearity from frequency-mixing, polarization-mixing, and frequency-polarization mixing, respectively. To separate DC component we apply a high pass filter. The initial phase value can be disregarded under nonlinearity circumstance. The measurement intensity I_m with no DC components and the disregarded initial phase value are expressed as follows.

$$\begin{aligned} I_m &\propto (E_{B1} + E_{B2})(E_{B1} + E_{B2})^* \\ &= \bar{A}\bar{B}\cos(2\pi\Delta ft + \phi) + (\bar{A}\beta + \bar{B}\alpha)\cos(2\pi\Delta ft) \\ &\quad + (\alpha\beta + \beta_{pf}\alpha_{pf})\cos(2\pi\Delta ft - \phi) \\ &\quad + (\bar{A}\beta_{pf} - \bar{B}\alpha_{pf})\sin(2\pi\Delta ft) + (\alpha\beta_{pf} - \beta\alpha_{pf}) \\ &\quad \times \sin(2\pi\Delta ft - \phi) \end{aligned} \qquad (5)$$

where $\alpha = \alpha_p + \alpha_f$ and $\beta = \beta_p + \beta_f$.

The nonlinearity is included in the second and the third cosine terms and the two sine terms. In Eq. (5) the sine terms indicate errors caused by imperfect alignment of PBS. As the nonlinearity from the optical components is generally smaller than that caused by laser sources [7], we can ignore the partial nonlinearity errors from mirrors and PBS. Therefore if the orientation of PBS is arranged carefully, the last two sine terms in (5) can be eliminated. We use a lock-in-amplifier to obtain a simplified expression with phase information.

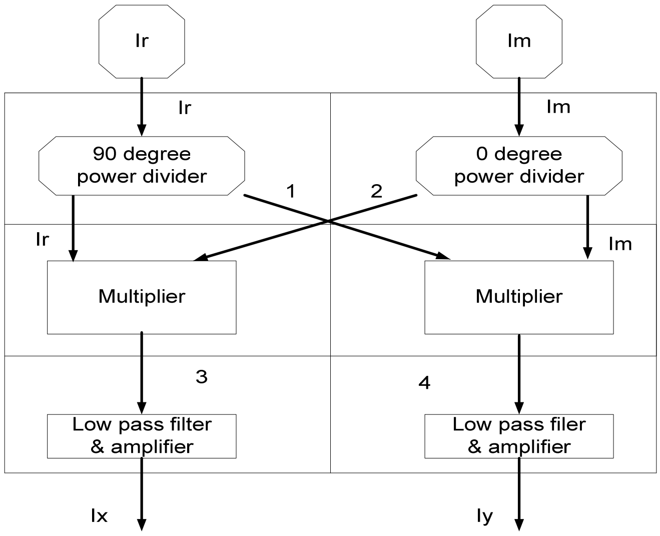

Figure 2. Block diagram of a lock-in-amplifier: (1) $I_r e^{j\pi/2}$, (2) I_m, (3) $I_m \times I_r$, (4) $I_m \times I_r e^{j\pi/2}$.

Figure 2 shows the configuration of a lock-in-amplifier. Each I_r and I_m from photo detector *A* and *B* is connected to the lock-in-amplifier as a signal input. In this process, to obtain the intensities of $I_x$and I_y, the 90 degree shifted signal of I_r and the original I_r is multiplied by another input signal I_m in (5).

$$\begin{aligned} I_m I_r \approx & \cos(2\pi\Delta ft)[\bar{A}\bar{B}\cos(2\pi\Delta ft+\phi)+(\bar{A}\beta+\bar{B}\alpha) \\ & \times\cos(2\pi\Delta ft)+(\alpha\beta+\beta_{pf}\alpha_{pf})\cos(2\pi\Delta ft-\phi)+ \\ & (\bar{A}\beta_{pf}-\bar{B}\alpha_{pf})\sin(2\pi\Delta ft)+(\alpha\beta_{pf}-\beta\alpha_{pf}) \\ & \times\sin(2\pi\Delta ft-\phi)] \\ I_m I_r e^{j\pi/2} \approx & \sin(2\pi\Delta ft)[\bar{A}\bar{B}\cos(2\pi\Delta ft+\phi)+(\bar{A}\beta+\bar{B}\alpha) \\ & \times\cos(2\pi\Delta ft)+(\alpha\beta+\beta_{pf}\alpha_{pf})\cos(2\pi\Delta ft-\phi) \\ & +(\bar{A}\beta_{pf}-\bar{B}\alpha_{pf})\sin(2\pi\Delta ft)+(\alpha\beta_{pf}-\beta\alpha_{pf}) \\ & \times\sin(2\pi\Delta ft-\phi)] \end{aligned} \tag{6}$$

After passing through a low pass filter the magnitudes of I_x and I_y are expressed as follows.

$$I_x = \left(\frac{\bar{A}\bar{B} + \alpha\beta + \beta_{pf}\alpha_{pf}}{2} \right)\cos\phi - \left(\frac{\alpha\beta_{pf} - \beta\alpha_{pf}}{2} \right)\sin\phi$$
$$+ \frac{\bar{A}\beta + \bar{B}\alpha}{2}$$
$$@\delta_1 \cos\phi - \delta_2 \sin\phi + \delta_3$$

$$I_y = \left(\frac{\alpha\beta + \beta_{pf}\alpha_{pf} - \bar{A}\bar{B}}{2} \right)\sin\phi - \left(\frac{\alpha\beta_{pf} - \beta\alpha_{pf}}{2} \right)\cos\phi$$
$$- \frac{\bar{A}\beta_{pf} - \bar{B}\alpha_{pf}}{2} \qquad (7)$$
$$@\mu_1 \sin\phi - \delta_2 \cos\phi - \mu_2$$

where the parameters of $\delta_1, \delta_2, \delta_3, \mu_1$ and μ_2 represent the constants of nonlinearity compensation considering the frequency mixing, polarization mixing, and frequency-polarization mixing. The sine term in I_x and the cosine term in I_y can be eliminated by adjusting PBS carefully.

3. Adaptive nonlinearity compensation using TLS algorithm

As an estimation method, the TLS algorithm finds a solution of a linear system influenced by errors. TLS algorithm has an excellent performance based on least squares (LS) methods. In the linear equations of $Ax \approx b$ the general LS method finds an optimal solution x^* which minimizes $\left\| b - \hat{b} \right\|$ subject to $\hat{A}x = \hat{b}$. Here $\hat{A}$ and $\hat{b}$ mean the matrix without errors. While the TLS algorithm finds a solution x^* which minimizes $\left\| \left[A; b \right] - \left[\hat{A}; \hat{b} \right] \right\|_F$ subject to $\hat{A}x = \hat{b}$. Here $\left\| \cdot \right\|_F$ indicates the Frobenius norm. The singular value decomposition (SVD) [9] is used through TLS method.

In this section we deal with nonlinearity compensation of heterodyne laser interferometer using TLS algorithm. To apply TLS algorithm, we utilize the displacement measurement from a capacitance displacement sensor (CDS) as a reference signal. The measured lengths from CDS are transformed to phase information according to Eq. (3). With the reference phase from CDS, the intensities of I_x and I_y with no nonlinearity errors are represented as follows.

$$I_x = \frac{AB}{2}\cos\phi$$
$$I_y = \frac{AB}{2}\sin\phi \qquad (8)$$

The intensities ($\hat{I}_x$, $\hat{I}_y$) of a laser interferometer under the nonlinearity errors can be expressed in the same way.

$$\hat{I}_x = \frac{AB}{2}\cos\hat{\phi}$$
$$\hat{I}_y = \frac{AB}{2}\sin\hat{\phi} \qquad (9)$$

Since the error terms from imperfect PBS arrangement can be ignored in Eq. (7), we rewrite $\hat{I}_x$ and $\hat{I}_y$ using the measured phase from CDS to remove the unwanted nonlinearity.

$$\hat{I}_x = \left(\frac{\bar{A}\bar{B}+\alpha\beta+\beta_{pf}\alpha_{pf}}{2}\right)\cos\phi + \frac{\bar{A}\beta+\bar{B}\alpha}{2}$$
$$@\delta_1\cos\phi+\delta_3$$
$$\hat{I}_y = \left(\frac{\alpha\beta+\beta_{pf}\alpha_{pf}-\bar{A}\bar{B}}{2}\right)\sin\phi \qquad (10)$$
$$@\mu_1\sin\phi$$

Here the compensation parameters of δ_1, δ_3, and μ_1 with unwanted nonlinearity are expressed as $\delta_1 = (\bar{A}\bar{B}+\alpha\beta+\beta_{pf}\alpha_{pf})/2$, $\delta_3 = (\bar{A}\beta+\bar{B}\alpha)/2$ and $\mu_1 = (\alpha\beta+\beta_{pf}\alpha_{pf}-\bar{A}\bar{B})/2$, respectively. The constants of δ_1, δ_3, and μ_1 terms are used to compensate the shifted elliptical Lissajous to a circular trajectory centered at the origin. To obtain the compensation parameters of δ_1, δ_3, and μ_1, TLS algorithm can be used. To use TLS algorithm $\hat{I}_x$ and $\hat{I}_y$ should be expressed in a linear matrix form as follows.

$$Kx = m$$
$$K=\begin{bmatrix}\cos\phi & 1 & 0\\ 0 & 0 & \sin\phi\end{bmatrix},\; m=\begin{bmatrix}\hat{I}_x\\ \hat{I}_y\end{bmatrix} \qquad (11)$$

Where 0 entries in Eq. (11) mean the eliminated $\cos\phi$ term and $\sin\phi$ term under ideal PBS arrangement, and the compensation parameters of x is $x^T = [\delta_1, \delta_3, \mu_1]$. To take a SVD, the above matrix in (11) can be transformed to an augmented one as

$$\bar{K} = \begin{bmatrix}\cos\phi & 1 & 0 & \vdots\, \hat{I}x\\ 0 & 0 & \sin\phi & \vdots\, \hat{I}y\end{bmatrix} \qquad (12)$$

Using the result of SVD, we can obtain the optimal compensation parameters using TLS algorithm from right singular vector [10].

$$x^* = -\frac{1}{(v_{n+1,n+1})}[v_{1,n+1,}\cdots v_{n,n+1}]^T \qquad (13)$$

The compensated intensities of $\hat{I}_x$ and $\hat{I}_y$ can be transformed to optimal one using the compensation values of δ_1^*, δ_3^* and μ_1^*

$$I_x^* = \frac{\hat{I}_x - \delta_3^*}{2\delta_1^*} AB, \quad I_y^* = \frac{\hat{I}_y}{2\mu_1^*} AB \tag{14}$$

The compensated intensities in (14) form approximately an ideal circle with a radius of *AB/2*. Finally the compensated phase (ϕ^*) and the measurement length using arc-tangent can be obtained as

$$\phi^* = \tan^{-1}\left(\frac{I_y^*}{I_x^*}\right)$$
$$L^* = \frac{\phi^* \lambda_m}{4\pi n} \tag{15}$$

4. Experimental results

In this section we demonstrate the effectiveness of TLS algorithm with some experimental results and computer simulation results. The laser head used in experiment is WT307b from Agilent Technologies with 0.63299112 μm wavelength (λ_m). The amplitude of *A* and *B* is set to 1 volt under the experimental circumstance and refractive index (*n*) is 1.00000002665. As a translational stage to be measured in nanometer-scale, the linear piezo-electric transducer (PI: p-621.1CL) is used. We utilize the capacitance displacement sensor (PI: D-100) as a reference signal.

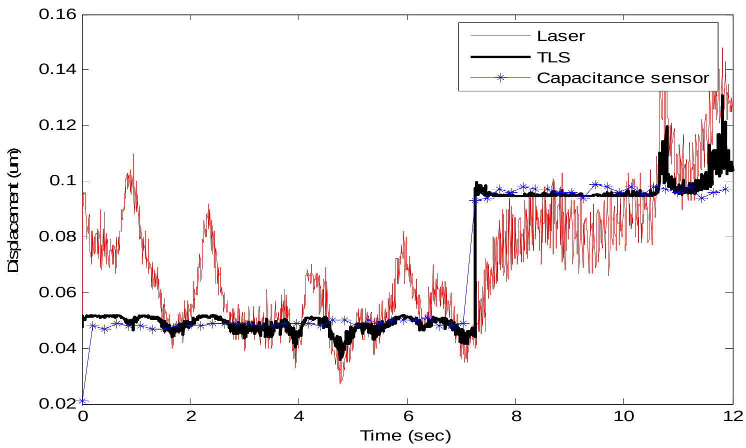

Figure 3. Comparison of fixed positions (*y* = 50, 100 *n*m)

Figure 3 shows the performance of the TLS method compared with CDS and laser measurement according to each fixed position, such as 50 *n*m and 100 *n*m, respectively. The solid line is a displacement measured from laser interferometer with no compensation and the starred line is a reference signal from CDS. The thick line is a compensated displacement measurement using TLS algorithm. As shown in Fig. 3 we can obtain more stable measurement values which have less chattering on each fixed position after applying the adaptive compensation algorithm.

Figure 4 shows the performance of the TLS method through the computer simulation for a moving stage within the range of y = 0 ~ 150 nm. Here, the sold line is a displacement measurement including nonlinearity errors, and the thick line is the compensated displacement measurement using TLS algorithm. The reference signal which indicates an ideal displacement measurement is represented as a starred line. As shown in Fig. 4 we can obtain the compensated measurement values with TLS method.

Figure 5 shows the difference between laser displacement measurement and compensated measurement using TLS method for a moving stage with experimental results. In the same way the reference signal from CDS is used as a reference. As shown in Fig. 5, the displacement measurement after TLS compensation is much similar to CDS displacement values.

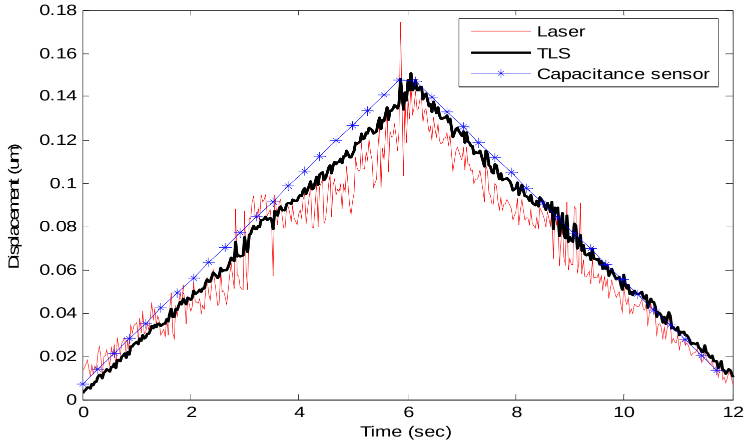

Figure 4. Nonlinearity compensation simulation for a moving stage.

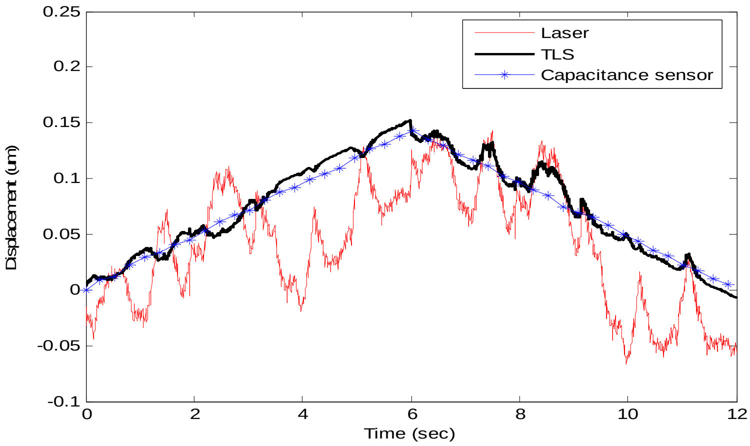

Figure 5. Comparison of a moving stage (y = 0 ~ 150 nm).

5. Conclusion

As a device of a nano-scale precise displacement measurement, the heterodyne laser interferometer is affected by unwanted nonlinearity errors which include frequency mixing, polarization mixing, frequency-polarization mixing, and ghost reflections. In this paper, an approach using the TLS algorithm for nonlinearity compensation is suggested. To make the problem solving more simple, the error caused by PBS arrangement is ignored. It is shown that the nonlinearity error can be compensated by optimal parameters from TLS method. Also it is represented that TLS method can reduce the chattering through some experimental results. Therefore, the TLS compensation algorithm enables the interferometer robust to nonideal environments.

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