

TDoA based UGV Localization using Adaptive Kalman Filter Algorithm

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Abstract

The measurement with a signal of time difference of arrival (TDoA) is a widely used technique in source localization. However, this method involves much nonlinear calculation. In this paper, we propose a method that needs less computation for UGV location tracking using extended Kalman filtering based on non linear TDoA measurements. To overcome the inaccurate results due to limited linear approximation, this paper suggests a position estimation algorithm based upon an adaptive fading Kalman filter. The adaptive fading factor enables the estimator to change the error covariance according to the real situation. Through the comparison with other analytical methods, simulation results show that the proposed localization method achieves an improved accuracy even with reduced computational efforts.

Keywords: Adaptive Kalman Filter Algorithm, TDoA, UGV

1. Introduction

The confirmation of a present position and the estimation of a future path are one of the most important techniques for unmanned ground vehicle (UGV) realization [1]. For many cases, the global positioning system (GPS), which is famous for the good performance in position estimation, has been widely used [2-3], [9]. However, there have been some problems to be applied for UGV. For example, GPS system needs separate receivers, and constantly it has to obtain signals from more than 3 satellites for position tracking. It is also vulnerable to the indoor case or the reflected signal fading.

There have been many efforts to solve the localization problem in a closed analytical method [4-6]. In case of analytical method, it demands many computations and most of them do not consider the external noises which happen in real process. Therefore there exists a limit to be used in a real time process directly such as UGV position estimation.

As a position tracking method, we apply the time difference of arrival (TDoA) signal which needs no special equipment. To reduce the computational efforts and to estimate more precisely even for noise added real process, the Kalman filter can be used effectively as an estimator. However, to measure the position of UGV based on TDoA signal, it needs to use an extended Kalman filter (EKF) [7] through linear approximation of localization and TDoA measurement process at each estimation iteration.

In this paper, we use the modified Kalman filter algorithm (adaptive fading Kalman filter: AFKF) [8] for precise position tracking. To do so, we do the system modeling for UGV localization process. Using the adaptive fading factors, the error covariance can be modified to follow the real system model. Finally some simulation results show the effectiveness of the proposed position estimation algorithm through the comparison with EKF.

2. System modeling for UGV localization

2.1. Analytical methods

The signals sent to each base station (BS: the known position) by a mobile station (MS: the unknown position) have a time difference because of the BS's scattered location. The basic concept of position estimation is to use the hyperbolic curves from the definition of TDoA as shown in figure 1. The analytic method to find the target position can be summarized as follows.

Let $\mathbf{p}_i = \text{col}\{x_i, y_i\}$, $i=1, 2, 3, \dots, m$ be the known locations of m receivers and let $\mathbf{u} = \text{col}\{x, y\}$ be the unknown location of the UGV. Using the norm of a difference between the receiver position and UGV, we define as

$$\begin{aligned} r_i^o &= \|\mathbf{u} - \mathbf{p}_i\| \\ r_{i1}^o &= ct_{i1} \end{aligned} \quad (1)$$

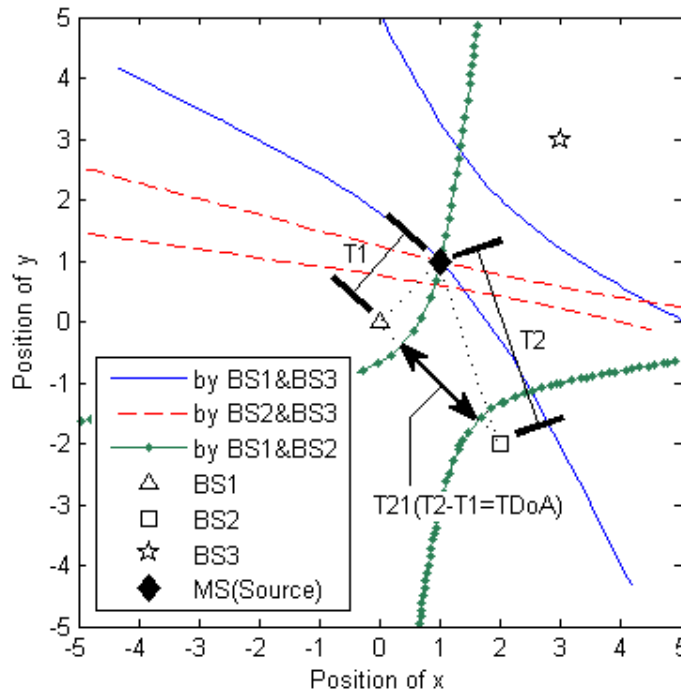


Figure 1. Geometric method using hyperbola.

where t_{i1} is the TDoA between the i -th receiver and the first one, c is the signal propagation velocity. Using the relation in (1), it is easily derived as

$$r_i^o = r_1^o + r_{i1}^o, \quad i = 2, 3, \dots, m. \quad (2)$$

After squaring Eq. (2) and using $(r_i^o)^2 = \langle \mathbf{u} - \mathbf{p}_i, \mathbf{u} - \mathbf{p}_i \rangle$, the equation regarding the UGV location can be expressed in the following nonlinear form.

$$\begin{aligned} \|\mathbf{u}\|^2 - 2\mathbf{p}_i^T \mathbf{u} + \|\mathbf{p}_i\|^2 &= \|\mathbf{u}\|^2 - 2\mathbf{p}_i^T \mathbf{u} + \|\mathbf{p}_i\|^2 \\ &+ 2r_1^o r_{i1}^o + (r_{i1}^o)^2, \quad i = 2, \dots, m \end{aligned} \quad (3)$$

Eq. (3) can be rewritten as

$$\begin{aligned} \|\mathbf{p}_i\|^2 - \|\mathbf{p}_i\|^2 + (r_{i1}^o)^2 &= 2\langle \mathbf{p}_i - \mathbf{p}_1, \mathbf{u} \rangle \\ &- 2r_1^o r_{i1}^o, \quad i = 2, \dots, m \end{aligned} \quad (4)$$

Since r_1^o is the distance from the UGV to receiver #1, the solution of $\mathbf{u}(x, y)$ must satisfy the following additional constraint .

$$(r_1^o)^2 = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 \quad (5)$$

The above equations of (4) and (5) can be simplified by placing the position of receiver #1 as the origin of the coordinate system, i.e., $\mathbf{p}_1 = 0$.

$$\begin{aligned} \frac{1}{2}(-\|\mathbf{p}_i\|^2 + (r_{i1}^o)^2) &= -\langle \mathbf{p}_i, \mathbf{u} \rangle - r_1^o r_{i1}^o, \quad i = 2, 3, \dots, m \\ x^2 + y^2 + z^2 - (r_1^o)^2 &= 0 \end{aligned} \quad (6)$$

The nonlinear localization equation can be rewritten in the form of linear matrix equation.

$$\begin{aligned} \boldsymbol{\rho} &= - \begin{bmatrix} r_{21}^o \\ \vdots \\ r_{m1}^o \end{bmatrix}, \mathbf{d} = \begin{bmatrix} \langle \mathbf{p}_2, \mathbf{p}_2 \rangle \\ \vdots \\ \langle \mathbf{p}_m, \mathbf{p}_m \rangle \end{bmatrix}, \mathbf{G} = \begin{bmatrix} \mathbf{p}_2^T \\ \vdots \\ \mathbf{p}_m^T \end{bmatrix}, \\ \mathbf{h} &= \frac{1}{2} \begin{bmatrix} \|\mathbf{p}_2\|^2 - (r_{21}^o)^2 \\ \vdots \\ \|\mathbf{p}_m\|^2 - (r_{m1}^o)^2 \end{bmatrix} = \frac{1}{2}(\mathbf{d} - \boldsymbol{\rho} \bullet \boldsymbol{\rho}) \end{aligned} \quad (7)$$

where \bullet denotes the Hadamar elementwise vector multiplication, $\boldsymbol{\rho}, \mathbf{h}$ and \mathbf{d} are $(m-1)$ -vectors, and \mathbf{G} is an $(m-1) \times 3$ matrix, respectively.

Now the vector \mathbf{u} is then a solution of the following UGV localization problem.

$$\begin{aligned} \mathbf{G}\mathbf{u} &= \mathbf{h} + \boldsymbol{\rho}r_1^o \\ (r_1^o)^2 &= \langle \mathbf{u}, \mathbf{u} \rangle \end{aligned} \quad (8)$$

2.2. System modeling

To implement the TDoA based localization method using hyperbolic curve under the noise added real situation, it is required to reduce the computational effort to solve in real time process. In real situation, the TDoA is affected by the external noises.

$$t = t_o + \Delta t \quad (9)$$

where t_o is the ideal TDoA and Δt is the added noise. As a robust solution, Kalman filter approach can be more efficient than hyperbolic method if we use TDoA as measurement data.

To apply Kalman filter in a localization problem, the state-space equation needs to be formulated. Therefore the UGV translational problem can be modeled in a discrete form as following.

$$s(k+1) = As(k) + Bu(k) + w(k)$$

$$A = \begin{bmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (10)$$

where $s(k) = [x \ y \ \dot{x} \ \dot{y}]^T$, $u(k)$ is the known velocity of a moving UGV, Δ is the sampling time, and $w(k)$ is the process noise in AWGN. The output equation can be formulated using the measurement of TDoA value.

$$z(k) = h(s(k), v(k))$$

$$= \frac{1}{c} (\|Ms(k) - p_i\| - \|Ms(k) - p_j\|) + v(k) \quad (11)$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

where p_i is the position of i -th receiver, and c is the propagation speed, and $v(k)$ is the measurement noise in AWGN. The output $z(k)$ is the TDoA value obtained from the definition. In this model, however, because of the nonlinear terms in the output equation, the modified Kalman filter (extended Kalman filter: EKF) should be used [10]. The EKF algorithm can be used through the linearization of partial differentiation .

$$H_k \approx \frac{\partial h_k}{\partial s}, \quad V_k \approx \frac{\partial h_k}{\partial v} \quad (12)$$

3. Localization using adaptive fading Kalman filter

The extended Kalman filter is a well-known method for position tracking system, but the divergence of estimated result from a modeling error is considered to be a crucial weaknesses. Generally, the dynamic properties and errors are considered together in Kalman filter. However the divergence could happen in any circumstance if the result of real system doesn't match to the ideal value of the estimated model. Especially, for the nonlinear system model based on TDoA data the divergence from the real value can

cause a serious problem. As a solution to prevent the divergence of Kalman filter is the adaptive fading Kalman filter (AFKF), which uses the fading factor in updating the Kalman gain. The basic theory is that the estimated result is regulated by the degree of divergence (DoD).

The EKF algorithm consists of two parts. The summary for EKF is given as

1) Time update (prediction part)

(a) The state projection:

$$\hat{s}_k^- = f(\hat{s}_k, u_k, w_k)$$

(b) The error covariance projection:

$$P_k^- = A_k P_{k-1} A_k^T + W_k Q_k W_k^T$$

2) Measurement update (correcting part)

(a) Kalman gain update:

$$K_k = P_k^- H_k^T [H_k P_k^- H_k^T + V_k R_k V_k^T]^{-1}$$

(b) The error covariance update:

$$P_k = [I - K_k H_k] P_k^-$$

(c) The estimate update with measurement z_k :

$$\hat{s}_k = \hat{s}_k^- + K_k [z_k - h(\hat{s}_k^-, u_k, v_k)]$$

$$\hat{z}_k = h(\hat{s}_k^-, u_k, v_k)$$

$$\phi_k = z_k - \hat{z}_k$$

As stated in (1) time update process, we evaluate the estimate of system state by time flow, and we calibrate the state estimate comparing the differences between a real measured value and an estimate through an estimation modeling in (2) measurement update process.

The EKF itself cannot guarantee a stable accuracy, since it has the possibility of divergence under real circumstances. This happens from the linearization of nonlinear system. The more accurate position tracking can be possible with a precision estimation algorithm. The estimation accuracy can be increased through AFKF by adding an adaptive fading factor. The suboptimal fading factor λ_k adjusts the variance of the predicted state vector.

$$P_k^- = \lambda_k A_k P_{k-1} A_k^T + w_k Q_k W_k^T \quad (13)$$

where $\lambda_k = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_m)$. If the estimated values get close to the real value, the fading factor becomes $\lambda_k \leq 1$. That is, it enters into a steady state process. In other cases, the fading factor λ_k is updated as follows:

$$\lambda_{k+1} = \max \left\{ 1, \frac{\alpha \cdot \text{tr}[N_k]}{\text{tr}[M_k]} \right\}, \quad (14)$$

$$N_k = C_0 - R_k - H_k Q_k H_k^T$$

$$M_k = H_k A_k P_k A_k^T H_k^T$$

$$C_0 = \begin{cases} \frac{\phi_0 \phi_0^T}{2}, & k = 0 \\ \frac{\lambda_k \phi_k \phi_k^T}{1 + \lambda_k}, & k \geq 1 \end{cases}$$

where α is a positive constant, and $tr[\cdot]$ means a trace of a matrix.

The process of (14) is called as an adaptive fading loop. Through the adaptive fading loop, the divergence degree can be determined and the error covariance P_k can be changed adaptively with the fading factor λ_k . Finally the estimate of the TDoA output (\hat{z}_k) can be updated. To confirm the divergence due to the modeling errors, we can define the degree of stable (DoS) as following. If $\|z_k - \hat{z}_k\| \leq \varepsilon$ and $\hat{z}_k - \hat{z}_{k-1} \leq 0$, then $\lambda_{k+1} \leq 1$. In other cases with $\lambda_k > 0$, then $\lambda_{k+1} = \lambda_k - 0.1$. The fading factor λ_k can be adjusted adaptively using DoS.

4. Simulation results

We suppose that the UGV is moving at a constant speed but it changes a direction every 2 sec. Figure 2 shows the UGV circumstance with two receivers. In Fig. 2, the dotted line is the expected moving trajectory of UGV, and the solid line is the real trajectory of UGV. The real trajectory is different from the ideal one because of the measurement noise $w(k)$ and the process noise $v(k)$.

Figure 3 shows the performance of the proposed localization algorithm through the comparison with EKF. The performance is measured in terms of the norm of positioning error, i.e., $\|s(k) - \hat{s}(k)\|$. As shown in Fig. 3, the positioning error is much smaller than that of EKF. It means that the position estimation with AFKF is tracking more precisely to the real value $s(k)$. Figure 4 shows the change of the fading factor λ_k . It changes very steeply to correct the position error from the beginning and the estimate $\hat{s}(k)$ gets close to the real value within ε -neighborhood after 1 sec since the fading factor becomes small.

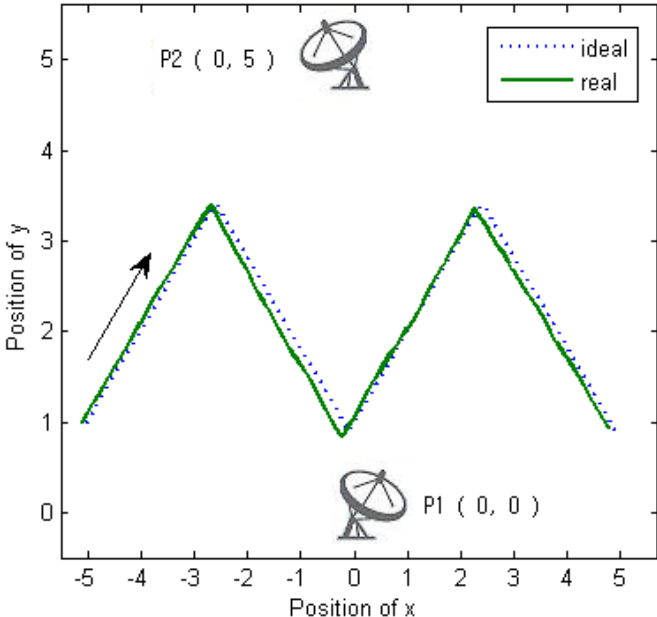


Figure 2. Simulation circumstance for UGV

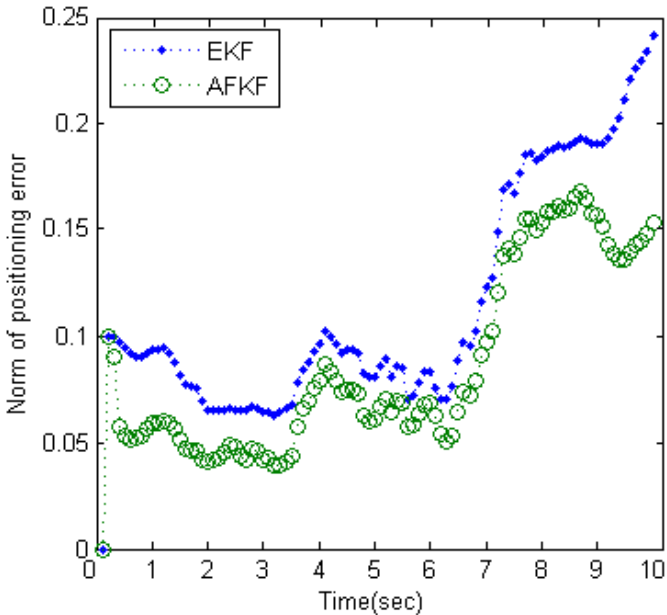


Figure 3. Performance comparison for EKF and AFKF

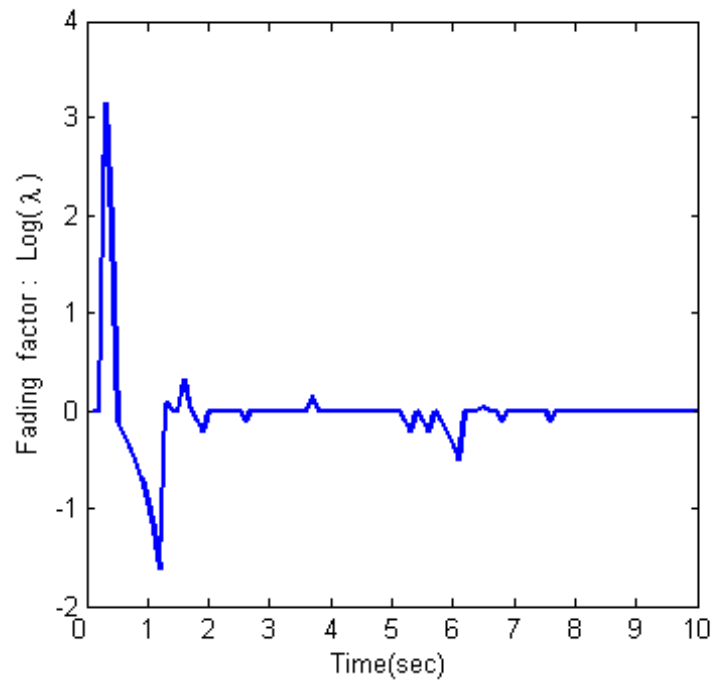


Figure 4. Change of fading factor, λ_k .

Figure 5 shows the results of path estimation for 2 different methods. In this figure, the solid line is a real UGV trajectory, the dotted line is a trajectory estimated by EKF, and the circle-dotted line is the one by AFKF. With the adaptive fading factor, the error covariance has been changed each estimation step. As shown in Fig. 5, the trajectory estimation using AFKF is close to the real value under noise added real circumstance.

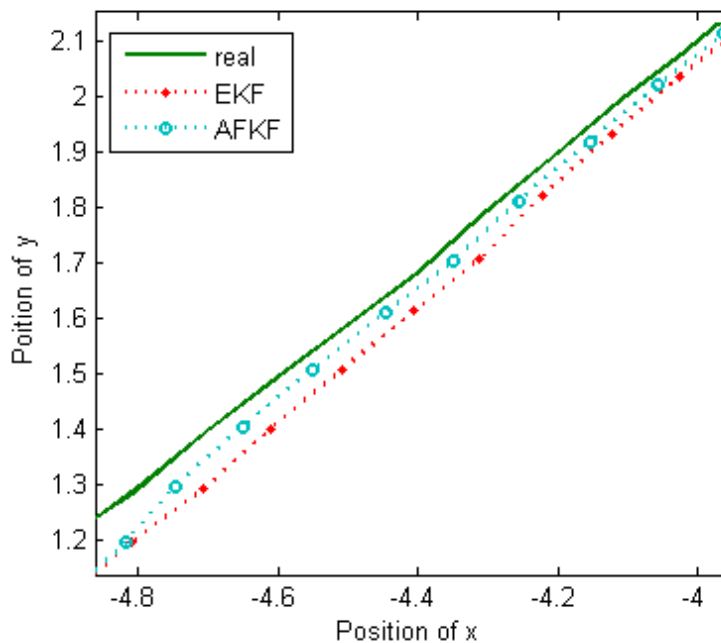


Figure 5. Comparison of trajectory estimation

5. Conclusion

In case of TDoA based position tracking system, it requires much calculation efforts to solve the nonlinear

process. In this paper, we applied EKF to estimate the precise position under real situation. The EKF gets the estimated value through linearization of a nonlinear process. However, there still remains the restriction of divergence if the result of real system is much different from the ideal estimated model which is approximated by partial differentiation.

To overcome the divergence problem in EKF, we applied AFKF algorithm which changes the error covariance using an adaptive fading factor. Through the simulation results, it is confirmed that the trajectory estimation using AFKF follows the real one more precisely. The positioning error from AFKF is less than that performed by EKF.

References

- [1] R. Madhavan and C. Schlenoff, "The effect of process models on short-term prediction of moving objects for unmanned ground vehicles," *International IEEE Conf. Intelligent Transportation Systems*, pp. 471-476, 2004.
- [2] D.J. Torrieri, "Statistical theory of passive location systems," *IEEE Tr. on Aerospace and Electronic Systems*, Vol. AES-20, No. 2, pp. 183-197, 1984.
- [3] H.K. Kwang, J.G. Lee, C.G. Park, "Adaptive two-stage EKF for INS-GPS loosely coupled system with unknown fault bias", *Jour. of Global Positioning System*, Vol. 5, pp. 62-69, 2006.
- [4] K.C. Ho and Y.T. Chan, "Solution and performance analysis of geolocation by TDoA," *IEEE Tr. Aerospace & Electronic Systems*, Vol. 29, No. 4, pp. 1311-1322, 1993.
- [5] M. Najar and J. Vidal, "Kalman tracking based on TDOA for UMTS mobile location," *IEEE International Symp. Personal, Indoor and Mobile Radio Communications*, Vol. 1, pp. B45-B49, 2001.

- [6] H.C. Schau and A.Z. Robinson, "Passive source localization employing intersecting spherical surfaces from Time-of-Arrival differences," *IEEE Tr. Acoustics, Speech, & Signal Processing*, Vol. ASSP-35, No. 8, pp. 1223-1225, 1987.
- [7] L.J. Levy, "The Kalman filter: navigation's integration workhorse", *Annual report in Applied Physics Laboratory*, Johns Hopkins University, 1997
- [8] Q. Xia, M. Rao, Y. Ying, and X. Shen "Adaptive fading Kalman filter with an application," *Automatica*, Vol. 30, No. 8, pp. 1333-1338, 1994.
- [9] C.P. Gleason, *Tracking human movement in an indoor environment using mobility profiles*, Master's thesis, University of Nebraska-Lincoln, August, 2006.
- [10] S.J. Julier and J.K. Uhlmann, "Unscented filtering and nonlinear estimation," *IEEE Review*, Vol. 92, No. 3, pp. 401-422, 2004.

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