Multivariable Integrated Model Predictive Control of Nuclear Power Plant

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Abstract. This paper presents the method of multivariable integrated model predictive control for the nuclear power plant, including designing controller and building the modeling of turbine and once-through steam generator. The simulation results show that the rotate speed of turbine and outlet pressure of steam generator under the multivariable integrated model predictive control is faster steady and smaller overshoot than under the PID control, when the power load of nuclear power plant is changed. Simulation test indicates that the multivariable integrated model predictive control can obtain better control performance for nuclear power plant.

Keywords: Nuclear power plant; Multivariable integrated model; Predictive control.

1 Introduction

Since fossil-fuel resources are lesser and lesser in the earth, the growing demand for nuclear energy as clean energy sources are more and more. Nuclear power plant is a class of important application of nuclear energy. Because nuclear power plant is a large complex systems composed of the primary loop, secondary loop and a number of auxiliary devices, the PID control has limitation, especially to solve the coupling problem of rotate speed of turbine and steam pressure of outlet of steam generator. In order to enhancing of the control performance for nuclear power plant, new control law is studied to improve the running safety and stability of nuclear power plant.

The predictive control is presented in 70s of the last century. Subsequently, it is applied successfully in industrial process control field by Richalet [1], and Rouhani [2]. Generalized predictive control (GPC) is one of the widely applying predictive control algorithm [3], which has shown its characteristic of overcoming time-varying, time-delay and nonlinearity remarkably in industrial field. Because there is a large amount of computation for elementary GPC algorithm, an improved GPC algorithm is presented in this paper, that is called multivariable integrated model predictive control.

This paper describes multivariable integrated model predictive control for the rotate speed of turbine and steam pressure of outlet of steam generator [4]-[7]. The algorithm of the multivariable integrated model predictive control is studied. The mathematical models of turbine and once-through steam generator and other equipments are built. The coordinated controller is designed. The simulation results show that control effect under multivariable integrated model predictive control is better than that under PID control.

2 Mathematical Models

2.1 Mathematical Models of Steam Turbine

The turbine system is divided into thermodynamic system and propulsive system based on the energy transition mode. In this system, turbine converts hot energy into mechanical energy; the propulsive system converts mechanical energy into pulsive energy. In this paper, the mathematical models of turbine system are built, such as steam quantity, steam expansion work and rotor model.

Steam Flow. When considering the variation of load, the flux changing of steam may be expressed as:

$$\frac{G}{G'} = \sqrt{\frac{P_i^2 - P_2^2}{P_i'^2 - P_2'^2}} \sqrt{\frac{T_i'}{T_i}} \sqrt{1 - \frac{\Delta\rho}{1-\rho}} \quad . \tag{1}$$

Where P_i, P_i' is the pressure in front of stage; P_2, P_2' is the pressure of after stage; T_i, T_i' is temperature; G, G' is flux; ρ is reaction， $\Delta\rho$ is the changing of reaction.

Steam Expansion Work. The formula of steam expansion work may be written as:

$$\begin{aligned} & s_1 = f_{s1}(p_1, h_1), s_2 = s_1 \\ & h_{2r} = f(p_2, s_2), \Delta h = \eta(h_1 - h_{2r}) \quad . \\ & W = G_{in} \times \Delta h \end{aligned} \tag{2}$$

Where s_1, s_2 is the entropy of stage inlet and outlet of turbine; p_1, p_2 is the pressure before stage and after stage; h_1 is the enthalpy before stage; h_{2r} is the enthalpy after stage; Δh is the changing of enthalpy; η is stage efficiency; W is stage power.

Rotor Model. The rotate speed of rotor and propeller shaft is written as:

$$\begin{aligned} & J \frac{d\varpi}{dt} = M_T - M_{SZ} \\ & J_S \frac{d\varpi_S}{dt} = M_{SZ} - M_S \end{aligned} \quad . \tag{3}$$

Where ϖ, ϖ_S is the angular velocity of turbine rotor and propeller; J, J_s is the moment of inertia of turbine rotor and propeller; M_T, M_{sz}, M_s is the moment of turbine rotor, shaft and propeller, respectively.

2.2 Mathematical Models of Once-through Steam Generator

In once-through steam generator, the secondary loop water is heated into steam by primary loop coolant through once-through tube. According to water changing status and diathermanous characteristics in secondary loop, the once-through steam generator is divided into three parts, such as preheating section, boiling section and superheated steam section, to see Fig. 1.

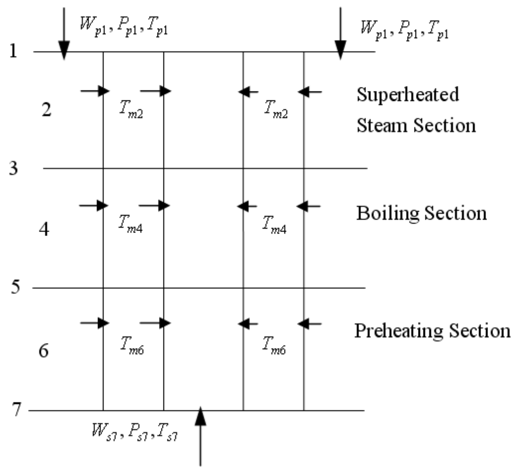

Fig. 1. Diathermanous illustration of once-through steam generator

Preheating Section. The dynamic mathematical model of length changing of preheating section of once-through steam generator is expressed as:

$$\frac{dl_6}{dt} = \frac{W_{s7} - W_{s5}}{F_s \gamma_{s6}} \quad . \tag{4}$$

Boiling Section. The dynamic mathematical model of length changing of boiling section of once-through steam generator is expressed as:

$$\frac{dl_4}{dt} = \frac{W_{s5} - W_{s3}}{F_s \gamma_{s4}} \frac{Bu_{fs4} + u_{gs4}}{B+1} \quad . \tag{5}$$

Superheated Steam Section. The dynamic mathematical model of length changing of superheated steam section of once-through steam generator is expressed as:

$$\frac{dl_2}{dt} = (\frac{W_{s3} - W_{s1}}{F_s l_2} - \frac{d\gamma_{s2}}{dt}) \cdot \frac{l_2}{\gamma_{s2}} \quad . \qquad \textbf{(6)}$$

The dynamic mathematical model of the other variables can be uniformly written as:

$$\frac{dT_{pi}}{dt} = \frac{c_{pi}(W_{pi-1}T_{pi-1} - W_{pi+1}T_{pi+1}) - Q_{pi}}{c_{pi}F_p\gamma_{pi}l_i} - \frac{T_{pi}}{l_i}\frac{dl_i}{dt} \quad . \qquad \textbf{(7)}$$

$$\frac{dT_{mi}}{dt} = \frac{Q_{pi} - Q_{mi} - Q_{si}}{c_{mi}\gamma_{mi}F_m l_i} - \frac{T_{mi}}{l_i}\frac{dl_i}{dt} \quad . \qquad \textbf{(8)}$$

$$\frac{dT_{si}}{dt} = \frac{W_{si+1}T_{si+1} - W_{si-1}T_{si-1} + Q_{si}}{c_{si}F_s\gamma_{si}l_i} - \frac{T_{si}}{l_i}\frac{dl_i}{dt} \quad . \qquad \textbf{(9)}$$

Where $i = 2,4,6$; l_2, l_4, l_6 is available length of superheated steam section, boiling section and preheating section, respectively; $W_{s1}, W_{s3}, W_{s5}, W_{s7}$ is medium flux of secondary loop, at node 1, 3, 5 and 7, respectively; $W_{p1}, W_{p3}, W_{p5}, W_{p7}$ is coolant flux of primary loop, at node 1, 3, 5 and 7, respectively; T_{p2}, T_{p4}, T_{p6} is temperature of coolant in primary loop side, respectively, in superheated steam section, in boiling section and in preheating section; T_{s2}, T_{s4}, T_{s6} is temperature of medium in the secondary loop side, respectively, in superheated steam section, in boiling section and in preheating section; T_{m2}, T_{m4}, T_{m6} is average temperature of tube in superheated steam section, in boiling section and in preheating section, respectively; F_s, F_p is cross section area of medium path in secondary loop and in primary loop, respectively; F_m is average cross section area; Q_{s2}, Q_{s4}, Q_{s6} is heat quantity of secondary loop, at node 1, 3, 5 and 7, respectively; $\gamma_{(\cdot)}$ represents density of preheating section, boiling section and superheated steam section, respectively; $c_{(\cdot)}$ represents specific heat of preheating section, boiling section and superheated steam section, respectively.

3. Multivariable Integrated Model Predictive Control

3.1 Multivariable Integrated Model Predictive Control System

The block diagram of multivariable integrated model predictive control for nuclear power plant is shown in Fig. 2.

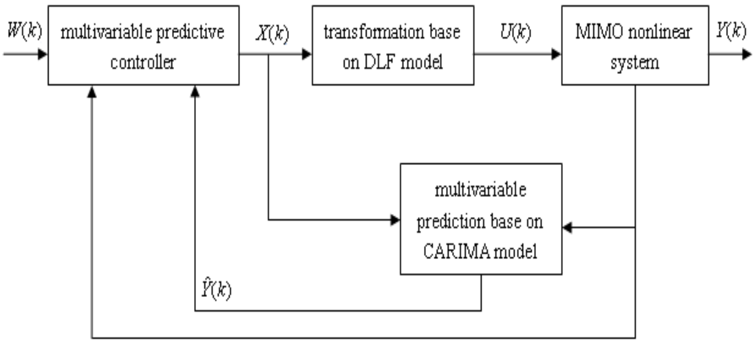

Fig. 2. Block diagram of multivariable integrated model predictive control for the nuclear power plant

In Fig. 2, the integrated model includes two parts: the nonlinear static DLF network and dynamic linear CARIMA model.

The nonlinear static DLF network model is written as:

$$X(k) = W_F U(k) + N_{BP}(U(k)) \quad . \qquad \textbf{(10)}$$

Where W_F is weight matrix of connecting input layer with output layer; $N_{BP}(\bullet)$ represents the mapping of multilayer BP network.

The dynamic linear CARIMA model is expressed as follow:

$$A_0(z^{-1})Y(k) = B_0(z^{-1})X(k) + C_0(z^{-1})\xi(k)/\Delta \quad . \qquad \textbf{(11)}$$

Where $Y(k)=[y_1(k)y_2(k)\cdots y_n(k)]^T$; $X(k)=[x_1(k)x_2(k)\cdots x_n(k)]^T$; $U(k)=[u_1(k)u_2(k)\cdots u_n(k)]^T$; $A_0(z^{-1})$ and $B_0(z^{-1})$ is $n\times n$ matrix.

3.2 Multivariable Predictive Control Algorithm

Following equations can be gotten by equation (11).

$$\begin{aligned} A_1(z^{-1})y_1(k) &= B_{11}(z^{-1})x_1(k-1) + B_{12}(z^{-1})x_2(k-1) \\ A_2(z^{-1})y_2(k) &= B_{21}(z^{-1})x_1(k-1) + B_{22}(z^{-1})x_2(k-1) \end{aligned} \quad .$$

Thus we get following equation base on Diophantine equation.

$$I = E_j^{'}(z^{-1})A(z^{-1})\Delta + z^{-j}F_j(z^{-1}) \quad .$$

Then the predictive equations are:

$$\begin{aligned} \hat{Y}_1 &= G_{11}(z^{-1})\Delta X_1 + G_{12}(z^{-1})\Delta X_2 + f_1 \\ \hat{Y}_2 &= G_{21}(z^{-1})\Delta X_1 + G_{22}(z^{-1})\Delta X_2 + f_2 \end{aligned} \quad . \qquad \textbf{(12)}$$

Where $\hat{Y}_j = \left[\hat{y}_j(k+1)\,\hat{y}_j(k+2)\cdots\hat{y}_j(k+n) \right]^T$,

$\Delta X_j = \left[\Delta x_j(k)\Delta x_j(k+1)\cdots\Delta x_j(k+n-1) \right]^T$,

$f_j = \left[f_j(k+1) f_j(k+2)\cdots f_j(k+n) \right]^T \; (j=1,2)$,

$$G_{11} = \begin{bmatrix} g_{110} & 0 & 0 & 0 \\ g_{111} & g_{110} & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ g_{11n-1} & g_{11n-2} & \cdots & g_{110} \end{bmatrix}, G_{12} = \begin{bmatrix} g_{120} & 0 & 0 & 0 \\ g_{121} & g_{120} & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ g_{12n-1} & g_{12n-2} & \cdots & g_{120} \end{bmatrix},$$

$$G_{21} = \begin{bmatrix} g_{210} & 0 & 0 & 0 \\ g_{211} & g_{210} & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ g_{21n-1} & g_{21n-2} & \cdots & g_{210} \end{bmatrix}, G_{22} = \begin{bmatrix} g_{220} & 0 & 0 & 0 \\ g_{221} & g_{220} & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ g_{22n-1} & g_{22n-2} & \cdots & g_{220} \end{bmatrix}.$$

The matrix $G_{11}, G_{12}, G_{21}, G_{22}$ can be identified by least square method.

Define the cost function as:

$$\begin{aligned} \min J_1 &= \sum_{i=1}^{n} [y_1(k+i) - w_1(k+i)]^2 + \sum_{j=1}^{l} \lambda_j \Delta x_1^2(k+j-1) \\ \min J_2 &= \sum_{i=1}^{n} [y_2(k+i) - w_2(k+i)]^2 + \sum_{j=1}^{l} \lambda_j \Delta x_2^2(k+j-1) \end{aligned}.$$

Where $w(k+j) = \begin{bmatrix} w_1(k+j) \\ w_2(k+j) \end{bmatrix} = \begin{bmatrix} \alpha y_1(k) + (1-\alpha) y_{r1} \\ \alpha y_2(k) + (1-\alpha) y_{r2} \end{bmatrix}$.

Thus, the optimal control law can be written as:

$$\begin{aligned} \Delta X_1 &= (G_{11}^T G_{11} + \lambda I)^{-1} G_{11}^T (W_1 - G_{12}\Delta X_2 - f_1) \\ \Delta X_2 &= (G_{22}^T G_{22} + \lambda I)^{-1} G_{22}^T (W_2 - G_{21}\Delta X_1 - f_2) \end{aligned}. \qquad \textbf{(13)}$$

After modifying the error of actual output and predictive output, the next time predictive value is: $\tilde{Y} = \hat{Y}_j + h e_j(k+1)$.

Where $\tilde{Y} = [\tilde{y}_j(k+1), \tilde{y}_j(k+2), \cdots, \tilde{y}_j(k+p)]^T$ $(j=1,2)$ is predictive output at $t=(k+1)T$ time. $h = [h_1, h_1, \cdots, h_p]^T$ is the vector of modifying coefficient, $h_1 = 1$.

Here, $\tilde{Y}$ is predictive initial value at next time. Because using predictive initial value at $t=(k+1)T$ time to predict output value at $t=(k+2)T, \cdots, (k+p+1)T$ time, so, $y_{0j}(k+i) = \tilde{y}_j(k+i+1)$.

Then the predictive initial value vector Y_0 at next time is as follow:

$$\begin{cases} y_{0j}(k+i) = \tilde{y}_j(k+i+1) + h_{i+1} e_j(k+1) \\ y_{0j}(k+i) = \tilde{y}_j(k+p) + h_p e_j(k+1) \end{cases} \quad j=1,2 .$$

The predictive vector, at next time, is:

$$f_j = \begin{bmatrix} f_j(k+1) \\ f_j(k+2) \\ \vdots \\ f_j(k+n) \end{bmatrix} = \begin{bmatrix} \hat{y}_j(k+2/k) \\ \hat{y}_j(k+3/k) \\ \vdots \\ \hat{y}_j(k+n+1/k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} e_j(k+1) \qquad j=1,2 \ .$$

After $G_{11}, G_{12}, G_{21}, G_{22}$ and f_1, f_2 are given, the control variable can be calculated as follow:

$$x_j(k) = x_j(k-1) + \vartriangle x_j(k) \ .$$

From Fig. 2, we can find that the relationship between $X(K)$ and $U(K)$ is expressed by static DLF network.

For $X(k) = W_F U(k) + N_{BP}(U(k-1))$

Then, the desired control law is:

$$U(k) = W_F^{-1}\left[X(k) - N_{BP}(U(k-1))\right] \ . \qquad \textbf{(14)}$$

4. Simulation Research

Based on the dynamic mathematical model of steam turbine and steam generator, this paper carries out simulation research on applying multivariable integrated model predictive control (MIPC). The predictive control parameters are $n=10, m=2$, λ=[1，0；0，1], α=0.399. The DLF network is 2-10-2 network structure. The sampling time is $T = 0.1s$.

We had done a lot of simulation research on this control method. Here, the simulation results of the load changing, from 100 percent to 83 percent, are given as follow, to see Fig. 3 and Fig. 4.

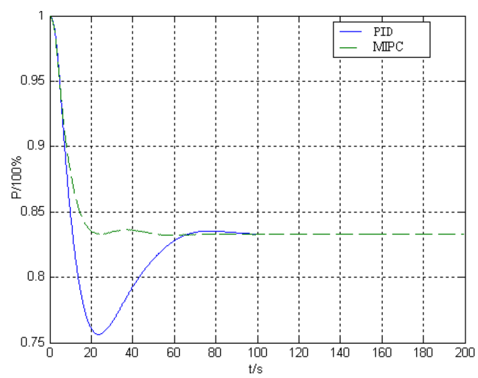

Fig. 3. Rotate speed changing of steam turbine

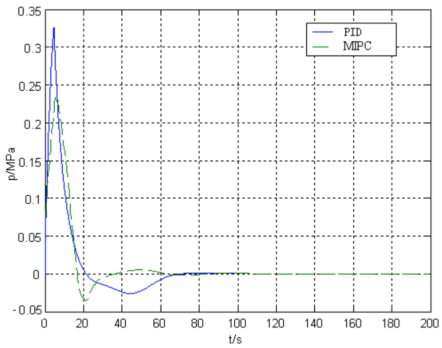

Fig. 4. Outlet pressure changing of steam generator

When the load of nuclear power plant is changed, the response characteristic of turbine rotate speed and steam pressure under the multivariable integrated model predictive control is faster than under PID control, and the overshoot and settling time under PID control is more 3 to 5 percent than under the multivariable integrated model predictive control.

5 Conclusion

In this paper, we discussed multivariable integrated model predictive control method. For complex nonlinear system we can design its multivariable integrated model predictive controller through nonlinear static DLF network model and dynamic linear CARIMA model. The simulation results indicate that multivariable integrated model predictive control method is to be suitable for the changing situations of load of nuclear power plant.

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