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A Numerical Analysis of 1D Angular Interferometry Based on Random Array

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Abstract

The spatial sampling the planar wave front by an array antennas is generally uniform where the spatial frequency is less or equal to half of the wavelength of the radiating source. In one dimensional geometry, this condition implies that the array is linear and uniform. If the inter-element distance is not constant and changes according to certain rule, the spatial sampling of the wave front is not uniform where it has an impact on estimating the parameters of the radiating sources. In this paper, we propose a configuration of one dimensional array of identical and isotropic sensors such as dipoles, where the inter-element distance is modeled as random variable sampled from continuous uniform distribution. We focus on estimating the angular parameters of far field and narrowband uncorrelated closely sources that have the same signal power. Using random configuration of the one dimensional array comparatively to uniform linear array, we present some numerical results where we show that on average, the angular separation of the punctual sources using high resolution techniques is more precise in the case of random array if the standard deviation of the inter-element distance is half the wavelength.

Keywords: *random array, uniform linear array, resolution power, source separation, uniform distribution, angle of incidence*

1. Introduction

Antenna processing [1] plays a crucial role in several fields including telecommunications [2], geophysics [3], underwater acoustics [4-8] and radio astronomy [9]. The advantage of array processing is based on high resolution estimation of the parameters of radiating sources such as theirs angular and statistical parameters [10]. Based on the principle of mutiple antennas, the interferometric relations permit to accurately estimate the parameters of the wave field by the progressive phase difference between the sensors, especially in the case of narrowband approximation of the wave equation [11].

The simplest methodology is the beam forming [12,13] which is equivalent to the spatial Fourier transform of the wave field, larger number of antennas allows to estimate the parameters of the sources. For example, in the field of acoustics, it is possible to separate different acoustic fields using large number of sensors. In different scenarios, such as the case of coherence or correlation of radiating sources [14], or in the case of closely sources, high resolution spectral techniques [10,11,14,15] are required to obtain an accurate estimation of the parameters of each source, closely sources means that the angular distance between these sources is less than Rayleigh angular resolution limit of

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the array of sensors, high resolution techniques such as multiple signal classification technique [16], minimum norm technique [17] or propagator operator [18] are useful in this case, however their performances depend on other parameters such as the number of acquired samples and the level of perturbation that is computed using the metric of signal to noise ratio [14].

High resolution techniques [10,11,14,15] depend on the geometry of the sensors, whether it is one or two dimensional. Concerning the one dimensional case, the most well known example is the array that consists of elements that are identical and separated by the same inter-element distance, this type is known as uniform linear array [10,14]. Other models were proposed including non uniform linear array [19], logarithm array [20] and random array [21]. In this paper, we propose a numerical evaluation of angular source separation using random array where the inter-element distance between the sensors is random variable sampled from uniform distribution [22], we evaluate the resolution power of some high resolution techniques with one dimensional random array comparatively to uniform linear array.

2. Random Array of Antennas

One dimensional array consists of sensors that are placed in line where the distance between two consecutive sensors may vary, the simplest example is the uniform linear array [10,14] where the distance is less or equal to half of the wavelength similarly to temporal sampling of Nyquist–Shannon theorem [23], the length of a uniform array of N elements is given by $\mathrm{Le} = (\mathrm{N}-1)\mathrm{d}$, where the Rayleigh angular resolution limit is given as the half power beam width of the array factor $\theta_{hpbw} \approx \lambda / Le$ assuming that the individual sensors are calibrated, identical and isotropic [24-25]. Given the condition of far field radiations generated by P punctual sources, the signal at the m^{th} sensor is given by the following equation:

$$x_m(t) = \sum_{u=1}^{P} s_u(t) e^{-jk(m-1)d\sin\theta_u} + n_m(t) \tag{1}$$

Where the time index after down conversion and sampling is $t = 1,...,K$, $s_u(t)$ is the envelope of the u^{th} source, k is the wave number defined by $k = 2\pi/\lambda$, θ_u is the angle of incidence of the planar wave of the u^{th} source relatively to the normal of the array, $n_m(t)$ is the additive noise at the m^{th} sensor modeled by complex normal distribution, the noise is spatially and temporally uncorrelated. The concatenation of the N waveforms yields a complex matrix of data $x \in C^{N\times K}$ as $x = [x_1,...,x_N(t)]^T$ where $(.)^T$ denotes the transpose operation. Using the compact form, the data can be written as $x(t) = As(t) + n(t)$ where $s(t) \in C^{P\times K}$, $n(t) \in C^{N\times K}$ and $A \in C^{N\times P}$ is the steering matrix [14] of the uniform linear array which has Vandermonde structure, it is given by $A = [a_1,...,a_P]$. Assuming that the phase reference is the first element of the array, each steering vector can be written as:

$$a_i = [1, e^{-jkd\sin\theta_i},..., e^{-jkd(N-1)\sin\theta_i}]^T \tag{2}$$

Given N elements and P sources, the steering matrix verifies the following property:

$$Tr(AA^+) = Tr(A^+A) = NP \tag{3}$$

Where $Tr(.)$ is the trace operator and $(.)^{+}$ is the conjugate transpose. Based on second order statistics of data $x(t)$, angular and statistical parameters of the sources including the angles of incidence θ_i and the powers of the waveforms $< s_i(t)s_i^*(t) >= \sigma_i^2$ can be estimated using high resolution techniques. We evaluate the estimation quality of the angular parameters using a random distribution of sensors. We consider a configuration where the inter-element distance δ is modeled as random variable sampled from uniform distribution that is defined by the probability density function:

$$p(\delta) = \begin{cases} (b-a)^{-1}, \text{ if } a \le \delta \le b \\ 0, \text{ if } \delta < a \text{ or } \delta > b \end{cases} \tag{4}$$

Where the standard deviation is defined by the following equation:

$$\Delta\delta = \left(<\delta^2> - <\delta>^2\right)^{1/2} = \left(\int \delta^2 p(\delta)d\delta - \left(\int \delta p(\delta)d\delta\right)^2\right)^{1/2} = \frac{(b-a)}{\sqrt{12}} \tag{5}$$

The configuration of the random array in this case is generated by the following methodology; let the first element r_1 to be placed in the origin of the reference $(0,0)$ such that the $N-1$ elements are placed on the x axis, the coordinates of the remaining elements are calculated by the following iterative relation:

$$r_{n+1} = r_n + \delta_n \tag{6}$$

Where r_n is the coordinate of the n^{th} sensor and δ_n is random variable sampled from uniform distribution described by $a = 0$ and $b > 0$, this process requires $N-1$ realizations of δ. The parameter of interest in this configuration is the wavelength, since for the uniform linear array, the inter-element distance must be less or equal to half of the wavelength, we compute the coordinates using the statistics of the inter-element distance δ such that the standard deviation must be less than or equal to half of the wavelength. To illustrate the geometrical distribution of the sensors, for standard deviation $\Delta\delta = \lambda/2 = 0.5$, we present in Figure 1 the first 25 realizations of the random array using $N = 20$ elements.

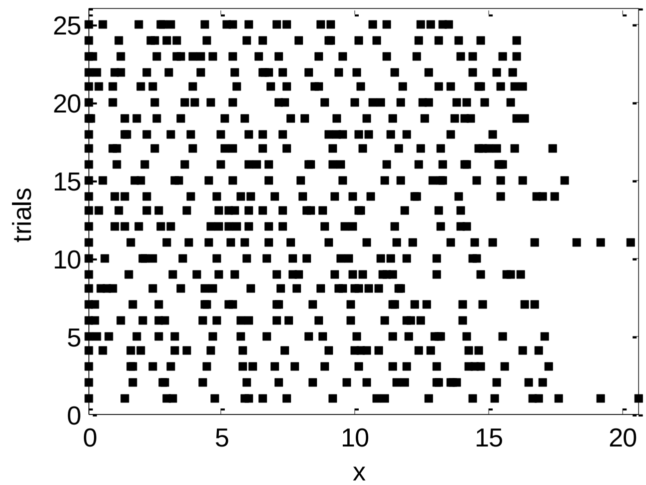

Figure 1. Realizations of the Random Array based on Uniform Distribution with $\Delta\delta = \lambda/2$

The parameter b of the random number generator is calculated using the relation $b=\sqrt{12}(\lambda/2)$. We note that all the realizations have the same values for the first sensor as previously mentioned. Next we evaluate the array factor to examine the variation of the global response of the array with respect to the change of the coordinates, for each realization of the random array, assuming that each element is isotropic with a gain of unity, we calculate the radiation pattern in the angular domain $\theta \in [0,\pi]$ where each element is described by current $I_i = 1$ and phase $\phi_i = 0$, the equation is given by:

$$F(\theta)=\sum_{n=1}^{N} e^{jk\delta_n \cos(\theta)} = 1+\sum_{n=2}^{N} e^{jk\delta_n \cos(\theta)} \tag{7}$$

In Figure 2 we present the different shapes of the normalized array factor, for each realization of random array, the result corresponds to the case of $\Delta\delta = \lambda/2$. We remark that the width of the main lobe is approximately the same for the different samples, however the random variations of the secondary lobes are different.

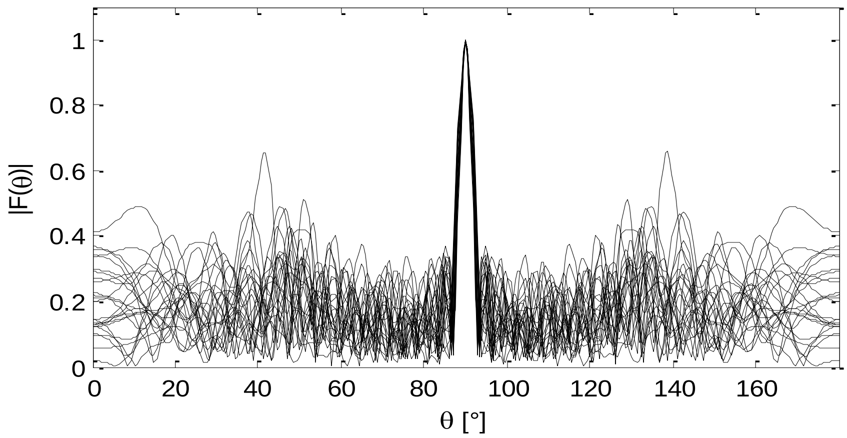

Figure 2. Normalized Array Factors $|F(\theta)|$ of the Random Array with $\Delta\delta = \lambda/2$ and $L = 25$ Samples

Note that the shape of the array factor changes according to the value of $\Delta\delta$, therefore it is possible to obtain an average of the different realizations of $F(\theta)$ to estimate its characteristics such as the half power beam width, the minimum value and the ratio of the maximum over the second maximum. We limit this study to the case of far field source separation in the horizontal plan, we consider that several narrowband sources that are oriented vertically radiate in plan (x, y) where the random array consists of dipoles that are placed vertically (parallel to z axis), we evaluate the resolution power of some high resolution techniques comparatively to the case of uniform linear array, the details of this comparison is given in the next section.

3. Computation and Discussion

In this part, we present some numerical results based on Monte Carlo trials, we evaluate the resolution power of some high resolution techniques including multiple signal classification [16], minimum variance distortionless response [14] and propagator operator [18] using uniform linear array and proposed random array. The array consists of $N = 16$ isotropic and identical sensors for both arrays where the standard deviation of the random array is set to $\Delta\delta = \lambda/2$, we consider a planar wave field that consists of $P = 4$

sources with equal signal power $\sigma_i^2 = 1$ W, in the same horizontal plan with angles of incidence $\theta = [10^\circ, 12^\circ, 15^\circ, 21^\circ]$.

The waveforms are sampled using $K = 400$ where the envelopes of the sources $s_i(t)$ are modeled by complex centered random processes. The high resolution methods are based on second order statistics where the localization functions are calculated in the angular domain $[0^\circ, 25^\circ]$ with angular step $d\theta = 0.1^\circ$. After generating one sample of random array, data $x(t)$ for both arrays are simulated and the estimation of the angular parameters are performed based on $L = 1000$ trials.

The evaluation is based on two levels of signal to noise ratio given by $SNR = 10\log_{10}(1/\sigma^2)$ where σ^2 is the power of the noise field. Staring with multiple signal classification method, we present in figure.3 the average of the localization functions obtained from linear and random arrays where $SNR = 2$ dB.

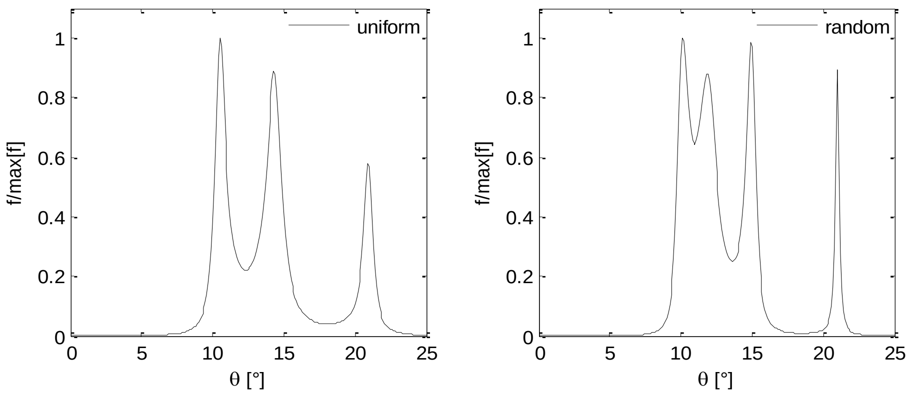

Figure 3. Average Localization Function of Multiple Signal Classification Technique with $SNR = 2$ dB and $\Delta\delta = \lambda / 2$

The second source given by $\theta = 12^\circ$ is note detected using uniform array, and the localization function of the random array could separate the four parameters of θ, for the second test, we present in Figure 4 the result using low level of noise perturbation where $SNR = 20$ dB.

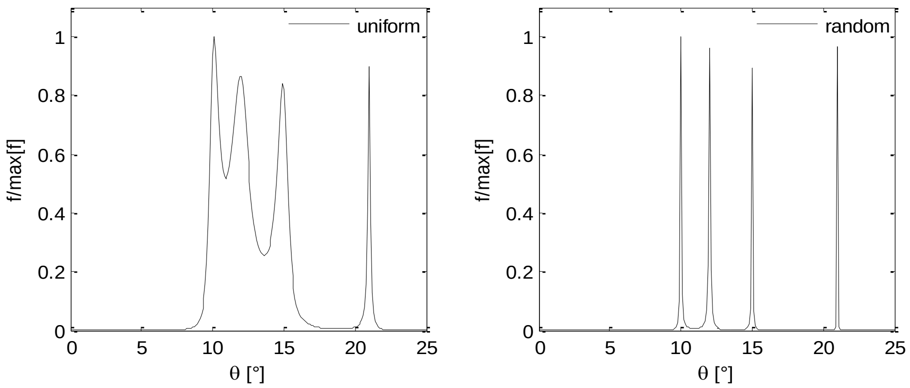

Figure 4. Average Localization Function of Multiple Signal Classification Technique with $SNR = 20$ dB and $\Delta\delta = \lambda / 2$

As we remark, if the signal to noise ratio is higher, the localization functions calculated using uniform and random arrays have almost the same resolution power, note that the sharpness of $f(\theta)$ using random array is more evident. For the second test, we compare the performance of the minimum variance distortionless response method where the spectrum is based on the inverse of the spectral matrix of the array, the angular spectrum [14] is computed based on the following relation:

$$f(\theta) = \frac{1}{a^{+}(\theta)\Gamma^{-1}a(\theta)} \tag{8}$$

Where $a(\theta)$ is the generated steering vector, for $SNR = 2$ dB, the result is given in Figure 5.

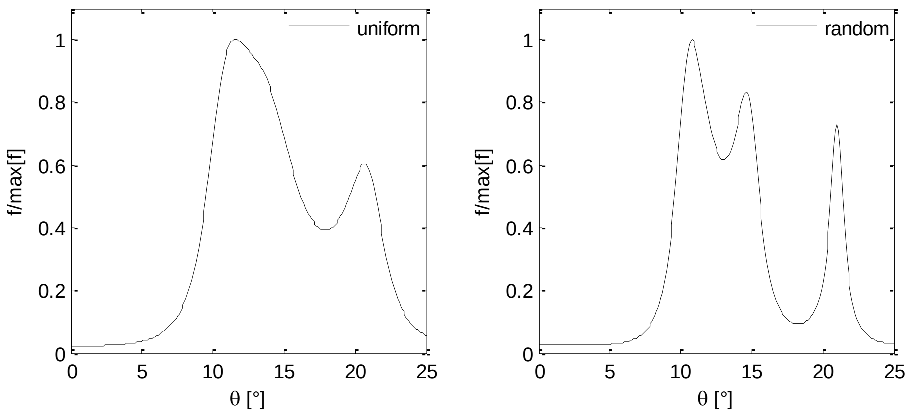

Figure 5. Average Localization Function of Minimum Variance Distortionless Response Technique with $SNR = 2$ dB and $\Delta\delta = \lambda/2$

For random array, the first and second sources are merged into single peak at around 10° while the function of the uniform array is characterized by low resolution, the peaks of the second and third sources are not detected. Using the level of $SNR = 20$ dB, we present the result of the estimation in Figure 6.

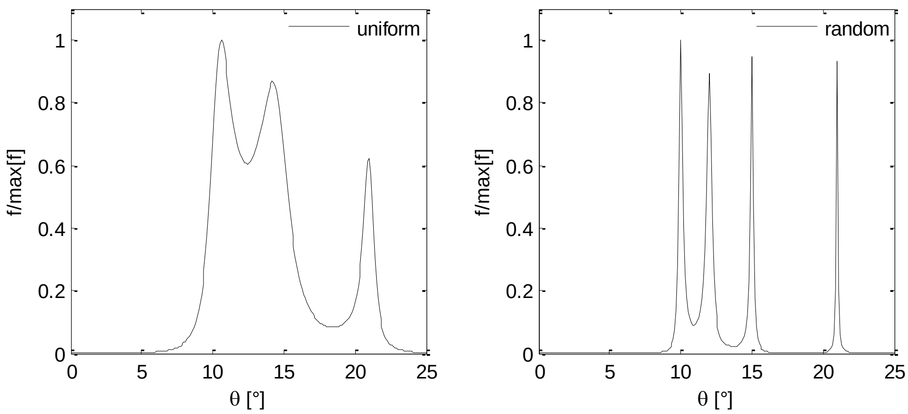

Figure 6. Average Localization Function of Minimum Variance Distortionless Response Technique with $SNR = 20$ dB and $\Delta\delta = \lambda/2$

The spectrum of the uniform array is of low resolution, first, the two sources at 10° and 12° are not separated and the third source is incorrectly estimated because the peak is translated to the left. The spectrum obtained using random array is of high resolution. For the third method, we implement the propagator operator, the result of the average localization function for $SNR = 2$ dB is presented in Figure 7.

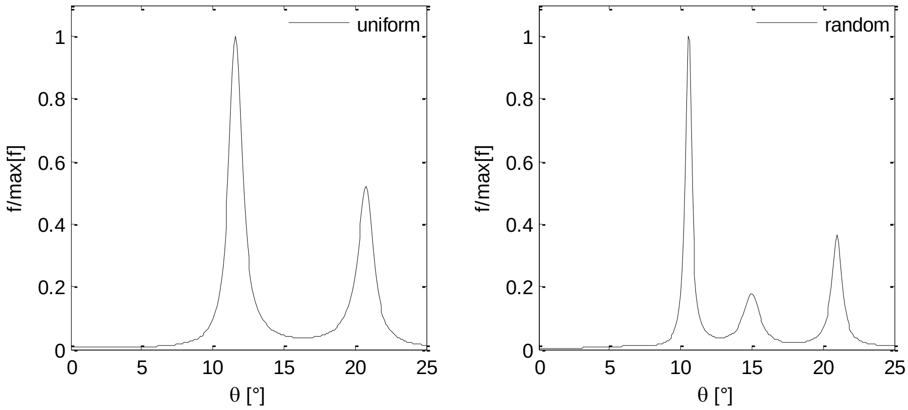

Figure 7. Average Localization Function of the Propagator with $SNR = 2$ dB and $\Delta\delta = \lambda/2$

We remark that the function $f(\theta)$ of the random array could estimate three sources while the generated function of the uniform array contained two peaks at approximately 12° and 21°, therefore the propagator method for this case of large array, low SNR level and closely sources is not convenient. For high level of SNR we present in Figure 8 the average of the functions.

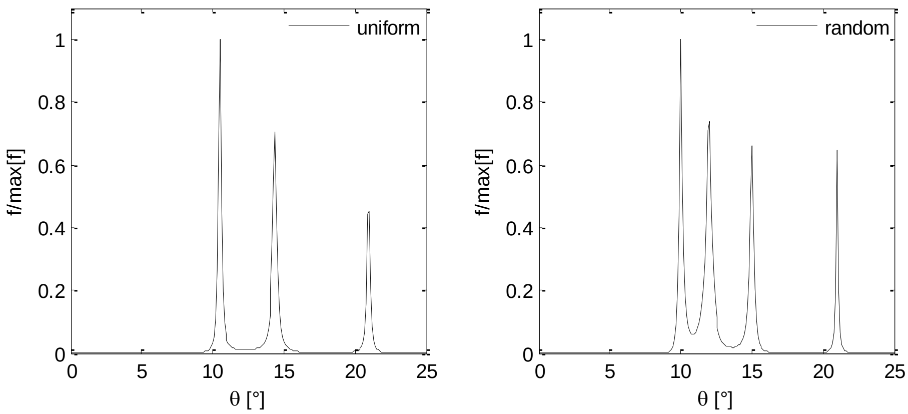

Figure 8. Average Localization Function of the Propagator with $SNR = 20$ dB and $\Delta\delta = \lambda/2$

The average function $f(\theta)$, in high SNR level, is of high resolution using random array comparatively to the uniform array where the second source is not detected. For the second part of the simulation, we evaluate the quality of noise power estimation using subspace decomposition for linear and random arrays. Given the covariance matrix:

$$\Gamma = <xx^{+}> = \frac{1}{K}\sum_{t=1}^{K} x(t)x^{+}(t) \tag{9}$$

Where $x(t) \in C^{N\times 1}$, the subspace decomposition is given by:

$$\begin{cases} \Gamma = U\Lambda U^{+} \\ U = [U_s, U_n] \end{cases} \tag{10}$$

Where U is unitary matrix such that the signal subspace is $U_s \in C^{N\times P}$, the noise subspace is $U_n \in C^{N\times N-P}$ and $P_n = U_n U_n^{+}$ is the projector into the noise subspace, the subspace decomposition can also be written in terms of the eigenvalues as the following:

$$\Gamma = \sum_{i=1}^{N} \lambda_i u_i u_i^{+} \tag{11}$$

Where λ_i is the ith eigenvalue that corresponds to the eigenvector u_i. The noise power estimation can be obtained by the following steps:

$$\begin{cases} F = P_n \Gamma \\ H = P_n^{\dagger} F \\ \hat{\sigma}^2 = Tr(H)/N \end{cases} \tag{12}$$

Where $(.)^{\dagger}$ is the pseudo-inverse operator. For different levels of SNR, we compute the absolute value of the difference between the estimated and true values of noise power $e = |\hat{\sigma}^2 - \sigma^2|$ for uniform and random arrays, the result is presented in figure.9. We remark that, in this configuration of antennas-sources, the quality of estimation of the noise power for both arrays is approximately the same.

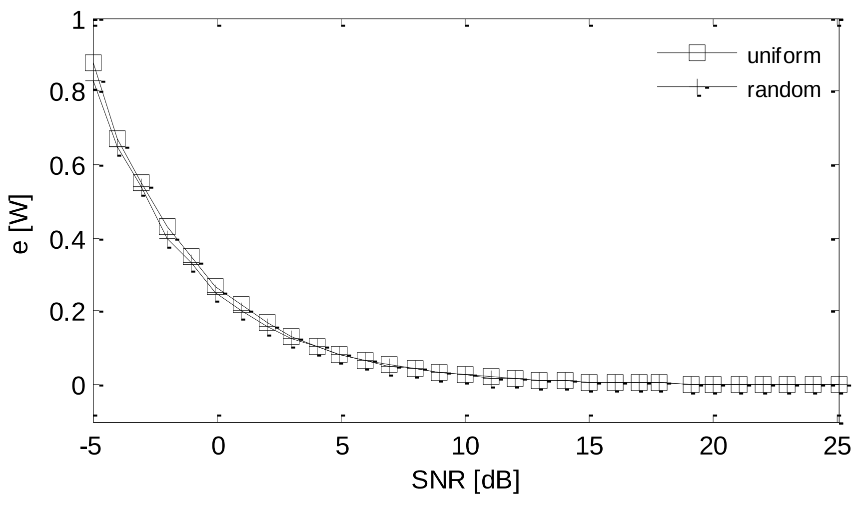

Figure 9. Absolute Value of the Difference $e = |\hat{\sigma}^2 - \sigma^2|$ for Uniform and Random Arrays with $\Delta\delta = \lambda/2$

The presented simulation results demonstrate that in order to characterize the angular and statistical parameters of far field radiating sources, by an array of sensors, it is possible to enhance the resolution power of spectral technique by changing the inter-distance between the sensors. For example, in laboratory, one can manually change the positions of the antennas, process the obtained data and compare the quality of source separation, changing the positions of the elements can be made using certain rules or functions. In the presented simulated case, the distribution of the antennas using random number generator obtained from uniform distribution has good impact on the resolution of angular spectra where $\Delta\delta = \lambda / 2$, therefore, as perspective, one can evaluate the quality of the angular resolution by changing the standard deviation as function of the wavelength $\Delta\delta = f(\lambda)$.

4. Conclusion

This paper was devoted to the study of one dimensional angular source separation, using non uniform condition of the array of antennas. Based on far field and narrowband conditions, we have proposed a distribution of the coordinates of the antennas where the inter-element distance is modeled by random variable sampled from uniform distribution. Using the assumption of identical and isotropic sensors, we have numerically evaluated the resolution power of spectral techniques in order to estimate the angular parameters of punctual sources using large array and closely sources, it was found that random distribution of sensors can enhance the process of source separation if the standard deviation of the distance between the sensors equals half of the wavelength.

Appendix

In this part, we present the Mathworks program to generate the radiation pattern of the random array based uniform distribution, as presented in Figures 1 and 2. In the first part, the coordinates of the $N = 20$ identical sensors are generated using $L = 25$ trials, the returned matrix is Ar where each line corresponds to one realization. The radiation pattern is computed in the domain $\theta \in [0, \pi]$ where the returned matrix is AF.

```
N=20;
L=25;
lambda=1;
k=2*pi/lambda;
theta=0:0.01:pi;
sd=sqrt(12)*lambda/2;
% coordinates
Ar=zeros(L,N);
for t=1:L
ar=zeros(1,N);
d=sd*rand(1,N-1);
for n=1:N-1
    ar(n+1)=ar(n)+d(n);
end
Ar(t,:)=ar;
end

% Array factor.
AF=zeros(L,length(theta));
for l=1:L
    F=0;
    for t=1:N
        F=F+exp(j*k*Ar(l,t)*cos(theta));
    end
    F=abs(F);

    AF(l,:)=F/max(F);
end
```

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