

Adaptive Multiple-sliding-mode Control of a VTOL UAV Based on the MRPs

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Abstract

An adaptive multiple-sliding-mode control strategy is proposed in this paper for trajectory tracking control of a vertical take-off and landing unmanned aerial vehicle. First, a non-redundant error model of the aircraft is established based on the Modified Rodrigues Parameters. The cascade property is analyzed, based on which the system is divided into position and attitude subsystems, respectively. Then, an adaptive law is designed to estimate the unknown parameters, based on which a multiple-sliding-mode controller is applied to deal with external disturbances. It is proved by Lyapunov theory that the tracking errors can converge to a small neighborhood of the origin point by choosing appropriate control parameters. Simulation results show that the proposed controller enables the VTOL UAV to track a desired trajectory quickly and accurately.

Keywords: Vertical take-off and landing unmanned aerial vehicle; Modified Rodrigues Parameters; Sliding Mode Control; Adaptive; Cascade System

1. Introduction

In recent years, the researches and applications of vertical take-off and landing unmanned aerial vehicle (VTOL UAV) have been widely concerned, such as unmanned inspection, rescue in disasters and road traffic supervision. The VTOL UAV is a kind of non-holonomic cascade system with complicated restrictions, which do not satisfy the necessary requirement of smooth feedback stabilization from Brockett, only the non-smooth or time-varying control law could stabilize this kind of systems [1]. Thus, new control methodologies should be explored for this kind of systems.

Many researches focus on the attitude control of the UAVs, however, trajectory tracking control is a more challenging work due to its strong coupling, unknown parameters, external disturbance, etc. In [2-4], backstepping control and feedback linearization are introduced for trajectory tracking problem. However, they cannot deal with the system uncertainties and external disturbances effectively. In [5-6], intelligent control strategies, such as neural networks, fuzzy control are adopted. However, the convergence rate will limit the applications of these methods in practice. In [7-10], sliding model control and model predictive control can increase the control accuracy with system uncertainties. Concerning with the cascade property of the VTOL UAV, hierarchical control strategy is usually applied, the position and attitude controllers are usually designed separately. However, the stability is usually analyzed based on subsystems, while strictly Lyapunov analysis for closed-loop system is important.

From the analysis above, we propose a novel adaptive multiple-sliding-mode control strategy in this paper. The system model is established based on Modified Rodrigues Parameters (MRPs). Then, the trajectory tracking problem is formulated and system error model is established. The system error model is divided into two parts, translational

subsystem and rotational subsystem, based on which the controller for each subsystem can be designed separately. For the controller, an adaptive law is designed to estimate the unknown parameters, based on which a multiple-sliding-mode controller is applied to deal with external disturbances. At last, according to the Rodrigues formula and the character of the MRPs, it's proved that the tracking errors of the aircraft are bounded through global stability analysis.

This paper is organized as follows. In Section 2, the system model of VTOL UAV is established based on Modified Rodrigues Parameters (MRPs). In Section 3, trajectory tracking problem is described, and the cascade property is analyzed. In Section 4, an adaptive multiple-sliding-mode control methodology is proposed, and stability of closed-loop system is analyzed based on cascade theory. Simulation in Section 5 is carried out to show the effectiveness of the proposed method, followed by the Conclusions in Section 6.

2. System Model

Define an earth frame, whose x-axis points northward, y-axis eastward and z-axis downward, namely a North-East-Down frame. We consider the VTOL UAV as a rigid body without deformation, and the system model is described as:

$$\begin{cases} \dot{\xi} = v \\ m\dot{v} = e_3 mg - Re_3 T + d_1 \\ \dot{\sigma} = G(\sigma)\omega \\ J\dot{\omega} = -\omega \times J\omega + \tau + d_2 \end{cases} \quad (1)$$

where $\xi, v \in \mathbf{R}^3$ are the position and velocity vectors under the earth frame. $\omega \in \mathbf{R}^3$ is the angle velocity of the rigid body. d_1 and d_2 are external disturbances acting on the translational and rotational subsystems satisfying assumption 1. $R \in \mathbf{R}^{3 \times 3}$ is attitude transition matrix, $e_3 = [0 \ 0 \ 1]^T$ is a unit vector. σ is the MRPs which is defined as $\sigma = r \tan \alpha/4$ [11]. r and α represent a rotation axis that does not coincide with one of the original coordinate axes and angle about the axis, respectively. m and $J \in \mathbf{R}^{3 \times 3}$ are the mass and inertia matrix of the rigid body, where J is considered to be a diagonal, positive definite matrix. J_x, J_y, J_z are moments of inertia about the three coordinate axes. g is the gravity acceleration. T and $\tau \in \mathbf{R}^3$ are the throttle and moment on the rigid body. The expressions of $R(\sigma)$ and $G(\sigma)$ are given in as:

$$R(\sigma) = I_3 - \frac{4(1 - \sigma^T \sigma)}{(1 + \sigma^T \sigma)^2} S(\sigma) + \frac{8}{(1 + \sigma^T \sigma)^2} S(\sigma)^2 \quad (2)$$

$$G(\sigma) = \frac{1}{2} \left(\frac{1 - \sigma^T \sigma}{2} I_3 + S(\sigma) + \sigma \sigma^T \right) \quad (3)$$

Assumption 1. the disturbances d_1 and d_2 are bounded with $\|D_1\| \leq \eta_1$ and $\|D_2\| \leq \eta_2$.

3. Problem Formulation

The trajectory tracking problem of a VTOL UAV is investigated, and the objective of this research is to design the control thrust T_d and τ_d , such that the VTOL UAV can track a desired trajectory quickly and accurately. The vector $[\mathbf{x}_d \ \dot{\mathbf{x}}_d \ \ddot{\mathbf{x}}_d]$ is the desired

trajectory which contains position, velocity and acceleration and the desired acceleration \ddot{x}_d satisfies the following assumption 2.

Assumption 2. The desired acceleration is bounded with $\|\ddot{x}_d\| \leq \nu$, where ν is a positive constant.

Define the system errors as: position error $\tilde{\xi} = \xi - x_d$, velocity error $\tilde{v} = v - \dot{x}_d$, attitude matrix error $\tilde{R} = RR_d^T$, $\tilde{\sigma}$ and $\tilde{\omega}$ are MRPs and angular velocity error defined as:

$$\tilde{\sigma} = \sigma \oplus \tilde{\sigma}_d, \tilde{\omega} = \omega - \tilde{R}\omega_d \quad (4)$$

So we get the error equations of the system:

$$\Sigma_1 : \begin{cases} \dot{\tilde{\xi}} = \tilde{v} \\ \dot{\tilde{v}} = g e_3 - \frac{1}{m} n_d T_d - \ddot{x}_d + \frac{1}{m} (I - \tilde{R})(n_d T_d) + d_1 \end{cases} \quad (5)$$

$$\Sigma_2 : \begin{cases} \dot{\tilde{\sigma}} = G(\tilde{\sigma})\tilde{\omega} \\ \dot{\tilde{\omega}} = J^{-1}(-\omega \times J\omega + \tau_d + d_2) - (\tilde{R}\dot{\omega}_d - \tilde{R}S(\tilde{\omega})\omega_d) \end{cases} \quad (6)$$

where equations (5) and (6) are the position and velocity subsystems. $\tilde{\sigma}$ and $\tilde{\sigma}_d$ are calculated from \tilde{R} and R_d by (2). $\tilde{\omega}$ and ω_d are calculated from $\tilde{\sigma}$ and $\tilde{\sigma}_d$. $(\tilde{R}\dot{\omega}_d - \tilde{R}S(\tilde{\omega})\omega_d)$ is the derivative of $\tilde{R}\omega_d$ with respect to time^[11]. $n_d = R_d e_3$ is the last row of the desired attitude matrix. T_d and τ_d are the controllers that converge the system errors to 0.

If $\tilde{\sigma} = 0$, then the coupled term between the two subsystems $\frac{1}{m}(I - \tilde{R})(n_d T_d) = 0$. So we can consider the position error subsystem as a system with the disturbance $\tilde{\sigma}$. The subsystem (5) constitutes a nominal system Σ'_1 and a coupled term g :

$$\Sigma'_1 : \begin{cases} \dot{\tilde{\xi}} = \tilde{v} \\ \dot{\tilde{v}} = g e_3 - \frac{1}{m} n_d T_d - \ddot{x}_d + d_1 \end{cases} \quad (7)$$

$$h = \frac{1}{m}(I - \tilde{R})(n_d T_d) \quad (8)$$

4. Controller Design and Stability Analysis

First, an adaptive multiple-sliding-mode control method is designed to stabilize the translational error subsystem Σ'_1 and rotational error subsystem Σ_2 . Then the uniformly ultimately bounded property of the position error subsystem is proven by analyzing the property of the coupled term.

4.1. Controller Design

STEP1. Controller design of the position error nominal subsystem Σ'_1 .

The purpose of controller design of the position error nominal subsystem is to acquire the controller T_d and virtual controller n_d , which make the states of Σ'_1 uniformly ultimately bounded.

Define two manifolds $s_1 = \tilde{\xi}$, $s_2 = \tilde{v} - \alpha_1(s_1)$, where $\alpha_1 = -k_1 s_1$ is the virtual controller to be solved. Based on the definition of manifold, we acquire $\dot{\alpha}_1 = -k_1(s_2 - k_1 s_1)$. \hat{m} is the estimated mass of the aircraft. The error of mass estimation $\tilde{m} = \hat{m} - m$ satisfies assumption 3.

Assumption 3. the error of mass estimation is bounded with $\|\tilde{m}\| \leq \varepsilon_1$, where ε_1 is a positive constant.

We acquire the controller T_d and virtual controller n_d as the following:

$$\begin{cases} T_d = \left\| \hat{m} \left(e_3 g - \ddot{x}_d + k_1(s_1 - k_1 s_2) + \frac{c \eta_1 s_2}{(s_2^T s_2 + \rho^2)^{1/2}} \right) + s_1 + k_2 s_2 \right\| \\ n_d = \left[\hat{m} \left(e_3 g - \ddot{x}_d + k_1(s_1 - k_1 s_2) + \frac{c \eta_1 s_2}{(s_2^T s_2 + \rho^2)^{1/2}} \right) + s_1 + k_2 s_2 \right] / T_d \end{cases} \quad (9)$$

where k_1, k_2 are the control parameters, c and ρ determine the neighborhood which the system errors converge to.

The adaptive law is

$$\dot{\hat{m}} = \gamma s_2^T \left(e_3 g - \ddot{x}_d + k_1(s_1 - k_1 s_2) + \frac{c \eta_1 s_2}{(s_2^T s_2 + \delta^2)^{1/2}} \right) \quad (10)$$

The desired input σ_d and MRPs error $\tilde{\sigma}$ of the attitude error subsystem Σ_2 can be calculated by the virtual controller n_d .

Remark 1. Consider that ω_d and $\dot{\omega}_d$ are required in STEP2, the controller (9) should not contain sign function, so the controller could only converge the trajectory tracking error into a small neighborhood.

STEP2. Controller design of the attitude error subsystem Σ_2 .

The purpose of controller design of the attitude error subsystem is to acquire the controller τ_d , which stabilizes the states of Σ_1' asymptotically.

Define two manifolds $s_3 = \tilde{\sigma}$, $s_4 = \tilde{\omega} - \alpha_2(s_3)$, where $\alpha_1 = -k_1 s_1$ is the virtual controller to be solved. Based on the definition of manifold, we acquire $\dot{\alpha}_2 = -k_3 G(s_3)(s_4 - k_3 s_3)$.

Define a linear operator $L(\cdot) : \mathbf{R}^3 \rightarrow \mathbf{R}^{3 \times 3}$ as

$$L(\mu) = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix} \quad (11)$$

So there exists $J\mu = L(\mu)\hat{\Xi}$ for a vector $\mu = [\mu_1 \ \mu_2 \ \mu_3]^T$, where $\Xi = [J_x \ J_y \ J_z]^T$. $\hat{\Xi}$ is the estimation of Ξ . The error of the estimation $\tilde{\Xi} = \hat{\Xi} - \Xi$ satisfies assumption 4.

Assumption 4. The estimation error of moments of inertia is bounded with $\|\tilde{\Xi}\| \leq \varepsilon_2$, where ε_2 is a positive constant.

Hence we get the controller as

$$\tau_d = L(\hat{\Xi}) \left((\tilde{R}\dot{\omega}_d - \tilde{R}S(\varepsilon_4)\omega_d) + k_3 G(s_3)(s_4 - k_3 s_3) \right) + \omega \times L(\hat{\Xi})\omega - k_4 s_4 - s_3 - \eta_2 \operatorname{sgn}(s_4) \quad (12)$$

where k_3, k_4 are control parameters.

Remark 2. the operator $\operatorname{sgn}(\cdot)$ of a vector denotes a sign function of each term in the vector.

The adaptive law is

$$\dot{\hat{\Xi}} = \Gamma^{-1} \left(-S(\omega)L(\omega) - L(\tilde{R}\dot{\omega}_d - \tilde{R}S(s_4)\omega_d) - L(k_3 G(s_3)(s_4 - k_3 s_3)) \right)^T s_4 \quad (13)$$

4.2. Stability Analysis

Theorem 1. Consider the position error nominal subsystem Σ'_1 of the VTOL UAV, under the control law (9) and adaptive law (10), its position and velocity errors are uniformly ultimately bounded.

Proof. According to the definition of s_1 and s_2 , manifolds s_1 and s_2 are diffeomorphisms of the position error $\tilde{\xi}$ and velocity error \tilde{v} , so the stability of $\tilde{\xi}$ and \tilde{v} is equivalent to that of manifolds s_1 and s_2 .

Define the Lyapunov function

$$V_1 = \frac{1}{2} s_1^T s_1 + \frac{m}{2} s_2^T s_2 + \frac{1}{2\gamma} \tilde{m}^2 \quad (14)$$

The derivate of the Lyapunov function is

$$\dot{V}_1 = -k_1 s_1^T s_1 + s_1^T s_2 + s_2^T \left(m(e_3 g - \ddot{x}_d + D_1 + k_1(s_2 - k_1 s_1)) - n_d T_d \right) + \frac{1}{\gamma} \tilde{m} \dot{\tilde{m}} \quad (15)$$

Substitute the control law (9) and adaptive law (10) into (15), we get

$$\begin{aligned} \dot{V}_1 &= -k_1 s_1^T s_1 + s_1^T s_2 + s_2^T \left(m(e_3 g - \ddot{x}_d + D_1 + k_1(s_2 - k_1 s_1)) \right. \\ &\quad \left. - m \left(e_3 g - \ddot{x}_d + k_1(s_1 - k_1 s_2) + \frac{c\eta_1 s_2}{(s_2^T s_2 + \delta^2)^{1/2}} \right) - s_1 - k_2 s_2 \right) \\ &\quad + \frac{1}{\gamma} \tilde{m} \left(\dot{\tilde{m}} - \gamma s_2^T \left(e_3 g - \ddot{x}_d + k_1(s_1 - k_1 s_2) + \frac{c\eta_1 s_2}{(s_2^T s_2 + \delta^2)^{1/2}} \right) \right) \\ &\leq -k_1 s_1^T s_1 - k_2 s_2^T s_2 - m\eta_1 \|s_2\| \left(\frac{c \|s_2\|}{(s_2^T s_2 + \delta^2)^{1/2}} - 1 \right) \end{aligned} \quad (16)$$

when $\|s_2\| \geq \delta / (c^2 - 1)^{1/2}$

$$\dot{V}_1 \leq -k_1 s_1^T s_1 - k_2 s_2^T s_2 \quad (17)$$

In can be concluded that s_1 and s_2 are uniformly ultimately bounded, so the position and velocity errors $\tilde{\xi}$ and \tilde{v} are uniformly ultimately bounded [12].

Theorem 2. Consider the attitude error subsystem Σ_2 of the VTOL UAV, under the control law (12) and adaptive law (13), the error of MRPs and error of angle velocity are uniformly asymptotically stable.

Proof. According to the definition of s_1 and s_2 , manifolds s_3 and s_4 are diffeomorphisms of the error of MRPs $\tilde{\sigma}$ and error of angle velocity $\tilde{\omega}$, so the stability of $\tilde{\sigma}$ and $\tilde{\omega}$ is equivalent to that of manifolds s_3 and s_4 .

Define the Lyapunov function

$$V_2 = 2\ln(1 + s_3^T s_3) + \frac{1}{2} s_4^T J s_4 + \frac{1}{2} \tilde{\Xi}^T \Gamma \tilde{\Xi} \quad (18)$$

Based on the property of MRPs [10]

$$\sigma^T G(\sigma) = \left(\frac{1 + \sigma^T \sigma}{4} \right) \sigma^T \quad (19)$$

Then we get

$$\begin{aligned} \dot{V}_2 &= -k_3 s_3^T s_3 + s_3^T s_4 + s_4^T J \dot{s}_4 + \tilde{\Xi}^T \Gamma \dot{\tilde{\Xi}} \\ &= -k_3 s_3^T s_3 + s_3^T s_4 + \tilde{\Xi}^T \Gamma \dot{\tilde{\Xi}} + s_4^T \left((-\omega \times J \omega + \tau_d + D_2) \right. \\ &\quad \left. - J \cdot (\tilde{R} \dot{\omega}_d - \tilde{R} S(s_4) \omega_d) - k_3 J \cdot G(s_3)(s_4 - k_3 s_3) \right) \\ &\leq -k_3 s_3^T s_3 + s_3^T s_4 + s_4^T \left((-\omega \times L(\hat{\Xi}) \omega + \tau_d) \right. \\ &\quad \left. - L(\hat{\Xi}) (\tilde{R} \dot{\omega}_d - \tilde{R} S(s_4) \omega_d) - k_3 L(\hat{\Xi}) G(s_3)(s_4 - k_3 s_3) \right) \\ &\quad + \tilde{\Xi}^T \Gamma \left(\dot{\tilde{\Xi}} - \Gamma^{-1} \left(-S(\omega) L(\omega) - L(\tilde{R} \dot{\omega}_d - \tilde{R} S(s_4) \omega_d) \right. \right. \\ &\quad \left. \left. - L(k_3 G(s_3) s_4) \right)^T s_4 \right) + \eta_2 |s_4| \end{aligned} \quad (20)$$

Substitute the control law (12) and adaptive law (13) into (20), we get

$$\dot{V}_2 \leq -k_3 s_3^T s_3 - k_4 s_4^T s_4 \quad (21)$$

According to the Barbalat lemma, $\lim_{t \rightarrow \infty} s_3 = \lim_{t \rightarrow \infty} s_4 = 0$, so the states of attitude error subsystem Σ_2 is globally uniformly asymptotically stable [12].

Theorem 3. Consider the original trajectory tracking error subsystem Σ_1 of the VTOL UAV, the control law (9), (12) and adaptive law (10), (13) can make sure the error of MRPs and error of angle velocity uniformly ultimately bounded with appropriate control parameters.

Proof: First, we analyze the property of the coupled term. The Euclidean norm of the coupled term is as follows:

$$\|h\| = \frac{T_d}{m} \sqrt{n_d^T (2I_3 - \tilde{R}^T - \tilde{R}) n_d} \quad (22)$$

Assume \tilde{R} is obtained by rotating α around the axis r which is a unit vector. Using the Rodrigues formula we get

$$\tilde{R}(\alpha, r) = I_3 + \sin \alpha S(r) + (1 - \cos \alpha) S(r)^2 \quad (23)$$

Hence we get

$$\|h\| = \frac{2T_d}{m} \left| \sin \frac{\alpha}{2} \right| \sqrt{n_d^T S(r)^2 n_d} \quad (24)$$

Obviously, $\mathbf{n}_d^T S(\mathbf{r})^2 \mathbf{n}_d$ is a quadratic form and $\mathbf{n}_d^T \mathbf{n}_d = 1$, hence we can get $\lambda_{\min}(S(\mathbf{r})^2) \leq \mathbf{n}_d^T (S(\mathbf{r})^2) \mathbf{n}_d \leq \lambda_{\max}(S(\mathbf{r})^2)$. The operators $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximum and minimum eigenvalues of a matrix. As the eigenvalues of $S(\mathbf{r})^2$ are $\lambda_1 = 0, \lambda_{2,3} = -1$, consequently:

$$0 \leq \|h\| \leq \frac{2T_d}{m} \left| \sin \frac{\alpha}{2} \right| \quad (25)$$

Since $s_1 - k_1 s_2 = \tilde{v}$, according to (10) and the assumptions 2 and 3

$$T_d \leq \hat{m} \left(g + \nu + \nu + \frac{c\eta_1}{2\delta} \right) + k (\|s_1\| + \|s_2\|) \quad (26)$$

where $k = \max\{1, k_2\}$, then

$$T_d^2 \leq 2\hat{m}^2 \left(g + \nu + \nu + \frac{c\eta_1}{2\delta} \right)^2 + 2k^2 (\|s_1\|^2 + \|s_2\|^2) \quad (27)$$

Define the generalized error states $\mathbf{z}_1^T = [s_1^T \quad s_2^T]$, according to the definition of Euclidean norm:

$$\|s_1\|^2 + \|s_2\|^2 = \|\mathbf{z}_1\|^2 \quad (28)$$

Hence, we get $T_d^2 \leq 2\hat{m}^2 \left(g + \nu + \nu + \frac{c\eta_1}{2\delta} \right)^2 + 2k^2 \|\mathbf{z}_1\|^2$, then we get:

$$T_d \leq \sqrt{2}\hat{m} \left(g + \nu + \nu + \frac{c\eta_1}{2\delta} \right) + \sqrt{2}k \|\mathbf{z}_1\| \quad (29)$$

From the definition of MRPs

$$\left| \sin \frac{\alpha}{2} \right| = \frac{\|\tilde{\sigma}\|}{1 + \|\tilde{\sigma}\|^2} \leq \|\tilde{\sigma}\| = \|s_3\| \quad (30)$$

According to the assumption 2, we get

$$\|g\| \leq \frac{\sqrt{2}\hat{m}}{m} \left(g + \nu + \nu + \frac{c\eta_1}{2\delta} \right) + \frac{\sqrt{2}k}{m} \|\mathbf{z}_1\| \quad (31)$$

From (30) and (27), we can get

$$\begin{aligned} \|L_g V_1(t, x)\| &\leq 2\sqrt{2} \|s_2\| \left(\hat{m} \left(g + \nu + \nu + \frac{c\eta_1}{2\delta} \right) + k \|\mathbf{z}_1\| \right) \|s_3\| \\ &\leq \left(2\sqrt{2}\hat{m} \left(g + \nu + \nu + \frac{c\eta_1}{2\delta} \right) \|s_1\| + 2\sqrt{2}k \|\mathbf{z}_1\|^2 \right) \|s_3\| \end{aligned} \quad (32)$$

Consequently, there exists

$$\|L_g V_1(t, x)\| \leq (a_1 \|s_1\| + a_2 \|\mathbf{z}_1\|^2) \|s_3\| \quad (33)$$

where $a_1 = 2\sqrt{2}\hat{m}\left(g + \nu + \nu + \frac{c\eta_1}{2\delta}\right)$ and $a_2 = 2\sqrt{2}k$.

According to theorem 2, under the control law (12) and adaptive law (13), there exists a κ_∞ function $\beta(\|\tilde{\sigma}\|, t-t_0)$ which satisfies the inequality as follows:

$$\|s_3(t, t_0, \|\tilde{\sigma}\|)\| \leq \beta(\|\tilde{\sigma}\|, t-t_0) \quad (34)$$

There exists t_1 when $t > t_1$, $\beta(\|\tilde{\sigma}\|, t-t_0) \leq \varepsilon$, then we get:

$$\begin{aligned} \dot{V}_1 &\leq -k_1 s_1^T s_1 - k_2 s_2^T s_2 - m\eta_1 \|s_2\| \left(\frac{c\|s_2\|}{(s_2^T s_2 + \delta^2)^{1/2}} - 1 \right) + a_1 \varepsilon \|s_2\| + a_2 \varepsilon \|z_1\|^2 \\ &\leq -a_3 \|z_1\|^2 - m\eta_1 \|s_2\| \left(\frac{c\|s_2\|}{(s_2^T s_2 + \delta^2)^{1/2}} - a_4 \right) \end{aligned} \quad (35)$$

where $a_3 = \min\{k_1, k_2\} - a_2 \varepsilon$ and $a_4 = \left(1 + \frac{a_1 \varepsilon}{m\eta_1}\right)$.

Since $\|z_1\| \leq a_4 \delta / (c^2 - a_4^2)^{1/2}$, consider that $\|s_2\| \leq \|z_1\|$, so there exists $\|s_2\| \leq a_4 \delta / (c^2 - a_4^2)^{1/2}$, then we get

$$\dot{V}_1 \leq -a_3 \|z_1\|^2 \quad (36)$$

The generalized error states z_1 are uniformly ultimately bounded. From the definition of z_1 , the position and velocity errors of the system are diffeomorphisms of z_1 , so they are uniformly ultimately bounded. The controller will make the trajectory tracking error converge to a compact set by choosing appropriate control parameters.

5. Simulations and Analysis

Assume that the disturbance force and moment are

$$D_1 = \begin{bmatrix} \sin(\pi t/10) + \sin(\pi t/100) \\ \sin(\pi t/10) + \sin(\pi t/100) \\ \sin(\pi t/10) + \sin(\pi t/100) \end{bmatrix} \times 10^{-1} N \quad (37)$$

$$D_2 = \begin{bmatrix} \sin(\pi t/10) + \sin(\pi t/100) \\ \sin(\pi t/10) + \sin(\pi t/100) \\ \sin(\pi t/10) + \sin(\pi t/100) \end{bmatrix} \times 10^{-2} N \cdot m \quad (38)$$

Assume that the initial position of the VTOL UAV is $[0.5 \ 0.5 \ 0]^T$ (unit: m). The trajectory is generated in MATLAB to simulate the process of vertical take-off, hover and landing, and test the effect of the controller proposed in this paper.

Firstly, we simulated the stability control of the VTOL UAV. Assume that the initial position of the aircraft is $[1 \ 1 \ 0]^T$ (unit: m), the desired position is $[0 \ 0 \ -5]^T$ (unit: m). Figure 1 and Figure 2 show the stability control effect of the position and attitude subsystems.

Remark 2. Because we take the North-East-Down frame as the earth frame, the desired altitude -5m means that the VTOL UAV aircraft should take off and stabilize itself at the height of 5m.

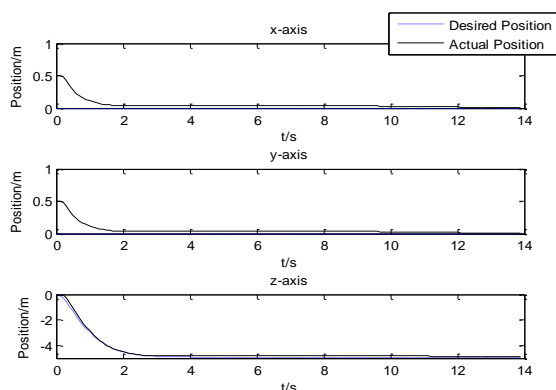


Figure 1. The Stabilization Control Effect of Position Subsystem

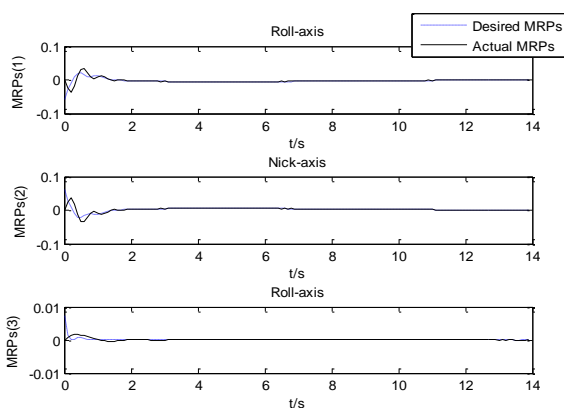


Figure 2. The Stabilization Control Effect of Attitude Subsystem

It is shown in Figure 1 that the controller has a good effect on the stability control of the VTOL UAV. The position error can converge to zero in less than 3 seconds. Figure 2 shows that the MRPs of the system will converge to desired attitude quickly.

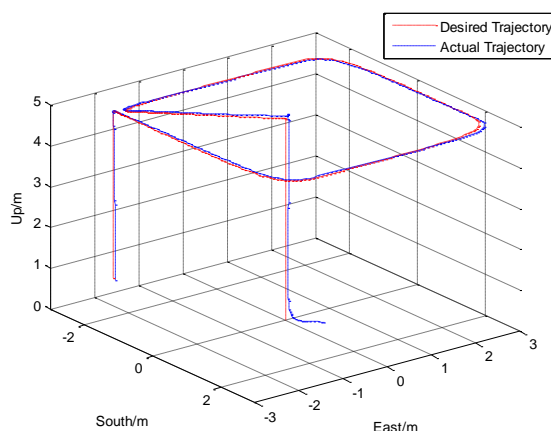


Figure 3. The Trajectory Tracking Effect of the VTOL UAV

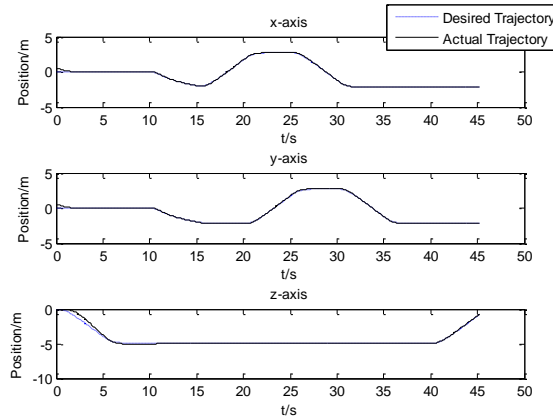


Figure 4. The Control Effect of the Position Subsystem

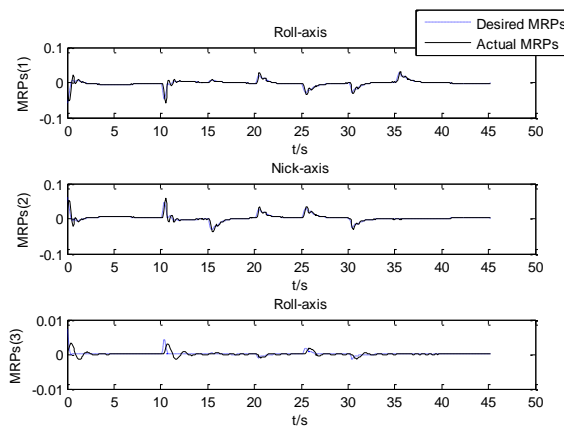


Figure 5. The Control Effect of the Attitude Subsystem

Figure 3 and Figure 4 show that the controller proposed in this paper has an ideal effect of trajectory tracking control, which enables the aircraft tracking an arbitrary trajectory. The trajectory of the aircraft will converge to the desired trajectory quickly even if the system has initial errors. It is shown in Figure 5 that the attitude subsystem has a good tracking effect while the desired MRPs keep changing.

5. Conclusions

In this paper, an adaptive based multiple-sliding-mode control methodology is proposed for trajectory tracking problem of a VTOL UAV. The system error model is established based on MRPs. The system error model is divided into two parts, translational subsystem and rotational subsystem, based on which the controller for each subsystem can be designed separately. For the controller, an adaptive law is designed to estimate the unknown parameters, based on which a multiple-sliding-mode controller is applied to deal with external disturbances. Simulations results show that the proposed control strategy can enable the VTOL UAV to track a desired trajectory quickly and accurately. The controller can deal with the uncertainties successfully to increase the control accuracy.

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