

Temporary Tracking Specifications Conversion to Obtain the Upper Frequency Limit

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Abstract

In robust control techniques where the control problem is in the frequency domain, it is required to specify process plants and disturbances in this domain, but also the limits and restrictions imposed originally in temporary form. A good technique to translate specifications from time to frequency will positively influence in the degree of overdesign and amount of iteration to achieve adequate results. The “inconvenient” is that for each type of specification is necessary to develop a specific conversion technique. The relevant technical for tracking specifications translation referred to the upper limit frequency is presented here. It will use approaches based on subcritical second order systems or an envelope approach with higher order systems.

Keywords: *Time domain, Frequency domain, Quantitative feedback theory, Tracking, Overdesign, Inverse interpolation*

1. Introduction

When transferring specifications from one domain to another is interested maximum precision and, in particular, from time to frequency, since in many control problems this conversion affects in later stages [16]. So if it is not done properly, it will generate a cumulative error, from the beginning of the problem. Parameter translation from frequency domain (*WD*) to time domain (*TD*) has direct application functions [3], so that, temporal verification of processed results in the *WD* can be carried out after frequency analysis without problem, in most cases. However, contrary, the treatment of the above functions is simply not possible in reverse form, so approximation techniques, not trivial at all, should be applied or classical hypotheses for simplification of the problem, with the consequent efficiency reduction in this case [13].

Control methodologies whose design phase is developed in the *WD*, as in “Quantitative Feedback Theory” (*QFT*) [7, 8, 9], require a translation in frequency of the specifications imposed in the *TD* as accurate as possible, since any deviation will affect to the bounds definition stage, critical when synthesize the controller [15].

Traditionally, the low accuracy contributed to the specifications conversion from *TD* to *WD* by techniques such as Krishnan and Cruickshanks’ [12], D’Azzo and Houppis models [4], or the Franchek and Herman’s procedure [5], is supplemented with specification, design and analysis processes of manual iteration, ending when the designer understands as acceptable the results achieved.

In control methodologies whose design stage is performed in frequency [17], but requirements are from temporal domain, a rigorous technical of transfer specifications from *TD* to *WD* is important for the following reasons:

- The desired controller can be set to more precise conditions, which generates less iteration until a solution is achieved, providing adequate results in both frequency and time domains.

- The “uncertainty” presented by the requirements demanded in the design phase of the controller will be minimal, so that, the degree of overdesign and thus the feedback cost are reduced [6].

However, the specifications conversion used is necessary to be a specific technical respect to the type of requirement is intended to translate. It will not be the same talking about tracking specifications, either of specifications for regulation (sensitivities); it will be necessary also to distinguish between coupling and decoupling of controlled variables for tracking and, even, if the limit to define is upper or lower. In any case, depending on the characteristics of the specification, so it should be the specific transfer process, thus a conversion is achieved with high accuracy and reliability, in every situation.

For tracking purposes, we can define the coupling of an output variable with respect to a reference input in two ways:

- From two limits, lower and upper, where the response should be between both two.
- From a single limit, upper or lower and, then, the response must take values lower or higher, respectively, from the limit considered.

Anyway, the specifications conversion for tracking between domains involves defining a specific one for every kind of limit, upper and lower, as the input parameters and output requirements are different in each case.

It is presented here an automatic transfer procedure for tracking specifications from time domain to frequency domain, specific to achieve the upper frequency limit. This limit is approximated to a 2nd order subcritical damped transfer function (*TF*) with characteristic parameters “ ω_n ” (natural frequency) and “ ζ ” (damping coefficient). To obtain rise and settlement times required is applied the necessary inverse interpolation, which will consist of a technique of successive approximations based on “*Newton's Method for 5th order Ascending and Descending Differences*”, described with detail in [10].

The proposed technique will work with step inputs, providing 2nd order transient responses with subcritical damping, representing the upper frequency limit of a tracking specification. Additionally, this technique can provide manual temporary tracking specifications translation to frequency domain with higher order system solution, through an envelope approach.

2. Technical for Tracking Specifications Translation of the Upper Frequency Limit

The method for tracking specifications translation from *TD* to *WD* presented here [10], is defined by the following relationship,

$$\begin{array}{c}
 TD_{final}(y_{max}, r_{max}, dev) \\
 \downarrow \\
 WD(Num_{max}, Den_{max})
 \end{array}
 \tag{1}$$

The *TD* input parameters describe, the maximum amplitude y_{max} of the step response, which tracks the step input with maximum amplitude r_{max} , where dev is the settlement channel of the response (admissible tolerance). *WD* equivalent parameters determine the *TF* of the upper limit $T_{Upp}(j\omega)$ given by,

$$T_{Upp}(j\omega) = \frac{Num_{max}}{Den_{max}}(j\omega)
 \tag{2}$$

The result will be characterized by a defined maximum overshoot Mp , function of the input parameters y_{max} and r_{max} , such that,

$$M_p = \frac{y_{\max} - r_{\max}}{r_{\max}} \quad (3)$$

If we consider the overshoot M_p defined for subcritical systems as in [14], *i.e.*:

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \quad (4)$$

Which corresponds a minimum damping coefficient ($\zeta = \zeta_{\min}$),

$$\zeta_{\min} = \left[\frac{\left(\ln Mp / \pi \right)^2}{1 + \left(\ln Mp / \pi \right)^2} \right]^{1/2} \quad (5)$$

On the other hand, for rise time tr and settlement time ts of the response, as what is sought is an upper frequency limit, the temporal equivalences are minimum values. However, for practical purposes must be imposed,

- Reference upper times tr_{upp} and ts_{upp} (for example, if the user does not indicate other, by default 5s and 20s, respectively), to seek associated ω_n with minimum times;
- To avoid a singularity by using inverse interpolation, minimum times must be limited, which can be nulls in theory, but not so in practice.

The sequence of steps for tracking specifications translation of the upper frequency limit is as follows,

1. Determination of minimum damping coefficient ζ_{\min} . From input parameters y_{\max} and r_{\max} , the maximum overshoot M_p (3) is obtained, which inserted into equation (5) provides ζ_{\min} .

2. Find out the relationship $\omega_n(tr, ts, \zeta_{\min}, dev)$. Considering that 2nd order systems with the same ζ , but different ω_n , are characterized by the same overshoot M_p , so they have the same relative stability. The time constant of systems with the same relative stability is given by,

$$T = \frac{1}{\zeta\omega_n} \quad (6)$$

It can be expressed for generic tr and ts , respectively, as

$$tr(\omega_n) = \frac{1}{\zeta\omega_n} k_r \quad (7)$$

$$ts(\omega_n) = \frac{1}{\zeta\omega_n} k_s \quad (8)$$

Parameters k_r and k_s depend on the tolerance band value imposed [14]. If we use the value $\omega_n = 1$, the above equations (7) and (8) can be rewritten as,

$$tr(\omega_n = 1) = \frac{1}{\zeta} k_r \quad (9)$$

$$ts(\omega_n = 1) = \frac{1}{\zeta} k_s \quad (10)$$

Combining (7) with (9) and (8) with (10), the pair of natural frequency values ω_{nr} and ω_{ns} , associated to t_r and t_s , respectively, are obtained in the following way,

$$\frac{tr(\omega_n = 1)}{tr(\omega_{nr})} = \omega_{nr} \quad (11)$$

$$\frac{ts(\omega_n = 1)}{ts(\omega_{ns})} = \omega_{ns} \quad (12)$$

Note that ω_{nr} is a specific ω_n for tr required and, so for ω_{ns} respect to ts .

2a. Applying equation (11) with tr and ζ_{min} , it is achieved,

$$\frac{tr(\omega_n = 1, \zeta_{min})}{tr(\omega_{nr}, \zeta_{min})} = \omega_{nr}(tr, \zeta_{min}) \quad (13)$$

Being $tr(\omega_{nr}, \zeta_{min})$ an input parameter required and $tr(\omega_n=1, \zeta_{min})$ calculated with a successive approximation technique, finally $\omega_{nr}(tr, \zeta_{min})$ is obtained from equation (13).

2b. Similarly, applying equation (12) for ts and ζ_{min} , it is achieved,

$$\frac{ts(\omega_n = 1, \zeta_{min})}{ts(\omega_{ns}, \zeta_{min})} = \omega_{ns}(ts, \zeta_{min}) \quad (14)$$

With the input parameter $ts(\omega_{ns}, \zeta_{min})$ and $ts(\omega_n=1, \zeta_{min})$ calculated using a successive approximation technique, $\omega_{ns}(ts, \zeta_{min})$ is obtained from equation (14).

The way proposed to determine $tr(\omega_n=1, \zeta_{min})$ in 2a) and $ts(\omega_n=1, \zeta_{min})$ in 2b) is applying “differences Newton method” [2]. For this, use as interpolation function the typical step sign $f_{step}(t)$, i.e.:

$$f_{step}(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \operatorname{sen}\left(\omega_n \sqrt{1-\zeta^2} t + \arccos\zeta\right) \quad (15)$$

For each tr and ts , values of $f_{step}(tr)$ and $f_{step}(ts)$ are given. Observe that we want to obtain temporal parameters applying inverse interpolation in (15), with $\omega_n=1$ and $\zeta = \zeta_{min}$ so, only there is one unknown parameter in the equation each time.

3. Choose for $\omega_n(\zeta_{min})$ the biggest value between $\omega_{nr}(tr, \zeta_{min})$ and $\omega_{ns}(ts, \zeta_{min})$, being this the most restrictive condition for the temporary joint requirements (tr , ts).

4. Calculate $\omega_n(\zeta_{min}, tr, ts, dev)$, varying tr and ts , with $tr_{upp} \geq tr \geq tr_{min}$ and $ts_{upp} \geq ts \geq ts_{min}$, which will be named as (ω_n, ζ_{min}) pairs or double vector $wd(\omega_n, \zeta_{min})$.

To do this, decrease the value of tr and ts from its initial input upper values, tr_{upp} and ts_{upp} , respectively, to a practical minimum limit, for example, using the following routine:

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tr=trupp; ts=tsupp
apply 2) and 3): obtain wd(ωn(trupp, tsupp), ζmin)
while tr>0.01 % trmin=0.01, practical limit imposed
    ts=ts-0.5
    {
        while (ts>tr) and (ts>0.02) % tsmin=0.02, practical limit
            apply 2) and 3): obtain wd(ωn, ζmin)
            ts=ts-0.5
        end
        tr=tr-0.5
    }
end

```

Observe that for a specific ts value, if you reduce ω_n value respect to the one given for the associated pair (ω_n, ζ_{min}) , to maintain ts value, ζ_{min} must be bigger (Mp decreases,

which is valid). So, allowed frequency response is given for the area underneath the curve (ω_n, ζ_{min}) . See Figure 1.

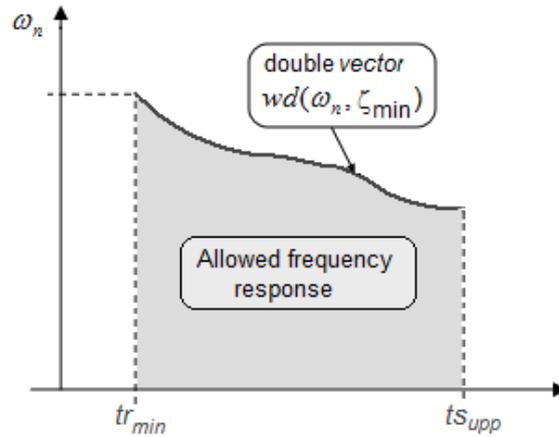


Figure 1. Graphical Representation of $wd(\omega_n, \zeta_{min})$

5. Define 2nd order curves $T(j\omega)$, such that,

$$T(j\omega) = \frac{\omega_n^2}{s^2 + 2\zeta_{min}\omega_n s + \omega_n^2} \quad \text{with } s = j\omega \quad (16)$$

Where, replacing the double vector $wd(\omega_n, \zeta_{min})$, the set of curves in the Bode diagram is obtained, which fulfill tracking specifications of the upper time limits and maximum overshoot, initially imposed.

6. Achieving the upper limit $T'_{Upp}(j\omega)$ in the frequency. On the Bode diagram above, the upper intersection (maximums in magnitude and in phase) of the different curves, defined by the double vector $wd(\omega_n, \zeta_{min})$, is searched.

7. As the upper frequency limit curve $T'_{Upp}(j\omega)$ is made by the intersection of several $T(j\omega)$ curves, the way of describing its TF is to approximate it using one of the following criteria, depending on the bandwidth and precision of interest :

- a) Set the upper limit in the WD with accuracy at low frequencies. That is, it is selected as $T_{Upp}(j\omega)$ the $T(j\omega)$ of maximum magnitude, at the lowest frequency in the frequency work band used. This approach will be named as *restriction at low frequencies*.
- b) Set the upper limit in the WD with accuracy at high frequencies. That is, it is selected as $T_{Upp}(j\omega)$ the $T(j\omega)$ of maximum magnitude, at the highest frequency of the frequency work band. This approach will be named as *restriction at high frequencies*.
- c) Use rationalization of the frequency responses obtained in 6) to get an approach at the envelope of the upper limit $T_{Upp}(j\omega)$. It generates higher order TFs using an order reduction method, such as defined in [1]. This approach will be named as *restriction at the envelope approach*.

8. Getting the final TD parameters, from the WD ones achieved, to match the original TD requirements. That is,

- Also, to maintain stability there must be no RHP poles or zeros in the final output TF , nor poles with zero value. The program allows eliminating manually this type of poles/zeros and even too, those with insignificant values to reduce order.

Additionally, the program offers to the user the following possibilities:

- It permits to apply gain adjustment over the output TF obtained.
- It presents numerical and graphical magnitude/phase differences between the input $T'_{Upp}(j\omega)$ and the output $T_{Upp}(j\omega)$. With this information, the user decides to stop the process or not.
- The process ends offering numerical and graphical temporary responses associated with $T_{Upp}(j\omega)$ obtained. With the visual information, the user decides if the approach is adequate or not.

4. Application of the Technical for Tracking Specifications Translation of the Upper Frequency Limit

Suppose tracking specifications given by the upper frequency limit, described by the set of temporal parameters $TD_{original}(y_{max}=2.3, r_{max}=1.8, dev=\pm 1.8\%)$, ie, an input step of amplitude 1.8 must generate a step response with a maximum amplitude value 2.3 (maximum overshoot equivalent of 27.78%), with maximum rise and settlement times, by default of 5s and 20s, respectively, and the admissible tolerance of $\pm 1.8\%$. Applying the technique described above, with the downloadable programs in Matlab as indicated in Section5, it is obtained:

- 2nd order responses $T(j\omega)$, from $wd(\omega_n, \zeta_{min})$, which are fitted to the $TD_{original}$ parameters, given by the upper time limit; these are described, in this case, with a total of 39 TFs .
- Bode diagram of the 2nd order responses $T(j\omega)$, from the double vector $wd(\omega_n, \zeta_{min})$. Results in Figure 2.
- Sets of TD responses, equivalent to those $T(j\omega)$ systems in the WD. Results are presented in Figure 3.

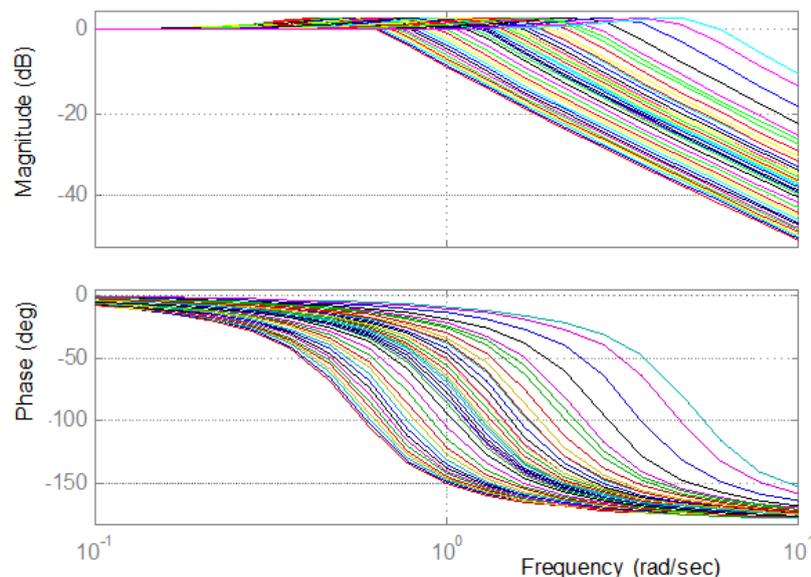


Figure 2. 2nd Order Responses $T(j\omega)$

- Upper limit $T'_{Upp}(j\omega)$ in the frequency domain. It is obtained looking for maximums in magnitude and phases, throughout the frequency work band, over the 39 TFs achieved. Results in Figure 4.

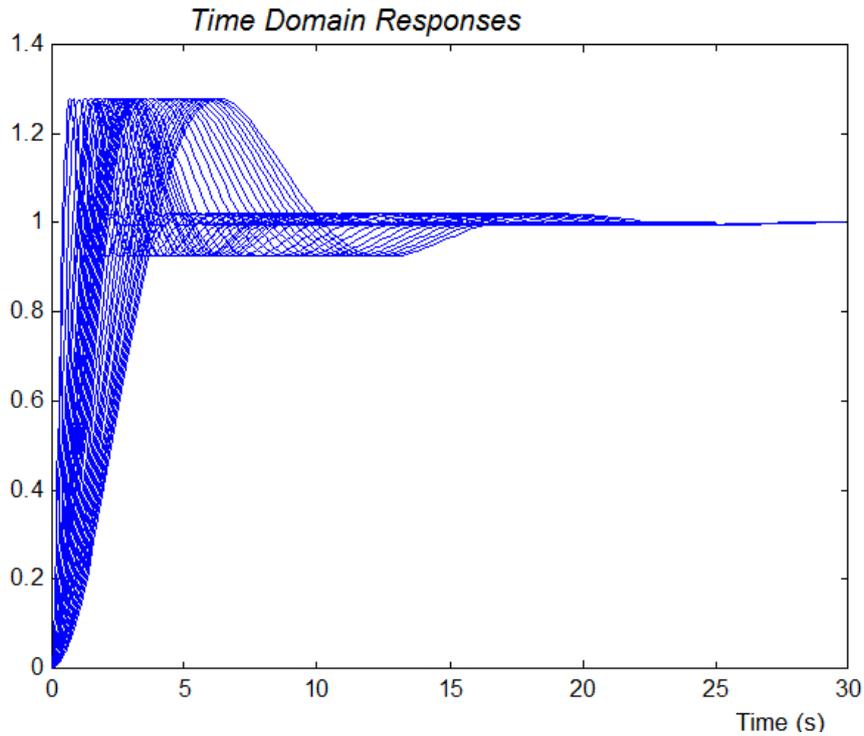


Figure 3. Set of Temporary Responses, Equivalent to the Systems $T(j\omega)$

- Upper limit $T_{Upp}(j\omega)$ in the frequency domain, considering *restriction at low frequencies*, ie, from the curves of maximum magnitude in the low frequency area. Results in Figure 5.
- Obtaining curve and *final TD* parameters that fit to the *original TD*, applying *restriction at low frequencies*. Temporary results in Figure 6.
 - Upper frequency limit defined as the most restrictive curve $T(j\omega)$, at low frequencies,

$$\left(\frac{Num_{max}}{Den_{max}} \right)_{Lo} = \frac{0.2915}{s^2 + 0.4077s + 0.2915} \quad (21)$$

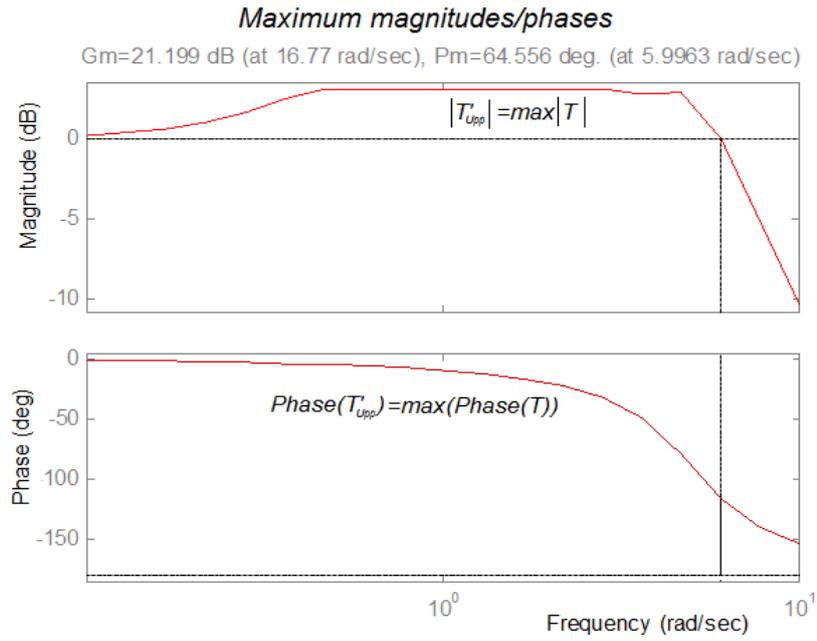


Figure 4. Upper Limit $T'_{upp}(j\omega)$ in the WD

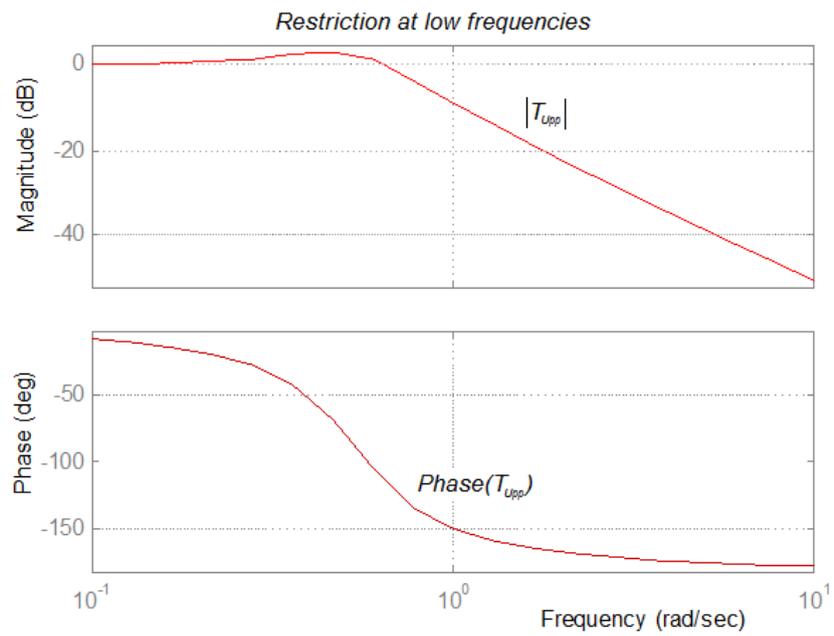


Figure 5. Upper Limit $T_{upp}(j\omega)$ in the WD, Considering Restriction at Low Frequencies

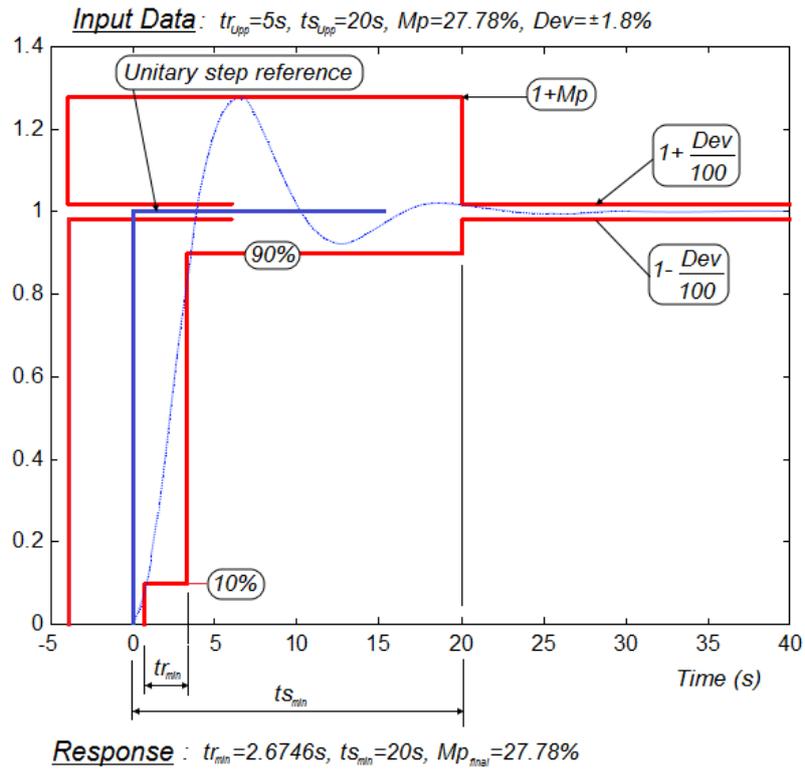


Figure 6. Final Temporary Response Associated with $T_{upp}(j\omega)$, Considering Restriction at Low Frequencies

- Upper limit $T_{upp}(j\omega)$ in the frequency domain, considering *restriction at high frequencies*, ie, from the curves of maximum magnitude in the high frequency area. Results in Figure 7.

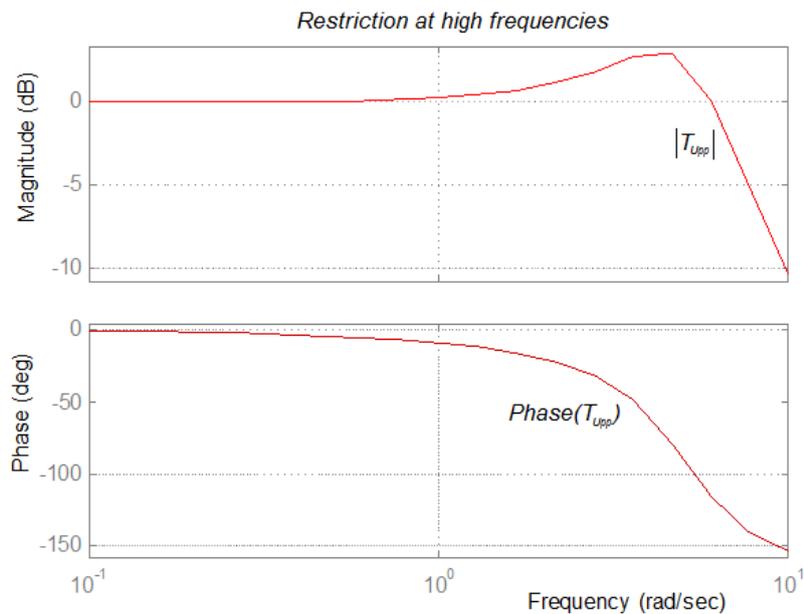


Figure 7. Upper Limit $T_{upp}(j\omega)$ in the WD, Considering Restriction at High Frequencies

- Curve and *final TD* parameters that fit to the *original TD*, applying *restriction at high frequencies*. Temporary results in Figure 8.
 - Upper frequency limit defined as the most restrictive curve $T(j\omega)$, at high frequencies,

$$\left(\frac{Num_{max}}{Den_{max}} \right)_{Hi} = \frac{25.15}{s^2 + 3.787s + 25.15} \quad (22)$$

- Upper limit $T_{Upp}(j\omega)$ in the frequency domain, considering *restriction at the envelope approach*, ie, rationalizing the real limits until *TF* has not got too high order. Results in Figure 9. Graphical magnitude/phase differences between $T'_{Upp}(j\omega)$ and $T_{Upp}(j\omega)$ are expressed in Figure 10. We are obtaining a maximum magnitude error of $0.3342rad$ and a maximum phase error of $8.38 degrees$.
- Curve and *final TD* parameters that fit to the *original TD*, applying *restriction at the envelope approach*. Temporary results in Figure 11.
 - Upper frequency limit obtained rationalizing the upper intersection in the set $T(j\omega)$, defined with two poles and one zero. There are other possible stable *TF*, but this one offers the closest maximum overshoot ($Mp_{final}=25.41\%$) respect the required, with minimum approach errors and not too high order.

$$\left(\frac{Num_{max}}{Den_{max}} \right)_{Env} = \frac{0.3099s + 21.35}{s^2 + 3.697s + 21.35} \quad (23)$$

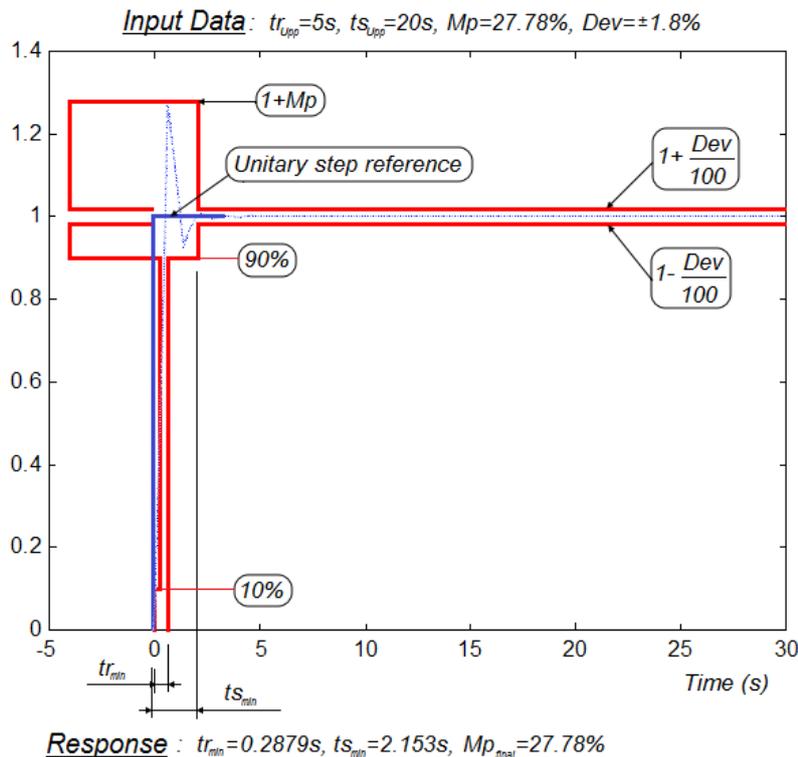


Figure 8. Final Temporary Response Associated with $T_{Upp}(j\omega)$, Considering Restriction at High Frequencies

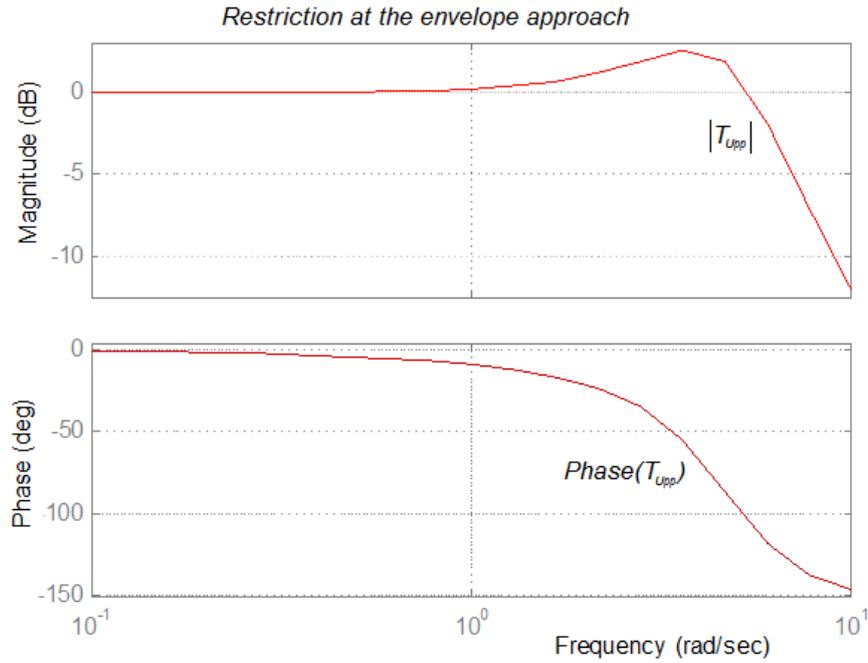


Figure 9. Upper Limit $T_{Upp}(j\omega)$ in the WD, Considering Restriction at the Envelope Approach

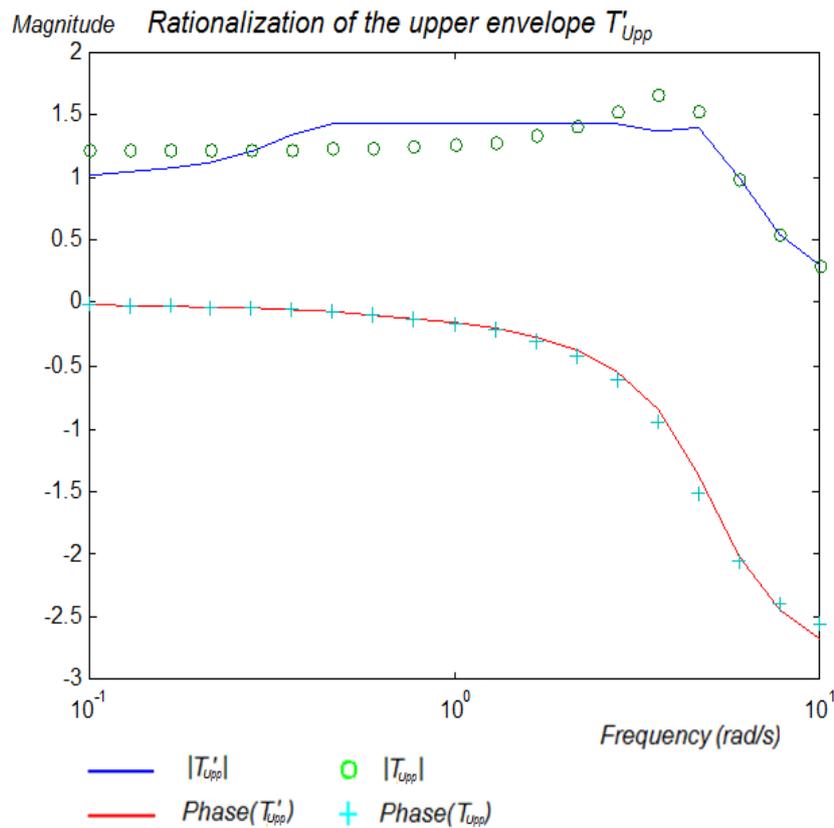


Figure 10. Graphical differences in Magnitude and Phase between $T'_{Upp}(j\omega)$ and the Upper Envelope Approach TF $T_{Upp}(j\omega)$

5. Programs Development

The necessary programs to verify the above example and any other, for tracking temporary specifications translation to frequency domain, have been developed in Matlab format.

“Temporary Tracking Specifications Conversion to Obtain the Upper Frequency Limit” can be downloaded from the next URL:

<https://www.dropbox.com/sh/16w7ujosn9yet7v/AADmlrplf1489cK7Uaa6ODvca?dl=0>

The application starts with the program STEPTRAC.M

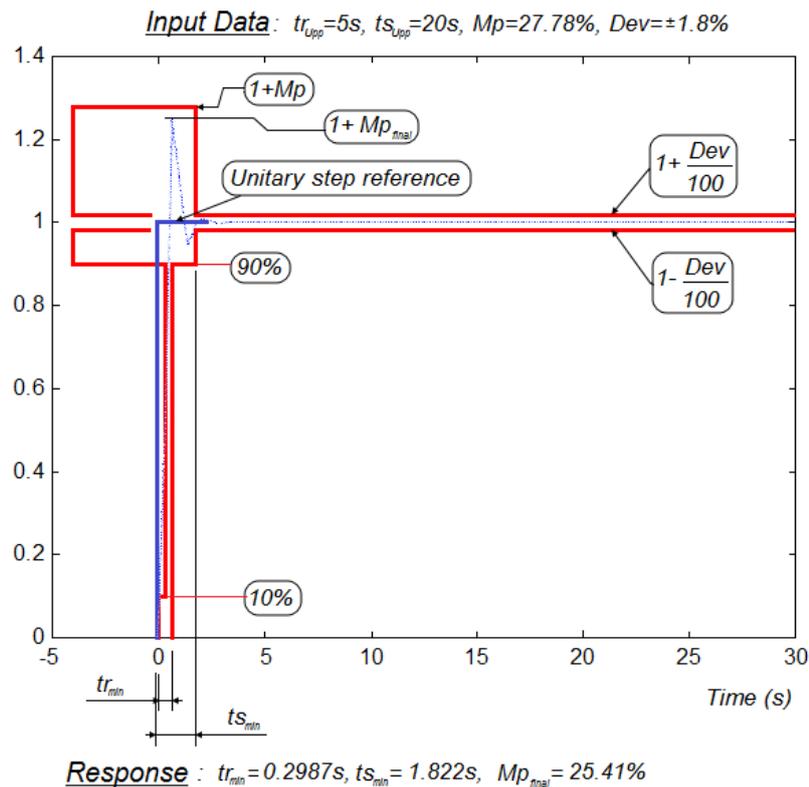


Figure 11. Final Temporary Response Associated with $T_{Upp}(j\omega)$, Considering Restriction at the Envelope Approach

6. Conclusions

Techniques for transferring rigorous specifications from TD to WD are more important than traditionally they have been considered: control methodologies with temporary requirements, but whose design stage is performed in the frequency domain, supplement the consequences of shortcomings in this field using iterative processes, long and complex sometimes, which still affect the final result as overdesign and relaxation of specifications, that cannot be fulfilled.

These conversion techniques between domains will allow having to work with less iteration until a suitable solution is achieved, if it exists, since the controller searched in the WD can match to more exactly frequency conditions from the beginning. Furthermore, being the deviation of the requirements demanded in the design stage minimum, the overdesign cost is reduced.

It has been developed here an automatic specific procedure for precision conversion of tracking specifications from time to frequency, to get the upper frequency limit using

approach to a 2nd order system with subcritical damping. On the other hand, it can be chosen the translation option with manually approach to a higher order system too.

The technique generates the *TF* associated to an upper frequency limit that, with a step input of value r_{max} , produces a subcritical damped response with maximum amplitude y_{max} , so that, the overshoot M_p will be a particular one. Also it uses, as additional input parameters, maximum reference times of rise and settlement, $t_{r_{upp}}$ and $t_{s_{upp}}$, respectively. From these times the search begins by inverse interpolation of the response frequency associated to the more appropriate upper limit. The process and final result depend on the selected approach, which determines the area of the bandwidth where maximum accuracy is required: low or high frequency (specific area in the bandwidth) or envelope (whole bandwidth).

The lower frequency limit required by some type of tracking specifications is often described in temporary terms of overshoot response to a given step input. Considering the principles of the technique presented in this paper, it can be developed a specific method to get this kind of tracking specification in the *WD*.

The methodology described basing on temporary parameters inverse interpolation, can be simplified applying classical hypothesis, although this implies reducing accuracy.

Sensitivity specifications described in time are characterized by a disturbance input which should produce a damped impulse response, with very specific amplitude and fading characteristics. The methodology for temporary specifications conversion to frequency using inverse interpolation can be applied in this case too, by timely adaptation.

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