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L_2-gain Analysis and Design of Discrete-time Switched Systems with Input Saturation

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Abstract

The problem of L_2-gain analysis and anti-windup compensation gains design is studied for a class of discrete-time switched systems subject to actuator saturation via the multiple Lyapunov function approach. When a set of anti-windup compensation gains is given, a sufficient condition on tolerable disturbances is obtained, under which the state trajectory starting from the origin will remain inside a bounded set. Then over this set of tolerable disturbances, we obtain the upper bound of the restricted L_2-gain. Furthermore, the anti-windup compensation gains and the switched law, which aim to determine the maximum disturbance tolerance capability and the minimum upper bound of the restricted L_2-gain, are presented by solving a convex optimization problem with linear matrix inequality (LMI) constraints. Finally we give a numerical example to demonstrate the effectiveness of the proposed method.

***Keywords**: L_2-gain; anti-windup; switched systems; actuator saturation; multiple Lyapunov function; tolerable disturbances*

1. Introduction

As an important class of a hybrid system, switched systems have attracted much attention in recent years for its theoretical [1-3] and practical [4-6] importance which consist of a set of continuous-time and/or discrete-time subsystems interacting with a logical or decision-making process. As [1] pointed out, stability of switched systems is of the most importance in analysis and design. Therefore, many approaches and techniques have been introduced to study the stability [7-11] and synthesis problem for such systems [12-17]. Among these methods, the common Lyapunov function is used to check the stability property under arbitrary switchings [8]. Although this property is a desirable property, not all or even most switched systems do not possess a common Lyapunov function. And even then, the switched system still is of stability under certain switching laws based on other methods. Among them, the multiple Lyapunov functions method [9]-[10], the single Lyapunov function method [11, 12] and the average dwell-time technique [13, 14] are effective tools of choosing such switching laws.

On the other hand, the system with exogenous disturbances is a familiar type of uncertain systems in practice. The L_2-performance analysis has also an important influence on systems subject to disturbance, because it provide a kind of measure of the certain extent of the influence of disturbance. [14] investigated the disturbance attenuation properties of time-controlled switched systems by using an average dwell time approach incorporated with a piecewise Lyapunov function. Using multiple Lyapunov functions method, [15] investigated the L_2-gain analysis of switched systems, which enables derivation of improved an L_2-gain characterization and a method on switching law design. All the results mentioned above study continuous-time switched systems. However, from

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a practical point of view, it makes more sense to study discrete-time switched systems with disturbances. Via multiple quadratic Lyapunov-like functions, [16] considered the exponential stability and L_2 induced gain performance for a class of discrete-time switched systems. The L_2-gain analysis and control synthesis for uncertain discrete-time switched systems were investigated by using the switched Lyapunov function method in [17].

In addition, due to physical constraints or safety limit input saturation appear in almost all practical systems. It can degrade performance of system and even make system unstable. Thus, study of control systems with input saturation has received a great deal of attention [18-22], which focus mainly on how to cope with saturation nonlinearity. There are many methods developed to study this kind of systems [23-26]. However, the anti-windup approach is a far more practical approach for dealing with saturation nonlinearity. This method is to firstly design a linear controller that meets the performance requirement of the closed-loop system without considering actuator saturation and then to design an anti-windup compensator to reduce the effects of the actuator saturation [26]. For switched systems subject to actuator saturation, studying its property become more difficult. It is because that the saturation nonlinearities and switching are interacted on each other. The existing results of switched systems with actuator saturation are relatively few [27-31]. By using multiple Lyapunov functions method, the L_2-gain analysis and control synthesis problem is addressed for a class of uncertain switched systems with saturating actuators in [27]. The design of switching scheme is considered for a class of switched systems in the presence of actuator saturation in [28]. In order to enlarge the domain of attraction of the linear systems, the idea of switching among multiple anti-windup gains was explored based on the min function of multiple quadratic Lyapunov functions in [29]. For a class of uncertain switched discrete systems subject to actuator saturation, [30] addressed the problem of disturbance attenuation was via the multiple Lyapunov functions method. Via the multiple Lyapunov functions approach, the problem of disturbance tolerance/rejection is considered for a class of switched systems with actuator saturation in [31]. About study of the L_2-gain analysis and anti-windup design problem of switched discrete systems on the strength of the multiple Lyapunov functions technique, to the best of my ability, there are nearly no results in the existing literature. That is our motivation.

Based on the multiple Lyapunov function approach, the L_2-gain analysis and anti-windup compensation gains design are considered for a class of discrete-time switched systems subject to actuator saturation in this paper. Firstly, we obtain a sufficient condition of disturbance tolerance under which the state trajectory starting from the origin will remain inside a bounded set. Then, over the set of tolerable disturbances we analyzed the restricted L_2-gain.Furthermore, in order to obtain the maximal disturbance tolerance capacity and the minimum upper bound of the restricted L_2-gain over the set of tolerable disturbances, the problem of designing the anti-windup compensation gains and the switched law is formulated and solved as a convex optimization problem with LMI constraints.

The rest of this paper is organized as follows: The system description and preliminaries are given in section 2. Section 3 and 4 analyze the disturbance tolerance capacity and L_2-gain respectively. The anti-windup synthesis problem is considered in section 5. An example is shown in section 6, followed by conclusions in section 7.

2. Problem Statement and Preliminaries

In this paper, the following discrete-time switched systems with actuator saturation are considered :

$$\begin{aligned} x(k+1) &= A_\sigma x(k) + B_\sigma sat(u(k)) + E_\sigma w(k), \\ y(k) &= C_{\sigma 1} x(k), \\ z(k) &= C_{\sigma 2} x(k), \end{aligned} \tag{1}$$

where $k \in Z^+$, $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the control input vector, $y(k) \in R^p$ is the measured output vector, $z(k) \in R^l$ is the controlled output and $w(k) \in R^q$ is the external disturbance input. $\sigma(k)$ is a switching signal which takes its values in the finite set $I_N = \{1, \ldots, N\}$; $\sigma(k) = i$ means that the *i*-th subsystem is active. A_i, B_i, E_i, C_{i1} and C_{i2} are real constant matrices of appropriate dimensions. Due to the presence of actuator saturation, the L_2-gain may not be well defined when the external disturbances is sufficiently large, because a sufficiently large external disturbance may drive the system state or output unbounded under any control input [20, 31]. Therefore, we assume that

$$W_\beta^2 := \left\{ w : R_+ \to R^q, \sum_{k=0}^{\infty} w^T(k) w(k) \le \beta \right\}, \tag{2}$$

where β is some positive number that is aimed at representing disturbance tolerance capability of system. $sat : R^m \to R^m$ is the vector valued standard saturation function defined as

$$sat(u) = \left[sat(u^1), \ldots, sat(u^m) \right]^T, \tag{3}$$

$$sat(u^j) = sign(u^j) \min\left\{1, \left|u^j\right|\right\}, \forall j \in Q_m = \{1, \ldots, m\}. \tag{4}$$

Notice that here we have slightly abused the notation by using "$sat(\cdot)$" to stand for both scalar and vector-valued saturation functions. It is generally known that it is without loss of generality to assume unity saturation level. The non-unity saturation level can always be transformed into unity saturation level by scaling the matrix B_i and u [25].

For system (1), suppose that a set of n_c-order dynamic output feedback controllers are of the form

$$\begin{aligned} x_c(k+1) &= A_{ci} x_c(k) + B_{ci} u_c(k), \\ v_c(k) &= C_{ci} x_c(k) + D_{ci} u_c(k), \forall i \in I_N, \end{aligned} \tag{5}$$

where $x_c(k) \in R^{n_c}$, $u_c(k) = y(k)$ and $v_c(k) = u(k)$ are the vector of state, input and controller output respectively. In this paper, we focus on L_2-gain analysis and anti-windup gains design, so we assume that the dynamic compensators have been designed for the system (1) without actuator saturation, as commonly adopted in the literature (see, for example [26]).

For the sake of weakening the undesirable effects of the windup caused by actuator saturation includes adding to the controller dynamics a "correction" term of the form $E_{ci}(sat(v_c(k)) - v_c(k))$. Then, the modified controller structure has the form

$$\begin{aligned} x_c(k+1) &= A_{ci} x_c(k) + B_{ci} u_c(k) + E_{ci}(sat(v_c(k)) - v_c(k)), \\ v_c(k) &= C_{ci} x_c(k) + D_{ci} u_c(k), \forall i \in I_N. \end{aligned} \tag{6}$$

Clearly, through adding such the correction terms, the dynamic controllers (6) go on operating in the linear domain without actuator saturation, which does not affect the systems performance. Then the controller state of the system with input saturation can be revised by using the anti-windup compensators which restore the system nominal performance as much as possible.

Then, when we adopt the above controllers and anti-windup tactic, the closed-loop system will be written as

$$\begin{aligned} & x(k+1) = A_i x(k) + B_i sat(v_c(k)) + E_i w(k), \\ & y(k) = C_{i1} x(k), \\ & z(k) = C_{i2} x(k), \\ & x_c(k+1) = A_{ci} x_c(k) + B_{ci} C_{i1} x(k) + E_{ci}(sat(v_c(k)) - v_c(k)), \\ & v_c(k) = C_{ci} x_c(k) + D_{ci} C_{i1} x(k), \ \forall i \in I_N. \end{aligned} \tag{7}$$

Now, define a new state vector

$$\zeta(k) = \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix} \in R^{n+n_c}, \tag{8}$$

and the matrices

$$\tilde{A}_i = \begin{bmatrix} A_i + B_i D_{ci} C_{i1} & B_i C_{ci} \\ B_{ci} C_{i1} & A_{ci} \end{bmatrix}, \tilde{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, G = \begin{bmatrix} 0 \\ I_{n_c} \end{bmatrix},$$

$$K_i = \begin{bmatrix} D_{ci} C_{i1} & C_{ci} \end{bmatrix}, \tilde{E}_i = \begin{bmatrix} E_i \\ 0 \end{bmatrix}, \tilde{C}_{i2} = \begin{bmatrix} C_{i2} & 0 \end{bmatrix}.$$

Therefore, in combination with (7) and (8), the closed-loop system can be rewritten as

$$\begin{aligned} \zeta(k+1) &= \tilde{A}_i \zeta(k) - (\tilde{B}_i + G E_{ci}) \psi(v_c) + \tilde{E}_i w(k), \\ z(k) &= \tilde{C}_{i2} \zeta(k), \ \forall i \in I_N, \end{aligned} \tag{9}$$

where $v_c = K_i \zeta(k), \psi(v_c) = v_c - sat(v_c)$.

In this paper, we design the switched law and the anti-windup compensation gains via multiple Lyapunov such that the largest disturbance tolerance level of the system (9) is obtained at the beginning , then the minimized upper bound of the restricted L_2-gain is achieved.

Definition 1 [20], [31]. Given $\gamma > 0$. The system (9) is said to have a restricted L_2-gain less than γ, if there exists a switching signal $\sigma(k)$ such that the following condition is satisfied under the zero initial condition,

$$\sum_{k=0}^{\infty} z^T(k) z(k) < \gamma^2 \sum_{k=0}^{\infty} w^T(k) w(k),$$

for all nonzero $w(k) \in W_\beta^2$.

To develop the main results, we need the following lemmas.

Lemma 1 [Schur's complements]. Given the symmetric matrix $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix}$, the following statements are equivalent:

1): $A < 0$;

2): $A_{11} < 0, A_{22} - A_{12}^T A_{11}^{-1} A_{12} < 0$;

3): $A_{22} < 0, A_{11} - A_{12} A_{22}^{-1} A_{12}^T < 0$.

For a positive definite matrix $P \in R^{(n+n_c)\times(n+n_c)}$ and a scalar $\rho > 0$, an ellipsoid $\Omega(P, \rho)$ is defined as

$$\Omega(P, \rho) = \left\{ \zeta \in R^{n+n_c} : \zeta^T P \zeta \le \rho \right\}.$$

Consider matrices $K_i, H_i \in R^{m\times(n+n_c)}$ and define the following polyhedral set:

$$L(K_i, H_i) = \left\{ \zeta \in R^{n+n_c} : \left| (K_i^j - H_i^j) \zeta \right| \le 1, i \in I_N, j \in Q_m \right\},$$

where K_i^j, H_i^j are the j-th row of matrices K_i and H_i respectively.

Lemma 2 [26]. Consider the function $\psi(v_c)$ defined above. If $\zeta \in L(K_i, H_i)$, then the relation

$$\psi^T(K_i \zeta) J_i [\psi(K_i \zeta) - H_i \zeta] \le 0, \ \forall i \in I_N, \tag{10}$$

holds for any matrix $J_i \in R^{m\times m}$ diagonal and positive definite.

3. Disturbance Tolerance

In this section, we derive a sufficient condition under the given anti-windup gain matrices E_{ci} via the multiple Lyapunov function method, which guarantees that the state trajectory of the system (9) starting from the origin will remain inside a bounded set for any disturbance satisfying (2). The approach obtaining the largest disturbance tolerance level by designing the switched law and the anti-windup compensation gains will be stated in Section5.

Theorem 1 Suppose there exist positive definite matrices P_i, matrices H_i and diagonal positive definite matrices J_i and a set of scalars $\beta_{ir} \geq 0$ such that

$$\begin{bmatrix} -P_i + \sum_{r=1,\, r\neq i}^{N} \beta_{ir}(P_r - P_i) & H_i^T J_i & 0 & \tilde{A}_i^T P_i \\ * & -2J_i & 0 & -(\tilde{B}_i + GE_{ci})^T P_i \\ * & * & -I & \tilde{E}_i^T P_i \\ * & * & * & -P_i \end{bmatrix} < 0, \quad \forall i \in I_N, \tag{11}$$

and

$$\Omega(P_i, \beta) \cap \Phi_i \subset L(K_i, H_i), \forall i \in I_N. \tag{12}$$

Then under the switched law

$$\sigma = \arg\min\{\zeta^T(k) P_i \zeta(k), i \in I_N\}, \tag{13}$$

where $\Phi_i = \{\zeta(k) \in R^{n+n_c} : \zeta^T(k)(P_r - P_i)\zeta(k) \geq 0, \forall r \in I_N, r \neq i\}$, any trajectory of the system (9) starting from the origin will remain inside the region $\cup_{i=1}^{N}(\Omega(P_i, \beta) \cap \Phi_i \subset L(K_i, H_i))$ for every $w \in W_\beta^2$.

Proof. By condition (12), if $\forall \zeta \in \Omega(P_i, \beta) \cap \Phi_i$, then $\zeta \in L(K_i, H_i)$. Therefore, in view of Lemma 2, for $\forall \zeta \in \Omega(P_i, \beta) \cap \Phi_i$ it follows that $\psi(K_i\zeta(k)) = K_i\zeta(k) - sat(K_i\zeta(k))$ satisfies the sector condition (10).

In view of the switching law (13), for $\forall \zeta(k) \in \Omega(P_i, \beta) \cap \Phi_i \subset L(K_i, H_i)$, the *i*-th subsystem is active.

Then, we choose the following quadratic Lyapunov function candidate for the system (9) as

$$V(\zeta(k)) = V_{\sigma(k)}(\zeta(k)) = \zeta^T(k) P_{\sigma(k)} \zeta(k). \tag{14}$$

We split the proof into two parts.

Case 1: when $\sigma(k+1) = \sigma(k) = i$, for $\forall \zeta(k) \in \Omega(P_i, \beta) \cap \Phi_i \subset L(K_i, H_i)$, the difference of $V(\zeta(k))$ along the solution of the closed-loop switched system (9) is

$$\begin{aligned} \Delta V(\zeta(k)) &= \zeta^T(k+1) P_i \zeta(k+1) - \zeta^T(k) P_i \zeta(k) \\ &= [\tilde{A}_i\zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) + \tilde{E}_i w(k)]^T \\ &\quad \times P_i[\tilde{A}_i\zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) \\ &\quad + \tilde{E}_i w(k)] - \zeta^T(k) P_i \zeta(k). \end{aligned} \tag{15}$$

Therefore, by using Lemma 2 and condition (12), we have

$$\begin{aligned} \Delta V(\zeta(k)) \leq\, & [\tilde{A}_i\zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) + \tilde{E}_i w(k)]^T \\ & \times P_r[\tilde{A}_i\zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) \\ & + \tilde{E}_i w(k)] - \zeta^T(k)(P_i)\zeta(k) - 2\psi^T(K_i\zeta(k)) J_i[\psi(K_i\zeta(k)) - H_i\zeta(k)], \end{aligned}$$

Case 2: $\sigma(k) = i, \sigma(k+1) = r$ and $i \neq r$, for $\forall \zeta(k) \in \Omega(P_i, \beta) \cap \Phi_i \subset L(K_i, H_i)$. Then using the switching law (13) gives

$$\Delta V(\zeta(k)) = \zeta^T(k+1)P_r\zeta(k+1) - \zeta^T(k)P_i\zeta(k) \le \zeta^T(k+1)P_i\zeta(k+1) - \zeta^T(k)P_i\zeta(k).$$

In view of Case 1 and Case 2, we get

$$\begin{aligned}\Delta V(\zeta(k)) \le {} & [\tilde{A}_i\zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) + \tilde{E}_i w(k)]^T \\ & \times P_r[\tilde{A}_i\zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) \\ & + \tilde{E}_i w(k)] - \zeta^T(k)(P_i)\zeta(k) - 2\psi^T(K_i\zeta(k))J_i[\psi(K_i\zeta(k)) - H_i\zeta(k)],\end{aligned}$$

or equivalently

$$\Delta V(x(k)) \le \begin{bmatrix}\zeta\\ \psi\\ w\end{bmatrix}^T \begin{bmatrix}\tilde{A}_i^T P_r \tilde{A}_i - P_i & -\tilde{A}_i^T P_r(\tilde{B}_i + GE_{ci}) + H_i^T J_i & \tilde{A}_i^T P_r \tilde{E}_i \\ * & (\tilde{B}_i + GE_{ci})^T P_r(\tilde{B}_i + GE_{ci}) - 2J_i & -(\tilde{B}_i + GE_{ci})^T P_r \tilde{E}_i \\ * & * & \tilde{E}_i^T P_r \tilde{E}_i\end{bmatrix} \begin{bmatrix}\zeta\\ \psi\\ w\end{bmatrix}, \quad (16)$$

$$\forall (i, r) \in I_N \times I_N.$$

Then, from Lemma 1, (11) is equivalent to

$$\begin{bmatrix}\tilde{A}_i^T P_r \tilde{A}_i - P_i + \sum_{r=1, r\ne i}^{N} \beta_{ir}(P_r - P_i) & -\tilde{A}_i^T P_r(\tilde{B}_i + GE_{ci}) + H_i^T J_i & \tilde{A}_i^T P_r \tilde{E}_i \\ * & (\tilde{B}_i + GE_{ci})^T P_r(\tilde{B}_i + GE_{ci}) - 2J_i & -(\tilde{B}_i + GE_{ci})^T P_r \tilde{E}_i \\ * & * & \tilde{E}_i^T P_r \tilde{E}_i - I\end{bmatrix} < 0. \quad (17)$$

Multiplying (17) from the left by $[x^T\ \psi^T\ w^T]$ and from the right by $[x^T\ \psi^T\ w^T]^T$, we have

$$\Delta V(k) = V(k+1) - V(k) < w^T(k)w(k) - \sum_{r=1, r\ne i}^{N} \beta_{ir}\zeta^T(k)(P_r - P_i)\zeta(k), \quad (18)$$

Again by the switching law (13), we obtain

$$\sum_{r=1, r\ne i}^{N} \beta_{ir}\zeta^T(k)(P_r - P_i)\zeta(k) \ge 0,$$

which in turn gives

$$\Delta V(k) = V(k+1) - V(k) < w^T(k)w(k). \quad (19)$$

Then, when we consider $V(k)$ as the overall Lyapunov function of system (9), It follows that

$$\Delta V(k) = V(k+1) - V(k) < w^T(k)w(k),\ \forall \zeta(k) \in \cup_{i=1}^{N}(\Omega(P_i, \beta) \cap \Phi_i). \quad (20)$$

Therefore, it follows

$$\sum_{t=0}^{k} \Delta V(t) < \sum_{t=0}^{k} w^T(t)w(t),$$

which indicates

$$V(k+1) < V(0) + \sum_{n=0}^{k} w^T(n)w(n),\ \forall k \ge 0.$$

Due to $x(0) = 0$ and $\sum_{k=0}^{\infty} w^T(k)w(k) \le \beta,$ we can obtain

$$V(k+1) < \beta, \quad (21)$$

which implies that the state trajectory of the system (9) starting from the origin will always remain inside the region $\cup_{i=1}^{N}(\Omega(P_i, \beta) \cap \Phi_i)$ for all times. Thus, this completes the proof.

In view of the above established result, we easily know that the disturbance tolerance capability is estimated firstly before we analyze the restricted L_2-gain for the closed-loop system (9). Clearly, constant β provides a kind of measure of the system's disturbance tolerance capability. Thus, the largest disturbance tolerance level β^* is able to be determined by solving the following optimization problem,

$$\begin{aligned} &\sup_{P_i,\, H_i,\, J_i,\, \beta_{ir}} \quad \beta \\ &s.t.\, (a)\ inequality\ (11),\ \forall i \in I_N, \\ &\qquad (b)\ \Omega(P_i, \beta) \cap \Phi_i \subset L(K_i, H_i),\ \forall i \in I_N. \end{aligned} \tag{22}$$

Then, pre- and post-multiplying both sides of inequality (11) by $block-diagonal$ $\{P_i^{-1}, J_i^{-1}, I, P_r^{-1}\}$ and letting $P_i^{-1} = X_i,\, H_i P_i^{-1} = M_i,\, J_i^{-1} = S_i$, it follows that

$$\begin{bmatrix} -X_i - \sum_{r=1,\, r\neq i}^{N} \beta_{ir} X_i & M_i^T & 0 & X_i \tilde{A}_i^T & X_i & X_i & X_i \\ * & -2S_i & 0 & -S_i(\tilde{B}_i + GE_{ci})^T & 0 & 0 & 0 \\ * & * & -I & \tilde{E}_i^T & 0 & 0 & 0 \\ * & * & * & -X_r & 0 & 0 & 0 \\ * & * & * & * & -\beta_{i1}^{-1} X_i & 0 & 0 \\ * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & -\beta_{iN1}^{-1} X_N \end{bmatrix} < 0. \tag{23}$$

By using a similar method as in [31], the condition (b) is guaranteed by

$$P_i - \sum_{r=1,\, r\neq i}^{N} \delta_{ir}(P_r - P_i) - \beta (K_i^j - H_i^j)^T (K_i^j - H_i^j) \geq 0, \tag{24}$$

where K_i^j, H_i^j are the j-th row of matrices K_i and H_i respectively and $\delta_{ir} > 0$.

Then from the lemma 1, (24) is equivalent to

$$\begin{bmatrix} P_i - \sum_{r=1,\, r\neq i}^{N} \delta_{ir}(P_r - P_i) & (K_i^j - H_i^j)^T \\ * & \mu \end{bmatrix} \geq 0, \tag{25}$$

where $\mu = \beta^{-1}$.

Thus, pre- and post-multiplying both sides of inequality (25) by $block-diagonal$ $\{P_i^{-1}, I\}$, we also have

$$\begin{bmatrix} X_i + \sum_{r=1,\, r\neq i}^{N} \delta_{ir} X_i & X_i K_i^{jT} - M_i^{jT} & X_i & X_i & X_i \\ * & \mu & 0 & 0 & 0 \\ * & * & \delta_{i1}^{-1} X_1 & 0 & 0 \\ * & * & * & \ddots & 0 \\ * & * & * & * & \delta_{iN}^{-1} X_N \end{bmatrix} \geq 0, \tag{26}$$

where M_i^j denotes the j-th row of M_i.

As a result, the optimization problem (22) can be formulated as

$$\begin{aligned} &\inf_{X_i,\, M_i,\, S_i,\, \beta_{ir},\, \delta_{ir}} \quad \mu \\ &s.t.\, (a)\ inequality\ (23),\ \forall i \in I_N, \\ &\qquad (b)\ inequality\ (26),\ \forall i \in I_N,\ \forall j \in Q_m. \end{aligned} \tag{27}$$

4. L_2-gain Analysis

The L_2-gain which can measure the disturbance rejection capability is one of the important performance index for control systems. However, due to the presence of actuator saturation, the disturbance rejection capability of the system with actuator saturation is measured by means of the restricted L_2-gain over a set of tolerable disturbances. Thus, we study the restricted L_2-gain problem for the system (9) via the multiple Lyapunov function method in this section. Similarly, we suppose that the anti-windup compensation gains E_{ci} are given beforehand.

Theorem 2. Consider switched systems (9). For given positive scalar $\beta \in (0, \beta^*]$ and constant γ, suppose there exist positive definite matrices P_i, matrices H_i, and diagonal positive definite matrices J_i and a set of scalars $\beta_{ir} \geq 0$ such that

$$\begin{bmatrix} -P_i + \sum_{r=1, r\neq i}^{N} \beta_{ir}(P_r - P_i) & H_i^T J_i & 0 & \tilde{A}_i^T P_r & \tilde{C}_{i2}^T \\ * & -2J_i & 0 & -(\tilde{B}_i + GE_{ci})^T P_r & 0 \\ * & * & -I & \tilde{E}_i^T P_r & 0 \\ * & * & * & -P_r & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (28)$$

$\forall i \in I_N$,

and

$$\Omega(P_i, \beta) \cap \Phi_i \subset L(K_i, H_i), \forall i \in I_N. \quad (29)$$

Then, under the switching law

$$\sigma = \arg\min\left\{\zeta^T P_i \zeta, i \in I_N\right\}, \quad (30)$$

the restricted L_2-gain from w to z over W_β^2 is less than γ.

Proof. Using the similar method as for proving Theorem 1, we choose the same the multiple Lyapunov function candidate for the system (10) as

$$V(\zeta(k)) = V_{\sigma(k)}(\zeta(k)) = \zeta^T(k) P_{\sigma(k)} \zeta(k). \quad (31)$$

We still split the proof into two parts.

Case 1: $\sigma(k+1) = \sigma(k) = i$, for $\forall \zeta(k) \in \Omega(P_i, \beta) \cap \Phi_i \subset L(K_i, H_i)$. Then, Computing the variation of $V(\zeta(k))$ along the trajectory of the switched system (9), we have

$$\begin{aligned} \Delta V(\zeta(k)) &= \zeta^T(k+1) P_i \zeta(k+1) - \zeta^T(k) P_i \zeta(k) \\ &= [\tilde{A}_i \zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) + \tilde{E}_i w(k)]^T \\ &\quad \times P_i[\tilde{A}_i \zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) \\ &\quad + \tilde{E}_i w(k)] - \zeta^T(k) P_i \zeta(k). \end{aligned}$$

Then, in view of Lemma 2 and condition (29), it follows that

$$\begin{aligned} \Delta V(\zeta(k)) \leq\ & [\tilde{A}_i \zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) + \tilde{E}_i w(k)]^T \\ & \times P_i[\tilde{A}_i \zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) \\ & + \tilde{E}_i w(k)] - \zeta^T(k) P_i \zeta(k) - 2\psi^T(K_i\zeta) J_i[\psi(K_i\zeta) - H_i\zeta], \end{aligned}$$

Case 2: $\sigma(k) = i, \sigma(k+1) = r$ and $i \neq r$, for $\forall \zeta(k) \in \Omega(P_i, \beta) \cap \Phi_i \subset L(K_i, H_i)$. Then applying the switching law (30), we obtain

$$\Delta V(\zeta(k)) = \zeta^T(k+1) P_r \zeta(k+1) - \zeta^T(k) P_i \zeta(k) \leq \zeta^T(k+1) P_i \zeta(k+1) - \zeta^T(k) P_i \zeta(k).$$

From Case 1 and Case 2, we have

$$\begin{aligned} \Delta V(\zeta(k)) \leq\ & [\tilde{A}_i \zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) + \tilde{E}_i w(k)]^T \\ & \times P_i[\tilde{A}_i \zeta(k) - (\tilde{B}_i + GE_{ci})\psi(K_i\zeta(k)) \\ & + \tilde{E}_i w(k)] - \zeta^T(k) P_i \zeta(k) - 2\psi^T(K_i\zeta) J_i[\psi(K_i\zeta) - H_i\zeta], \end{aligned}$$

or equivalently

$$\Delta V(\zeta(k)) \leq \begin{bmatrix} \zeta \\ \psi \\ w \end{bmatrix}^T \begin{bmatrix} \tilde{A}_i^T P_i \tilde{A}_i - P_i & -\tilde{A}_i^T P_i(\tilde{B}_i + GE_{ci}) + H_i^T J_i & \tilde{A}_i^T P_i \tilde{E}_i \\ * & (\tilde{B}_i + GE_{ci})^T P_i(\tilde{B}_i + GE_{ci}) - 2J_i & -(\tilde{B}_i + GE_{ci})^T P_i \tilde{E}_i \\ * & * & \tilde{E}_i^T P_i \tilde{E}_i \end{bmatrix} \begin{bmatrix} \zeta \\ \psi \\ w \end{bmatrix}, \quad \forall i \in I_N. \tag{32}$$

Then, in view of Lemma 1, (28) is equivalent to

$$\begin{bmatrix} \tilde{A}_i^T P_i \tilde{A}_i - P_i + \gamma^{-2} \tilde{C}_{i2}^T \tilde{C}_{i2} + \sum_{r=1, r\neq i}^{N} \beta_{ir}(P_r - P_i) & -\tilde{A}_i^T P_i(\tilde{B}_i + GE_{ci}) + H_i^T J_i & \tilde{A}_i^T P_i \tilde{E}_i \\ * & (\tilde{B}_i + GE_{ci})^T P_i(\tilde{B}_i + GE_{ci}) - 2J_i & -(\tilde{B}_i + GE_{ci})^T P_i \tilde{E}_i \\ * & * & \tilde{E}_i^T P_i \tilde{E}_i - I \end{bmatrix} < 0. \tag{33}$$

Multiplying (33) from the left by $[\zeta^T \ \psi^T \ w^T]$ and from the right by $[\zeta^T \ \psi^T \ w^T]^T$, we obtain

$$\Delta V(k) = V(k+1) - V(k) < w^T(k)w(k) - \gamma^{-2} z^T(k)z(k) - \sum_{r=1, r\neq i}^{N} \beta_{ir} \zeta^T(k)(P_r - P_i)\zeta(k). \tag{34}$$

Again from the switching law (30), we have

$$\sum_{r=1, r\neq i}^{N} \beta_{ir} \zeta^T(k)(P_r - P_i)\zeta(k) \geq 0,$$

which imply that

$$\Delta V(k) = V(k+1) - V(k) < w^T(k)w(k) - \gamma^{-2} z^T(k)z(k). \tag{35}$$

Then, considering $V(k)$ as the overall Lyapunov function of system (9), we obtain

$$\Delta V(k) = V(k+1) - V(k) < w^T(k)w(k) - \gamma^{-2} z^T(k)z(k), \forall \zeta(k) \in \cup_{i=1}^{N}(\Omega(P_i, \beta) \cap \Phi_i). \tag{36}$$

Therefore,

$$\sum_{k=0}^{\infty} \Delta V(k) < \sum_{k=0}^{\infty} w^T(k)w(k) - \gamma^{-2} \sum_{k=0}^{\infty} z^T(k)z(k). \tag{37}$$

Then,

$$V(\infty) < V(0) + \sum_{k=0}^{\infty} w^T(k)w(k) - \gamma^{-2} \sum_{k=0}^{\infty} z^T(k)z(k). \tag{38}$$

Due to $V(0) = 0$ and $V(\infty) \geq 0$, we obtain

$$\sum_{k=0}^{\infty} z^T(k)z(k) < \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k), \tag{39}$$

which implies that the system (9) has its restricted L_2-gain from w to z over W_β^2 less than γ. Thus the proof is completed.

In order to minimize the upper bound of the restricted L_2-gain of the system (9), the optimization problem can be solved for given $\beta \in (0, \beta^*]$ as follows:

$$\begin{aligned} &\inf_{P_i,\,H_i,\,J_i,;\beta_{ir}} \quad \gamma^2 \\ &s.t.\ (a)\ inequality\ (28),\ \forall i \in I_N, \\ &(b)\ \Omega(P_i,\beta) \cap \Phi_i \subset L(K_i, H_i),\ \forall i \in I_N. \end{aligned} \tag{40}$$

Applying a similar method as used in changing (22) into (27), we can convert the optimization problem (40) into an optimization problem consisting of LMIs. Therefore, the constraints (a) in (40) is equivalent to

$$\begin{bmatrix} -X_i - \sum_{r=1,r\neq i}^{N} \beta_{ir} X_i & M_i^T & 0 & X_i \tilde{A}_i^T & X_i \tilde{C}_{i2}^T & X_i & X_i & X_i \\ * & -2S_i & 0 & -S_i(\tilde{B}_i + GE_{ci})^T & 0 & 0 & 0 & 0 \\ * & * & -I & \tilde{E}_i^T & 0 & 0 & 0 & 0 \\ * & * & * & -X_r & 0 & 0 & 0 & 0 \\ * & * & * & * & -\theta I & 0 & 0 & 0 \\ * & * & * & * & * & -\beta_{i1}^{-1} X_1 & 0 & 0 \\ * & * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & * & -\beta_N^{-1} X_N \end{bmatrix} < 0, \tag{41}$$

where $\theta = \gamma^2$ and (b) in (40) is guaranteed by

$$\begin{bmatrix} X_i + \sum_{r=1,\,r\neq i}^{N} \delta_{ir} X_i & X_i K_i^{jT} - M_i^{jT} & X_i & X_i & X_i \\ * & \mu & 0 & 0 & 0 \\ * & * & \delta_{i1}^{-1} X_1 & 0 & 0 \\ * & * & * & \ddots & 0 \\ * & * & * & * & \delta_{iN}^{-1} X_N \end{bmatrix} \geq 0, \tag{42}$$

Then, the optimization problem (40) can be formulated as

$$\begin{aligned} &\inf_{X_i,\,M_i,\,S_i,\,\beta_{ir},\,\delta_{ir}} \quad \theta \\ &s.t.\ (a)\ inequality\ (41),\ \forall i \in I_N, \\ &(b)\ inequality\ (42),\ \forall i \in I_N,\ \forall j \in Q_m. \end{aligned} \tag{43}$$

5. Anti-windup Synthesis

In fact, anti-windup compensation gains can be designed in order to further improve the closed-loop system (9) performance. Thus, the optimum solutions in section 3 and 4 can be obtained by anti-windup compensation gains design.

Let $N_i = E_{ci} S_i$. Then, (23) and (41) are respectively equivalent to

$$\begin{bmatrix} -X_i - \sum_{r=1,\,r\neq i}^{N} \beta_{ir} X_i & M_i^T & 0 & X_i \tilde{A}_i^T & X_i & X_i & X_i \\ * & -2S_i & 0 & -S_i \tilde{B}_i^T - N_i^T G^T & 0 & 0 & 0 \\ * & * & -I & \tilde{E}_i^T & 0 & 0 & 0 \\ * & * & * & -X_i & 0 & 0 & 0 \\ * & * & * & * & -\beta_{i1}^{-1} X_i & 0 & 0 \\ * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & -\beta_{iN1}^{-1} X_N \end{bmatrix} < 0. \tag{44}$$

and

$$\begin{bmatrix} -X_i - \sum_{r=1,r\neq i}^{N} \beta_{ir} X_i & M_i^T & 0 & X_i\tilde{A}_i^T & X_i\tilde{C}_{i2}^T & X_i & X_i & X_i \\ * & -2S_i & 0 & \begin{matrix} -S_i\tilde{B}_i^T \\ -N_i^T G^T \end{matrix} & 0 & 0 & 0 & 0 \\ * & * & -I & \tilde{E}_i^T & 0 & 0 & 0 & 0 \\ * & * & * & -X_i & 0 & 0 & 0 & 0 \\ * & * & * & * & -\theta I & 0 & 0 & 0 \\ * & * & * & * & * & -\beta_{i1}^{-1} X_1 & 0 & 0 \\ * & * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & * & -\beta_N^{-1} X_N \end{bmatrix} < 0. \tag{45}$$

Therefore, the optimization problem which aims to obtain the largest disturbance tolerance level β^* is formalized as follows:

$$\begin{aligned} &\inf_{X_i,\, M_i,\, N_i,\, S_i,\, \beta_{ir},\, \delta_{ir}} \quad \mu \\ &s.t.\ (a)\ inequality\ (44),\ \forall i \in I_N, \\ &\qquad (b)\ inequality\ (26),\ \forall i \in I_N,\ \forall j \in Q_m, \end{aligned} \tag{46}$$

and then, when any $\beta \in (0, \beta^*]$ is given, the minimum upper bound of the restricted L_2-gain will be obtained by solving the following optimization problem,

$$\begin{aligned} &\inf_{X_i,\, M_i,\, N_i,\, S_i,\, \beta_{ir},\, \delta_{ir}} \quad \theta \\ &s.t.\ (a)\ inequality\ (45),\ \forall i \in I_N, \\ &\qquad (b)\ inequality\ (42),\ \forall i \in I_N,\ \forall j \in Q_m. \end{aligned} \tag{47}$$

When these optimization problems (46) and (47) are solved, we can compute the anti-windup compensation gains $E_{ci} = N_i S_i^{-1}$.

6. An Illustrative Example

In order to illustrate the effectiveness of the proposed method, we give the following example in the section.

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i sat(v_c(k)) + E_i w(k), \\ y(k) &= C_{i1} x(k), \\ z(k) &= C_{i2} x(k), y \end{aligned} \tag{48}$$

and the dynamic output feedback controllers with the anti-windup terms are given as

$$\begin{aligned} x_c(k+1) &= A_{ci} x_c(k) + B_{ci} C_{i1} x(k) + E_{ci}(sat(v_c(k)) - v_c(k)), \\ v_c(k) &= C_{ci} x_c(k) + D_{ci} C_i x(k), \end{aligned} \tag{49}$$

where $\sigma(k) \in I_2 = \{1, 2\}$,

$$A_1 = \begin{bmatrix} 1.25 & 0 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0.339 & 0 \\ 0 & 1.487 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0.75 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -1.3 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.3 & 0.02 \\ 0.44 & 0.04 \end{bmatrix}, E_2 = \begin{bmatrix} 0.6 & 0.35 \\ 0.55 & 0.1 \end{bmatrix}, C_{11} = \begin{bmatrix} 0.345 \\ 0.69 \end{bmatrix}^T, C_{21} = \begin{bmatrix} 0.17 \\ -0.3 \end{bmatrix}^T,$$

$$C_{12} = \begin{bmatrix} 0.058 \\ 0.030 \end{bmatrix}^T, C_{22} = \begin{bmatrix} -0.019 \\ 0.017 \end{bmatrix}^T, A_{c1} = \begin{bmatrix} 0.1133 & 0 \\ 0.0138 & -0.1143 \end{bmatrix},$$

$$A_{c2} = \begin{bmatrix} -0.0515 & 0 \\ 0.0043 & -0.0309 \end{bmatrix}, B_{c1} = \begin{bmatrix} -0.0209 \\ -0.0904 \end{bmatrix}, B_{c2} = \begin{bmatrix} -0.0525 \\ 0.0286 \end{bmatrix},$$

$$C_{c1}=\begin{bmatrix}2.3191\\-0.4768\end{bmatrix}^T, C_{c2}=\begin{bmatrix}-2.9468\\-1.5688\end{bmatrix}^T, D_{c1}=-0.5437, D_{c2}=-1.5199.$$

Firstly, we design the set of anti-windup compensation gains by using the proposed method in section 5 such that the capability of disturbance tolerance of the system (48)-(49) is maximized via the multiple Lyapunov function method. Thus, solving the optimization problem (46), we obtain the optimal solutions as follows:

$$\mu^*=0.0512, \beta^*=\mu^{*-1}=19.5443, S_1=100.6026, S_2=49.0746,$$

$$X_1=\begin{bmatrix}40.3865 & -3.7704 & -2.6631 & -3.5830\\ * & 85.7636 & -7.8685 & 1.2099\\ * & * & 7.4896 & 0.0544\\ * & * & * & 49.6998\end{bmatrix},$$

$$X_2=\begin{bmatrix}43.3526 & -4.0234 & -2.8885 & -3.8178\\ * & 85.7749 & -8.6503 & 1.3275\\ * & * & 7.4351 & 0.0541\\ * & * & * & 49.6978\end{bmatrix},$$

$$N_1=\begin{bmatrix}-6.3077\\2.4807\end{bmatrix}, N_2=\begin{bmatrix}7.3592\\-1.1133\end{bmatrix},$$

$$M_1=\begin{bmatrix}-3.9531 & -37.9724 & 15.3169 & -0.2201\end{bmatrix},$$

$$M_2=\begin{bmatrix}-1.6823 & -11.9026 & -18.5000 & 0.3105\end{bmatrix},$$

$$E_{c1}=N_1S_1^{-1}=\begin{bmatrix}-0.0627\\0.0247\end{bmatrix}, E_{c2}=N_2S_2^{-1}=\begin{bmatrix}0.1500\\-0.0227\end{bmatrix}.$$

In addition, if we let $E_{c1}=E_{c2}=0$, the obtained optimal solution is $\beta^*=3.1756$, which implies the disturbance tolerance capacity of the system expanded under the effect of the anti-windup compensators.

Finally, for any given $\beta\in(0,\beta^*]$, we can obtain the minimum upper bound of the restricted L_2-gain of the switched system (48)-(49) by solving optimization problem (47).The fig. 1 shows the relation of the restricted L_2-gain γ and different values $\beta\in(0,\beta^*]$ of the corresponding system.

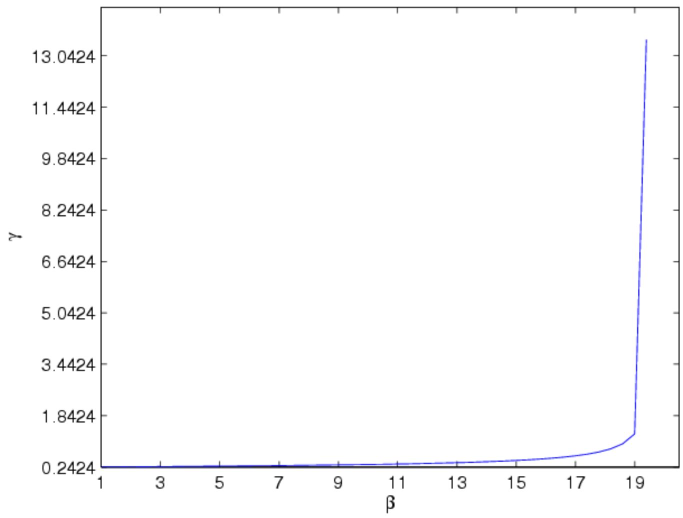

Figure 1.The restricted L_2-gain of the switched system (48)-(49) for any $\beta\in(0,\beta^*]$.

On the other hand, we apply the method in [32] to the considered system and find that all the optimization problems have no solutions, which is because the problem of disturbance tolerance/rejection is required to be solvable for every subsystem in [32]. However, it is easy to verify that in this example, the problem of disturbance tolerance/rejection for each subsystem is not solvable.

7. Conclusions

The problem of L_2-gain analysis and anti-windup design has been investigated for a class of discrete-time switched systems subject to actuator saturation. We derive some sufficient conditions of disturbance tolerance and restricted L_2-gain by using the multiple Lyapunov function method. Furthermore, we propose a method of designing the anti-windup compensators of the considered system such that the disturbance tolerance capacity is maximized and the upper bound of the restricted L_2-gain over the set of tolerable disturbances is minimized respectively.

Compared with the existing results for switched systems subject to actuator saturation, there are three features of our results. First of all, the L2-gain analysis and anti-windup design problem are simultaneously addressed for discrete-time the switched systems with saturating actuator, while most existing works considered only the problem of stability; second, the multiple Lyapunov functions method is used to study the disturbance tolerance/rejection problem for discrete-time switched systems with actuator saturation for the first time and no solvability of the problem for subsystem is required, while in the existing literature, the problem has been investigated by using the switched Lyapunov function method which requires the solvability for each subsystem; third, the anti-windup compensators are designed within an LMI framework which possess better performance for switched systems with input saturation, while the existing literature either only conducts the analysis, not design, or studies the design issue using the switched Lyapunov function.

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