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Controlling the Dynamics and Stabilization of the Altitude and Attitude of An Underactuated Tri-rotor UAV

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Abstract

In this article, an Adaptive Sliding Mode Control (ASMC) method is proposed for controlling the dynamics and stabilization of the altitude and attitude of an underactuated tri-rotor Unmanned Aerial Vehicle (UAV). The model of UAV is highly nonlinear and complex in nature with the six degree of freedom (6-DOF) and subdivided in to fully actuated and underactuated subsystems which is controlled by the sliding surface of ASMC. Furthermore, the stability of a system is validated using Lyapunov stability theory. Simulation results showed that the proposed control scheme enabled the tri-rotor UAV to fly effectively when compared with Adaptive Robust Backstepping (ARB) control. Lastly, an external disturbance is added in the system to check the effectiveness of the proposed scheme in which a wind disturbance test is done with the proposed controller that shows the robustness, fast error convergence and good transient behavior.

Keywords: *Tri-rotor UAV, underactuated system, Adaptive Sliding Mode Control (ASMC)*

1. Introduction

Unmanned Aerial Vehicle (UAV) is the most remarkable inventions in the 21st century [1]. The UAV is divided in to two categories, one is fixed wing and second is rotor craft UAV's. However, rotor craft UAV's have an advantage it does not need a runway to execute a flight because of its Vertical Takeoff and Landing (VTOL) ability [2-4]. Furthermore, the rotor craft UAV's are also classified in to bi-rotor, tri-rotor, quad-rotor and hex-rotor crafts, having Multiple Input and Multiple Output (MIMO) underactuated systems. These days the usage of UAV is quite common in our society, they are commonly used in surveillance, rescue and searching tasks and *etc.* [5-7].

Due to its compact size and VTOL ability, it hovers with very simple construction and maintenance. A rotor craft has four input control commands called as altitude, lateral, longitudinal and angular moments (Col., Lon., Lat., Ped.,). The linear and angular velocity subcomponents are (u, v, w) and (p, q, r). (φ, θ, ψ) are the Euler angles of the aerial vehicle. A number of physical effects that UAV can exhibit includes inertial torque, effects of the gyroscope and aerodynamics forces making it difficult for UAV to perform the successful flight operation in the real world [8].

In this article, the model of tri-rotor UAV is used due to its properties of strongly coupled, multivariable, nonlinear, and underactuated in nature along with (6-DOF) the six degree of freedom [9-11]. To establish the nonlinear control scheme to control the model

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of 6-DOF organize with linear and angular velocities. The enquiry phase directs a fault which is established in the moment of yaw control induced due to the reaction and torque which is produced by the rotor. To fix, this fault, a series of mechanical designs of tri-rotor UAV's has been prepared to their own parameters. After, that a Brushless Direct Current (BLDC) motor is mounted, which tilt the yaw control to nullify the moments [12]. The key role of this mechanical design is better turning, movement by utilizing the tilt rotor axes.

Before the real flight simulation, to simulate the nonlinear model of tri-rotor UAV in this article, its Euler angles, linear and angular velocity subcomponents, control commands and track the reference path. The dynamic model of the UAV is highly coupled with a lot of parameter uncertainties to which a prospective controller has to be robust [13-15]. While, the efficiency and performance will possibly be reduced in the situation of a collapse, it requires that the controller must stabilize the system and bears compact functions, in which steady hovering and safe flight is done.

Formerly, different control algorithms were used to control the dynamics of underactuated UAV. PID, the most classical method that was used in [16], backstepping control of underactuated system used in [17], Sliding Mode Control (SMC) was already used in [18]. In this article, we used an Adaptive Sliding Mode Controller (ASMC) along with the wind disturbance test, which was not used in any of the aforementioned work.

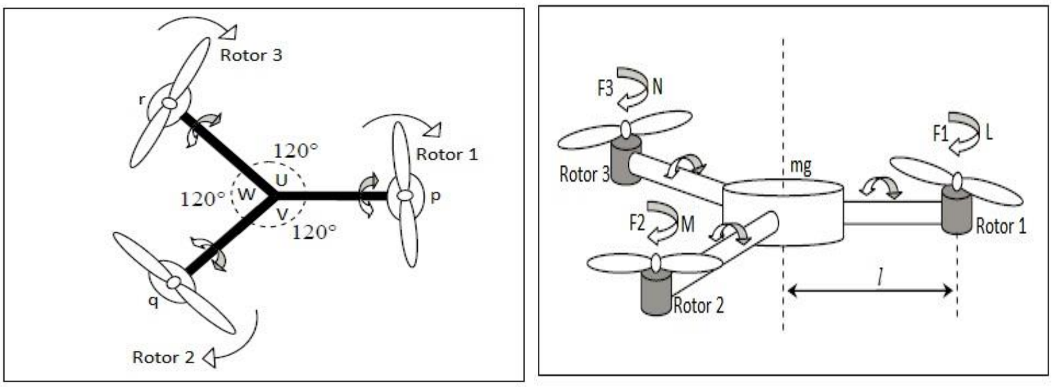
To validate the effectiveness of our proposed ASMC, it is compared with Adaptive Robust Backstepping (ARB) Controller of [19] without disturbance and with the wind disturbance test to cross check the efficiency and robustness.

The key contributions of this article are: (1) an adaptive sliding surface is proposed to remove the steady state error and for the rejections of model uncertainties (2) the designed ASMC uses linear and angular velocity subcomponents as an input providing practical inside of the real world scenario (3) in spite of the model uncertainties, close loop errors in the UAV system converges to zero, which is proved in the simulation and result section (4) stability of the system is proven by Lyapunov stability function.

The breakup of the article is structured as follows. Section 2 provides the preliminaries about tri-rotor UAV, which is followed by its dynamics while problem is described in Section 3. Section 4 discuss the control algorithm and Section 5 defines the simulation and result analysis. Finally, Section 6 concludes the whole article.

2. The Preliminaries of Tri-Rotor UAV

Figure 1(a) describe the 2-D view of UAV, while Figure 1(b) defines the 3-D view of the aerial robot.



(a) (b)

Figure 1 (a). The Upper Vision of UAV along with Angular and Linear Velocity Rates (b): The 3-D Vision of UAV with Respect to the Center of Gravity

The mathematical model can be written as,

$$\begin{cases} m\dot{\upsilon} = mg - T \\ \dot{\Gamma} = W(\Gamma)\delta \\ j\dot{\delta} = \delta * j\delta + \tau \end{cases} \tag{1}$$

In the above equation (01) $\upsilon \in \mathrm{R}$, the velocity of the UAV is dependent on its rotational matrix "R" which defines the position of the vehicle. The angular velocity "δ" of the vector coordinates is also dependent on the rotational matrix [20-22]. Where "m" is the mass and $\mathrm{j} \in R$ inertial points,"T" is the rotational torque, $\tau \in R$. Now the Euler angles $\Gamma = (\varphi, \theta, \psi)$ is used to derive the attitude of the UAV around x, y, z axes. Now the movement around x, y, z axis are designated as roll, pitch and yaw respectively [23] [24]. Now the rotation matrix can be written as,

$$\begin{cases} W(\Gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & sin\varphi \\ 0 & -sin\varphi & cos\varphi \end{bmatrix} \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ R(\Gamma) = \begin{bmatrix} cos\theta cos\psi & cos\theta sin\psi & -sin\theta \\ sin\varphi sin\theta cos\psi - cos\psi sin\psi & sin\varphi sin\theta sin\psi - cos\varphi cos\psi & sin\varphi cos\theta \\ cos\varphi cos\theta cos\psi + sin\varphi sin\psi & cos\varphi sin\theta sin\psi - sin\theta cos\psi & cos\psi cos\theta \end{bmatrix} \end{cases} \tag{2}$$

The tri-rotor aerial vehicle has three motors which is fixed in its triangular frame of the body, the input and output force and torque is written as,

$$\begin{cases} f_i = kt\ u_i^2 \rightarrow kt\ u_i|u_i| \\ \tau_i = k\tau\ u_i^2 \rightarrow k\tau\ u_i|u_i| \end{cases} \tag{3}$$

Where $\mathrm{i} = 1, 2, 3$ for all three rotors of the aerial vehicle.

The tri-rotor UAV system is a combination of cascades and the complex dynamics of the underactuated system. Due to their complex coupling relationship exists in the system, the design and stability analysis of the vehicle is also quite complex [25]. Initially, the system model decoupled into fully actuated subsystem and an underactuated subsystem. Thus by simplifying the controller design and its stability analysis by using Lyapunov stability analysis. Now according to the principle of rotor craft UAV, adjusting the rotational speed of the motor controls the attitude of the vehicle [26]. By changing the rotor speed the direction of the vehicle can be controlled. Now with respect to Newton's second law, the sum of external forces acted on the aircraft is equivalent to the frame of time that changes the body momentum. Additionally, external moments acting on the UAV is equivalent to change the rate of rotational movements [27-29].

Translational and rotational moment variation is referred as an inertial reference frame. The UAV aircraft has a proficiency of identical forces, aerodynamic moments and velocity components which can make into a six-degree of freedom (6-DOF) equation of motion those are nonlinear in nature [30]. Now, according to the rotors of the aircraft, to get the vehicle to keep in hover the basic speed of the motor taking into account that the aircraft is in hovering state. The total force in the aircraft exerted on the body which can be expressed as,

$$\sum_{i}^{3} F_i = mg \tag{4}$$

3. Dynamics of Tri-Rotor UAV & Its Problem Description

The dynamic equations of the aircraft is defined in the equation (05) to (12) which are described below. The linear, angular velocity components and Euler angles are defined by (p, q, r), (u, v, w) and (φ, θ, ψ). Therefore, the control variables are described as follows: $(\nabla\delta_T, \nabla\delta_\varphi, \nabla\delta_\theta, \nabla\delta_\psi)$, respectively for the basic speed of the aircraft to maintain hovering [31].

Linear system dynamics are,

$$\begin{cases} \dot{u} = (1/m)(F_1 cos\theta cos\psi + F_2 cos\theta sin\psi - F_3 sin\theta) \\ \dot{v} = (1/m)(F_1(sin\varphi sin\theta cos\psi - cos\varphi sin\psi) + F_2(sin\varphi sin\theta sin\psi + cos\theta cos\psi) + F_3(sin\varphi cos\theta)) \\ \dot{w} = (1/m)(F_1(cos\varphi sin\theta cos\psi + sin\varphi sin\psi) + F_2(cos\varphi sin\theta sin\psi - sin\varphi cos\psi) + F_3(cos\varphi cos\theta)) \end{cases} \tag{5}$$

Angular system dynamics are,

$$\begin{cases} \dot{p} = (I_y - I_z/I_x)(r * q) + (1/I_x)(\sqrt{3} * L/2)(F_2 - F_3) \\ \dot{q} = (I_z - I_x/I_y)(r * p) + (1/I_y)(1/2)(F_2 + F_3) - L * f_1 cos\nabla\delta_\psi + \tau_1 sin\nabla\delta_\psi \\ \dot{r} = (I_x - I_y/I_z)(q * p) + (1/I_z)(1/2)(F_2 + F_3)L * f_1 sin\nabla\delta_\psi - \tau_1 cos\nabla\delta_\psi - \tau_2 - \tau_3 \end{cases} \tag{6}$$

According to the definition of tri-rotor the speed control of three motors and its motion relationship can be written as,

$$\begin{cases} F_1 = C_\pi \rho A r^2 (\delta_T + \nabla\delta_T - \nabla\delta_\theta + \nabla\delta_\psi)^2 \\ F_2 = C_\pi \rho A r^2 (\delta_T + \nabla\delta_T + \nabla\delta_\varphi - \nabla\delta_\psi)^2 \\ F_3 = C_\pi \rho A r^2 (\delta_T + \nabla\delta_T - \nabla\delta_\theta + \nabla\delta_\psi)^2 \end{cases} \tag{7}$$

Table 1. Tri-Rotor UAV Dynamics & Its Constants

Axis system (x , y ,z)	Linear velocity components	Rotational velocity components	Inertial Coordinates
Roll (φ)	u	p	I_x
Pitch (θ)	v	q	I_y
Yaw (ψ)	w	r	I_z

Hovering: The state of hover or hovering of tri-rotor is based on the blades of the propeller which increases the blade numbers on the propeller and restricts the speed of forward flight. Now employing the theory of momentum for the dynamics of helicopter which is also related to the tri-rotor UAV theory [32]. Whereas, "C_π" is the coefficient of thrust, "ρ" is the density of air, "A" is the blade area, "r" is the blade radius, "δ_T" is the angular velocity of thrust and "C_Q" is the rotor shaft constant.

Case for hovering: If the aircraft is hovering the following conditions must be true. $(1){:}\ \varphi = \dot{\varphi} = 0, (2){:}\ \theta = \dot{\theta} = 0, (3){:}\ \psi = \dot{\psi} = 0$ and $(4){:}\ Cos\varphi = Cos\theta = 1$. The tri-rotor UAV flight in a hover state in the equilibrium point condition at that time by compared with δ_T than $\nabla\delta_T, \nabla\delta_\varphi, \nabla\delta_\theta, \nabla\delta_\psi$ are high order small qualities so $3C_\pi \rho A r^2\, \delta_T^2 = mg$, than we get: $\delta_T = (mg/3C_\pi \rho A r^2)^{1/2}$.

The dynamics of the tri-rotor UAV is obtained as follows,

$$\begin{cases} \ddot{x} = (1/m)(\cos\varphi\cos\psi\sin\theta + \sin\psi\sin\varphi)\sum_i^3 F_i \\ \ddot{y} = (1/m)(\sin\theta\sin\psi\cos\varphi - \sin\varphi\cos\psi)\sum_i^3 F_i \\ \ddot{z} = -g + (1/m)(\cos\theta\cos\varphi)\sum_i^3 F_i \end{cases} \quad (8)$$

By using equation (5) and (7) and taking into account the effect of high-order small quantities which can be expressed as $\sum_i^3 F_i = mg + 8C_\pi \rho A r^2\, \delta_T \nabla\delta_T$ and then after getting the location of subsystem simplification system dynamics equations are ,

$$\begin{cases} \ddot{x} = g(\theta Cos\,\psi + \varphi sin\psi) \\ \ddot{y} = g(\theta\, sin\psi - \varphi\, cos\,\psi) \\ \ddot{z} = 8C_\pi \rho A r^2\, \delta_T \nabla\delta_T/m \end{cases} \quad (9)$$

The attitude kinematics equations of tri-rotor UAV is obtained as,

$$\begin{cases} \delta_x = \dot{\varphi} Cos\theta - \dot{\psi} Cos\varphi Sin\theta \\ \delta_y = \dot{\theta} + \dot{\psi} Sin\varphi \\ \delta_z = \dot{\varphi} Sin\theta + \dot{\psi} Cos\theta Cos\varphi \end{cases} \quad (10)$$

The attitude dynamic equations of tri-rotor UAV are,

$$\begin{cases} I_x\dot{\delta}_x = C_\pi \rho A r^2 d + (I_y - I_z)\delta_y\delta_z \\ I_y\dot{\delta}_y = C_\pi \rho A r^2 d + (I_z - I_x)\delta_z\delta_x \\ I_z\dot{\delta}_z = C_\pi \rho A r^2 d + (I_x - I_y)\delta_x\delta_y \end{cases} \quad (11)$$

Let us suppose that the motion model of the system can be simplified as,

$$\begin{cases} \delta_x = \dot{\varphi} \\ \delta_y = \dot{\theta} \\ \delta_z = \dot{\psi} \end{cases} \quad (12)$$

$$\begin{cases} \dot{\delta}_x = 3d\,C_\pi \rho A r^2 \delta_T \nabla\delta_\varphi/I_x \\ \dot{\delta}_y = 3dC_\pi \rho A r^2 \delta_T \nabla\delta_\theta/I_y \\ \dot{\delta}_z = 3dC_\pi \rho A r^2 \delta_T \nabla\delta_\psi/I_z \end{cases} \quad (13)$$

The model simplification of a tri-rotor UAV system is a typical underactuated system and 6- degree of freedom (6-DOF) via four control inputs typically named as altitude, lateral, longitudinal and angular moments ($\nabla\delta_T$,$\nabla\delta_\varphi$,$\nabla\delta_\theta$, $\nabla\delta_\psi$) [33-35]. But in fact, in aircraft systems there are two non-holonomic constraints that is:

$$\begin{cases} Sin\varphi = (-\ddot{x} Sin\varphi + \ddot{y} Cos\varphi)/(\ddot{x}^2 + \ddot{y}^2 + (\ddot{z} + g))^{1/2} \\ Tan\theta = (\ddot{x} Cos\varphi + \ddot{y} Sin\varphi/\ddot{z} + g) \end{cases} \quad (14)$$

Due to the presence of two non-holonomic constraints, causes the vehicle to realize the full meaning of six degrees of freedom. Whereas the roll and pitch are attitude which is bounded by the position error of the aircraft [36-37].

$$\begin{cases} \ddot{z} = 8C_\pi \rho A r^2\, \delta_T \nabla\delta_T/m \\ \ddot{\psi} = 8C_Q \rho A r^3\, \delta_T \nabla\delta_T/I_z \end{cases} \quad (15)$$

The underactuated system dynamic equations are,

$$\begin{cases} \ddot{x} = g(\theta Cos\,\psi + \varphi\sin\psi) \\ \ddot{y} = g(\theta\sin\psi - \varphi\cos\psi) \end{cases} \quad (16)$$

$$\begin{cases} \ddot{\varphi} = 3dC_\pi \rho A r^2 \delta_T \nabla\delta_\varphi/I_x \\ \ddot{\theta} = 3dC_\pi \rho A r^2 \delta_T \nabla\delta_\theta/I_y \end{cases} \quad (17)$$

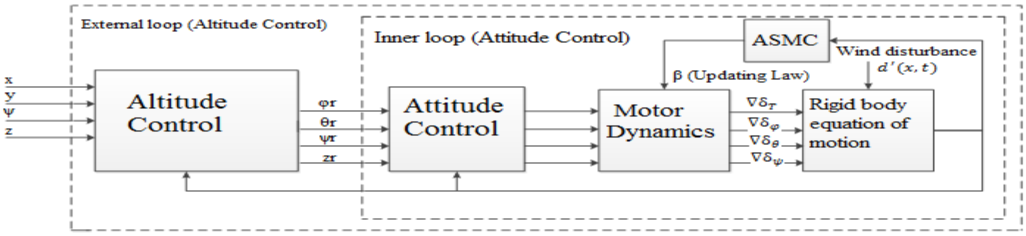


Figure 2. Complete Block Diagram of Control System

4. Controller Designing

In this part, of the article the complete hierarchy of ASMC is defined. In which observer is designed for the inner loop to control the attitude of tri-rotor UAV, which is shown in Figure 2. The height or altitude and yaw control equation in the absence of disturbance is written as,

$$\begin{cases} \nabla\delta_T = (m/8C_\pi\rho Ar^2\,\delta_T)(k_{z1}(z_d - z) - k_{z2}\dot{z} + k_{z3}\int(z_d - z)dt) \\ \nabla\delta_\psi = (I_z/8C_Q\rho Ar^3\,\delta_T)(k_{\psi1}(\psi_d - \psi) - k_{\psi2}\dot{\psi} + k_{\psi3}\int(\psi_d - \psi)dt) \end{cases} \tag{18}$$

The design process of the adaptive sliding mode controller for the underactuated subsystem is presented below. Now $x_1 = [x\ y]^T, x_2 = [\dot{x}\ \dot{y}]^T, x_3 = [\varphi\ \theta]^T, x_4 = [\dot{\varphi}\ \dot{\theta}]^T, u = [\nabla\delta_\varphi\ \nabla\delta_\theta]^T$, the system can be written as the following standard form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = g_1(x_3) + d'(x,t) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = g_2(u) + d'(x,t) \end{cases} \tag{19}$$

In which, $g_1(x_3) = \begin{bmatrix} gSin\psi & gCos\psi \\ -gCos\psi & gSin\psi \end{bmatrix} x_3$, $g_2(u) = \begin{bmatrix} 3dC_\pi\rho Ar^2\delta_T/I_x & 0 \\ 0 & 3dC_\pi\rho Ar^2\delta_T/I_y \end{bmatrix} u,$

Lemma 1: *"$d'(x,t)$" is the external disturbance. Now adaptive base SMC error vector can be written as $e_1 = x_1 - x_{1d}$, $e_2 = x_2$, $e_3 = g_1(x_3)$, $e_4 = (\partial g_1(x_3)x_4)/\partial x_3$, $E_{rr} = [e_1 e_2 e_3]$, the UAV positional control "x_{1d}" is the constant, then $x_1 = 0$. Defines a system $S = C_1e_1 + C_2e_2 + C_3e_3 + C_4$.*

Proposition 1: Let $d'(x,t)$ is the bounded disturbance and satisfy: $\|E_{rr}\|\bar{d}' \geq \bar{d}'$.

Supposition: *The adaptive sliding mode variable structure control is obtained, so that the points lie on the sliding surface $S = 0$ are the end points which satisfy the following condition: $S = 0, \dot{S} = 0$, and $\ddot{S} = 0$. Ensures that the system state is always in the sliding surface when $S = 0$, the controller is equivalent to "u_{eq}". Through the position $\dot{S} = 0$ the equivalent control of the adaptive sliding mode controller can be obtained as,*

$$u_{eq} = -[\partial g_1(x_3)g_2/\partial x_3]^{-1}[C_1x_2 + C_2g_1(x_3) + C_3(\partial g_1(\partial x_3)x_4/x_3)] \tag{20}$$

Switching control u_{swc}, can make the adaptive sliding control function uniform "S" converges to zero.

$$u_{swc} = -[\partial g_1(x_3)g_2/\partial x_3]^{-1}[\beta sgn(S) + \eta S] \tag{21}$$

Proposition 2: *The updating law of adaptive sliding mode control is $\beta = (C_2\bar{d}'\|E_{rr}\| + \sqrt{2} * g\bar{d}'\|E_{rr}\|) + \alpha$.*

At that time, the adaptive sliding mode controller is obtained as,

$$u = u_{eq} + u_{swc} \tag{22}$$

Now, the derivative of $g_1(x_3)$ is,

$$\frac{\partial g_1(x_3)}{\partial x_3} = \begin{bmatrix} gSin\psi & gCos\psi \\ -gCos\psi & gSin\psi \end{bmatrix} \tag{23}$$

$\|\partial g_1(x_3)/\partial x_3\| = \sqrt{2} * g$, which states that $\sqrt{2} * g\bar{d}'\|E_{rr}\| \geq \mathrm{d}' \geq \|\partial g_1(x_3)/\partial x_3\|$.

Proof: The stability of the UAV system is defined by using Lyapunov candidate function which is, $V = (1/2)S^T S$

Invoking the lemma (1) and equation (23) than the Lyapunov function derivative with respect to time is written as,

$$\dot{V} = \|S\|\alpha - \eta S^T S \geq S^T[C_1 e_2 + C_2(e_3 + d') + C_3 e_4 + \partial g_1(x_3)/\partial x_3(g_2(u) + \mathrm{d}'(x,t))] \tag{24}$$

The derivative of Lyapunov candidate function is negative with respect to time, therefore the adaptive sliding mode function "S" converges to a zero, in the infinite state of time. In this case, the system can be reduced to an agitation system as follows:

$$\dot{E}_{rr} = \mathrm{d}'(x,t) + AE_{rr} \tag{25}$$

$A = \begin{bmatrix} 0 & I_2 & 0 \\ 0 & 0 & I_2 \\ -C_1 I_2 & -C_1 I_2 & -C_1 I_2 \end{bmatrix}$, $d = \begin{bmatrix} 0 \\ d' \\ 0 \end{bmatrix}$, $\|E_{rr}\|\bar{d}' \geq \|d\|$. Where "$A$" is a Hurwitz matrix.

So for the given condition, $Q = Q^T > 0$. there is a unique solution for this $P = P^T > 0$, for Lyapunov stability. Now define a Lyapunov function:

$$\dot{V}_1 \leq -\eta_{min}(Q)\|E_{rr}\|^2 + 2\eta_{max}(P)\bar{d}'\|E_{rr}\| \tag{26}$$

Lemma 2: *As long as inequality* $\eta_{min}(Q)/2\eta_{max}(P) > \bar{d}'$, *holds then* $V_1 \leq 0$, *the system is asymptotically stable. The system errors* e_1, e_2, e_3 *will converge to zero. According to* $S = C_1 e_1 + C_2 e_2 + C_3 e_3 + e_4 = 0$, *we knew that system will converge to zero.*

5. Simulation and Result Analysis

In this part of the article, tri-rotor dynamic model equations (05) to (08) are used to prove the effectiveness and stability of our proposed adaptive sliding mode controller scheme with wind disturbance. Furthermore, the efficiency of the proposed Adaptive Sliding Mode Controller (ASMC) is compared with the Adaptive Robust Back-stepping (ARB) control method for the underactuated UAV [19]. According to the model of the tri-rotor aerial vehicle, the designing parameters, set the following simulation conditions, in which UAV model parameters shown in the Table 2.

Table 2. The Tri-Rotor UAV Model Parameters

Variables	Value	S.I Units
(Mass) "m"	0.75	kg
(G.F) "g"	9.8	m/s^2
L	0.35	M
I_x	0.41	Kgm^2
I_y	0.41	Kgm^2
I_z	0.73	Kgm^2
k_{z1}	1.66	Ns/m
k_{z2}	0.33	Ns/m
k_{z3}	0.33	Ns/m
$k_{\psi 1}$	1.66	Ns/m
$k_{\psi 2}$	1	Ns/m
$k_{\psi 3}$	0.01	Ns/m

The control system block diagram is shown in Figure 2, having reference or the desired values of the UAV. The system performance have inner and outer loops such that ASMC control both the loops. Given values using as an input which is constantly changes the position and attitude of UAV by using its linear and angular velocities in the absence of disturbance in simulation case I and in the presence of wind disturbance test in case II, that is exposed to be real application in the real world. In order to achieve required altitude, angular velocity is used to control the error while linear velocity along with angular velocity is used to control the position. The control parameters for the underactuated UAV system is shown in the Table 3.

Table 3. The Tri-Rotor UAV Controller Parameters

Variables	Values
C_1	10
C_2	11
C_3	4
$\acute{\eta}$	0.1
β	0.1
ρ	$1/kg.m^{-3}$
A	$0.652/m^2$
α	0.1
r	$0.125/m^2$
C_π	0.00451
C_Q	0.000212
d	$0.35/m^2$

Case I: In this case of simulation, the comparison of ARB with ASMC is presented without disturbance. Moreover, the simulated results are shown in Figure 3 to 8. The starting position of UAV is [0, 0, 0]m, initial attitudes are [0, 0.45, 0]rad the reference altitude and attitude in this case are [10, 10, 10]m and [0, 0, 0]rad which is shown in Figure 3 and 4 respectively. Now by comparing our proposed controller with the ARB controller shows that the ASMC controller has better convergence rate and good transient performance than ARB. Figure 5 and 6 shows the linear and angular velocities subcomponents that converge to zero to fulfill the referred position. Figure 7, shows the input control commands and Figure 8 shows the sliding variables which converge to zero.

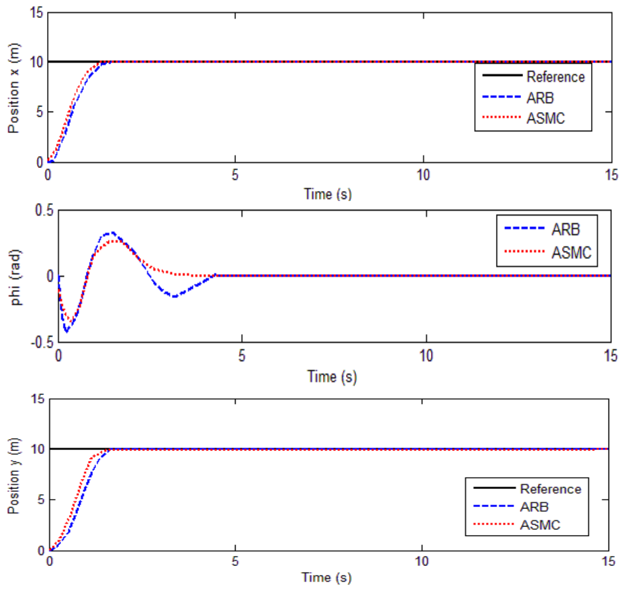


Figure 3. Positions (x, y, z) of UAV

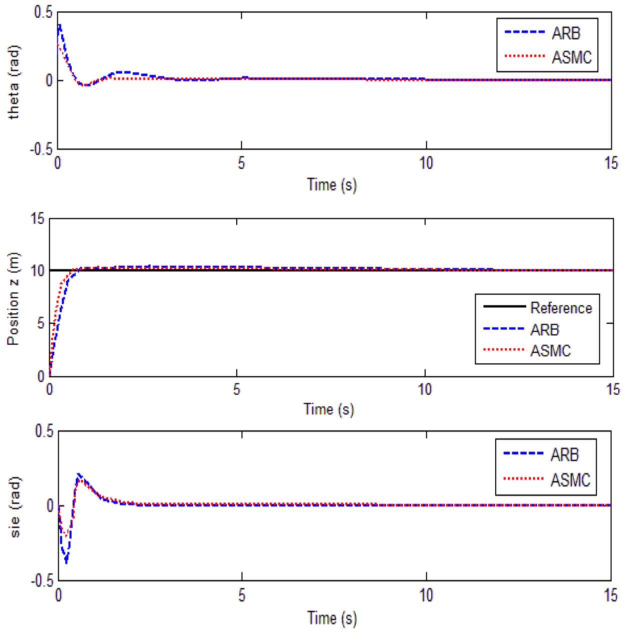


Figure 4. The Euler Angle Responses

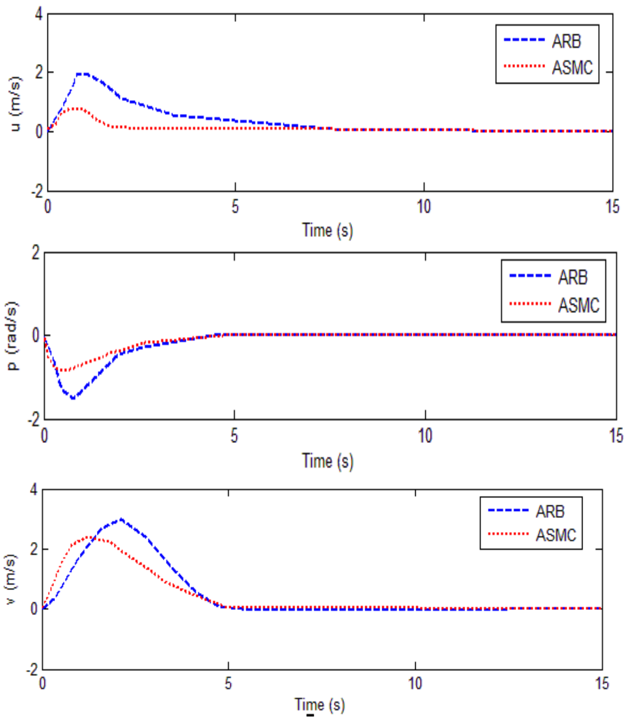


Figure 5. Linear Velocity Components

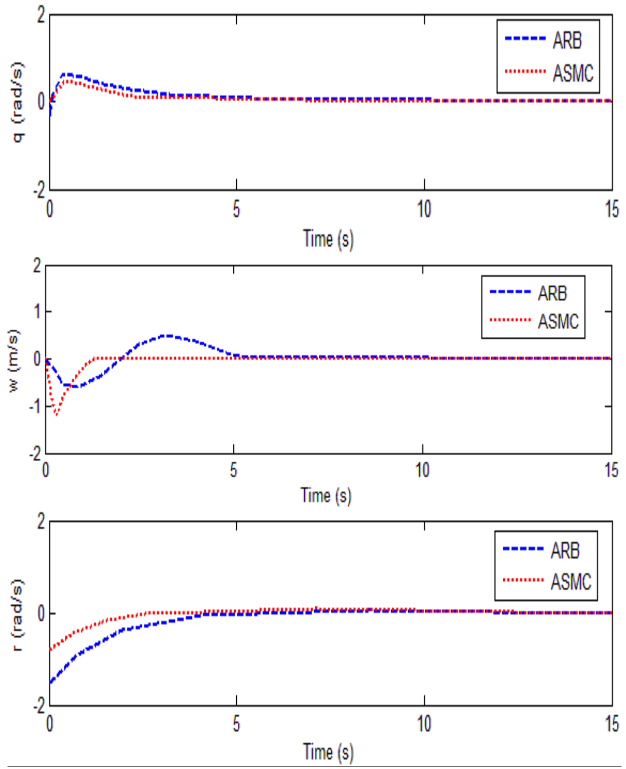


Figure 6. Angular Velocity Components

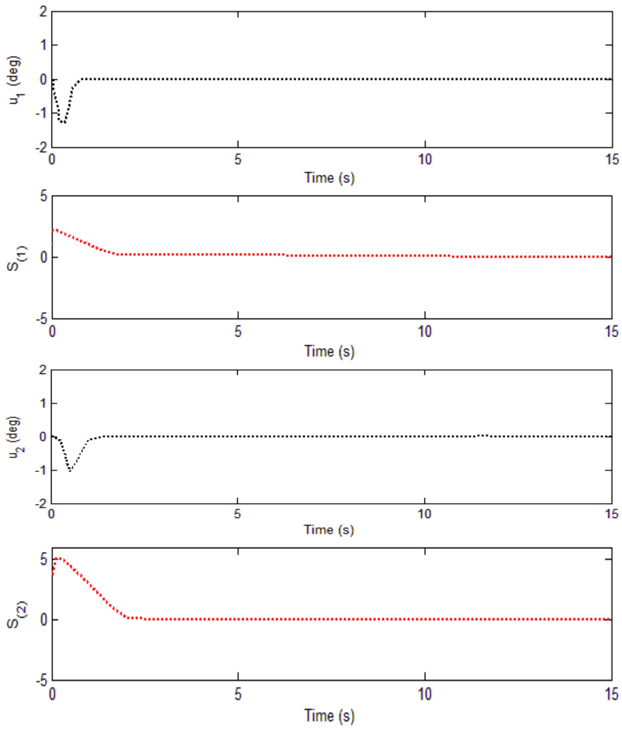


Figure 7. Input Control Commands

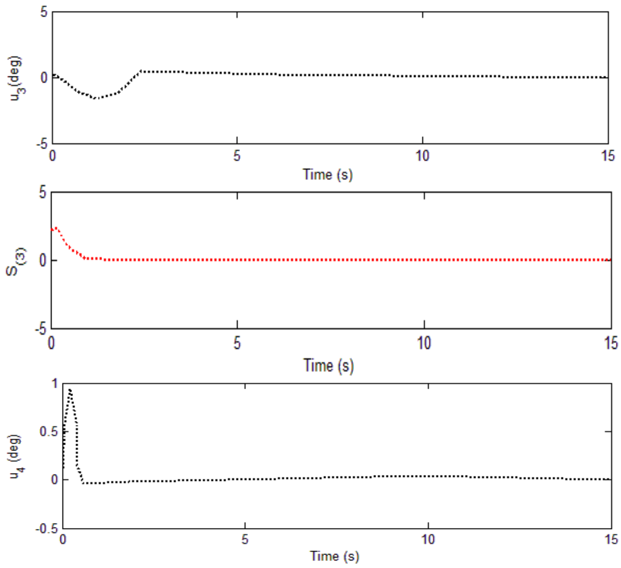


Figure 8. Sliding Variables

Case II: In this case of simulation, to validate the robustness of the ASMC method, the altitude and attitude tracking of UAV is performed. The simulations are performed, which is shown in Figure 9 to 14, that illustrates that ASMC will fulfill the control commands of input to reach at the reference position in a reasonable interval of time. The starting position and angles of the tri-rotor UAV for this simulation tests are [0, 0, 0]m and [0, 0, 0]rad. The reference position and Euler angles are used for the simulation tests are: [10, 10, 10]m and [0, 0, 0]rad which is shown in Figure 9 and 10.

Figure 11 and 12 shows the linear and angular velocity subcomponent which displays the similar behavior as the corresponding altitude and attitude angles. Definitely, these state variables are driven to their steady states as estimated and once again, results demonstrates the effectiveness of the ASMC.

Moreover, the controller "u_1" reaches its reference state at about 3 seconds and proves that the matrix Q is time invariant. In Figure 13, all the input control commands are shown, which are followed by the sliding variables of the controller which are S_1, S_2 and S_3 that converges to zero at a finite interval of time and is shown in Figure 14. Furthermore, control input $u_1 = 0$, proves that "Q" matrix is time invariant, at the start u_1 to u_4 have high oscillations, but it will converge to zero at a fair amount of time.

(Wind disturbance test): When the aircraft starts at the initial position, the disturbance of wind is along with it, in the direction of flight that will deviate the flight reference path. In the worst case scenario, the deviation of a reference path in the real world will directly affect the failure of flight. That's why in this article, a wind disturbance test is done to the check the performance. For this wind disturbance test, initially a wind is created artificially, in which the generated wind velocity polynomial is described as follows.

$$W_V = -1.1299x^5 + 4.25x^4 - .622x^3 + 3.45x^2 - 2.17x + 3.11 \quad (27)$$

To check the efficiency of the proposed controller along with external wind disturbance, an external velocity of wind is taken for the x-axis direction of UAV which is 1.25 m/sec, however, the roll angle of the aircraft change to 12.5 degree to hovering state. Furthermore, the gravity "g_e" is same as hovering state"u_x". Thus, wind disturbance is applied to the tri-rotor roll angle in the direction of UAV along x-axis,

$$tan\varphi = g_e / u_x \quad (28)$$

Same, as the wind disturbance in y-direction change the angle of pitch with the same degree. Moreover, tri-rotor focuses to slow time changing aerodynamic interface and noises. To check the validity of the proposed controller nonlinearity, noises and wind disturbance use as a constraint, in which the wind disturbance to the aircraft are as follows,

$$\begin{cases} \overline{u_x} = u_x Sin(2\pi t) + d_{F1} + \breve{A}.rands() \\ \overline{u_y} = u_y Cos(2\pi t) + d_{F2} + \breve{A}.rands() \\ \overline{u_z} = d_{F3} + \breve{A}.rands() \end{cases} \quad (29)$$

Where "$\overline{u_x}$", "$\overline{u_y}$" and "$\overline{u_z}$" are the wind disturbance external forces along with the nonlinearity applied to the aerial vehicle in the direction of x, y, and z axes respectively, "u_x" and "u_y" are the constant components of wind disturbance. The magnitude of random noise is denoted as "$\hat{A}$" and the time varying interface force is $d'(x,t) = [d_{F1}, d_{F2}, d_{F3}]$. The external wind disturbance is applied to the aircraft are $u_x = 0.75N$ and $u_y = 0.75N$ respectively. Now rewrite the aerodynamic interface force which is $d'(x,t) = [0.1Sin(0.1\pi t), 0.1Cos(0.1\pi t), 0.1Cos(0.1\pi t)]$, and $\tilde{A} = 0.19$.

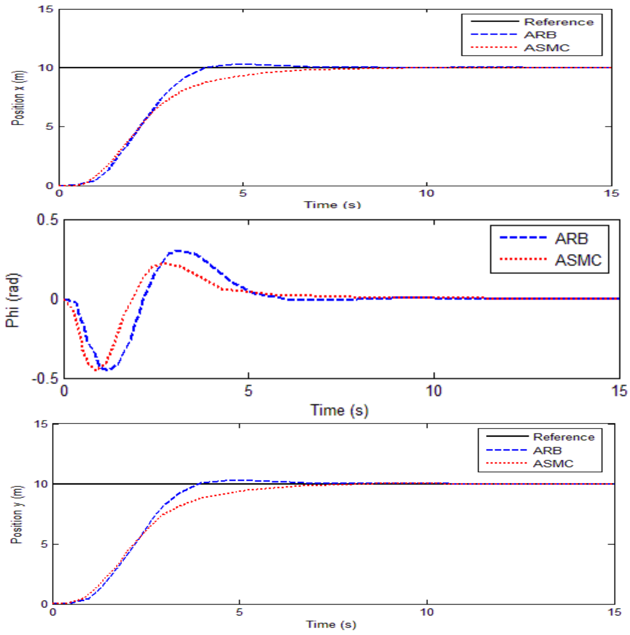


Figure 9. Positions (x, y, z) of UAV

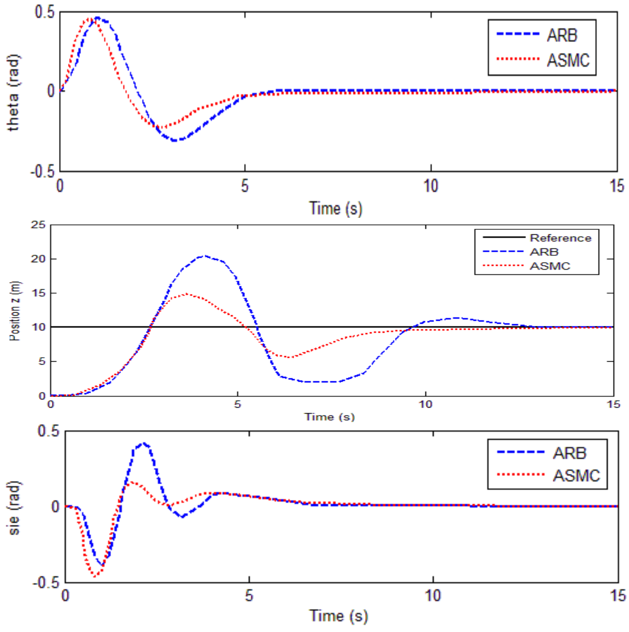


Figure 10. The Euler Angle Responses

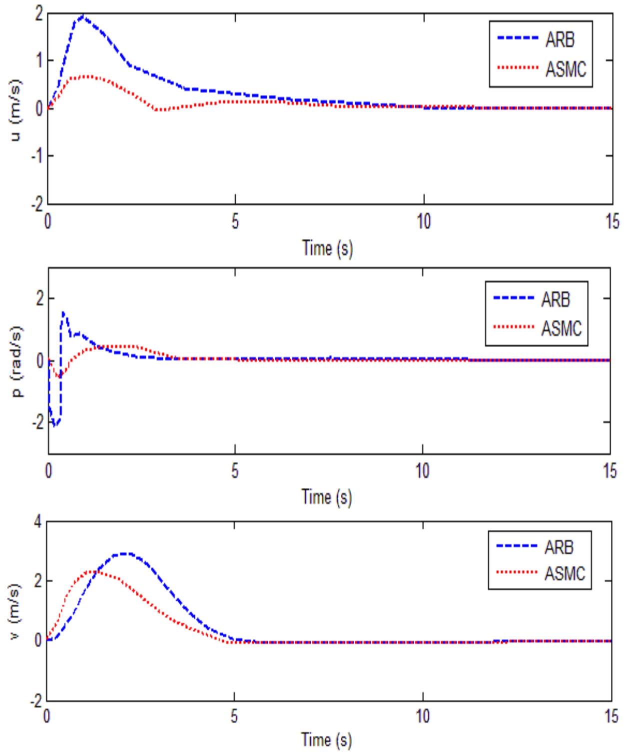


Figure 11. Linear Velocity Components

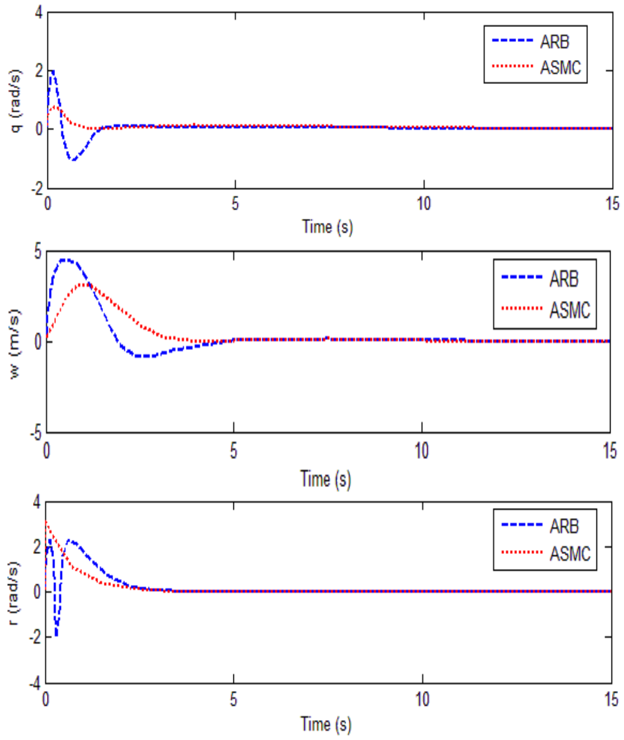


Figure 12. Angular Velocity Components

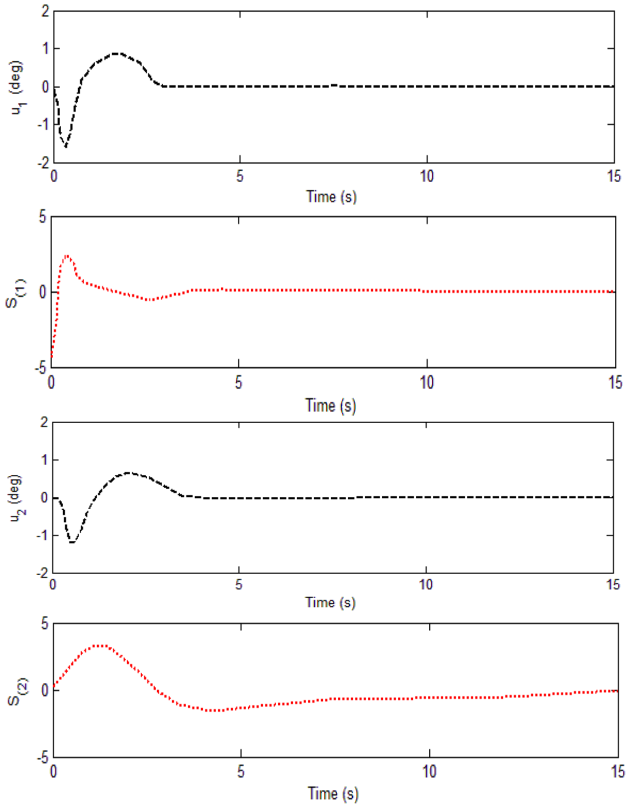


Figure 13. Input Control Commands

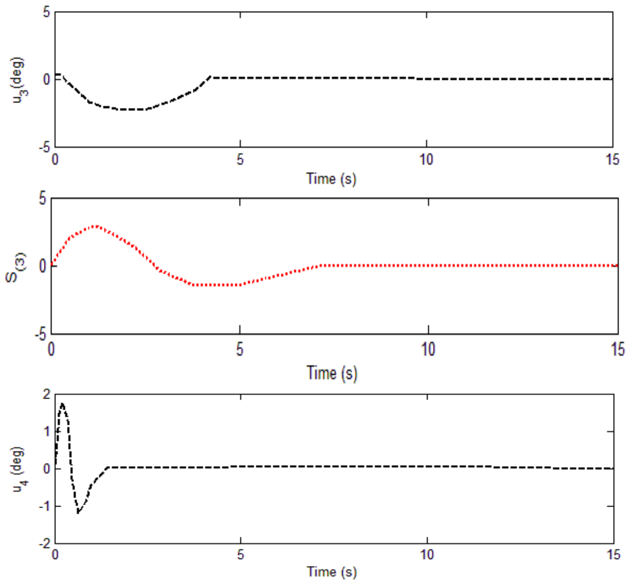


Figure 14. Sliding Variables

6. Conclusion

In the presented work, the ASMC is used for controlling the dynamics and stabilization of the altitude and attitude of tri-rotor aerial vehicle along with position control. The proposed system is simulated with two conditions, first ARB and ASMC comparison is given without disturbance and the controller successfully track the desired path. Secondly, the wind disturbance is introduced in the system such that the proposed controller offers better performance than ARB.

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