International Journal of Control and Automation
Vol. 10, No. 12 (2017), pp.51-66
http//dx.doi.org/10.14257/ijca.2017.10.12.05

Reduced Order Proportional Integral Observer for Disturbance Rejection

Manoj Swargiary[1*], Jayati Dey[2] and Tapas Kumar Saha[3]

Department of Electrical Engineering, National Institute of Technology Durgapur 713209, India
[1]manojsargu@gmail.com, [2]deybiswasjayati@gmail.com, [3]tapassahanit@gmail.com

Abstract

Reduced order observers are used effectively to observe the unmeasured states of a system from the knowledge of the remaining measurable states. However, reduced order proportional (P) observers developed in literatures, suffer from the limitations that, they fail to observe the states accurately in presence of disturbances. The present work proposes a reduced order proportional integral (PI) observer to overcome the limitations of the reduced order P observer. The veracity of the proposed observer is established theoretically as well as with examples. The sensor less speed control of dc motor is considered as a practical example to prove the veracity of the claim. It is seen that; the speed of the motor is estimated accurately from the measured armature current by the proposed PI observer even in presence of load disturbance.

Keywords*: Reduced order observer, Proportional integral observer, DC motor, Speed control, Sensorless*

1. Introduction

The observers are designed to reconstruct all the state variables. In practice, some of the state variables may be accurately measured. Such accurately measurable state variables need not to be estimated at all by employing an observer. Rather, an observer is required to estimate only the remaining states which are not available for measurement. Such is the case which arises for machine drives where sensorless speed control is preferred. This concept leads to the development of reduced order observers.

The observers with proportional gain only, suffers from the limitation that it is not effective in presence of disturbance in the plant inputs or outputs. There exists a sustained error between the steady state values of the actual and the observed states. Different types of such observers, like Luenberger observers, reduced order Luenberger observers [6]-[13], sliding mode observers [14]-[18] are mentioned in literatures. To minimize the steady state observer error in presence of disturbances, full order observers with integral action have been proposed in literatures [1]-[5]. PI type observer was first brought into attention by Beal and Shafai [1]. As the name suggests, it differs from the conventional observers by an additional integral path. Due to the integral path, an additional degree of freedom is available for observer design, which was used for recovering the traditional stability margins.

Reduced order PI observers have not yet been received significant attention in literature. The proportional (P) type reduced order observer, available in literature [23], suffers from the limitation that it fails to observe the states accurately in presence of disturbances. To overcome this, the present work develops a reduced order proportional-

Received (January 25, 2017), Review Result (October 31, 2017), Accepted (November 10, 2017)
* Corresponding Author

integral observer and it is theoretically proved to estimate the unobserved states successfully in presence of disturbances.

The effectiveness of the proposed PI type observer is demonstrated with respect to some numerical problems and is also demonstrated for sensor less speed control of DC motor. For ages, DC motors have been utilized as major variable speed drive in industries. A few of the DC motor speed control techniques are discussed in literatures [19]-[22]. The verification is carried out in MATLAB environment and the actual speed is only shown for comparison purpose.

The paper is organized as follows. Section 2 presents the mathematical background of reduced order proportional observer and its lack of capability of disturbance rejection. Section 3 establishes mathematically that the proposed PI observer has the capability of disturbance rejection. Section 4 considers numerical examples of double integral system and a third order system to establish the theoretical results. Section 5 presents sensor less speed control of DC motor with load torque disturbance. Conclusions are drawn in Section 6.

2. Overview of Reduced Order Proportional (P) Observer

Suppose that the state vector $x(t)$ is an $n\times1$-vector and the output vector $y(t)$ consists of m measurable states. Since m output variables are linear combinations of the state variables, m state variables need not to be estimated. Only, remaining $(n-m)$ states need to be estimated. Then the observer becomes an $(n-m)^{th}$ order observer. To present the basic idea of the reduced order observer, the case where the output is a scalar (*i.e.* $m=1$), is considered and the state equation for the reduced-order observer for observing the remaining $(n-1)$ states is derived.

The nominal plant has the mathematical model in state space form as follows.

$$\begin{aligned} \begin{bmatrix} \dot{x}_a(t) \\ \dot{x}_b(t) \end{bmatrix} &= \begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix} \begin{bmatrix} x_a(t) \\ x_b(t) \end{bmatrix} + \begin{bmatrix} B_a \\ B_b \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ W \end{bmatrix} w(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_a(t) \\ x_b(t) \end{bmatrix} \end{aligned} \tag{1}$$

Where

$x_a(t)$: Accurately measured states or output $y(t)$.

$x_b(t)$: States which cannot be measured.

$u(t)$: Control input.

$w(t)$: Disturbance.

The observer equation is given by

$$\dot{\tilde{x}}_b(t) = A_{bb}\tilde{x}_b(t) + A_{ba}x_a(t) + B_b u(t) + K_p\left(\dot{x}_a(t) - A_{aa}x_a(t) - B_a u(t) - A_{ab}\tilde{x}_b(t)\right) \tag{2}$$

Where, $\tilde{x}_b(t)$: $(n-1)\times1$ States to be observed,

K_p: $(n-1)\times1$ Proportional Observer gain.

Hence, putting differential terms in one side

$$\dot{\tilde{x}}_b(t) - K_p\dot{x}_a(t) = \left[A_{bb} - K_pA_{ab}\right]\tilde{x}_b(t) + \left[A_{ba} - K_pA_{aa}\right]x_a(t) + \left[B_b - K_pB_a\right]u(t) \tag{3}$$

Let

$$\eta(t) = x_b(t) - K_p y(t) = x_b(t) - K_p x_a(t) \tag{4}$$

And

$$\tilde{\eta}(t) = \tilde{x}_b(t) - K_p y(t) = \tilde{x}_b(t) - K_p x_a(t) \tag{5}$$

Differentiating $\eta(t)$ and $\tilde{\eta}(t)$ with respect to t, one obtains

$$\begin{aligned}\dot{\tilde{\eta}}(t) &= \dot{\tilde{x}}_b(t) - K_p\dot{x}_a(t)\\ &= \left[A_{bb} - K_pA_{ab}\right]\tilde{\eta}(t) + \left[A_{bb} - K_pA_{ab}\right]K_px_a(t) + \left[A_{ba} - K_pA_{aa}\right]x_a(t) + \left[B_b - K_pB_a\right]u(t)\\ &= \left[A_{bb} - K_pA_{ab}\right]\tilde{\eta}(t) + \left[B_b - K_pB_a\right]u(t) + \left[A_{ba} - K_pA_{aa} + K_p\left(A_{bb} - K_pA_{ab}\right)\right]x_a(t) \end{aligned} \quad (6)$$

And

$$\begin{aligned}\dot{\tilde{\eta}}(t) &= \dot{\tilde{x}}_b(t) - K_p\dot{x}_a(t)\\ &= \left[A_{bb} - K_pA_{ab}\right]\tilde{\eta}(t) + \left[A_{bb} - K_pA_{ab}\right]K_px_a(t) + \left[A_{ba} - K_pA_{aa}\right]x_a(t) + \left[B_b - K_pB_a\right]u(t)\\ &= \left[A_{bb} - K_pA_{ab}\right]\tilde{\eta}(t) + \left[B_b - K_pB_a\right]u(t) + \left[A_{ba} - K_pA_{aa} + K_p\left(A_{bb} - K_pA_{ab}\right)\right]x_a(t) \end{aligned} \quad (7)$$

The estimate state $\tilde{x}_b(t)$ can be obtained from the observer system as

$$\tilde{x}_b(t) = \tilde{\eta}(t) + K_py(t) \quad (8)$$

Now, introduce the error function $e(t)$ as follows.

$$e(t) = \eta(t) - \tilde{\eta}(t) \quad (9)$$

Differentiating $e(t)$ with respect to t, one can obtain

$$\begin{aligned}\dot{e}(t) &= \dot{\eta}(t) - \dot{\tilde{\eta}}(t)\\ &= \left(A_{bb} - K_pA_{ab}\right)\left(\dot{\eta}(t) - \dot{\tilde{\eta}}(t)\right) + Ww(t)\\ &= \left(A_{bb} - K_pA_{ab}\right)e(t) + Ww(t)\\ \therefore \dot{e}(t) &= \tilde{A}e(t) + Ww(t)\end{aligned} \quad (10)$$

If $\left(A_{bb}, A_{ab}\right)$ is observable, then one can choose K_p such that $\left(A_{bb} - K_pA_{ab}\right)$ is stable and Equation (10) is asymptotic. However, the effect of the disturbance $w(t)$ remain present in observation. For step or slowly varying $w(t)$, the estimation error is not diminishing for all time $t \geq 0$. Therefore, disturbance decoupling would not be attainable when perfect tracking is the primary objective. To get rid of the effect of disturbance in observation, a reduced order proportional integral (PI) observer is proposed in the next section.

3. Proposed Reduced Order PI Observer

This section proposes a reduced order PI observer specially to achieve disturbance decoupling in observation. The proposed proportional integral observer is given by

$$\dot{\tilde{x}}_b(t) = A_{bb}\tilde{x}_b(t) + A_{ba}x_a(t) + B_bu(t) + K_p\left(\dot{x}_a(t) - A_{aa}x_a(t) - B_au(t) - A_{ab}\tilde{x}_b(t)\right) + p(t) \quad (11)$$

Where $\tilde{x}_b(t)$: states to be observed

And $p(t)$: a new state vector with

$$\begin{aligned}p(t) &= K_I\int\left[\dot{x}_a(t) - A_{aa}x_a(t) - B_au(t) - A_{ab}\tilde{x}_b(t)\right]dt\\ &= K_I\int A_{ab}\left[x_b(t) - \tilde{x}_b(t)\right]dt\end{aligned}$$

And

$$\begin{aligned}\dot{p}(t) &= K_I\left[\dot{x}_a(t) - A_{aa}x_a(t) - B_au(t) - A_{ab}\tilde{x}_b(t)\right]\\ &= K_IA_{ab}\left[x_b(t) - \tilde{x}_b(t)\right]\end{aligned} \quad (12)$$

K_I : Integral Observer gain of dimension $(n-1)\times 1$

Hence observational error,

$$\dot{\tilde{x}}_b(t)-K_p\dot{x}_a(t)=\left[A_{bb}-K_pA_{ab}\right]\tilde{x}_b(t)+\left[A_{ba}-K_pA_{aa}\right]x_a(t)+\left[B_b-K_pB_a\right]u(t)+p(t) \quad (13)$$

And

$$\dot{p}(t)-K_I\dot{x}_a(t)=K_I\left[-A_{aa}x_a(t)-A_{ab}\tilde{x}_b(t)-B_au(t)\right] \quad (14)$$

Let's introduce two new variables

$$\eta(t)=x_b(t)-K_py(t)=x_b(t)-K_px_a(t) \quad (15)$$

And

$$\eta_p(t)=p(t)_{\tilde{x}_b=x_b}-K_Iy(t)=0-K_Ix_a(t)=-K_Ix_a(t) \quad (16)$$

Differentiating $\eta(t)$ and $\eta_p(t)$ with respect to t, one obtains

$$\begin{aligned}\dot{\eta}(t)&=\dot{x}_b(t)-K_p\dot{x}_a(t)\\&=\left[A_{bb}-K_pA_{ab}\right]\eta(t)+\left[A_{bb}-K_pA_{ab}\right]K_px_a(t)+\left[A_{ba}-K_pA_{aa}\right]x_a(t)+\left[B_b-K_pB_a\right]u(t)+Ww(t)\\&=\left[A_{bb}-K_pA_{ab}\right]\eta(t)+\left[B_b-K_pB_a\right]u(t)+\left[A_{ba}-K_pA_{aa}+K_p\left(A_{bb}-K_pA_{ab}\right)+K_I\right]x_a(t)\\&\quad+\eta_p(t)+Ww(t)\end{aligned}$$

And

$$\begin{aligned}\dot{\eta}_p(t)&=-K_I\dot{x}_a(t)=-K_I\dot{x}_a(t)\\&=K_I\left[-A_{aa}x_a(t)-B_au(t)-A_{ab}\left(x_b(t)-K_px_a(t)\right)\right]-K_IA_{ab}K_px_a(t)\\&=K_I\left[-A_{ab}\eta(t)-\left(A_{aa}+A_{ab}K_p\right)x_a(t)-B_au(t)\right]\end{aligned}$$

Hence, one can rewrite the above equations in matrix form as follows.

$$\begin{aligned}\begin{bmatrix}\dot{\eta}(t)\\ \dot{\eta}_p(t)\end{bmatrix}&=\begin{bmatrix}A_{bb}-K_pA_{ab} & I\\ -K_IA_{ab} & 0\end{bmatrix}\begin{bmatrix}\eta(t)\\ \eta_p(t)\end{bmatrix}+\begin{bmatrix}B_b-K_pB_a\\ -K_IB_a\end{bmatrix}u(t)\\&\quad+\begin{bmatrix}A_{ba}-K_pA_{aa}+K_p\left(A_{bb}-K_pA_{ab}\right)+K_I\\ -K_I\left(A_{aa}+A_{ab}K_p\right)\end{bmatrix}x_a(t)+\begin{bmatrix}W\\ 0\end{bmatrix}w(t)\end{aligned} \quad (17)$$

Further, define two variables $\tilde{\eta}(t)$ and $\tilde{\eta}_p(t)$ as follows.

$$\tilde{\eta}(t)=\tilde{x}_b(t)-K_py(t)=\tilde{x}_b(t)-K_px_a(t) \quad (18)$$

And

$$\tilde{\eta}_p(t)=p(t)-y(t)=p(t)-x_a(t) \quad (19)$$

Differentiating $\tilde{\eta}(t)$ and $\tilde{\eta}_p(t)$ with respect to t, one can obtain

$$\begin{aligned}\dot{\tilde{\eta}}(t)&=\dot{\tilde{x}}_b(t)-K_p\dot{x}_a(t)\\&=\left[A_{bb}-K_pA_{ab}\right]\tilde{\eta}(t)+\left[A_{bb}-K_pA_{ab}\right]K_px_a(t)+\left[A_{ba}-K_pA_{aa}\right]x_a(t)+\\&\quad\left[B_b-K_pB_a\right]u(t)+\tilde{\eta}_p(t)+K_Ix_a(t)\\&=\left[A_{bb}-K_pA_{ab}\right]\tilde{\eta}(t)+\tilde{\eta}_p(t)+\left[B_b-K_pB_a\right]u(t)+\begin{bmatrix}A_{ba}-K_pA_{aa}+\left(A_{bb}-K_pA_{ab}\right)K_p\\ +K_I\end{bmatrix}x_a(t)\end{aligned}$$

And

$$\begin{aligned}\dot{\tilde{\eta}}_p(t)&=\dot{p}(t)-K_I\dot{x}_a(t)\\&=-K_I\left[A_{aa}x_a(t)+B_au(t)\right]-K_I\left[A_{ab}\left(\tilde{x}_b(t)-K_px_a(t)\right)+A_{ab}K_px_a(t)\right]\\&=-K_IA_{ab}\tilde{\eta}(t)-K_I\left(A_{aa}+A_{ab}K_p\right)x_a(t)-K_IB_au(t)\end{aligned}$$

Hence,

$$\begin{bmatrix} \dot{\tilde{\eta}}(t) \\ \dot{\tilde{\eta}}_p(t) \end{bmatrix} = \begin{bmatrix} A_{bb} - K_p A_{ab} & I \\ -K_I A_{ab} & 0 \end{bmatrix} \begin{bmatrix} \tilde{\eta}(t) \\ \tilde{\eta}_p(t) \end{bmatrix} + \begin{bmatrix} B_b - K_p B_a \\ -K_I B_a \end{bmatrix} u(t) + \begin{bmatrix} A_{ba} - K_p A_{aa} + \left(A_{bb} - K_p A_{ab}\right) K_p + K_I \\ -K_I \left(A_{aa} + A_{ab} K_p\right) \end{bmatrix} x_a(t) \tag{20}$$

The estimated state $\tilde{x}_b(t)$ can be obtained from the observer system as

$$\tilde{x}_b(t) = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} \tilde{\eta}(t) \\ \tilde{\eta}_p(t) \end{bmatrix} + K_p y(t) \tag{21}$$

Now, introduce the estimating error function $e(t)$ as follows.

$$e(t) = \begin{bmatrix} \eta(t) - \tilde{\eta}(t) \\ \eta_p(t) - \tilde{\eta}_p(t) \end{bmatrix} = \begin{bmatrix} x_b(t) - \tilde{x}_b(t) \\ A_{ab} \int \left[x_b(t) - \tilde{x}_b(t) \right] dt \end{bmatrix} \tag{22}$$

Differentiating $e(t)$ with respect to t, one can obtain

$$\dot{e}(t) = \begin{bmatrix} \dot{\eta}(t) - \dot{\tilde{\eta}}(t) \\ \dot{\eta}_p(t) - \dot{\tilde{\eta}}_p(t) \end{bmatrix}$$

$$= \begin{bmatrix} \left(A_{bb} - K_p A_{ab}\right) & I \\ -K_I A_{ab} & 0 \end{bmatrix} \begin{bmatrix} \eta(t) - \tilde{\eta}(t) \\ \eta_p(t) - \tilde{\eta}_p(t) \end{bmatrix} + \begin{bmatrix} W \\ 0 \end{bmatrix} w(t)$$

$$\dot{e}(t) = \begin{bmatrix} \left(A_{bb} - K_p A_{ab}\right) & I \\ -K_I A_{ab} & 0 \end{bmatrix} e(t) + \begin{bmatrix} W \\ 0 \end{bmatrix} w(t)$$

$$\therefore \dot{e}(t) = \tilde{A} e(t) + \begin{bmatrix} W \\ 0 \end{bmatrix} w(t) \tag{23}$$

Theorem: If the disturbance $w(t)$ is a step or slowly varying function, and both $\left(A_{bb}, A_{ab}\right)$ and $\left(\left(A_{bb} - K_p A_{ab}\right), A_{ab}\right)$ are observable, then (20) is an asymptotic observer which allows one to decouple the effect of disturbance $w(t)$ for all $t \geq 0$.

Proof: If $\left(A_{bb}, A_{ab}\right)$ is observable, one can find a solution of proportional gain K_p to ensure a Hurwitz $\left(A_{bb} - K_p A_{ab}\right)$. Once K_p is chosen, $\left(A_{bb} - K_p A_{ab}\right)$ becomes automatically fixed up. Further, if $\left(\left(A_{bb} - K_p A_{ab}\right), A_{ab}\right)$ is observable, then one can find out a stable $\tilde{A}$ for the solution of integral gain K_I. Hence, it is obvious from (23) that if $\tilde{A}$ is Hurwitz, the error $e(t) = x_b(t) - \tilde{x}_b(t)$ would asymptotically tend to zero for step or slowly varying disturbance $w(t)$, and it is possible to decouple the effect of disturbance for all $t \geq 0$.

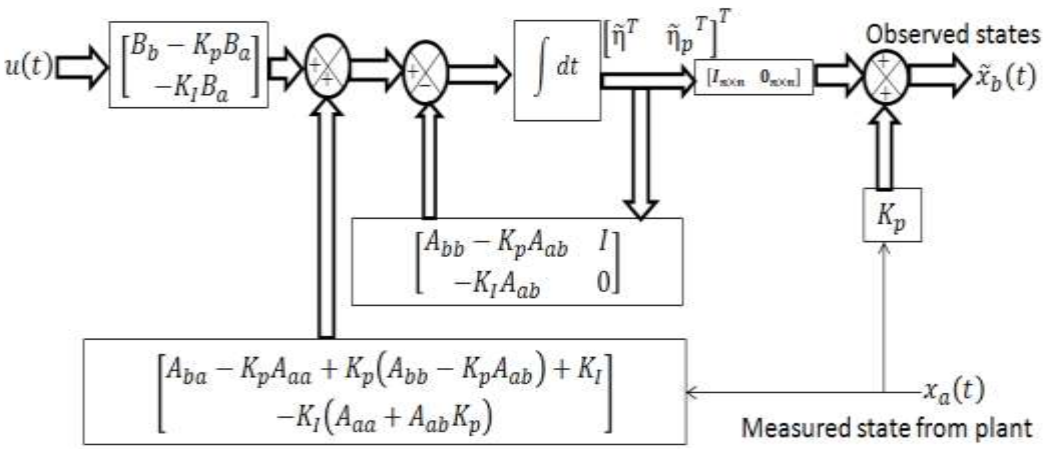

Figure 1. Block Diagram for Reduced Order Proportional Integral (PI) Observer

4. Numerical Examples

To illustrate that how a reduced order PI observer does the task of disturbance decoupling, two numerical examples are considered from the reference book [23].

Example 1: Consider a double integrator system described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 2 \end{bmatrix} w$$

And the output equation is given by $y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

One may visualize the double integrator system as a cart moving on a friction less rail for an input force $u(t)$. Hence, $x_1(t)$ and $x_2(t)$ would be the position and velocity of the cart and expressed in m and m/sec, respectively.

Solution: With reference to the proposed PI observer, $A_{aa} = 0, A_{ab} = 1$, $A_{ba} = 0, A_{bb} = 0$, $B_a = 0, B_b = 1$. The system is unstable having double pole at origin. The control input $u(t)$ is designed in such a way that the closed loop poles would be placed at suitable locations in LHP. To achieve this, a linear full state feedback control law $u = -[k_1 \quad k_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is considered. The controller gains $[k_1 \quad k_2] = [5 \quad 6]$ are obtained by the pole placement at $-2, -3$. However, the output $y(t)$ contains only x_1. This implies that the other state x_2 is required to be observed with the help of an observer. Therefore, a reduced order PI observer is designed next. The observer equation is given by

$$\begin{bmatrix} \dot{\tilde{\eta}}(t) \\ \dot{\tilde{\eta}}_p(t) \end{bmatrix} = \begin{bmatrix} -k_p & 0 \\ -k_I & 0 \end{bmatrix} \begin{bmatrix} \tilde{\eta}(t) \\ \tilde{\eta}_p(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} -k_p{}^2 + k_I \\ -k_I k_p \end{bmatrix} x_1(t)$$

$$\tilde{x}_2(t) = [1 \quad 0] \begin{bmatrix} \tilde{\eta}(t) \\ \tilde{\eta}_p(t) \end{bmatrix} + k_p x_1(t)$$

$$u(t) = -[k_1 \quad k_2] \begin{bmatrix} x_1(t) \\ \tilde{x}_2(t) \end{bmatrix}$$

Or, $u(t) = -\left[k_1 + k_2 k_p \quad k_2\right] \begin{bmatrix} x_1(t) \\ \tilde{\eta}(t) \end{bmatrix}$

The observer poles are chosen to be at least three to five times faster than the poles of the closed-loop system. Hence, they are placed at $-5,-5$.

The characteristic equation of the observer is obtained as

$$\det\left[sI-\tilde{A}\right]=\det\left[sI_{2x2}-\begin{bmatrix}-k_p & 1\\ -k_I & 0\end{bmatrix}\right]=s^2+k_ps+k_I=\left(s+5\right)^2$$

Thus, one obtains, $k_p=10,\ \ k_i=25$.

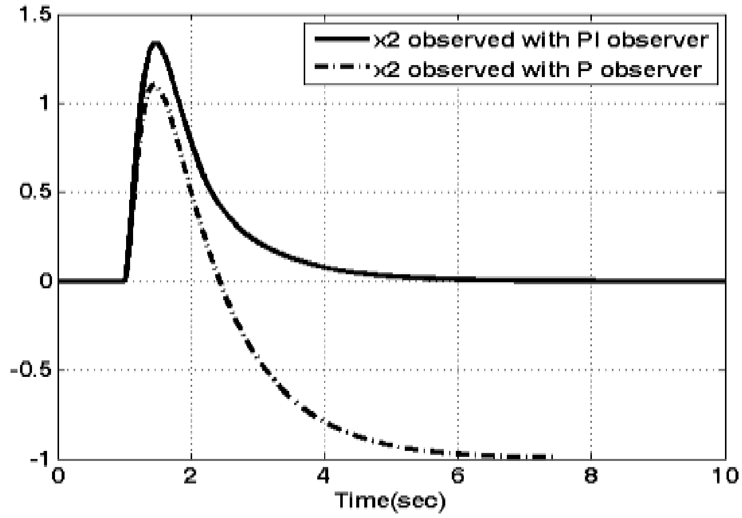

Figure 2. Response of the Double Integrator System with State Feedback Employing Reduced Order PI Observer with an Input Disturbance at 1 Sec

The double integrator system is simulated in MATLAB-SIMULINK environment with full state feedback control & proposed reduced order PI observer. A disturbance $w(t)$ of 5 unit is applied at 1 sec in the control input. The state x_2 is anticipated to attain zero value at steady state. However, it is clearly seen from Figure. 2 that the reduced order the proportional observer fails to observe accurately x_2 after 1 sec when the disturbance comes into play and a steady state error in observation is sustained. On the other hand, as expected, PI observer is estimating the unmeasured states accurately even when the disturbance affects the system.

Example 2: Consider a third order system described by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u+\begin{bmatrix}0\\ 0\\ 0.5\end{bmatrix}w$$

And the output equation is given by $y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$.

Solution: Let the closed-loop poles are to be placed at $-2\pm j2\sqrt{3}$, -6. Then the necessary state-feedback gain matrix can be obtained as $K=\begin{bmatrix}90 & 29 & 4\end{bmatrix}$. Next, assume that the output $y(t)$ can be measured accurately so that state variable x_1 (which is equal to $y(t)$ need not be estimated. Let design a reduced-order observer. The reduced-order observer is of third order. With the observer gains are as $K_p = \begin{bmatrix} k_{p1} & k_{p1} \end{bmatrix}^T$ and $K_I = \begin{bmatrix} 0 & k_{I2} \end{bmatrix}^T$.

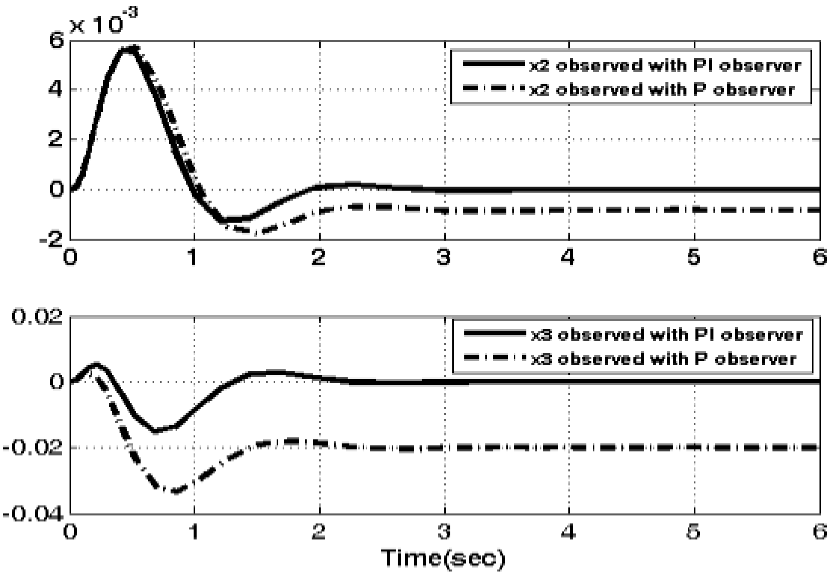

Figure 3. Response of the Third Order System with State Feedback Employing PI Observer with an Input Disturbance of 0.5 Unit

Let the observer poles be placed at $-10,-10,-10$. Thus, one obtains $K_p = \begin{bmatrix} 24 & 145 \end{bmatrix}^T$ and $K_I = \begin{bmatrix} 0 & 1000 \end{bmatrix}^T$.

The closed-loop system with state feedback control and PI observer is simulated in MATLAB-SIMULINK environment applying disturbance of 0.5 unit.

It is depicted in Figure. 3 that the unobserved states are satisfactorily being observed with the reduced order PI observer even in presence of disturbance. However, when P observer is applied, there is an undiminished steady state error clearly seen in the observed states in presence of disturbance. Therefore, one can conclude that the PI observer is seen to be attenuating the effect of disturbance and observing the unmeasured states satisfactorily.

5. Sensor less Speed Control of DC Motor with Reduced Order PI Observer

This section considers the problem of sensor less control of DC motor where the speed, instead of being measured, is observed by the reduced order PI observer. Hence, the observed speed is used as feedback for necessary control action.

5.1. DC Motor System Description & Control Topology

A separately excited dc motor can be mathematically expressed by the armature voltage and torque equations as follows.

$$v(t) - e_b(t) = Ri_a(t) + L\frac{di_a}{dt} \tag{24}$$

Where the back EMF is given by

$$e_b(t) = K_b\omega(t) \tag{25}$$

$$T(t) - T_L(t) = K_T i_a(t) - T_L(t) = B\omega(t) + J\frac{d\omega(t)}{dt} \tag{26}$$

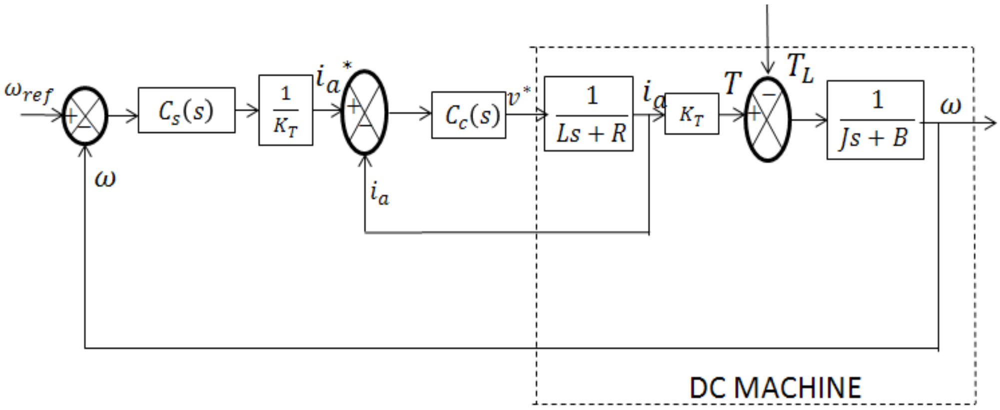

Figure 4. Control Block Diagram for the Speed Control of DC Motor

Where

$v(t)$: Motor terminal voltage

$i_a(t)$: Armature current

$\omega(t)$: Motor speed

R: Armature resistance

L: Armature inductance

J: Motor inertia

B: Motor damping constant

$T_L(t)$: Load torque

K_b: Back EMF constant

K_T: Torque constant

The equations (24), (25) and (26) can be rewritten as,

$$\frac{d}{dt}\begin{bmatrix} i_a(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} -R/L & -K_b/L \\ K_t/J & -B/J \end{bmatrix}\begin{bmatrix} i_a(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v(t) + \begin{bmatrix} 0 \\ -\frac{1}{J} \end{bmatrix} T_L(t) \tag{27}$$

The objective of the system controller is to achieve speed control of the dc motor even in presence of load torque disturbance. This is accomplished by controlling the voltage $v(t)$ using a current control loop along with speed control as in Figure. 4.

The speed controller and current controller are, respectively given by (28) and (29).

$$C_s(s) = K_{ps} + \frac{K_{is}}{s} \tag{28}$$

$$C_c(s) = K_{pc} + \frac{K_{ic}}{s} \tag{29}$$

Let the motor speed $\omega(t)$ is not measurable, then one has no option other than estimating the speed with the help of an observer which takes the current measurement as the input. Since the effect of load torque needs to be rejected for the sake of perfect speed tracking, PI observer is the definite choice. The reduced order observer equation is given by

$$\begin{aligned} \begin{bmatrix} \dot{\tilde{\eta}}(t) \\ \dot{\tilde{\eta}}_p(t) \end{bmatrix} &= \begin{bmatrix} -(B/J)+k_p(k_b/L) & k_I \\ (k_b/L) & 0 \end{bmatrix}\begin{bmatrix} \tilde{\eta}(t) \\ \tilde{\eta}_p(t) \end{bmatrix} + \begin{bmatrix} -k_p/L \\ -1/L \end{bmatrix} u(t) \\ &+ \begin{bmatrix} (k_t/J)+k_p(R/L)+k_p\left(-(B/J)+k_p(k_b/L)\right)+k_I \\ (R/L)+k_p(k_b/L) \end{bmatrix} i_a(t) \end{aligned} \tag{30}$$

Or,

$$\begin{bmatrix} \dot{\tilde{\eta}}(t) \\ \dot{\tilde{\eta}}_p(t) \end{bmatrix} = \tilde{A} \begin{bmatrix} \tilde{\eta}(t) \\ \tilde{\eta}_p(t) \end{bmatrix} + \tilde{B}u(t) + \tilde{B}_i i_a(t) \tag{31}$$

$$\tilde{\omega}(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{\eta}(t) \\ \tilde{\eta}_p(t) \end{bmatrix} + k_p i_a(t) \tag{32}$$

The observed speed $\tilde{\omega}(t)$ is fed back in the control scheme in absence of actual speed.

5.2. Design of Controller & Proposed PI Observer for the DC motor system

Both the PI controllers are designed keeping in mind that the current overshoot should not cross its rated value to protect the motor. The dc motor under consideration is of the rating 5HP, 1250 rpm, 240V, 16A. The current limiter is put in the forward path which sets the current limit between -16A to 16A. The limit is taken based on the rated value of the motor under consideration. The motor under consideration has the following data: armature resistance $R = 0.6\,\Omega$, armature inductance L=112.9 mH , motor inertia $J = 1Kg - m^2$, motor damping constant $B = 0$, torque constant $K_t = 1.79 Nm/Amp$, back EMF constant $K_b = 1.8025 V/rad/\sec$.

The PI controllers are designed based on root locus technique. The PI controller for the current loop is designed in such way that it's zero occurring at $-K_{ic}/K_{pc}$ is placed left to the controller pole at origin, and the original plant pole at $-R/L$ *i.e.* at -5.3571 . The gain K_{pc} is chosen to have a faster response with peak overshoot as small as possible.

By trial and error, the choice of $-K_{ic}/K_{pc}$ is made as -100, and $K_{pc} = 10$. This design ensures acceptable response of the current with stable closed loop poles.

Next, the PI controller for the speed loop is designed based on the same control technique. Since both the controller and the plant poles at origin, the controller zero which is occurring at $-K_{is}/K_{ps}$ will be placed left to them. By trial and error, the choice of $-K_{is}/K_{ps}$ is made as -50 , and $K_{ps} = 5$. To ensure that the current controller would have faster response, the speed controller cut off frequency is kept smaller than that of current controller.

The PI controllers presented in eqns. (28) and (29) become as follows.

$$C_s(s) = 5 + \frac{250}{s} \tag{33}$$

And

$$C_c(s) = 10 + \frac{1000}{s} \tag{34}$$

The observer poles are chosen to ensure at least five times faster response than the speed controller. The closed-loop equation of the observer is given by

$$s^2 - \left(A_{bb} - K_p A_{ab}\right) + A_{ab} K_I = 0 \tag{35}$$

Where, $A_{bb} = -B/J = 0$, $A_{ab} = -K_b/L = -16.0935$.

With a choice of $K_i/K_p = 175$ and $K_p = 2$, the observer poles are placed at $-16.09 \pm j73.306$.

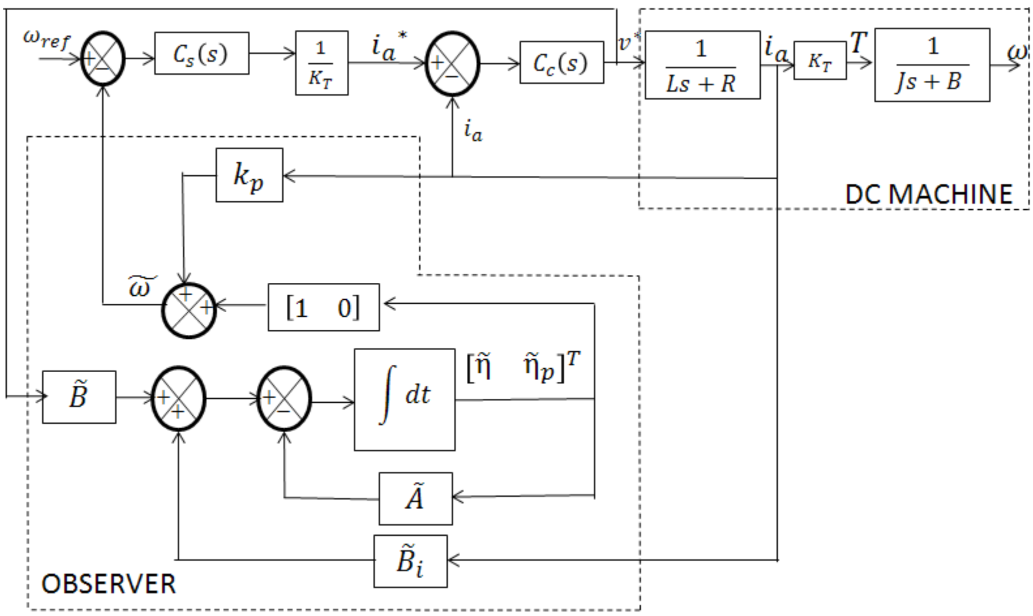

Figure 5. DC Motor Speed Control Scheme with Observed Speed

5.3. Results & Discussion

The designed PI observer along with the speed and current PI controllers are tested for the considered DC motor prototype in MATLAB-SIMULINK environment. The scheme incorporates the current sensors for the feedback to the observer and current controllers, and demonstrates the behaviour of actual and observed speed of the dc motor. The armature voltage of the motor is provided by a controlled four quadrant chopper. The observed speed is fed to the controller as speed feedback.

The experimental set-up is first run at initial speed 400 rpm with a load torque of 10% of the rated value *i.e.* 2.9 N-m. Then the command speed changes to 800 rpm and 1000 rpm, respectively, at 10 Sec and 20 Sec. The speed curve shows that the observer estimates perfectly the actual speed, rejects the load disturbance, and speed tracking with zero steady state error is achieved. The settling time taken is about 5 Sec. The speed characteristic is shown in Figure.6.

The corresponding voltage and current responses are, respectively, depicted in Figure.7 and Figure 8. The current attains the value 1.62A at steady-state up to 50 sec corresponding to a constant load torque of 2.9N-m.

Next, the load torque is changed to 80% of the rated load torque at 50 sec. The observer attenuates the effect of load disturbance, and observes the speed successfully with a settling time of few milliseconds only. The observed and actual speeds are found to be in good agreement to each other. This signifies the fact that the PI observer is capable of disturbance decoupling. Consequently, the current settles down to 11.16A.

The sensor less speed control of DC motor is performed with reduced order P and PI observer. As expected, a non-diminishing error exists between the actual and the observed speed in case of P observer. The speed curve with P observer is shown in Figure. 9. It is clearly seen that the error increases with increase in load torque. On the other hand, the speed curve with PI observer, shown in Figure. 10, depicts that the observed speed exactly matches with the actual one under load disturbance.

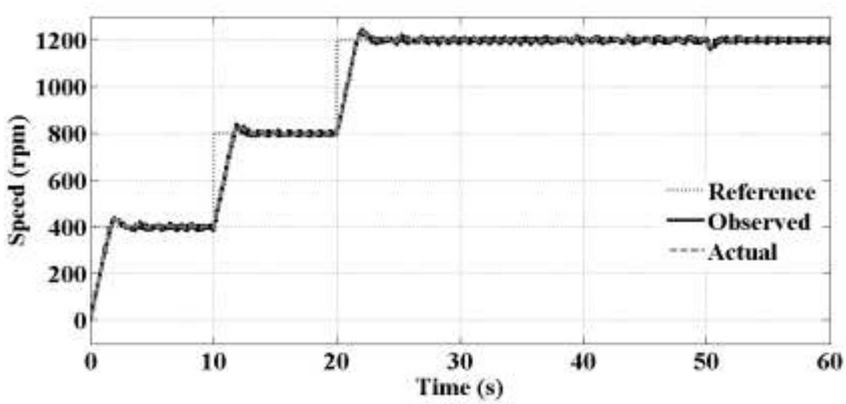

Figure 6. Response of the DC Motor with Reduced Order PI Observer for A Speed Reference of 400, 800 and 1200 rpm at 10% Rated Load Torque and with a Speed Reference of 1200 rpm at 80% Rated Load Torque Applied at 50 Sec

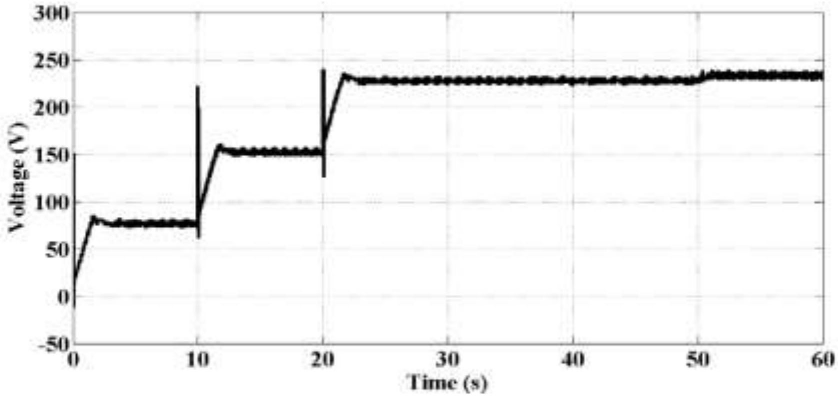

Figure 7. The Voltage Applied to the DC Motor in Case of Reduced Order PI Observer for A Speed Reference of 400, 800 And 1200 rpm at 10% Rated Load Torque and with a Speed Reference of 1200 Rpm at 80% Rated Load Torque Applied at 50 Sec

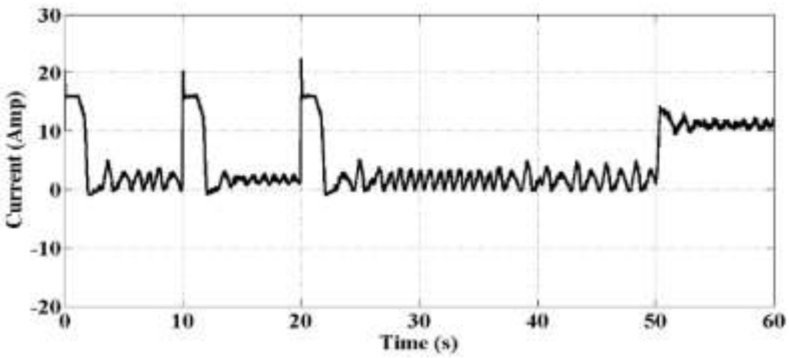

Figure 8. Armature Current Measured with Reduced Order PI Observer for a Speed Reference of 400, 800 And 1200 rpm at 10% Rated Load Torque and for a Speed Reference of 1200 Rpm at 80% Rated Load Torque Applied at 50 Sec

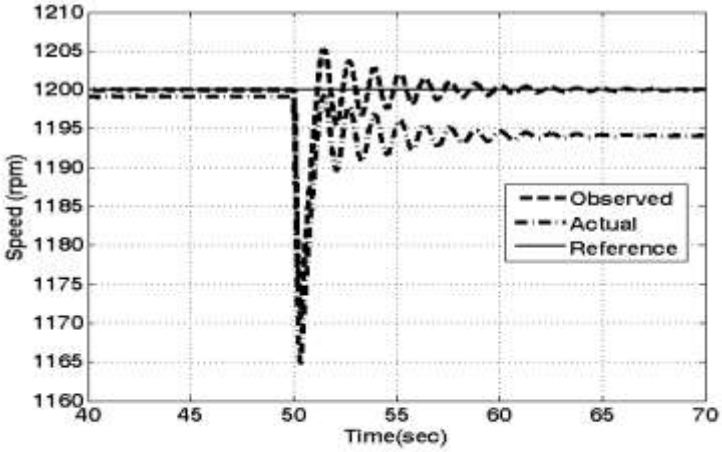

Figure 9. Response of The DC Motor with Reduced Order P Observer [Section 2] for a Speed Reference of 1200 Rpm at 10% and 80% Rated Load Torque Applied at 50 Sec

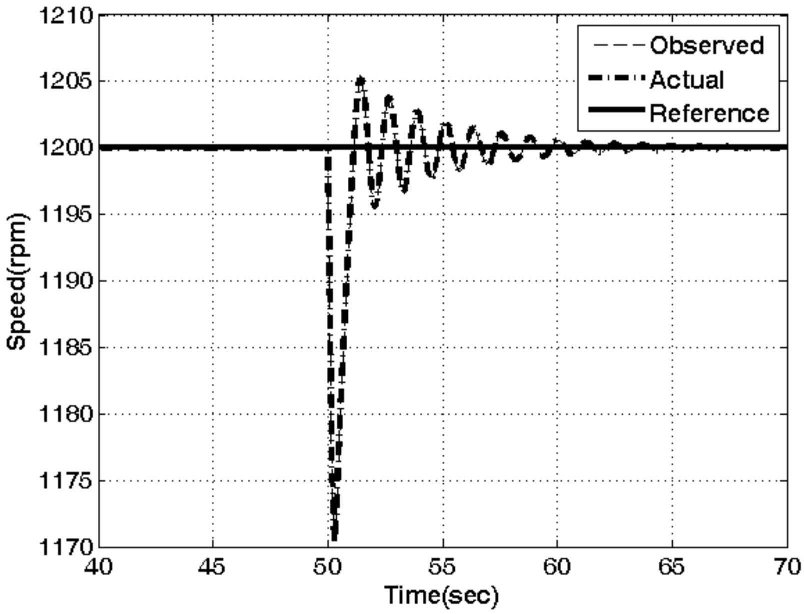

Figure 10. Response of the DC Motor with Reduced Order PI Observer for a Speed Reference of 1200 Rpm and 10% and 80% Rated Load Torque Applied at 50 Sec

6. Conclusion

In this paper, a reduced order PI observer is proposed with an objective to achieve disturbance rejection. It is established that the proportional reduced order observer, available in literature and presented in Section 2, fails to decouple the effect of disturbance in the state estimation. To overcome this, the present work introduces a reduced order proportional integral observer which is shown to achieve perfect estimation even in presence of disturbances. The veracity of the claim is established with the help of two mathematical examples. It is observed from the simulation results that; the proposed observer decouples the disturbance effect. Also, a practical example of sensor less speed controlled DC motor is considered to establish the claim. With the knowledge of the armature current, which is measured as the system output, the proposed observer accomplishes accurately the task of estimating the motor speed even in presence of load disturbance.

References

[1] S. O. Beale and B. Shafai, "Robust control system design with a proportional integral observer", International Journal of Control, vol-1, **(1989)**, pp. 554- 557.

[2] B. K. Krishna and K. Pousga, "Disturbance attenuation using proportional integral observers", International Journal Control, vol. 74, no. 6, **(2001)**, pp. 618 - 627.

[3] A.-G. Wu and G.-R. Duan, "Design of PI Observers for Continuous-Time Descriptor Linear Systems", IEEE Transactions on Systems, Man and Cybernetics—Part B: Cybernetics, vol. 36, no. 6, **(2006)**.

[4] Y.X. Yao and A.V. Radun, "Proportional integral observer design for linear systems with time delay", IET Control Theory and Application, **(2007)**, pp. 887 –892.

[5] J. Xu, C. C. Mi,J. Deng, Z. Chen and S. Li, "The State of Charge Estimation of Lithium-Ion Batteries Based on a Proportional-Integral Observer", IEEE Transactions on Vehicular Technology, vol. 63, no. 4, **(2014)**.

[6] T. Tuovinen, M. Hinkkanen and J. Luomi, "A Comparison of an Adaptive Full-Order Observer and a Reduced-Order Observer for Synchronous Reluctance Motor Drives", symposium on Sensor less control of electrical drives (SLED), **(2011)**.

[7] P. K Nandam and P.C Sen, "A comparative study of a Luenberger observer and adaptive observer-based variable structure speed control system using a self-controlled synchronous motor", IEEE Transactions on Industrial Electronics, vol. 37, no 2, **(1990)**, pp. 127 – 132.

[8] A. Savoia, M. Mengoni, L. Zarri and D. Casadei, "A nonlinear Luenberger observer for sensorless vector control of induction motors", International Aegean Conference on Electrical Machines and Power Electronics and Electromotion Joint Conference (ACEMP), **(2011)**, pp. 544 - 549.

[9] A. Saberi and P. Sannuti, "Observer design for loop transfer recovery and for uncertain dynamical systems", IEEE Transactions on Automatic Control, vol 35, no 8, **(1990)**, pp. 878 - 897

[10] M. Hasegawa, "Robust-adaptive-observer design based on γ-positive real problem for sensorless induction-motor drives", IEEE Transactions on Industrial Electronics, vol 53, no 1, **(2005)**, pp. 76 - 85.

[11] A. J. Mehta, B. Bandyopadhyay and A. Inoue, "Reduced Order Observer Design for Servo System Using Duality to Discrete-Time Sliding-Surface Design", IEEE Transactions on Industrial Electronics, vol. 57, no 11, **(2010)**, pp. 3793 - 3800.

[12] D. Zhengtao, "Observer Design in Convergent Series for a Class of Nonlinear Systems", IEEE Transactions on Automatic Control, vol. 57, no 7, **(2012)**, pp. 1849 - 1854.

[13] K. Robenack and A. F. Lynch, "High-gain nonlinear observer design using the observer canonical form", IET Control Theory & Applications, vol. 1, no 6, **(2007)**, pp. 1574 - 1579.

[14] M. Venkatesan and V. R. Ravi, "Sliding Mode Observer Based Sliding Mode Controller For Interacting Nonlinear System", 2nd International Conference on Current Trends in Engineering and Technology (ICCTET), **(2014)**, pp. 1 - 6.

[15] A. E. Rundell, S. V. Drakunov and R. A. DeCarlo, "A Sliding Mode Observer and Controller for Stabilization Of Rotational Motion of a Vertical Shaft Magnetic Bearing", IEEE Transactions on Control Systems Technology, vol. 4, no 5, **(1996)**, pp. 598 - 608.

[16] T. Bernardes, V. Montagner Foletto, H. A. Grundling and H. Pinheiro, "Discrete Time Sliding Mode Observer for Sensorless Vector Control of Permanent Magnet Synchronous Machine", IEEE Transactions on Industrial Electronics, vol. 61, no 4, **(2014)**, pp. 1679 - 1691.

[17] W.-F. Xie "sliding-Mode-Observer-Based Adaptive Control for Servo Actuator With Friction", IEEE Transactions on Industrial Electronics, vol. 54, no 3, **(2007)**, pp. 1517 - 1527.

[18] F. Chen and M. W. Dunnigan, "Comparative study of a sliding-mode observer and Kalman filters for full state estimation in an induction machine", IEE Proceedings on Electric Power Applications, vol. 149, no 1, **(2002)**, pp. 53 - 64.

[19] H. F. Weber, "Pulse-Width Modulation DC Motor Control", IEEE Transactions on Industrial Electronics and Control Instrumentation, vol. 12, no 1, **(1965)**, pp. 24 - 28.

[20] S. N. Singh and D. R. Kohli, "Analysis and Performance of a Chopper Controlled Separately Excited DC Motor", IEEE Transactions on Industrial Electronics, vol. 29, no. 1, **(1982)**.

[21] R. Silva-Ortigoza, V. M. Hernandez-Guzman, M. Antonio-Cruz and D. Munoz-Carrillo, "DC/DC Buck Power Converter as a Smooth Starter for a DC Motor Based on a Hierarchical Contro", IEEE Transactions on Power Electronics, vol. 30, no 2, **(2015)**, pp. 1076 - 1084.

[22] H. Sira-Ramirez and M.A. Oliver-Salazar, "On the Robust Control of Buck-Converter DC-Motor Combinations", IEEE Transactions on Power Electronics, vol. 28, issue 8, **(2013)**, pp. 3912-3922.

[23] K. Ogata, "Modern Control Engineering", 5th Edition, Prentice Hall.

Authors

Manoj Swargiary, he received the B.E. and M.Tech. in 2010 and 2014 respectively. He is currently pursuing Ph.D. in the Department of Electrical Engineering, National Institute of Technology Durgapur, India. His research interests are control system and machine drives.

Jayati Dey (M'09), she received the B.E.E., M.E., and Ph.D. in 2000, 2003, and in 2007 respectively. She is currently an Assistant Professor in the Department of Electrical Engineering, National Institute of Technology Durgapur, India. Her research interests are periodic compensation, switching power converters, robotics and mechatronics with special interest in robust control.

Tapas Kumar Saha (M'09), he received the B.E., M.E.E., and Ph.D. in 1997, 1999, and in 2009 respectively. He is currently an Associate Professor in the Department of Electrical Engineering, National Institute of Technology Durgapur, India. His current research interests are machine drives, power electronics, renewable energy and the grid integrated distributed generations.

