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A Novel Fuzzy Robust H_∞ Tracking Control Approach of Uncertain Nonlinear Systems via Output Feedback

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Abstract

A novel fuzzy robust H_∞ tracking control scheme via output feedback is investigated for a class of uncertain nonlinear systems. The nonlinear system is modeled in fuzzy Takagi-Sugeno (T-S) form, and a fuzzy output feedback controller based on fuzzy T-S model and fuzzy logic systems is determined to guarantee the desired H_∞ tracking performance. Based on Lyapunov stability theorem, a new tracking control criterion is formulated as linear matrix inequalities (LMIs) and adaptive law. Simulation results demonstrate the effectiveness and the feasibility of the developed tracking control scheme.

Keywords*: fuzzy T-S model; fuzzy logic systems; nonlinear systems; uncertainties; tracking control*

1. Introduction

As usual, tracking problem is more difficult than stability problem especially for nonlinear systems. Therefore, the problem of tracking controller design for nonlinear systems is a challenging issue.

Fuzzy control approach offers a control methodology to handle the nonlinear systems. By using fuzzy Takagi–Sugeno (T–S) model, a nonlinear system could be expressed as a weighted sum of some linear subsystems [1]. This model provides a fixed structure to some nonlinear systems which is favorable for system analysis under consideration. Therefore, many important results have been reported in [2-4] in T–S fuzzy systems.

Due to the existence of uncertainties, which is frequently a source of instability or degraded performance, the robust fuzzy control problem has been discussed for uncertain nonlinear systems based on fuzzy T-S model in [5-17].These control schemes are effective and feasible for uncertain nonlinear systems, where the matching condition and the upper bound are used to model and the uncertainties in [5-13] and [14-17], respectively. The matching condition and the upper bound are powerful tools for dealing with the uncertainties, but there still exists some conservatism. For example, it is difficult to determine the matching condition and the upper bound. Owing to the universal approximation property, fuzzy logic systems are used to model uncertain nonlinear systems in [18-21].Thus, we can use fuzzy logic system to replace the matching condition and the upper bound in modelling the uncertainties in systems.

On the other hand, in many engineer systems, the system states are often unavailable. Thus, it is necessary to design an output feedback controller based on an observer.

Based on the above discussions, in this paper, fuzzy T-S model and fuzzy logic systems are combined to design a fuzzy robust H_∞ tracking controller via output feedback for a class of uncertain nonlinear systems. Fuzzy T-S model is employed to represent the nonlinear system, fuzzy logic systems are used to compensate the

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uncertainties. Based on Lyapunov stability theorem, the proposed fuzzy control scheme guarantees the desired H_∞ tracking performance. Finally, simulation examples demonstrate that the proposed control method is feasible.

The main contributions and advantages are summarized as follows. (1)A less conservative fuzzy tracking controller via output feedback is proposed, where the matching condition and the upper bound are avoided and the dimension of LMIs is reduced. (2) The proposed control scheme is effective for the tracking control based on state feedback, and feasible for the output feedback-based H_∞ control.

The rest of the paper is organized as follows. Section 2 provides the formulation of the problem. Sections 3 develop a procedure of the controller design. Sections 4 gives the main result of the tracking control design. Section 5 presents an example to illustrate the effectiveness of the proposed method.

2. Problem Formulation

Consider the following uncertain nonlinear systems:

$$\begin{aligned}
&\dot{x}_1 = x_2,\\
&\cdots\\
&\dot{x}_{(\beta_1-1)} = x_{\beta_1},\\
&\dot{x}_{\beta_1} = f_1(x) + \tilde{f}_1(x) + (g_1(x) + \tilde{g}_1(x))u + d_1,\\
&\dot{x}_{(\beta_1+1)} = x_{(\beta_1+2)},\\
&\cdots\\
&\dot{x}_n = f_m(x) + \tilde{f}_1(x) + (g_m(x) + \tilde{g}_m(x))u + d_m,\\
&y = Cx,
\end{aligned} \tag{1}$$

where x , u and y are the system state, control input, and system output, respectively; x is assumed to be unavailable . $x = [x_1, \cdots, x_1^{(\beta_1-1)}, \cdots, x_{(n-\beta_m+1)}, \cdots, x_{(n-\beta_m+1)}^{(\beta_m-1)}]^T \in R^n$, $u \in R^m$,

$\beta_1 + \beta_2 + \cdots + \beta_m = n$; f_i $(i = 1,2,\cdots,m)$ are the smooth nonlinear functions, g_i $(i = 1,2,\cdots,m)$ are the smooth nonlinear vector functions, g_i $(i = 1,2,\cdots,m)$ are the smooth nonlinear vector functions, $\tilde{f}_i$ $(i = 1,2,\cdots,m)$ and $\tilde{g}_i$ $(i = 1,2,\cdots,m)$ are the unknown uncertainties of the system, $C \in R^{m\times n}$ is a constant matrix , and d_i $(i = 1,\cdots,m)$ are external bounded disturbances.

Control objective: Determine a fuzzy robust H∞ tracking controller to ensure that the system states of nonlinear system (1) follow those of the stable reference model.

3. Fuzzy Model, Observer Model, Reference Model and Fuzzy Controller

With a fuzzy model, an observer model, a reference model and a fuzzy controller in a closed-loop, a fuzzy-model-based control system is introduced.

3.1. Fuzzy Model

Consider the following fuzzy T-S model, which will be employed to express the nonlinear system.

Plant Rule i *: If* $z_1(t)$ *is* F_1^i *and*$,\cdots,$ *and* $z_s(t)$ *is* F_s^i , *Then*

$$\dot{x}(t) = A_i x(t) + B_i u(t) + d, \quad i = 1,\cdots,L, \tag{2}$$

$$y(t) = Cx(t),$$

where $z_1(t),\cdots,z_s(t)$ are the premise variables, $F_j^i\,(j=1,2,\cdots,s)$ are the fuzzy sets, L is the number of IF-THEN rules, A_i and B_i are some constant compatible matrices, $B_i=[0,\cdots,b_{i1}^T,\cdots,0,\cdots,b_{im}^T]^T\in R^{n\times m}$ with $b_{i1}\in R^m,\cdots,b_{im}\in R^m$, and $d=[0,\cdots,d_1,\cdots,0,\cdots,d_m]^T$. The final output of the fuzzy system is inferred as

$$\dot{x}(t)=\sum_{i=1}^{L}\mu_i A_i x(t)+\sum_{i=1}^{L}\mu_i B_i u(t)+d, \tag{3}$$

$y(t)=Cx(t),$

where

$$\mu_i=v_i(z(t))\Big/\sum_{i=1}^{L}v_i(z(t))\,,\ v_i(z(t))=\prod_{j=1}^{s}F_j^i(z_j(t))\,, \tag{4}$$

and $F_j^i(z_j(t))$ is the grade of membership of $z_j(t)$ in F_j^i. Suppose that $\mu_i\ge 0$ ($i=1,2,\cdots,L$). Obviously, for any $t\ge 0$, $\sum_{i=1}^{L}\mu_i=1$.

Hence, the nonlinear system (1) is equivalent to the following system:

$$\dot{x}(t)=\sum_{i=1}^{L}\mu_i A_i x(t)+\sum_{i=1}^{L}\mu_i B_i u(t)+B\Delta(x,u)+d\,, \tag{5}$$

$$y(t)=Cx(t) \tag{6}$$

where

$$B\Delta(x,u)=\begin{bmatrix}0\\ \cdots\\ \Delta_1\\ \cdots\\ \Delta_m\end{bmatrix}=\begin{bmatrix}x_2\\ \cdots\\ f_1+\tilde{f}_1+(g_1+\tilde{g}_1)u\\ \cdots\\ f_m+\tilde{f}_m+(g_m+\tilde{g}_m)u\end{bmatrix}-(\sum_{i=1}^{L}\mu_i A_i x(t)+\sum_{i=1}^{L}\mu_i B_i u(t)) \tag{7}$$

denotes the uncertainties of the system, $B=diag[C_1,\cdots,C_m]$, $C_i=[0,\cdots,0,1]^T\in R^{\beta_i}$, and $\Delta(x,u)=[\Delta_1,\cdots,\Delta_m]^T$.

3.2. Observer Model

Because the system states are unavailable, an observer is designed to estimate the system state $x(t)$. For the fuzzy model represented by (3) or (5), the observer shares the same IF parts with the following structure. The overall fuzzy observer is given as follows:

$$\dot{\hat{x}}(t)=\sum_{i=1}^{L}\mu_i A_i\hat{x}(t)+\sum_{i=1}^{L}\mu_i B_i u(t)+\sum_{i=1}^{L}\mu_i L_i(y(t)-\hat{y}(t))\,, \tag{8}$$

$$\hat{y}(t)=C\hat{x}(t)\,, \tag{9}$$

where $\hat{x}=[\hat{x}_1,\cdots,\hat{x}_1^{(\beta_1-1)},\cdots,\hat{x}_{(n-\beta_m+1)},\cdots,\hat{x}_{(n-\beta_m+1)}^{(\beta_m-1)}]^T\in R^n$, L_i $(i=1,\cdots,L)$ are matrices with appropriate dimensions.

3.3. Reference Model

Consider the following stable reference model

$$\dot{x}_r(t)=A_r x_r(t)+r(t)\,, \tag{10}$$

where $x_r(t)$ is a reference state, $r(t)$ is a bounded reference input, and A_r is an asymptotically stable matrix.

3.4. Fuzzy Controller

In order to guarantee the implementation of tracking, we design the following fuzzy tracking controller

$$u(t) = u_l(t) - u_f(t), \tag{11}$$

where $u_l(t)$ is a fuzzy output feedback controller based on T-S model, and $u_f(t)$ denotes an adaptive output feedback compensator based on fuzzy logic systems.

Based on the observer (8) and (9), $u_l(t)$ is designed as

$$u_l(t) = \sum_{i=1}^{L} \mu_i K_i(\hat{x}(t) - x_r(t)), \tag{12}$$

where K_i $(i = 1,\cdots,L)$ are the feedback gain matrices with appropriate dimension, and K_i $(i = 1,\cdots,L)$, L_i $(i = 1,\cdots,L)$ satisfy the inequalities

$$\overline{A}_{ij}^T P + P\overline{A}_{ij} + \frac{1}{\rho^2} PP + \overline{Q} < 0,\ i, j = 1,\cdots, L, \tag{13}$$

where $\overline{A}_{ij} = \begin{bmatrix} A_i + B_i K_j & -B_i K_j & L_i C \\ 0 & A_r & 0 \\ 0 & 0 & A_i - L_i C \end{bmatrix}$, ρ is a positive constant, $<$ denotes negative definite, P is a symmetric and positive definite matrix .

The adaptive compensator $u_f(t)$ is of the following form:

$$u_f(t) = \begin{cases} E^{-1}\hat{u}(\hat{x}, u \mid \Theta), & \text{if } E \text{ is nonsigular} \\ E^T(I + EE^T)^{-1}\hat{u}(\hat{x}, u \mid \Theta), & \text{if } E \text{ is sigular} \end{cases}, \tag{14}$$

where $E_i = [b_{i1}^T, \cdots, b_{im}^T]^T \in R^{m\times m}$, $E = \sum_{i=1}^{L} \mu_i E_i$. $\hat{u}(\hat{x}, u \mid \Theta)$ is constructed by fuzzy logic systems with the adaptive law for the weight Θ

$$\dot{\Theta} = \eta_1(\Psi(\hat{x}, u))^T \tilde{y}, \tag{15}$$

where η_1 is a positive constant, $\tilde{y} = y - \hat{y}$ is the output error of the system, $\Psi(\hat{x}, u) = diag[\xi_1^T(\hat{x}, u), \cdots, \xi_m^T(\hat{x}, u)]$, and $\xi_i(x, u)(i = 1,\cdots, m)$ are Gaussian basis-functions.

Remark 1: In order to obtain the feasible solution of the inequalities (13), by use of Schur complements, (13) could be transformed into the LMIs.

Denote $P = diag\{P_1, P_2, P_3\}$, $\overline{Q} = diag\{2Q, 2Q, 0\}$,where P_1, P_2 , P_3 and Q are some symmetric and positive definite matrices. The inequalities (13) are equivalent to the matrix inequalities (16).

$$\begin{bmatrix} S_1 & -P_1 B_i K_j & 0 & L_i C \\ -(B_i K_j)^T P_1 & S_{22} & P_2 & 0 \\ 0 & P_2 & -\rho^2 I & 0 \\ (L_i C)^T & 0 & 0 & S_3 \end{bmatrix} < 0\ ,\ i, j = 1, 2, \cdots, L\ , \tag{16}$$

where $S_1 = P_1(A_i + B_i K_j) + (A_i + B_i K_j)^T P_1 + \frac{1}{\rho^2} P_1 P_1 + 2Q$, $S_2 = P_2 A_r + A_r^T P_2 + 2Q$, and

$S_3 = P_3(A_i - L_i C) + (A_i - L_i C)^T P_3 + \frac{1}{\rho^2} P_3 P_3 + 2Q$.

Let $W = P_1^{-1}$ and $Y_j = K_j W$ .The matrix inequalities (16) imply $S_1 < 0$,which is equivalent to the following LMIs

$$\begin{bmatrix} S_{11} & W \\ W & -(2Q)^{-1} \end{bmatrix} < 0\ \ i, j = 1, 2, \cdots, L \tag{17}$$

with $S_{11} = A_i W + W A_i^T + B_i Y_j + (B_i Y_j)^T + (\rho^2)^{-1} I$.

Denote $Z_i = P_3 L_i$ ($i = 1,2,\cdots,L$).The matrix inequalities (16) imply $S_3 < 0$,which is equivalent to the following LMIs

$$\begin{bmatrix} S_{33} & P_3 \\ P_3 & -\rho^2 I \end{bmatrix} < 0 \quad i, j = 1,2,\cdots,L \tag{18}$$

with $S_{33} = P_3 A_i + A_i^T P_3 - Z_i C - (Z_i C)^T + 2Q$.

P_1 and K_j ($j = 1,2,\cdots,L$) could be obtained by solving the LMIs (17). We can obtain P_3 and L_i ($j = 1,2,\cdots,L$) by use of the LMIs (18). And then, substituting P_1, P_3 , K_j ($j = 1,2,\cdots,L$) and L_i ($j = 1,2,\cdots,L$) into (16), P_2 is also given.

Remark 2: The construction of fuzzy logic systems (19) and the approximation form (21) are given. In order to guarantee $\|\Theta\| \le \mathrm{M}$, the adaptive law (15) is modified by the projection algorithm [18].

Fuzzy logic systems are of the following form:

$$\hat{\Delta}(x, u \mid \Theta) = \Psi(x, u)\Theta \tag{19}$$

where $\Psi(x,u) = diag[\xi_1^T(x,u), \cdots, \xi_m^T(x,u)]$, $\Theta = [\theta_1^T, \theta_2^T, \cdots, \theta_m^T]^T$ .The kth element of $\Delta(x,u)$ is of the following form:

$\hat{\Delta}_k(x, u \mid \theta) = \xi_k^T(x,u)\theta_k$,

where $\xi_k^T(x,u) = (\xi_k^1, \cdots, \xi_k^p) \in R^p$,

$$\xi_k^l = \prod_{i=1}^{n}\prod_{j=1}^{m}\mu_{F_i^l}(x_i)\mu_{F_{u_j}^l}(u_j) \Bigg/ \sum_{l=1}^{p}\prod_{i=1}^{n}\prod_{j=1}^{m}\mu_{F_i^l}(x_i)\mu_{F_{u_j}^l}(u_j),$$

$\mu_{F_i^l}(x_i, x_i(t-\tau)) = \mu_{F_i^l}(x_i)\prod_{j=1}^{r}\mu_{F_i^l}(x_i(t-\tau_j))$, $\theta_k = (\theta_k^1, \cdots, \theta_k^p)^T \in R^p$, and $\mu_{F_i^l}(x_i)$ ($i = 1, 2, \cdots, n$) are the membership functions.

Define the optimal parameter Θ^*

$$\Theta^* \underline{\underline{\Delta}} \arg\min_{\Theta \in \Omega_1}[\sup_{x \in U_1} \| \hat{u}(x, u \mid \Theta) - \Delta(x,u) \|], \tag{20}$$

where $U_1 = \{x \in R^n\}$, $\Omega_1 = \{\Theta | \Theta \in R^{pm}, \|\Theta\| \le \mathrm{M}\}$. U_1, Ω_1 denote the sets of suitable bounds on x , Θ respectively.

Then the approximation error for $\Delta(x,u)$ is as follows:

$$\hat{\Delta}(\hat{x}, u \mid \Theta) - \Delta(x,u) = \Psi(\hat{x}, u)\tilde{\Theta} + w, \tag{21}$$

where $\tilde{\Theta} = \Theta - \Theta^* = [(\theta_1 - \theta_1^*)^T, \cdots, (\theta_m - \theta_m^*)^T]^T$ is the estimation error, $w = [w_1, \cdots, w_m]^T$ is a residual term.

4. Main Result

Substituting (11) into (4), we conclude

$$\dot{x}(t) = \sum_{i=1}^{L}\mu_i A_i x(t) + \sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j B_i K_j(\hat{x}(t) - x_r(t)) - B(\hat{u}(\hat{x}, u \mid \Theta) - \Delta(x,u)) + d\,. \tag{22}$$

Let $\tilde{x}(t) = [\hat{x}^T(t), x_r^T(t), e^T(t)]^T$,where $e(t) = x(t) - \hat{x}(t)$, $\overline{B} = [0 \;\; 0 \;\; B^T]^T$ and $d' = \begin{bmatrix} 0 & r^T(t) & d^T \end{bmatrix}^T$ .By using (5), (8), (9),(10) and (22), we get a new closed-loop system

$$\dot{\tilde{x}}(t)=\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j\bar{A}_{ij}\tilde{x}(t)+d' +\bar{B}(\hat{u}(\hat{x},u\,|\,\Theta)-\Delta(x,u))\,. \tag{23}$$

By use of (21), (23) is rearranged as

$$\dot{\tilde{x}}(t)=\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j\bar{A}_{ij}\tilde{x}(t) +\bar{B}(-\Psi(\hat{x},u)\tilde{\Theta}) +\bar{w}\,, \tag{24}$$

$\tilde{y}=y-\hat{y}=Ce(t)$,

where $\bar{w}=[0 \quad r^T(t) \quad \bar{w}_1^{\,T}]^T$, $\bar{w}_1=d-w'$, $w'=[0,\cdots,w_1,\cdots,0,\cdots,w_m]^T$. P_3, B and C satisfy

$$B^T P_3=C\,. \tag{25}$$

If P_3 does not exist for (25), then make a transformation for (24). $L(s)$ is chosen such that $L^{-1}(s)$ is a proper stable transfer function, where $L(s)=\{L_1(s),\cdots,L_m(s)\}$ with $L_i(s)=s^{\gamma_i}+a_{i1}s^{\gamma_i-1}+\cdots+a_{i\gamma_i}$ ($\gamma_i<\beta_i$, $i=1,2,\cdots,m$). Then a new state space realization for the output error dynamics of $e(t)$ can be given

$$\dot{e}_s(t)=\sum_{i=1}^{L}\mu_i[(A_i-L_iC)e_s(t) +B_s(-\Psi_1(\hat{x},u)\tilde{\Theta})+\bar{w}_1'\,, \tag{26}$$

$y_s(t)=C_s e_s(t)$,

where $e_s=[e_1,\cdots,e_1^{(\beta_1-1)},\cdots,e_{(n-\beta_m+1)},\cdots,e_{(n-\beta_m+1)}^{(\beta_m-1)}]^T\in R^n$, $C_s=C$ ($i=1,2,\cdots,m$), $B_s=diag[B_{1s},\cdots,B_{ms}]$, $B_{is}=[1,a_{i1},\cdots,a_{i\gamma_i}]^T$,($i=1,2,\cdots,m$) , $\Psi_1(\hat{x},u)=L^{-1}(s)\Psi(\hat{x},u)$, $w_1'=L^{-1}(s)w'$, $\bar{w}_1'=d-w_1'$. P_3, B_s and C_s satisfy

$$P_3B_s=C_s\,. \tag{27}$$

Theorem1. Consider the uncertain nonlinear systems (1).Assume that the symmetric and positive definite matrices P and Q, and a positive scalar ρ satisfy the inequalities (13), and the updating law for the parameter is chosen as (15). Then there exists a controller (11) with the fuzzy output feedback controller (12) and the adaptive compensator (14) such that the H∞ tracking performance is achieved

$$\int_0^T(\hat{x}(t)-x_r(t))^TQ(\hat{x}(t)-x_r(t))dt\le \tilde{x}^T(0)P\tilde{x}(0) +\frac{1}{\eta_1}\tilde{\Theta}^T(0)\tilde{\Theta}(0) +\rho^2\int_0^T(\bar{w}^T\bar{w})dt\,. \tag{28}$$

Proof：Consider the following Lyapunov candidate to investigate the system stability

$$V=\frac{1}{2}\tilde{x}^TP\tilde{x}+\frac{1}{2\eta_1}\tilde{\Theta}^T\tilde{\Theta}\,.$$

We calculate the time derivative of V along the trajectories the closed-loop system (6)
as follows:

$$\dot{V}=\frac{1}{2}\dot{\tilde{x}}^T(t)P\tilde{x}(t)+\frac{1}{2}\tilde{x}^T(t)P\dot{\tilde{x}}(t) +\frac{1}{\eta_1}\tilde{\Theta}^T\dot{\tilde{\Theta}}$$

$=\dot{V}_1+\dot{V}_2$,

where

$$\dot{V}_1=\frac{1}{2}(\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j(\bar{A}_{ij}\tilde{x}(t))^TP\tilde{x}(t)+\frac{1}{2}\tilde{x}^T(t)P(\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j\bar{A}_{ij}\tilde{x}(t)) +\frac{1}{2}\bar{w}^TP\tilde{x}(t)+\frac{1}{2}\tilde{x}^T(t)P\bar{w}\,, \tag{29}$$

$$\dot{V}_2=[\tilde{x}^TP\bar{B}(-(\Psi(x,u)\tilde{\Theta}) +\frac{1}{\eta_1}\tilde{\Theta}^T\dot{\tilde{\Theta}}]\,. \tag{30}$$

$$\dot{V}_1 \le \frac{1}{2}(\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j[\tilde{x}^T(t)\overline{A}_{ij}^T P\tilde{x}(t) + \tilde{x}^T(t)P\overline{A}_{ij}\tilde{x}(t)] - \frac{1}{2}(\frac{1}{\rho}P\tilde{x}(t) - \rho\overline{w})^T(\frac{1}{\rho}P\tilde{x}(t) - \rho\overline{w})$$

$$+\frac{1}{2}\rho^2\overline{w}^T\overline{w} + \frac{1}{2\rho^2}\tilde{x}^T(t)PP\tilde{x}(t)$$

$$\le \frac{1}{2}\sum_{i=1}^{L}\sum_{j=1}^{L}\mu_i\mu_j\tilde{x}^T(t)(\overline{A}_{ij}^T P + P\overline{A}_{ij} + \frac{1}{\rho^2}PP)\tilde{x}(t) + \frac{1}{2}\rho^2\overline{w}^T\overline{w}\,. \tag{31}$$

By using (13), we have

$$\dot{V}_1 \le -\frac{1}{2}\tilde{x}^T(t)\overline{Q}\tilde{x}(t) + \frac{1}{2}\rho^2\overline{w}^T\overline{w}\,. \tag{32}$$

Substituting (15) into (30) results in

$$\dot{V}_2 = 0\,. \tag{33}$$

Thus,

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \le -\frac{1}{2}\tilde{x}^T(t)\overline{Q}\tilde{x}(t) + \frac{1}{2}\rho^2\overline{w}^T\overline{w}\,. \tag{34}$$

When $\|\tilde{x}\| > \frac{\rho}{\lambda_{\min}(\overline{Q})}\|\overline{w}\|$, $\dot{V} < 0$. Thus, the closed-loop system (24) is stable.

Due to

$$\int_0^T (\hat{x}(t) - x_r(t))^T Q(\hat{x}(t) - x_r(t))dt \le \int_0^T \tilde{x}^T(t)\overline{Q}\tilde{x}(t)dt\,, \tag{35}$$

integrating the inequality (34) from $t = 0$ to T yields the H∞ tracking performance.

If there is no reference model and the system states are available, then we have the following Corollary 1, where a fuzzy robust H∞ control scheme for uncertain nonlinear systems is given.

Corollary 1. Consider the uncertain nonlinear systems (1). If there exist a common matrix $W > 0$, matrices Y_j, $j = 1,2,\cdots,L$, and a scalar $\rho > 0$ satisfying the following LMIs:

$$\begin{bmatrix} A_i W + W A_i^T + B_i Y_j + (B_i Y_j)^T + (\rho^2)^{-1} I & W \\ W & -Q^{-1} \end{bmatrix} < 0\,, i, j = 1,2,\cdots,L\,, \tag{36}$$

where $W = P^{-1}, Y_j = K_j W$, and $Q > 0$ is prescribed, and exists an updating law (37) of the parameter Θ for fuzzy adaptive system

$$\dot{\Theta} = \eta_1 \Psi^T(x(t), u(t)) B^T P x(t)\,, \tag{37}$$

where η_1 is a positive constant, then there exists a fuzzy state feedback controller (4) with

$$u_l(t) = \sum_{i=1}^{L}\mu_i K_i x(t)\,, u_f(t) = \begin{cases} E^{-1}\hat{u}(x,u\,|\,\Theta), & \text{if } E \text{ is nonsigular} \\ E^T(I + EE^T)^{-1}\hat{u}(x,u\,|\,\Theta), & \text{if } E \text{ is sigular} \end{cases} \tag{38}$$

such that the following H∞ performance

$$\int_0^T x^T(t)Qx(t)dt \le x^T(0)Px(0) + \frac{1}{\eta_1}\tilde{\Theta}^T(0)\tilde{\Theta}(0) + \rho^2\int_0^T(\overline{w}^T\overline{w})dt \tag{39}$$

is achieved.

5. Simulation Examples

In this section, two examples, a duffing chaos system and a mass-spring-damper system are given to illustrate the effectiveness and the feasibility of fuzzy control design.

Example 1：Consider the following duffing chaos system

$$x_1 = x_2,$$
$$x_2 = -0.1x_2 - x_1^3 + 12\cos(t) + u + d, \tag{40}$$

where d is the external disturbance, which is uncertain and bounded.

Step1: A two-rule fuzzy T-S model is used to approximate the nonlinear chaos system, where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -17.64 & -0.1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -16 & -0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The membership functions are defined as

$$\mu_{F_1^1}(x_1) = \begin{cases} 1 & x_1 \le -4 \\ (4.2 - x_1)/8.2 & -4 < x_1 \le 8.2 \\ 0 & x_1 > 8.2 \end{cases}, \text{ and } \mu_{F_1^2}(x_1) = 1 - \mu_{F_1^1}(x_1).$$

Step 2: On the basis of Theorem1, with $\rho = 0.8$ and $Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$, we have fuzzy state feedback gains $K_1 = [6.5147 \text{ - } 7.3035]$, $K_2 = [6.5149 \text{ - } 7.3039]$, and the symmetric and positive definite matrix $P = \begin{bmatrix} 0.2936 & 0.0264 \\ 0.0264 & 0.0326 \end{bmatrix}$.

Step 3: In the fuzzy adaptive compensator, the membership functions are selected as

$$\mu_{F_i^1}(x_i) = 1/(1 + \exp[5(x_i + 2)]), \mu_{F_i^2}(x_i) = \exp[-(x_i + 1.5)^2], \mu_{F_i^3}(x_i) = \exp[-(x_i + 0.5)^2],$$
$$\mu_{F_i^4}(x_i) = \exp[-(x_i)^2], \mu_{F_i^5}(x_i) = \exp[-(x_i - 0.5)^2], \mu_{F_i^6}(x_i) = \exp[-(x_i - 1.5)^2],$$
$$\mu_{F_i^7}(x_i) = 1/(1 + \exp[-5(x_i - 2)]), i = 1,2.$$

Step 4:The initial values are chosen
$\eta_1 = 50$, $\Theta(0) = [0.01 \quad 0.01 \quad 0.01 \quad 0.01 \quad 0.01 \quad 0.01 \quad 0.01]$, and $(x_1(0), x_2(0)) = (2,-2)$.

By using Corollary 1, we can get the state responses x_1, x_2, the control law u, which are shown in Figure 1- Figure 2.

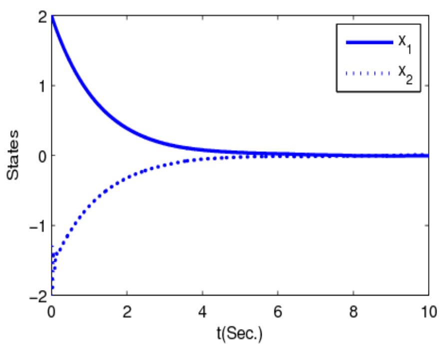

Figure 1. State Responses x_1, x_2

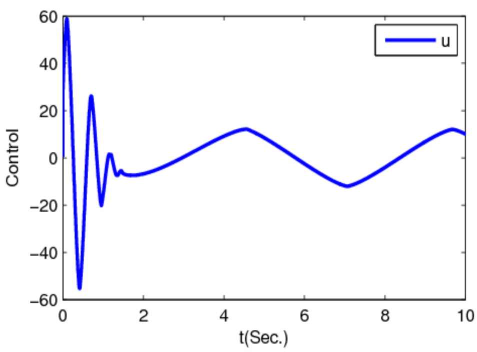

Figure 2. Control Input u

Simulation results illustrate the fuzzy control design is effective and feasible.

Example 2：Consider the following mass-spring-damper system in [5]

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -c(t)x_2 - 0.02x_1 - 0.67x_1^3 + u + d, \tag{41}$$

where d is the external bounded disturbance, $c(t)$ is uncertain and bounded.

The reference model is as follows:

$$\dot{x}_r(t) = A_r x_r(t) + r(t), \tag{42}$$

where

$A_r = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}, r(t) = \begin{bmatrix} 0 \\ r_1(t) \end{bmatrix}, r_1(t) = 4\sin(t)$.

Step1: A two-rule fuzzy T-S model is used to model the nonlinear mass-spring-damper system, where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.02 & -0.1125 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -1.527 & -0.1125 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

and the membership functions are adopted as

$$\mu_{F_1^1}(x_1(t)) = 1 - x_1^2(t)/2.25, \ \mu_{F_1^2}(x_1(t)) = x_1^2(t)/2.25 .$$

Step 2: On the basis of Theorem1, with $\rho = 0.8$ and $\mathrm{Q} = \begin{bmatrix} 0.001 & \\ & 0.001 \end{bmatrix}$, we have the fuzzy output feedback gains $K_1 = [-8.4445 \ \ -6.6377]$, $K_2 = [-8.4445 \ \ -6.6377]$,the observer gains

$L_1 = \begin{bmatrix} 2.5267 \\ 2.3605 \end{bmatrix}, L_2 = \begin{bmatrix} 2.5267 \\ 0.8535 \end{bmatrix}$.

Step 3: The fuzzy adaptive compensator of the membership functions are as follows:

$$R^{(j)}: \text{if } x_1 \text{ is } F_1^j \ \ x_2 \text{ is } F_2^j, \quad \text{then } y \text{ is } G^j \quad (j = 1,2,\cdots,5),$$

$\mu_{F_i^1}(x_i) = \exp[-(x_i + 0.6)^2] \quad (i = 1,2)$, $\mu_{F_i^2}(x_i) = \exp[-(x_i + 0.2)^2] \quad (i = 1,2)$, $\mu_{F_i^3}(x_i) = \exp[-(x_i)^2] \ (i = 1,2)$, $\mu_{F_i^4}(x_i) = \exp[-(x_i - 0.2)^2] \ (i = 1,2)$, $\mu_{F_i^5}(x_i) = \exp[-(x_i - 0.6)^2]$ $(i = 1,2)$.

Let $S_1 = \sum_{j=1}^{5} \prod_{i=1}^{2} \mu_{F_i^j}(x_i)$, then

$$\Psi(x,u) = [\prod_{i=1}^{2} \mu_{F_i^1}(x_i)/S_1, \cdots, \prod_{i=1}^{2} \mu_{F_i^5}(x_i)/S_1] = [\xi_1, \cdots, \xi_5] .$$

Step 4: The initial values

$(x_1(0), x_2(0), \hat{x}_1(0), \hat{x}_2(0), x_{r1}(0), x_{r2}(0)) = (-1, -1.3, -0.5, -0.8, -0.9, -1.2)$,

$\Theta(0) = [0.1, 0.1, 0.1, 0.1, 0.1]$, $\eta_1 = 15$ $B_s = [1 \ \ 16.0808]^T$, $c(t) = -0.125 + 0.1f$, where f is a square signal.

By use of the proposed method in Theorem1, simulation results are shown in Figures 3-7.

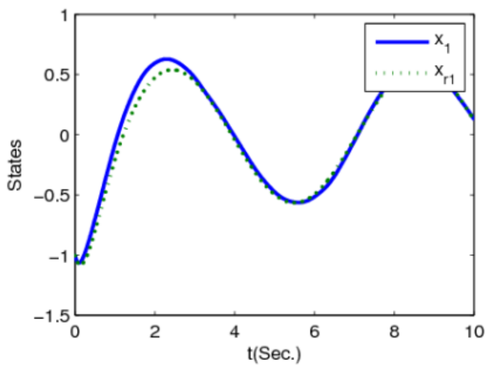

Figure 3. State Responses: x_1, x_{r1}

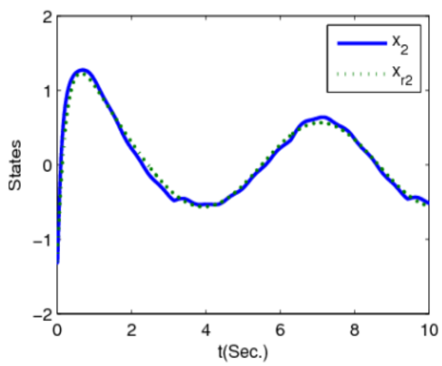

Figure 4. State Responses: x_2, x_{r2}

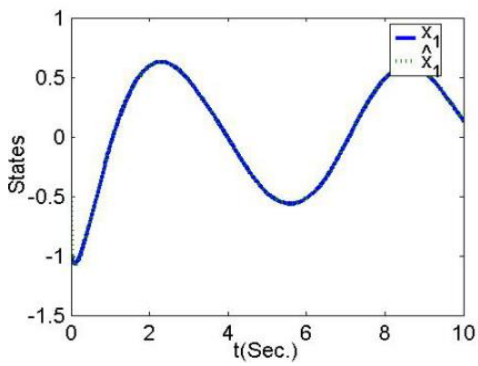

Figure 5. State x_1, Estimation State $\hat{x}_1$

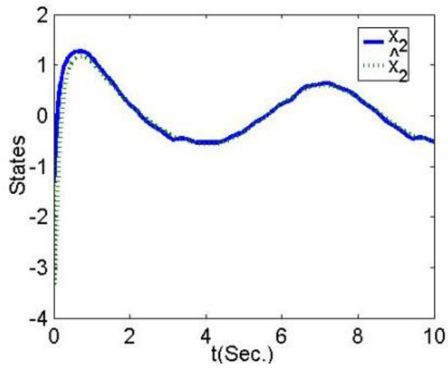

Figure 6. State x_2, Estimation State $\hat{x}_2$

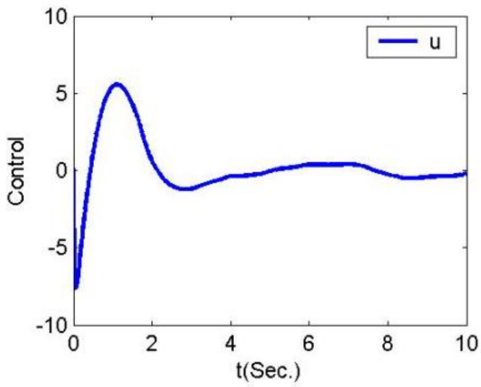

Figure 7. Control Input u

By using the methods of [5] and Theorem, the dimensions of the LMIs are given in Table 1and state responses are shown in Figure 8-Figure 9, where the matching condition is used to model the uncertainties.

Table 1. The Comparison for the Dimensions of Lmis

Method	[5]	Theorem
Dimension	12	8

It is seen from Table 1 that the dimension of the LMIs is simplified in the proposed method of this paper, which adds the existence of feedback gains.

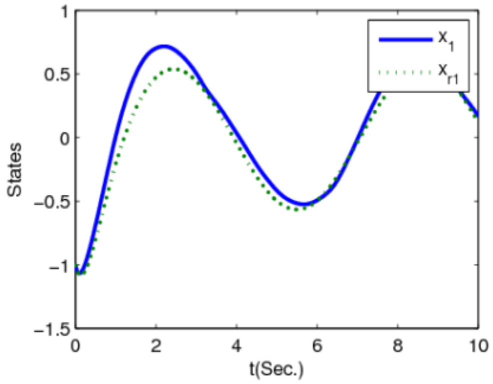

Figure 8. State Responses: x_1, x_{r1}

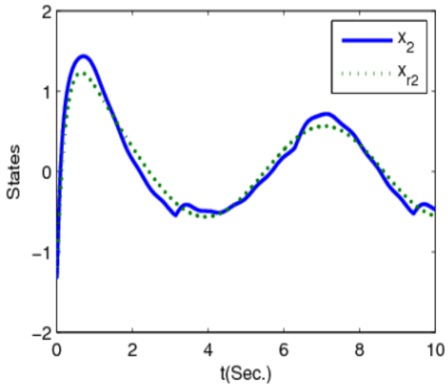

Figure 9. State Responses: x_2, x_{r2}

Comparing Figures 3-4 with Figures 8-9, we find that the developed control scheme in this paper could ensure that the system states rapidly track the reference states. This implies that the proposed method achieves a better performance.

6. Conclusion

By using fuzzy T-S model and fuzzy logic systems, a novel tracking control scheme was presented for uncertain nonlinear systems. The developed fuzzy controller makes full use of the advantages of fuzzy T-S model and fuzzy logic systems, where the matching condition and the upper bound are avoided and the dimension of LMIs is reduced. The proposed control scheme can guarantee to rapidly track the given reference signals. Furthermore, the tracking control design for discrete nonlinear systems will be also developed.

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