Development and Experiment on Digital Pre-Distortion for Analog Beamforming Systems

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Abstract

This paper proposes a digital pre-distortion (DPD) technique for analog beamforming systems. The considering system has M PAs (M is the number of antennas) and one digital chain. In contrast to the conventional DPD problem where one DPD linearizes one PA, a single DPD should linearize M PAs simultaneously in the analog beamforming system. In this paper, we design a new DPD by minimizing the sum of linearization errors of M PAs. To verify the proposed DPD, computer simulation and experiment are conducted. The results confirm that the proposed DPD can successfully linearize the multiple PAs in analog beamforming systems.

Keywords*: Digital Pre-Distortion (DPD); Analog beamforming; Adaptive algorithm; Power amplifier; Polynomial*

1. Introduction

MIMO (multiple input multiple output) techniques based on multiple antennas have been widely used these days [1]-[4]. While MIMO systems significantly improve the multiplexing and diversity gains in wireless communications, they require larger cost and size due to multiple signal paths. Those burdens increase as the number of antennas grow. Among MIMO techniques, analog beamforming (BF) has relatively low cost and small size since it is composed of one digital chain, *M* phase shifters, power amplifiers (PAs) and antennas, where M is the number of antennas. In general, PAs have non-linearity, which causes spectral regrowth and in-band signal distortion. To mitigate these problems, many linearization techniques have been studied. Among them, the DPD technique is known for excellent linearization performance. The DPD technique compensates for non-linearity of PAs by designing the combined characteristic of DPD and PA linear [5] [6].

This paper proposes a digital pre-distortion (DPD) technique for analog BF systems. The main difficulty for the conventional DPD to be applied to the analog BF systems is that the single DPD must linearize multiple PAs simultaneously since the system has only one digital chain. To design such DPD, we introduce a cost function based on the sum of the square errors for all the PAs. Specifically, the proposed DPD technique combines all the feedback samples from multiple PAs after phase de-rotation, and then the conventional indirect learning method finds the DPD parameters. The computer simulation and experiment results show that the proposed DPD can successfully linearize multiple PAs simultaneously in the analog BF systems.

2. System Model

Figure 1 shows the baseband equivalent system model for the analog BF transmitter. The transmitter comprises one digital chain and *M* antennas. The analog

BF coefficients are multiplied with the PA inputs by using the phase shifters. The output of m-th PA, denoted by $z_m(n)$, is $\varphi_m(exp(j\phi_m)y(n))$ where ϕ_m is the rotation phase of m-th antenna and $exp(j\phi_m)y(n)$ is the input of m-th PA. The output of DPD, $y(n)$, is $\psi(x(n))$ where $x(n)$ is the transmitted signal. $\varphi(\cdot)$ and $\psi(\cdot)$ are the characteristic functions of PA and PD, respectively.
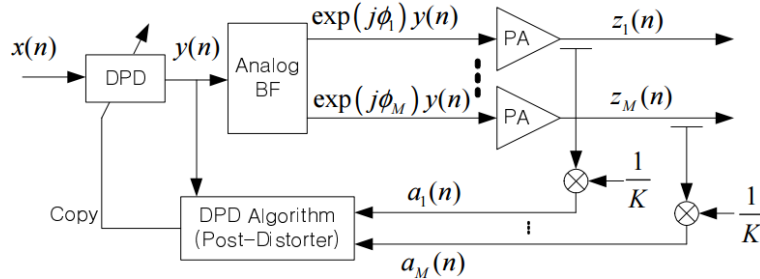

Figure 1. Digital Pre-Distortion Structure in Analog Beamforming Systems

As shown in Figure 1, one DPD must linearize M PAs simultaneously. If the characteristics of PAs are completely the same, constructing one feedback path among M PAs is enough to linearize all the PAs. However, in practice, the PAs have different characteristics. Hence, we need a new DPD strategy considering all the PAs. Assuming that the PA is modeled by a memory polynomial, the m-th feedback signal can be written as

$$
\begin{aligned}
a_m(n) &= \frac{\varphi_m\left(e^{j\phi m}y(n)\right)}{K} \\
&= \sum_{l=0}^{L_a}\sum_{p=1}^{P} h_{l,p,m}\left|e^{j\phi m}y(n-l)\right|^{2(p-1)} e^{j\phi m}y(n-l) \\
&= e^{j\phi m}\sum_{l=0}^{L_a}\sum_{p=1}^{P} h_{l,p,m}|y(n-l)|^{2(p-1)}y(n-l)
\end{aligned}
$$

(1)

where K is the PA gain, L_a is the memory depth, and $2P-1$ is the maximum nonlinear order of the PA. We introduce memory polynomial model for characterizing the DPD function as well. The DPD output can be expressed as

$$
\begin{aligned}
y(n) &= \psi\big(x(n)\big) \\
&= \sum_{l=0}^{L_p}\sum_{q=1}^{Q} w_{l,q}^{*} |x(n-l)|^{2(q-1)}x(n-l) \\
&= \mathbf{w}^{H}\mathbf{x}(n)
\end{aligned}
$$

(2)

where $2Q-1$ is the maximum polynomial order of DPD, L_p is the memory depth, $\mathbf{w}$ is the polynomial coefficient vector given by $\mathbf{w} = \left[w_{0,1}, w_{0,2}, \cdots, w_{0,P}, w_{1,1}, \cdots, w_{Q,L_P}\right]^{T}$, and $\mathbf{x}(n) = \left[x(n), x(n)|x(n)|^2, \cdots, x(n)|x(n)|^{2(Q-1)}, x(n-1), \cdots, x(n-L_P)|x(n-L_P)|^{2(Q-1)}\right]^{T}$. The number of coefficients to be found is $Q(L_P+1)$. The problem is to find $\mathbf{w}$ that linearizes all the PAs, *i.e.*, $\varphi_m\left(e^{j\phi m}\psi(x(n))\right) = Ke^{j\phi m}x(n)$ for $m = 1, \cdots, M$.

3. Proposed DPD

To design the proposed DPD, the DPD parameters are estimated using an indirect learning method [5]. In this method, the parameters linearizing the PA-postdistorter chain is first found and are then copied to the DPD. In order to design a single post-distorter that linearizes M PA-post-distorter chains, least squares (LS) criterion is used. The cost function is written as

$$
\begin{aligned}
\boldsymbol{\varepsilon} &= \sum_{n=1}^{N} \lambda^{N-n} |y(n) - e^{-j\phi_1} \mathbf{w}^H \mathbf{a}_1(n) + \cdots + y(n) - e^{-j\phi_M} \mathbf{w}^H \mathbf{a}_M(n)|^2 \\
&= \sum_{n=1}^{N} \lambda^{N-n} \left| My(n) - \mathbf{w}^H (e^{-j\phi_1} \mathbf{a}_1(n) + \cdots + e^{-j\phi_M} \mathbf{a}_M(n) \right|^2 \\
&= \sum_{n=1}^{N} \lambda^{N-n} |My(n) - \mathbf{w}^H \mathbf{a}(n)|^2
\end{aligned}
$$

(3)

where

$$
\mathbf{a}_m(n) = \begin{bmatrix} a_m(n), a_m(n)|a_m(n)|^2, \cdots, a_m(n)|a_m(n)|^{2(Q-1)}, a_m(n-1), \\ \cdots, a_m(n-L_p)|a_m(n-L_p)|^{2(Q-1)} \end{bmatrix}^T
$$

(4)

$\lambda \in [0,1]$ is a forgetting factor, and $\mathbf{a}(n) = e^{-j\phi_1} \mathbf{a}_1(n) + \cdots + e^{-j\phi_M} \mathbf{a}_M(n)$. The cost function in (3) is based on the sum of M errors, each of which is the difference between m-th PA input and the post-distorter output. Finding $\mathbf{w}$ that minimizes $\boldsymbol{\varepsilon}$ is the solution by considering all the PAs. The optimum $\mathbf{w}$ is found through the recursive least squares (RLS) algorithm [7], summarized in Table I.

Table I. Summary of RLS Algorithm

Initialization:
$\mathbf{w} = \mathbf{0}_Q(L_P + 1)$
$\mathbf{P} = \delta^{-1} \mathbf{I}_Q(L_P + 1)$
(δ: a small positive constant)
For time $n = 1, 2, \cdots, N$, compute
$\mathbf{b} = \mathbf{P}\mathbf{a}(n)$
$\mathbf{c} = \mathbf{b}/(\lambda + \mathbf{a}^H(n)\mathbf{b})$
$\xi(n) = My(n) - \mathbf{w}^H \mathbf{a}(n)$
$\mathbf{w} = \mathbf{w} + \mathbf{c}\,\xi^*(n)$
$\mathbf{P} = \lambda^{-1}\mathbf{P} - \lambda^{-1}\mathbf{c}\mathbf{a}^H(n)\mathbf{P}$

4. Computer Simulation

To verify the performance of the proposed DPD, we carried out computer simulation with MATLAB. The details of the simulation environments are as follows. The transmitted signal is Long Term Evolution (LTE) Downlink signal with 20MHz bandwidth. Saleh model, analytical formulation of a general structure for nonlinear distortion, is used for PA in the simulation [8]. This formulation is shown as follows:

$$
y(n) = x(n) \times \frac{\alpha K_1}{1 + K_2|x(n)|^2} e^{i\frac{\alpha K_3|x(n)|^2}{1+K_4|x(n)|^2}}
$$

(5)

We introduce α to change the PA characteristics among *M* PAs. The number of antennas is 4 (*M* = 4), and *α*s are in turn 1, 1.04, 1.08 and 1.12, respectively.

We assume that the conventional DPD works for only one PA among four. Specifically, in the simulation, the DPD characteristics are obtained from the first PA's feedback. Therefore, the linearization performance is expected to be good for the first PA but bad for the other PAs. Figure 2 and Figure 3 show the AM/AM (amplitude-to-amplitude modulation) and AM/PM (amplitude-to-phase modulation) characteristics after applying the conventional DPD and proposed DPD, respectively. The Figures (a), (b), (c) and (d) are AM/AM of the first PA and AM/PM of the first PA, AM/AM of the third PA and AM/PM of the third PA, respectively. Each figure has four curves: DPD characteristics (blue); PA characteristics without DPD (red); PA characteristics with DPD (green). According to Figures 2 and 3, both methods can linearize the PAs and it is not easy to determine which method is better than the other. To see the difference clearly, the PA output spectrums are compared.
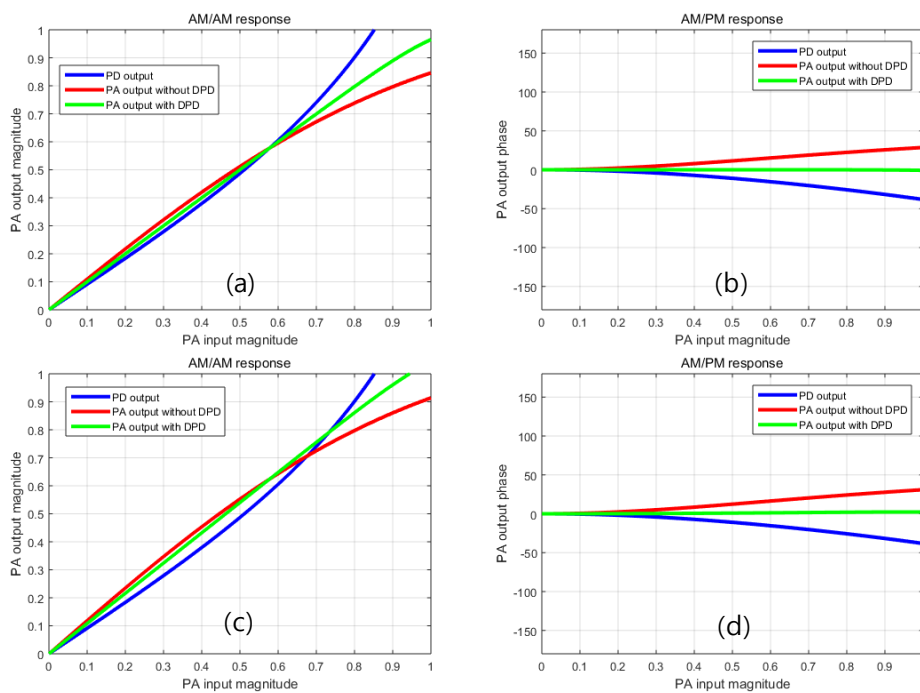

Figure 2. AM/AM & AM/PM Response for Conventional DPD
(a): AM/AM at 1ˢᵗ PA; (b): AM/PM at 1ˢᵗ PA; (c): AM/AM at 3ʳᵈ PA; (d): AM/PM at 3ʳᵈ PA
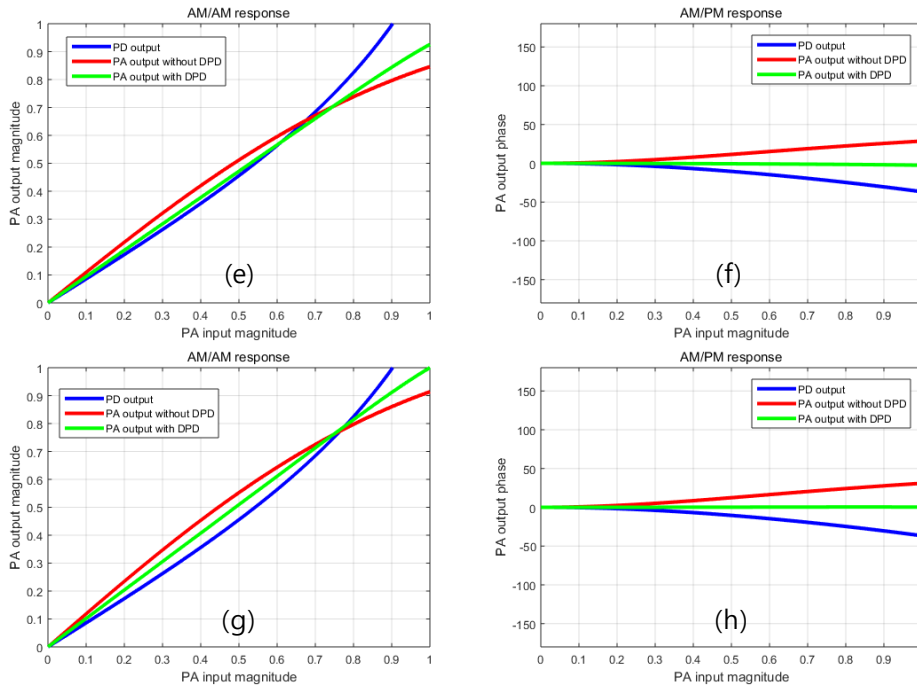
**Figure 3. AM/AM & AM/PM response for proposed DPD
(a): AM/AM at 1st PA; (b): AM/PM at 1st PA; (c): AM/AM at 3rd PA; (d): AM/PM at 3rd PA**

Figures 4 and 5 show the output spectrum of the PAs. The graphs are as follows: (a) DPD input (ideal signal); (b) PA output without DPD; (c) PA output with conventional DPD; (d) PA output with proposed DPD. Comparing (a) and (b), it can be seen that the spectral regrowth occurred by the non-linear PAs. Since the conventional DPD is designed to linearize the first PA only, ignoring the other PAs, the conventional DPD can linearize the first PA better than the proposed one. However, the proposed DPD performs better at the rest of the PAs. TableⅡ summarizes the spectral regrowth reductions of DPD techniques. Those results indicate that the proposed DPD is effective for the analog beamforming systems when the PA characteristics are quite different across the PAs.
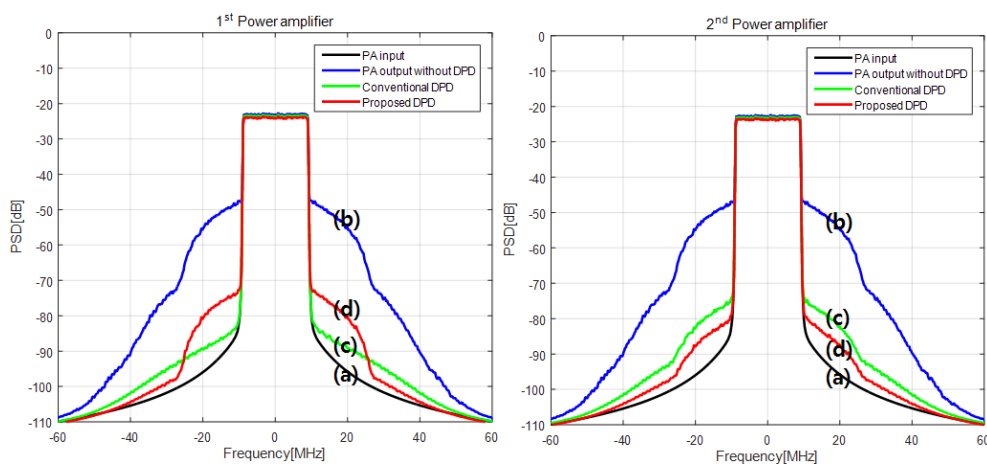

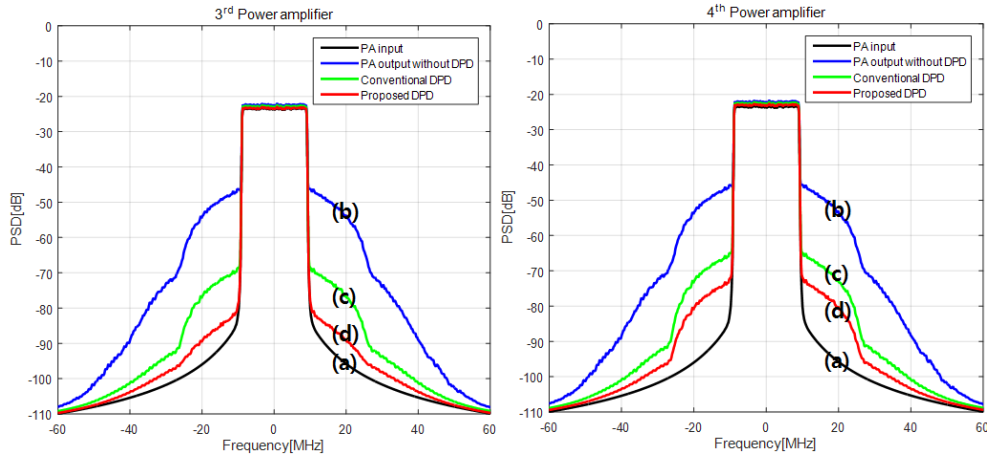**Figure 4. Spectrums at 1st and 2nd PA Outputs**

Figure 5. Spectrums at 3rd and 4th PA Outputs

Table Ⅱ. Performance of Spectral Regrowth Reduction

	Proposed DPD(dB)	Conventional DPD(dB)
1st PA	25	37
2nd PA	34	28
3rd PA	35	23
4th PA	28	20

5. Experiment Results

The proposed DPD is verified through experiment with commercial PAs. Figure 6 shows the experimental setup. We used long term evolution (LTE) downlink signals with 20MHz bandwidth, the same as in the simulation. The number of antennas is 2 ($M = 2$). The signal generator (E4438C by Agilent) converts the digital waveform into analog waveform. The center frequency of the waveform is 2605MHz. The analog waveform is divided into two signals by the power divider, and supplied to commercial PAs. The PA has 48.5dB gain. To implement phase shifters, we used two cables with different lengths. Our experiment environment provides 122 degree phase difference between two paths. The PA output is 40dBm. The BPFs at the PA outputs attenuate the harmonics. The BPF output are directly sampled by the digital oscilloscope (TDS7254 by Tektronix) and supplied to the PC. At the initial phase, the DPD is bypassed and the DPD parameters are obtained from the Tx waveform and the PA output waveform by the adaptive algorithm described in Table I.
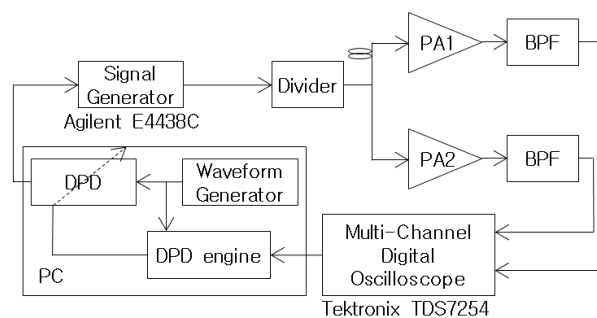

Figure 6. Digital Pre-Distortion Structure In Analog Beamforming Systems

Learning curves for the mean square errors (MSEs) are examined first. Figure 7 shows the learning curve for the proposed DPD. According to the results, the adaptive algorithms converge within 300 samples, and the MSE after convergence is around 10^{-4}.
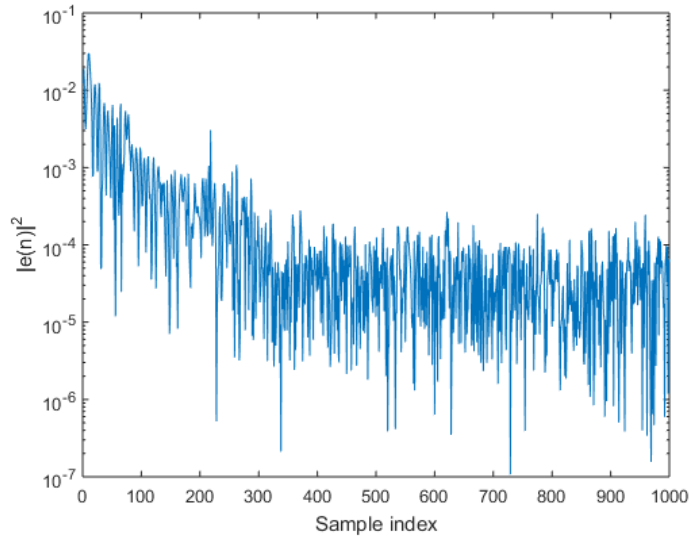

Figure 7. Learning Curves with Proposed DPD

Figures 8 and 9 show the AM/AM and AM/PM characteristics for the first PA and the second PA, respectively. The red curves represent the PA characteristics without DPD and the blue curves represent the DPD characteristics. It seems that the PD characteristics are the inverse characteristics of the PA.
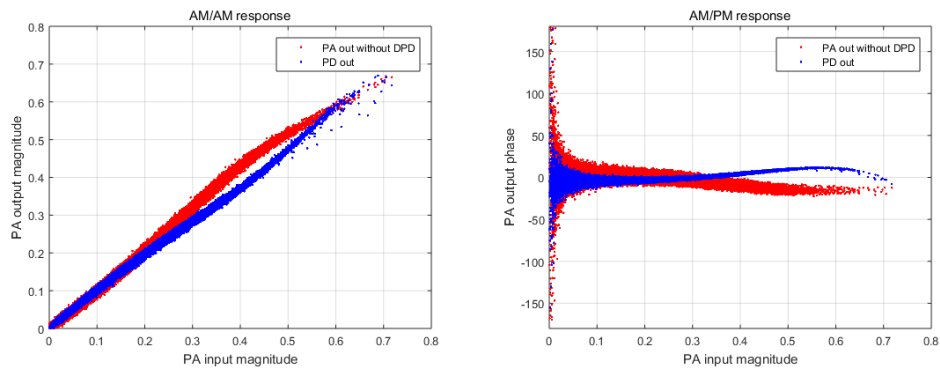

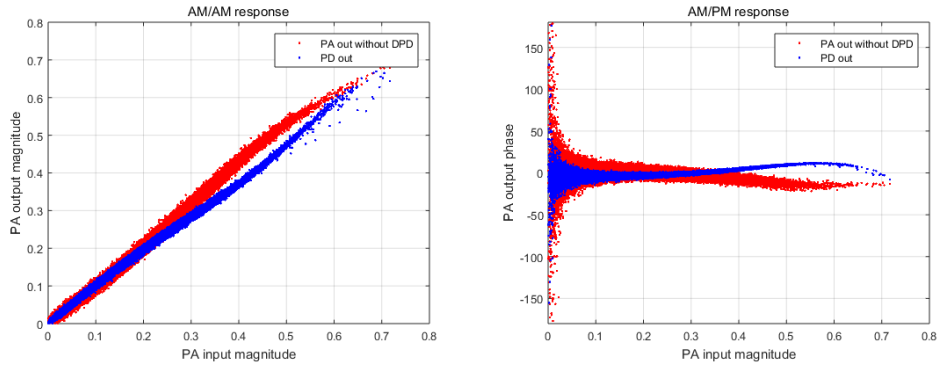**Figure 8. AM/AM and AM/PM Response at 1st PA**

Figure 9. AM/AM and AM/PM Response at 2nd PA

To examine the linearization performance in detail, the PA output spectrums are observed as shown in Figures 10 and 11. The curves represent as follows: PA output without DPD (blue); PA output with conventional DPD (red); PA output with proposed DPD (green). TableⅢ summarizes how much spectral regrowth can be reduced by the DPDs.
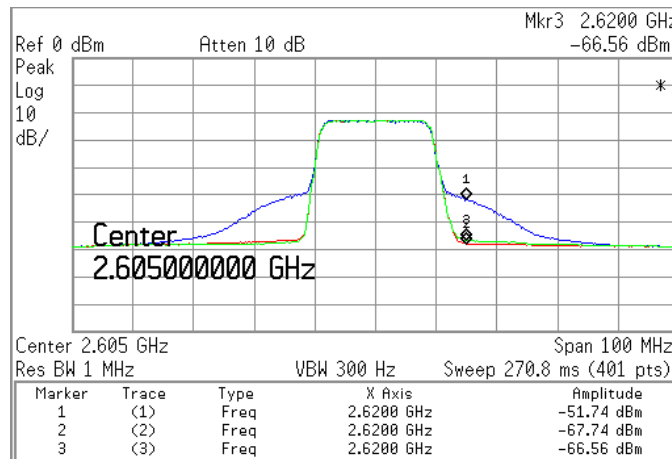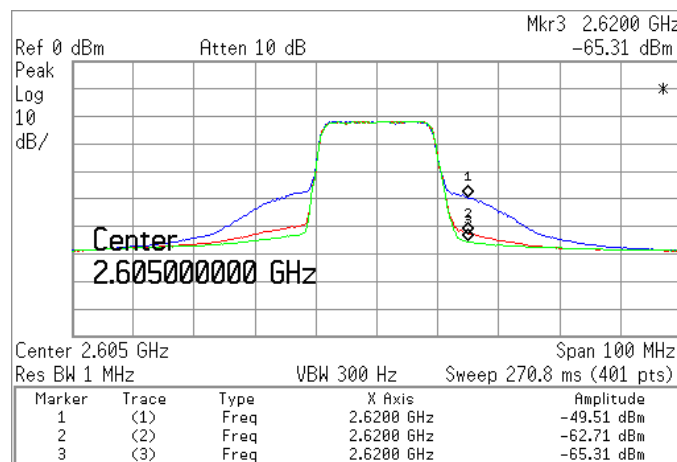

Figure 10. Spectrum at 1st PA Output

Figure 11. Spectrum at 2nd PA Output

TableⅢ. Performance to Reduce Spectral Regrowth

	Proposed DPD (dB)	Conventional DPD (dB)
1st PA	14.82	16
2nd PA	15.8	13.2

According to the results, the conventional DPD at the PA shows the best spectrum. The proposed DPD reduces about 14.8dB while the conventional DPD reduces 16dB. On the contrary, the conventional DPD at 2nd PA reduces the spectral regrowth by only 13.2dB while the proposed one reduces 15.8dB. Those results indicate that the proposed DPD can successfully linearize the two PAs simultaneously.

Conclusion

A DPD technique the analog BF system was proposed in this paper. Via computer simulation and experiment, it was verified that the proposed DPD could successfully linearize the multiple PAs while the conventional DPD has a large discrepancy in linearization performance across the PAs. The result indicates that the proposed DPD can be applied to analog BF systems.

Acknowledgments

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