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Unsteady Hydromagnetic Flow of Dusty Fluid over a Stretching Cylinder with Variable Viscosity and Thermal Conductivity

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Abstract

This paper deals with the study of unsteady flow of a conducting dusty fluid due to linearly stretching cylinder with variable viscosity and variable thermal conductivity. Present work investigates effects of temperature dependent viscosity and thermal conductivity on the velocity, temperature and species concentration profiles. The partial differential equations governing the boundary value problem are transformed into ordinary differential equations with the help of similarity transformations. Resultant equations are solved numerically using fourth order Runge–Kutta method with shooting technique. The influences of various parameters that characterize the flow on velocity, temperature and species concentration profile have depicted graphically and analyzed for both the fluid and dust phases. Numerical values of skin-friction coefficient, Nusselt number and Sherwood number are tabulated for various parameters.

Keywords: *Variable viscosity, variable thermal conductivity, unsteady flow, stretching cylinder and Shooting technique.*

Nomenclature:

a_0 constant
c_m specific heat of dust particles at constant pressure
c_p specific heat of fluid at constant pressure
D_m mass diffusivity
K Stokes' resistance(drag co-efficient)
l mass concentration
m mass of the dust particle
N umber density of the particle phase
(u,w) velocity components in the r and z directions of the fluid phase
Sc Schmidt number
T temperature of the fluid inside the boundary layer
T_p temperature of the dust particles inside the boundary layer
(u_p,w_p) velocity components in the r and z directions of the dust phase
z and r cylindrical polar coordinates measured in axial and radial directions,
β constant of expansion/ contraction strength
β^* volumetric coefficient of thermal expansion
θ_c thermal conductivity variation parameter
θ_r viscosity variation parameter,
μ coefficient of dynamic viscosity
ρ_p density of the particle phase

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$T_{p\infty}$ temperature of the dust particles in the free-stream

T_{∞} temperature of the fluid at free Stream

υ_{∞} kinematic viscosity of the fluid in the free stream

ρ density of the fluid

μ_{∞} coefficient of dynamic viscosity of the ambient fluid

σ electrical conductivity

τ relaxation time of particle phase

ω constant density ratio

1. Introduction

The hydromagnetic flow, heat and mass transfer of a dusty fluid over a stretching cylinder has gained considerable attention due to its applications in various fields of engineering and environmental sciences. In recent years, engineers and scientists have interested in gas-solid particle flows which arise in many industrial applications. It has several applications in the fields of combustion, fluidization, gas cooling systems, petroleum industry, hot rolling, electrostatic precipitation, purification of crude oil, polymer technology, cement process industry, steel manufacturing industry, fluid droplets sprays etc. In such situations, the quality of the product depends on the rate of cooling process and the process of stretching/shrinking (Bachok *et al.* [1]).

Sparrow and Gregg [2] have found the first approximate solution for the boundary layer flow over a vertical cylinder with heat flux using the power series expansion and similarity method. The boundary layer axi-symmetric flow of a viscous fluid towards a stretching circular cylinder with slip condition at the boundary has been presented by Mukhopadhyay [3]. The effect of first order thermal slip and second order momentum slip on the flow of viscous incompressible fluid over a shrinking cylinder was investigated by Mishra and Singh [4]. Hakiem and Rashad [5] analyzed the non-Darcy natural convection flow over a vertical cylinder in saturated porous medium with temperature-dependent viscosity. Chamkha *et al.* [6] discussed the effect of temperature dependent viscosity on the heat and mass transfer by non-Darcy free convection flow of viscous fluid over a vertical circular cylinder embedded in a porous medium. Abbas *et al.* [7] studied laminar MHD flow and heat transfer over a stretching cylinder in porous medium with thermal radiation. The numerical solutions of steady stagnation point flow of an incompressible viscous electrically conducting nano fluid towards a stretching cylinder with thermal radiation and heat exchanges at the surface was obtained by Akbar *et al.* [8].

It is interesting that in the above-mentioned works, fluids are considered in pure form. But, it is well known that air and water, by nature, contain impurities like dust particles and foreign bodies. In fact, the problem of two-phase flows in which solid particles are distributed in clean fluid has several practical applications such as the environmental pollution, sedimentation, centrifugal separation of particles, blood rheology, purification of crude oil and physiological flows. There are many investigations on this topic. Saffman [9] carried out pioneering work on the motion of dust particles in a laminar flow. He derived the equations for the flow satisfied by small disturbances of a laminar steady flow and discussed the effects of the dust on the motion of a gas carrying fluid. Nayfeh [10] studied oscillating dusty flow through a rigid pipe. Gupta and Gupta [11] have discussed flow of a gas containing solid particles in a channel with arbitrary time varying pressure. Flow of a dusty fluid in boundary layer over a semi-infinite flat plate was analyzed by Datta and Mishra [12]. Ramamurthy [13] investigated the effects of free convection on the Stokes problem for the flow of dusty fluid in an infinite vertical plate. Attia [14] has analyzed unsteady MHD Couette flow and heat transfer of dusty fluid with variable physical properties. Palani and Kim [15] obtained the approximate solution for the flow of a dusty-gas along a semi-infinite vertical cylinder. Gireesha *et al.* [16-18] discussed

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interesting results on the flow of dusty fluid due to linear and exponential stretching of porous and non porous sheet with various effects like radiation, source/sink parameter, viscous dissipation etc. Recently Manjunatha [19] was studied effect of radiation on MHD flow and heat transfer of dusty fluid over a stretching cylinder in a porous medium.

In most of the studies, mentioned above, authors restricted their studies to steady flows of a dusty fluid over stretching sheet. But recently, boundary layer unsteady flow of a dusty fluid over stretching cylinder has received considerable importance due to its many applications. It is also seen that, a number of investigators restricted their investigations by assuming viscosity and thermal conductivity of the fluid as constant. However, it is known from the work of Herwig and Wicken [20] that these properties may change with temperature. A theoretical investigation of the effects of temperature dependent viscosity on forced convection over a flat plate in porous medium has been presented by Ling and Dybbs [21]. So, more accurate prediction for the flow, heat transfer and mass transfer can be achieved by taking into account the variation of such properties with temperature. Hazarika and Konch [22] and Konch and Hazarika [23] have studied the influence of varying viscosity and thermal conductivity on convective heat and mass transfer in the flow of dusty fluid with radiation and viscous dissipation.

In this paper, we consider unsteady flow of a dusty fluid over a stretching cylinder and taking viscosity and thermal conductivity are functions of temperature. Here we focus on the effects induced by governing parameters on the flow, heat and mass transfer phenomena in presence of magnetic field.

The partial differential equations of the flow problem are reduced into ordinary differential equations employing similarity transformations. The resultant nonlinear ordinary differential equations are solved numerically by fourth order Runge-Kutta method together with shooting technique. Numerical results are presented as graphs and scrutinized. Effects of the parameters governing the flow on the velocity and temperature fields for both fluid and dust particle phase are discussed. Moreover, numerical results for the skin-friction coefficient, Nusselt number and Sherwood number are presented in a table.

2. Mathematical Formulation

Consider an unsteady laminar boundary layer flow of viscous incompressible dusty fluid over a stretching cylinder. The z-axis is measured along the axis of cylinder and r-axis is measured in the radial direction as shown in Fig. 1. Diameter of the cylinder is assumed to be a function of time with radius $r = a(t) = a_0\sqrt{1-\beta t}$. Further, a magnetic field of uniform strength $\vec{B}$ is introduced along radial direction.

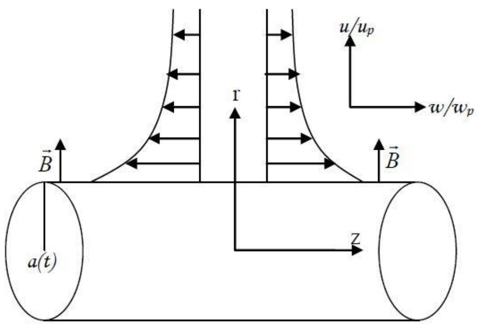

Figure 1. Schematic Diagram of the Flow Problem

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The analysis of this study is based on following assumptions:

- Physical properties are assumed as constant except for the fluid viscosity and thermal conductivity.
- Magnetic Reynolds number is assumed to be small therefore the induced magnetic field is negligible (Sutton [24]).
- Dust particles are assumed as electrically non-conducting, spherical in shape having the same radius and mass, and un-deformable.
- Number density of dust particles is taken as constant throughout the flow.

Using these assumptions together with usual boundary layer approximations and following Saffman [9] we get the equations of motion as:

For the fluid phase:

Equation of continuity:

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial w}{\partial z}=0 \tag{1}$$

Equation of momentum:

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial w}{\partial r}\right)+KN(w_p-w)-\sigma B_0^2 w \tag{2}$$

Equation of energy:

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r\frac{\partial T}{\partial r}\right)+\frac{Nc_p}{\tau_T}(T_p-T)+\frac{N}{\tau_v}(w_p-w)^2 \tag{3}$$

Equation of species concentration:

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial r}+w\frac{\partial C}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(rD_m\frac{\partial C}{\partial r}\right)+\frac{mN}{\rho\tau_c}(C_p-C) \tag{4}$$

For the dust phase:

Equation of continuity:

$$\frac{1}{r}\frac{\partial(r\rho_p u_p)}{\partial r}+\frac{\partial(\rho_p w_p)}{\partial z}=0 \tag{5}$$

Equations of momentum:

$$\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial r}+w_p\frac{\partial u_p}{\partial z}=\frac{K}{m}(u-u_p) \tag{6}$$

$$\frac{\partial w_p}{\partial t}+u_p\frac{\partial w_p}{\partial r}+w_p\frac{\partial w_p}{\partial z}=\frac{K}{m}(w-w_p) \tag{7}$$

Equation of energy:

$$Nc_m\left(\frac{\partial T_p}{\partial t}+u_p\frac{\partial T_p}{\partial r}+w_p\frac{\partial T_p}{\partial z}\right)=-\frac{Nc_p}{\tau_T}(T_p-T) \tag{8}$$

Equation of species concentration:

$$\frac{\partial C_p}{\partial t}+u_p\frac{\partial C_p}{\partial r}+w_p\frac{\partial C_p}{\partial z}=-\frac{mN}{\rho\tau_c}(C_p-C) \tag{9}$$

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The boundary conditions for the flow problem are given by:

$$\left.\begin{aligned}
&\text{At}\quad r=a(t):\quad u=0\,,\; w=\frac{1}{a_0^2}\frac{4\upsilon_\infty z}{1-\beta t}\,, T=T_w(z,t)=T_\infty+\frac{Az}{a_0\upsilon_\infty(1-\beta t)}\,,\\
&\qquad C=C_w(z,t)=C_\infty+\frac{Bz}{a_0\upsilon_\infty(1-\beta t)}\\
&\text{As}\quad r\to\infty:\quad u_p\to u\,, w\to 0\,, w_p\to 0\,, \rho_p\to\omega\rho\,,\; T\to T_\infty\,,\; T_p\to T_\infty\,,\; w_p\to 0\,,\\
&\qquad \rho_p\to\omega\rho\,,\; T\to T_\infty\,,\; T_p\to T_\infty\,,\; C\to C_\infty\,,\; C_p\to C_\infty
\end{aligned}\right\}\qquad (10)$$

where A and B are positive constants.

In order to reduce the nonlinear partial differential equations, governing the flow, into ordinary differential equations we can use the following similarity transformations:

$$\left.\begin{aligned}
&\eta=\left(\frac{r}{a_0}\right)^2\frac{1}{1-\beta t}\,,\; u=-\frac{1}{a_0}\frac{2\upsilon_\infty}{\sqrt{1-\beta t}}\frac{f(\eta)}{\sqrt{\eta}}\,,\; w=\frac{1}{a_0^2}\frac{4\upsilon_\infty z}{1-\beta t}f'(\eta)\,,\\
&u_p=\frac{1}{a_0}\frac{2\upsilon_\infty}{\sqrt{1-\beta t}}\frac{F(\eta)}{\sqrt{\eta}}\,,\; w_p=\frac{1}{a_0^2}\frac{4\upsilon_\infty z}{1-\beta t}G(\eta)\,,\;\; \rho_r=H(\eta)\,,\; \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\,,\\
&\theta_p(\eta)=\frac{T_p-T_\infty}{T_w-T_\infty}\,,\; \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}\,,\;\; \varphi_p(\eta)=\frac{C_p-C_\infty}{C_w-C_\infty}
\end{aligned}\right\}\qquad (11)$$

where $f(\eta)$, $\theta(\eta)$ and $\varphi(\eta)$ are the dimensionless stream function, temperature and species concentration of the fluid phase, respectively. $F(\eta)$, $\theta_p(\eta)$ and $\varphi_p(\eta)$ are the dimensionless velocity, temperature and species concentration of the dust phase, respectively. η is the similarity variable, $\rho_r=\frac{\rho_p}{\rho}$ is the relative density. The prime (') denotes derivative with respect to η.

Viscosity of the fluid is assumed to be an inverse linear function of temperature, and it can be expressed as following Lai and Kulacki [25]:

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\delta\left(T-T_\infty\right)\right]\qquad (12)$$

or, $\frac{1}{\mu}=\alpha\left(T-T_r\right)$, where $\alpha=\frac{\delta}{\mu_\infty}$ and $T_r=T_\infty-\frac{1}{\delta}$

Moreover, thermal conductivity of the fluid varies with temperature. Following Choudhury and Hazarika [26], we assumed thermal conductivity of the fluid as:

$$\frac{1}{\lambda}=\frac{1}{\lambda_\infty}\left[1+\xi\left(T-T_\infty\right)\right]\qquad (13)$$

or, $\frac{1}{\lambda}=\zeta\left(T-T_c\right)$, where $\zeta=\frac{\xi}{\lambda_\infty}$ and $T_c=T_\infty-\frac{1}{\xi}$.

Here α, δ, ξ, ζ, T_r and T_c are constants and their values depend on the reference state and thermal properties of the fluid i.e., υ (kinematic viscosity) and λ (thermal conductivity).

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Let us introduce two dimensionless parameters as: $\theta_r = \dfrac{T_r - T_\infty}{T_w - T_\infty}$ is the dimensionless reference temperature corresponding to viscosity, called the viscosity variation parameter and $\theta_c = \dfrac{T_c - T_\infty}{T_w - T_\infty}$ is the dimensionless reference temperature corresponding to thermal conductivity, called the thermal conductivity variation parameter. It is also important to note that θ_r and θ_c are negative for liquids and positive for gases (Kuppalapalle *et al*. [27]).

Using these two parameters in Eqs.(12) and (13) we get coefficient of viscosity and thermal conductivity as:

$$\mu = -\frac{\mu_\infty \theta_r}{\theta - \theta_r} \text{ and } \lambda = -\frac{\lambda_\infty \theta_c}{\theta - \theta_c}. \tag{14}$$

Substituting Eqs. (11)-(14) into Eqs. (2)-(10), we get the following nonlinear ordinary differential equations:

$$\frac{\theta_r}{\theta-\theta_r}\eta f''' - \frac{\theta_r}{(\theta-\theta_r)^2}\eta\theta' f'' + \frac{\theta_r}{\theta-\theta_r} f'' - ff'' + f'^2 + S(\eta f'' + f') - l\beta_v H(G - f') + Mf' = 0 \tag{15}$$

$$(S\eta + F)F' - \frac{1}{2\eta}F^2 + \beta_v(f - F) = 0 \tag{16}$$

$$(S\eta + F)G' + SG + G^2 - \beta_v(f' - G) = 0 \tag{17}$$

$$FH' + F'H + GH = 0 \tag{18}$$

$$\frac{\theta_c}{\theta-\theta_c}\eta\theta'' - \frac{\theta_c}{(\theta-\theta_c)^2}\eta\theta'^2 + \Pr(S\theta' - f\theta') - \Pr Sa_1(\theta_p - \theta) - \Pr EcSa_2(G - f')^2 = 0 \tag{19}$$

$$(S\eta + F)\theta_p' + \gamma a_3(\theta_p - \theta) = 0 \tag{20}$$

$$\frac{\theta_r}{\theta-\theta_r}\eta\varphi'' - \frac{\theta_r}{(\theta-\theta_r)^2}\eta\theta'\varphi' + \frac{\theta_r}{\theta-\theta_r}\varphi' + Sc(S\eta - f)\varphi' - Scl\beta_c H(\varphi_p - \varphi) = 0 \tag{21}$$

$$(S\eta + F)\varphi_p' + l\beta_c H(\varphi_p - \varphi) = 0 \tag{22}$$

The boundary conditions Eq. (10) reduces to:

$$\left.\begin{array}{l} f = 0,\ f' = 1,\ \theta = 1,\ \varphi = 1 \text{ at } \eta = 1, \\ f' = 0,\ F = -f,\ G = 0,\ H = \omega,\ \theta = 0,\ \theta_p = 0,\ \varphi = 0,\ \varphi_p = 0 \text{ as } \eta \to \infty \end{array}\right\} \tag{23}$$

where the dimensionless parameters are defined as follows:

$\beta_v = \dfrac{a_0^2(1-\beta t)}{4\upsilon_\infty \tau_v}$ is fluid particle interaction parameter for velocity, $\beta_T = \dfrac{a_0^2}{4\upsilon_\infty \tau_T}(1-\beta t)$ is fluid particle interaction parameter for temperature, $a_1 = \dfrac{N}{\rho\beta\tau_T}(1-\beta t)$ and $a_2 = \dfrac{N}{\rho\beta\tau_v}(1-\beta t)$ are constants, $Ec = \dfrac{w_w^2}{c_p(T_w - T_\infty)}$ is Eckert

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number, $M = \frac{\sigma B_0^2}{\rho c}$ is magnetic field parameter, $\Pr = \frac{\mu_\infty c_p}{\lambda_\infty}$ is Prandtl number, $S = \frac{a_0^2 \beta}{4\upsilon_\infty}$ is unsteadiness parameter, $l = mN/\rho_p$ is mass concentration, $\tau = m/K$ is relaxation time of the particle phase, $\rho_r = \rho_p/\rho$ is relative density, $\gamma = \frac{c_p}{c_m}$ is a constant.

2.1. Skin-friction Coefficient, Nusselt Number and Sherwood Number

Skin-friction coefficient (C_f), Nusselt number (Nu) and Sherwood number (Sh) are the parameters of physical and engineering interest for the present problem, which physically indicate the wall shear stress, rate of heat transfer and rate of mass transfer, respectively.

Skin-friction coefficient is defined as

$$C_f = \frac{\tau_w}{\rho w_w^2 / 2}, \text{ where } \tau_w = \mu \left.\frac{\partial w}{\partial r}\right)_{r=a} \text{ is the shearing stress.}$$

Using the non-dimensional variables we get the skin-friction coefficient as

$$C_f z / a = -\frac{\theta_r}{1-\theta_r} f''(1).$$

The Nusselt number is defined as

$$Nu = \frac{a(t) q_w}{\lambda_\infty (T_w - T_\infty)}, \quad \text{where } q_w = -\lambda \left.\frac{\partial T}{\partial r}\right)_{r=a} \text{ is the heat transfer from the sheet.}$$

Using the non-dimensional variables we get

$$Nu = \frac{2\theta_c}{1-\theta_c} \theta'(1).$$

The Sherwood number which is defined as

$$Sh = \frac{x m_w}{Dm_\infty (C_w - C_\infty)}, \text{ where } m_w = -Dm \left.\frac{\partial C}{\partial r}\right)_{r=a} \text{ is the mass flux at the surface and}$$

Dm_∞ is the diffusion constant at free stream.

Using the non-dimensional variables we get

$$Sh = Sc^{-1} \frac{2\theta_r}{1-\theta_r} \varphi'(1).$$

3. Results and Discussion

The Eqs. (15)–(22) governing the flow subject to boundary conditions Eq. (23) are solved numerically using fourth order Runge-Kutta method together with shooting technique. Numerical values are computed by developing suitable codes in MATLAB for the method.

In order to analyze the problem physically, a representative set of numerical results is shown graphically for velocity and temperature fields in Figures 2–11, to illustrate the influence of physical parameters embedded in the flow system. Numerical values of the parameters used for simulation are: $M=0.5$, $Pr=0.71$, $\beta=8$, $Ec=0.05$, $S=-2$, $Sc=0.22$, $Sc_p=0.22$, $\theta_r = 5$ and $\theta_c = 3$, unless otherwise stated.

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The effects of viscosity variation parameter θ_r on velocity and temperature profiles are shown through Figures 2 and 3, respectively. From Figure 2 it is observed that the velocity decreases with the increase of the viscosity variation parameter for both the fluid and dust phases. This is because of the fact that when temperature increases, viscosity of gas increases and hence velocity decreases. It is in good agreement with the work of Rashed [28]. On the other hand an increase in the value of θ_r results in an increase in the thermal boundary layer thickness. Hence the temperature increases (Figure 3) as viscosity variation parameter increases.

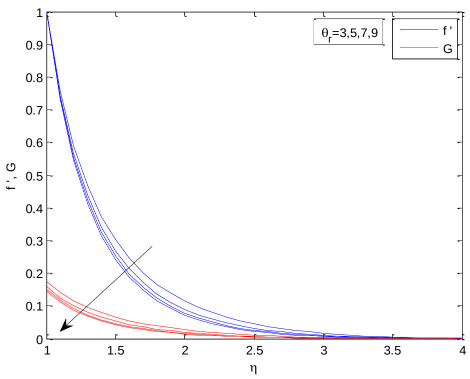

Figure 2. Velocity Profile for Different Θ_r

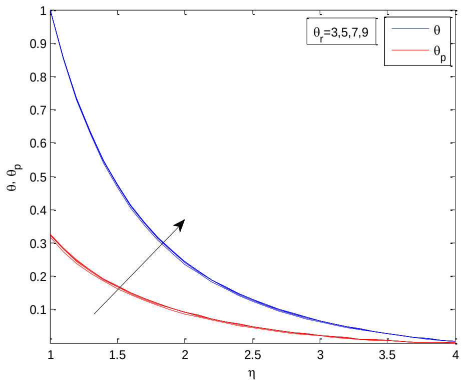

Figure 3. Temperature Profile for Different Θ_r

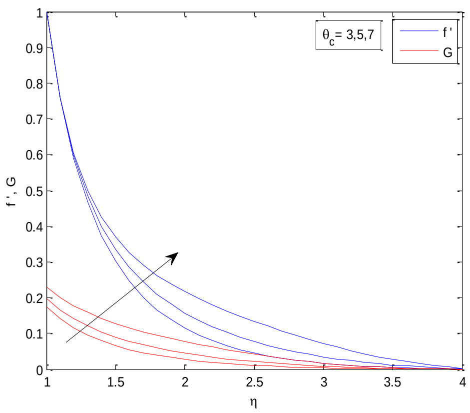

Figure 4. Velocity Profile for Different Θ_c

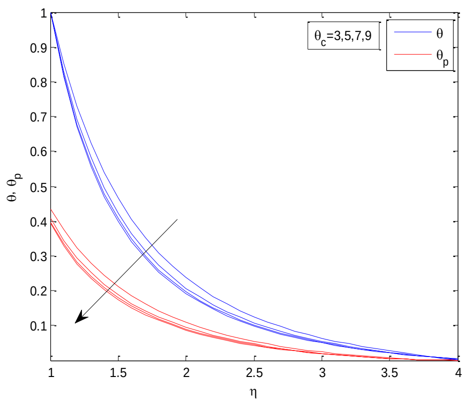

Figure 5. Temperature Profile for Different Θ_c

Figure 4 shows the influence of the thermal conductivity variation parameter θ_c on velocity profiles of both fluid and dust phase. It is seen from this figure that velocity increases with the increase of the thermal conductivity variation parameter. It is due to the fact that temperature decreases with the increasing values of θ_c and as a result viscosity decreases and so velocity increases.

From Figure 5 it is observed that temperature of both the fluid and dust phases decreases with the increasing values of θ_c. It is due to the reason that the thermal conductivity decreases when θ_c increases and as result temperature decreases.

Velocity and temperature fields for different values of the magnetic parameter *M* are depicted in Figures 6 and 7, respectively. From these figures it is seen that velocity decreases for increasing values of the parameter *M* while temperature increases as *M*

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increases for both the fluid and dust particles. This is because of the fact that when a transverse magnetic field is introduced to an electrically conducting fluid, the fluid experiences a resistive type of force known as the Lorentz force. Due to this force, friction between adjacent layers increases and as a result motion of the fluid decreases and temperature increases of both the fluid and dust phases in the boundary layer.

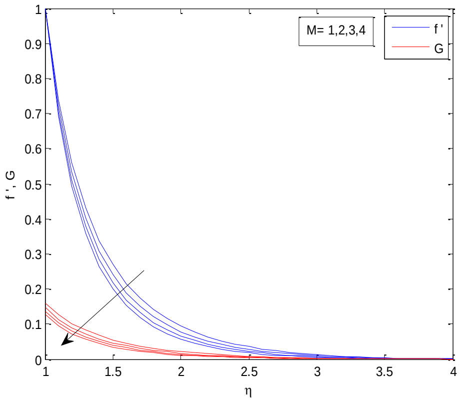

Figure 6. Velocity Profile for Different *M*

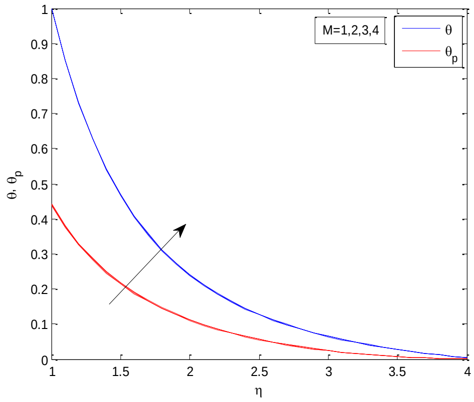

Figure 7. Temperature Profile for Different *M*

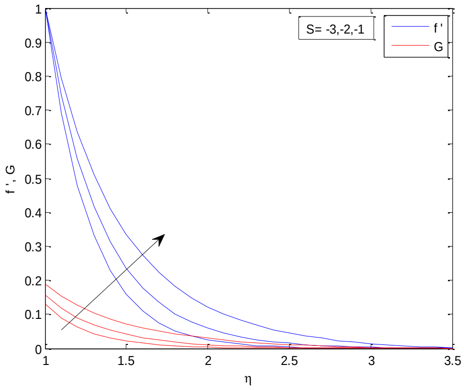

Figure 8. Velocity Profile for Different *S*

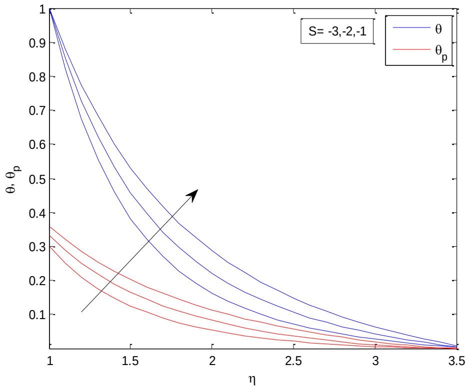

Figure 9. Temperature Profile for Different *S*

Figure 8 represents the velocity profile of both fluid and dust phases for different values of unsteady parameter *S*. From this figure one can seen that velocity increases with the increase of *S*. It is also observed that thickness of boundary layer increases with increasing values of *S*.

Figure 9 shows that temperature of the fluid increases with increasing values of *S*. It is due to the reason that rate of heat transfer decreases with increasing the value of *S*, as a result temperature increases.

Figure 10 indicates the velocity field of fluid and dust particles for various values of Prandtl number *Pr*. Here it is noticed that the effect of Prandtl number values greatly affects the velocity of the fluid and dust phase. The velocity of the fluid and dust phases increases as the Prandtl number *Pr* increases.

Figure 11 demonstrates the effects of Prandtl number (*Pr*) on temperature profile. We infer from this figure that temperature of the fluid and dust particle decreases significantly with the increase in *Pr*. This is because of the fact that increase in *Pr* indicates the increase of fluid heat capacity or decrease of the thermal diffusivity, which causes a diminution of the influence of the thermal expansion to the flow. As a result temperature decreases in the boundary layer.

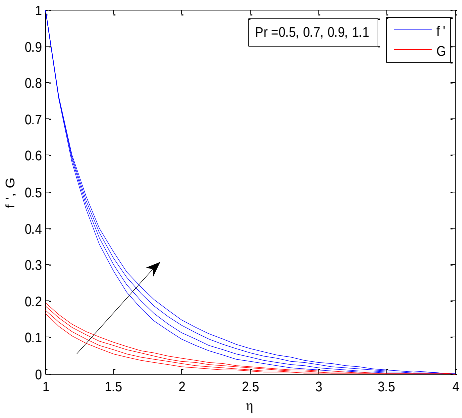

Figure 10. Velocity Profile for Different *Pr*

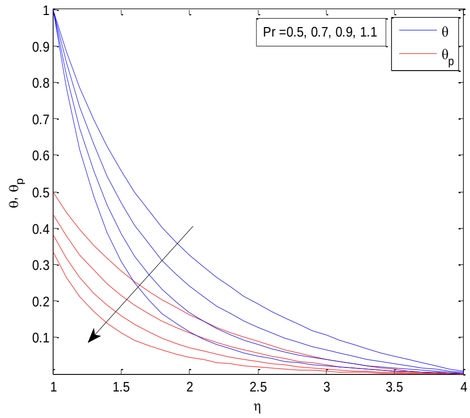

Figure 11. Temperature Profile for Different *Pr*

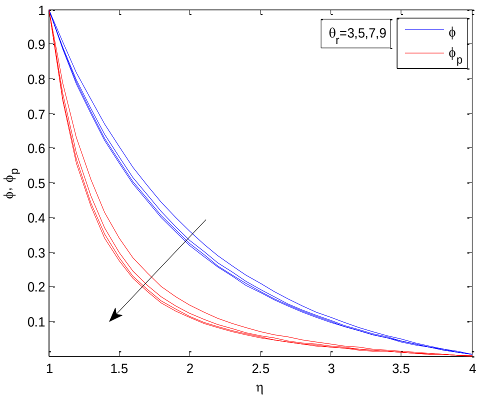

Figure 12. Concentration Profile for Different θ_r

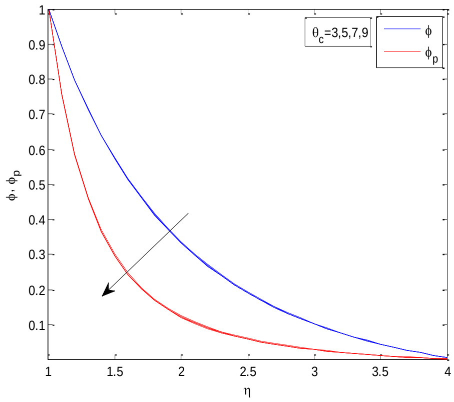

Figure 13. Concentration Profile for Different θ_c

Figures 12 and 13 display the effect of θ_r and θ_c on species concentration profile within the boundary layer. We observed that species concentration decreases with the increasing values of both θ_r and θ_c. It is due to the fact that increasing value of θ_r and θ_c , mass diffusivity increases and as a result species concentration decreases for both the fluid and dust phases.

The calculated values of skin-friction coefficient, Nusselt number and Sherwood number for various values of flow governing parameters are presented in Table 1. It is observed that an increase in the viscosity variation parameter θ_r leads to increase in the values of skin-friction coefficient, where as a decrease in the values of the Nusselt number. This effect arises because as the viscosity variation parameter increases, fluid boundary layer thickness decreases. Hence the value of the wall velocity gradient to increase yielding decreases in the skin friction coefficients and increases in the Nusselt number.

It is also noticed that with increase in the parameters θ_c and S the skin-friction coefficient decreases but it increases with M increased. A significant decrease is remarked in case of Nusselt number when there is an increase in the value of the parameters M and S. From the table it is also observed that an increase in the value of θ_c leads to a decrease in the Nusselt number. It is due to the reason that thickness of the thermal boundary layer

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decreases with the raise of θ_c and as a result wall temperature gradients decreases. Physically negative values of C_f mean that surface exerts a drag force on the fluid, so that stretching surface will induce the flow. From the table it is also observed that Sherwood number decreases significantly for increasing values of *S*.

Table 1. Effects of θ_r , θ_c , *M* and *S* on Local Skin-friction Coefficient(C_f), Local Nusselt Number(Nu) and Sherwood Number (*Sh*)

θ_r	θ_c	M	S	C_f	Nu	Sh
3	3	0.5	-2	-3.7757	4.269967	13.9999
5				-3.70327	4.249719	13.4527
7				-3.67615	4.241769	13.2368
5	3	0.5	-2	-3.70327	4.249719	13.4527
	5			-3.69998	4.243165	13.4073
	7			-3.69869	4.243022	13.3902
5	3	1	-2	-3.86581	4.243082	13.4460
		2		-4.16282	4.231584	13.4344
		3		-4.43051	4.221891	13.4247
5	3	0.5	-3	-4.57301	5.056291	15.46842
			-2	-3.70327	4.249719	13.45271
			-1	-2.93163	3.607416	11.52169

4. Conclusions

The unsteady flow of an incompressible, viscous and electrically conducting dusty fluid over a stretching cylinder in the presence of a magnetic field has been studied numerically. The effects of various physical parameters such as viscosity variation parameter (θ_r), magnetic parameter (*M*), thermal conductivity variation parameter (θ_c) and unsteady parameter (*S*) on flow, heat and mass transfer characteristics are discussed with the help of graphs and table. Some of the significant findings have been made for this study as presented below:

- An increase in the value of viscosity variation parameter retards the velocities of fluid and dust phase while it enhances the temperature profiles of both the phases.
- Velocities of fluid and dust phase increases for increasing values of thermal conductivity variation parameter whereas temperature of fluid and dust phase decreases in the boundary layer with it.
- The effect of magnetic parameter is to decrease the fluid and particle velocities, which is due to Lorentz force.
- The effect of unsteady parameter is seen to increase in both the velocity and temperature profile of fluid and dust particles.
- Velocity of fluid and dust particles increases with increasing Prandtl number whereas temperature of fluid and dust particles decreases with it.

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- Increasing values of viscosity variation parameter, thermal conductivity variation parameter and unsteady parameter increases skin-friction coefficient of dusty fluid but decreases rate of heat transfer.
- Rate of heat transfer decreases with increasing values viscosity variation parameter and thermal conductivity variation parameter
- Velocity and skin-friction coefficient of dust particles behaves same as dusty fluid.
- Species concentration decreases with increasing variable viscosity parameter and thermal conductivity parameter.

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