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Convective Heat Transfer in MHD Flow between Rotating Plates with Nonlinear Thermal Radiation

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Abstract

In this study, we analyzed the effect of nonlinear thermal radiation and non-uniform heat source/sink on MHD fluid flow between rotating horizontal plates with convective boundary conditions. By making use of the appropriate self-similarity transformations, the equations which govern the boundary layer equations are reduced into a set of nonlinear ordinary differential equations. Further, the resultant equations are solved numerically using the Runge-Kutta based shooting technique. Also, found the effect of different pertinent parameters on velocity and temperature profiles along with the friction factor and the local Nusselt number. Results displays that the radiation and non-uniform heat source/sink parameters have tendency to enhance the temperature profiles.

Keywords: *Convective conditions, non-uniform heat source/sink, non-linear thermal radiation, MHD, Rotation parameter*

Nomenclature:

u, v, w : Velocity components of the fluid in x, y and z directions respectively
ρ : Density of the fluid
Ω : Angular velocity
P : Pressure
μ : Dynamic viscosity of the fluid
σ : Electrical conductivity
B_0 : Uniform magnetic field
T : Temperature of the fluid
T_∞ : Ambient temperature of the fluid
T_w : Temperature of the fluid near the plate
k : Thermal conductivity
c_p : Specific heat at constant pressure
c : Constant
a_0 : Positive dimensional coefficient
S : Viscosity parameter
M : Magneticfield parameter
K_r : Rotation parameter
$\Pr$: Prandtl number
Bi : Biot number
A^*, B^* : Non-uniform heat source/sink parameter
R : Radiation parameter
θ_w : Temperature ratio parameter

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C_f : Skin friction coefficient

Nu : Local Nusselt number

1. Introduction

The Newtonian fluid flow with combined radiation and convection is of great reputation in many engineering and industrial application, including high-temperature heat exchangers furnaces, solar collectors, nuclear reactors and many other discipline. Many recognized models in the literature survey, the distribution of both velocity and temperature as well as the heat transfer coefficient between the plates of the flow. Although different shapes of flow are normally used in engineering problems, most of the attention of researchers has been given to flow inside a pipe or between two plates. Thermal radiation has a major role in the total surface heat transfer when the convection heat transfer coefficient is small. Chen [1] was explained the simultaneous radiative heat transfer in an absorbing, emitting and scattering medium in slug flow between parallel plates. The effects of heat transfer in an absorbing, emitting and scattering slug flow between parallel plates problem have been studied by Lii and Ozisik [2]. Fathalah and Elsayed [3] discussed the Natural convection due to solar radiation over a non-absorbing plate with and without heat losses. Singh [4] have studied hydro magnetic free convection flow past an impulsively started vertical infinite plate in a rotating fluid and he showed that fluid velocity is more influenced by rotation.

MHD is the mechanical property of fluids, which discuss the motion of extremely conducting fluid in the occurrence of magnetic field. The conducting fluids are produces an electric current in the fluid and this force can be boost up the mechanical properties of fluid. It has various attentions in the radiation treatment, space technology, Cancer homeo-therapy, marine engineering, rotating thin plates, nuclear design and processing. Radiation effect on mixed convection along a vertical plate with uniform surface temperature has been examined by Hossain and Takhar [5]. Bakier [6] investigated the thermal radiation effects on mixed convection from vertical surface in saturated porous media. Mbeledogu *et al.*, [7] presented the MHD effects on free convective flow of a compressible fluid past a moving vertical plate and concluded the temperature boundary layer increase as the radiation parameter and the time period are increased. The radiation effects on MHD Couette flow with heat transfer between two parallel plates have been illustrated by Mebine [8]. Radiation effects on combined convection over a vertical flat plate embedded in a porous medium of variable porosity have been studied by Pal and Mondal [9].

The combined effects of rotation on MHD flow past an impulsively started vertical plate with variable temperature was investigated by Rajput and Kumar [10]. Domairry *et al.* [11] have been explained the effects natural convection flow of a non-Newtonian Nanofluid between two vertical flat plates. The effects of heat transfer of Cu–water Nanofluid flow between parallel plates were examined by Sheikholeslami and Ganji [12]. The researchers [13-17] studied heat transfer characteristics of MHD flows by considering various channels. Radiation and soret effects of MHD Nanofluid flow over a moving vertical plate in presence of porous medium was explained by Raju *et al.*, [18] and found that soret number and Buoyancy parameter are helps to enhance the heat transfer rate. Raju *et al.*, [19] have studied the influence of non-uniform heat source/sink on MHD nanofluid flow over a moving vertical plate. Very recently, the researchers [20-24] investigated the heat transfer behaviour of various flows through different geometries.

In this study, we analyzed the effect of nonlinear thermal radiation and non-uniform heat source/sink on MHD fluid flow between rotating horizontal plates with convective boundary conditions. By making use of the appropriate self-similarity transformations, the equations which govern the boundary layer equations are reduced into a set of nonlinear ordinary differential equations.

2. Formulation of Problem

Consider a steady magneto hydrodynamic flow between two horizontal parallel plates. It is consider that the x-axis is along the plate, y-axis is perpendicular to the plate and the z -axis is normal to the xy-plane. It is also assumed that, both fluid and the plates are rotating together around the y-axis. The plates are placed at $y=0$ and $y=h$ as shown in Figure 1.

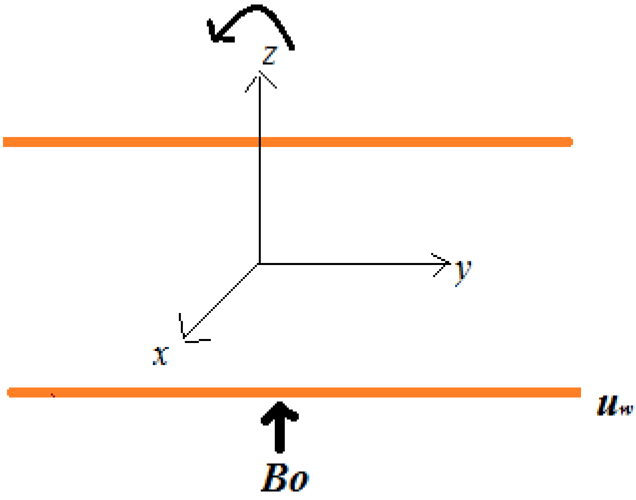

Figure 1. Flow Analysis

The lower plate is being stretched by two equal and opposite forces such that the position of origin remains unchanged. A uniform magnetic field of strength B_0 is applied in y-direction. The influences of induced and electric magnetic fields are neglected in the present study. The boundary layer equations governing the flow can be written as follows.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+2\Omega w=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}-2\Omega u=\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho}w, \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{q'''}{\rho c_p}+\frac{16\sigma^*}{3k\rho c_p}\frac{\partial T}{\partial y}\left(T^3\frac{\partial T}{\partial y}\right) \tag{5}$$

Subjected to the boundary conditions

$$\begin{aligned} & u = u_w(x) = cx, v = 0, w = 0, -kf\left(\frac{\partial T}{\partial y}\right) = h_f\left(T_h - T\right), \quad at\, y = 0, \\ & u = 0, v = 0, w = 0, T = T_0, \qquad at\, y = h, \end{aligned} \tag{6}$$

In the above equations u, v and w are velocity components along x, y and z -directions respectively, ρ is the effective density, Ω is the angular velocity, p is the modified fluid pleasure, μ is the dynamic viscosity, σ is the electrical conductivity, T is the temperature, k is the thermal conductivity, c_p is the specific heat at constant pressure, c is a constant, T_0, T_h are temperature at the plates which are placed at $y = 0$ and $y = h$ respectively and a_0 is the positive dimensional coefficient.

The time dependent non-uniform heat source/sink q''' defined as

$$q''' = \frac{k_f u_w(x)}{x\nu}\left(A^*(T_w - T_\infty) f' + B^*(T - T_\infty)\right), \tag{7}$$

The above equation positive values of A^*, B^* corresponds to heat generation and negative values are corresponds to heat absorption. We now introducing the similarity transformations as

$$\begin{aligned} & \eta = \frac{y}{h}, u = cxf'(\eta), v = -chf(\eta), \\ & w = cxg(\eta), \frac{T - T_h}{T_0 - T_h} = \theta(\eta), \end{aligned} \tag{8}$$

Then equation (8) will be automatically satisfied the continuity equation (1). Substituting the above variables into equations (2) and (3) and then eliminating the pressure gradient from the resulting equations gives

$$f^{iv} + S\left(ff''' - f'f''\right) - Mf' - 2K_r g' = 0, \tag{9}$$

Substitution of equation (8) in equations (4)-(5) gives

$$g'' + S\left(fg' - f'g\right) - Mg + 2K_r f' = 0, \tag{10}$$

$$\theta''\left(1 + R\left(1 + (\theta_w - 1)\theta\right)^3\right) + f\theta' - \theta' f + 3R\left(\theta_w - 1\right)\theta'^2\left(\left(1 + \left(\theta_w - 1\right)\theta\right)\right)^2 + A^* f' + B^*\theta = 0 \tag{11}$$

The corresponding transformed boundary conditions are,

$$\begin{aligned} & f = 0, f' = 1, g = 0, \theta'(\eta) = -Bi(1 - \theta(\eta)) \; at\, \eta = 0, \\ & f = 0, f' = 0, g = 0, \theta = 0, \qquad at\; \eta = 1, \end{aligned} \tag{12}$$

Where, S is the viscosity parameter, M is the magnetic field parameter, K_r is the rotation parameter, $\Pr$ is the Prandtl number, Bi is Biot number, A^*, B^* is Non-uniform heat source/sink parameter, R is Radiation parameter, θ_w is temperature ratio parameter. Which are given by

$$S = \frac{ah^2}{\upsilon} \; M = \frac{\sigma B_0^2 h^2}{\rho\upsilon} \; K_r = \frac{\Omega h^2}{\upsilon} \; \Pr = \frac{\mu c_p}{k} \tag{13}$$

For physical quantities of engineering interest are the skin-friction coefficient (C_f) and local Nusselt number (Nu) along the stretching wall are given by

$$C_f{}^* = \frac{Rx}{h} C_f = f''(0), Nu = -\theta'(0), \tag{14}$$

3. Results and Discussion

The coupled non-linear ordinary differential equation (9), (10) and (11) subjected to the boundary conditions (12) are solved numerically using Runge-Kutta based shooting technique. Further the effects of various governing parameters namely Viscosity parameter (S),Magnetic parameter (M), Rotation parameter (K_r), Radiation parameter (R), Ratio of temperature (θ_w), Non-uniform heat source/sink (A^*, B^*) on the velocity and temperature profiles along with friction factor and local Nusselt numbers are discussed with the help of graphs and tables. For numerical results we considered $M = 3$, $K_r = 2$, $S = 0.5$, $\Pr = 7$, these values are treated as common for the entire study except the varied values as displayed in the respective figures and tables.

Figure 2 illustrates the temperature profiles for the different values of the Radiation parameter (R). It is clear that the temperature profile of the flow is increasing with increasing the value of the Radiation parameter. The effect of ratio of temperature parameter on the temperature field is displayed in Figure 3. It is noticed that increasing the value of ratio temperature parameter and it enhances the temperature parameter. The effect of Biot number on temperature profile is displayed in the Figure 4. It is observed that increasing values of Biot number, it improves the temperature profile. Figures 5-7 depicts the effects of viscosity parameter on the $\theta(\eta)$, $f(\eta)$ and $g(\eta)$. As viscosity parameter increases, temperature profile and velocity profile decreases between the plates. Generally, viscous forces have less domination in the flow. Due to this reason we have seen a decrement in the velocity profiles of the flow.

Figures 8-10 display the influence of rotation parameter (K_r) on velocity and temperature profile. It noticed that increases in rotation parameter decline the velocity profile and it enhances the temperature profile. The influence of non- uniform heat source/sink parameter on temperature profiles is shown in Figures 11 and 12. It is evident that increasing the value of heat source sink parameter and it is enhancing the temperature profile. Table 1 shows the variation in friction factor and heat transfer rate for different values of physical parameters.

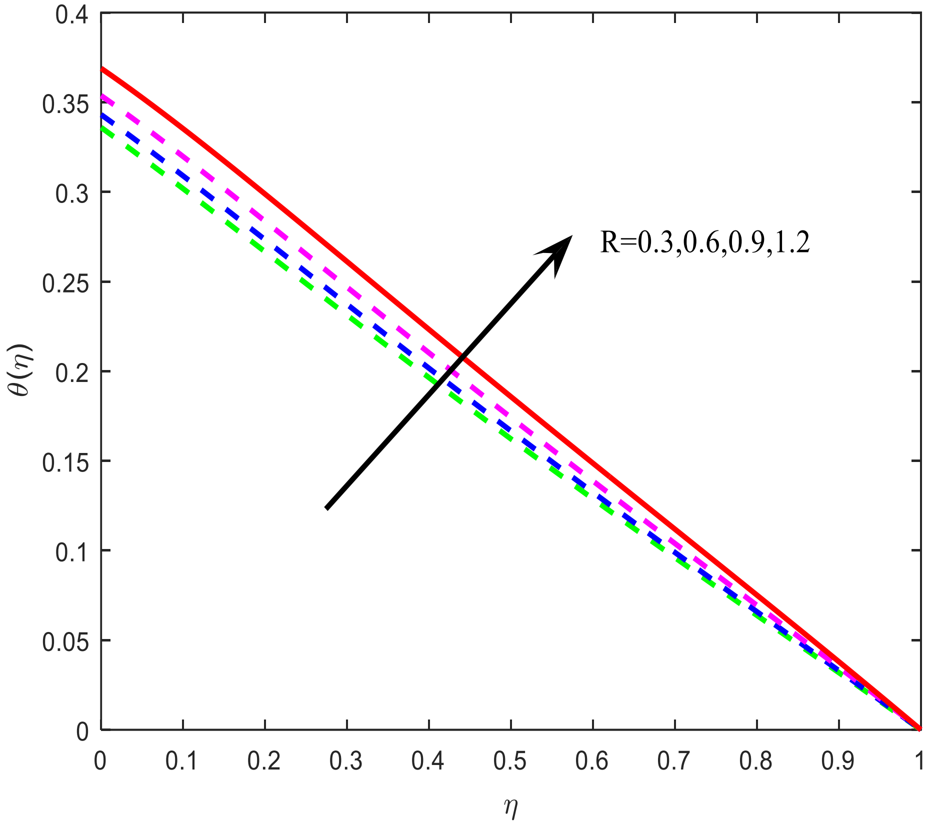

Figure 2. Temperature Profile for Different Values of R

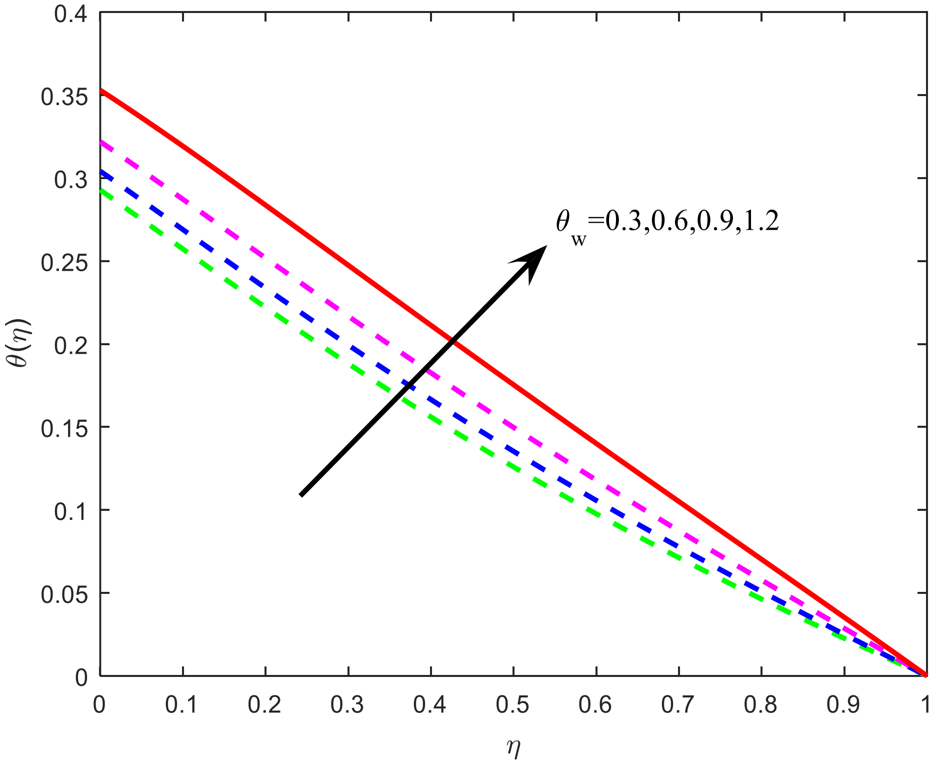

Figure 3. Temperature Profile for Different Values of θ_w

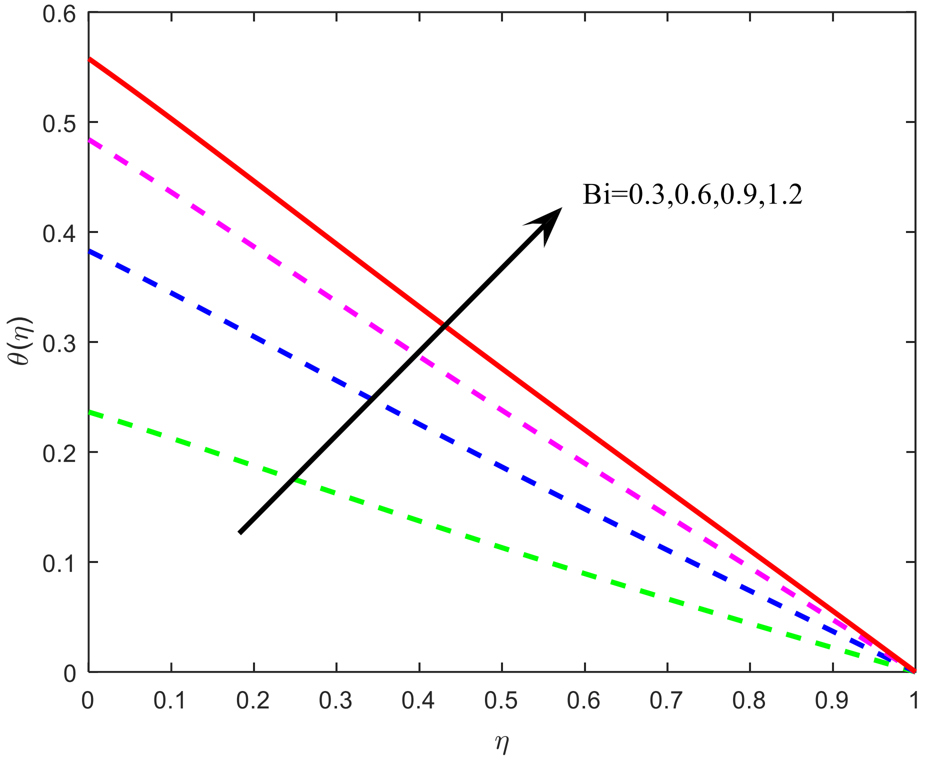

Figure 4. Temperature Profile for Different Values of Bi

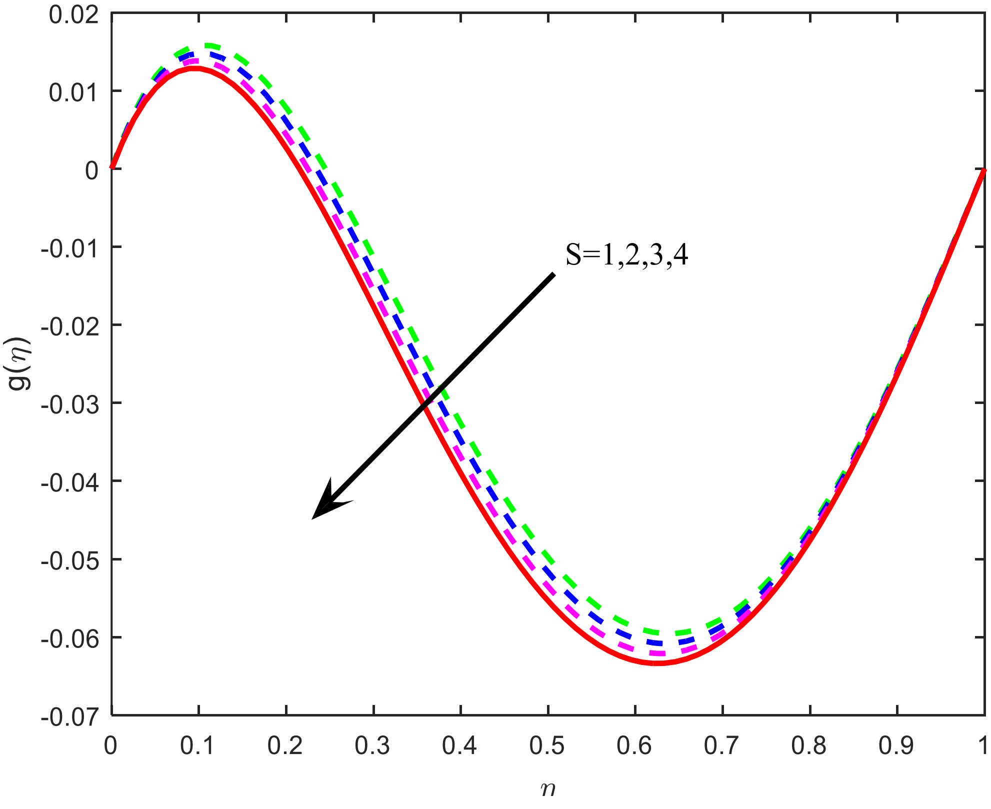

Figure 5. Velocity Profile for Different Values of S

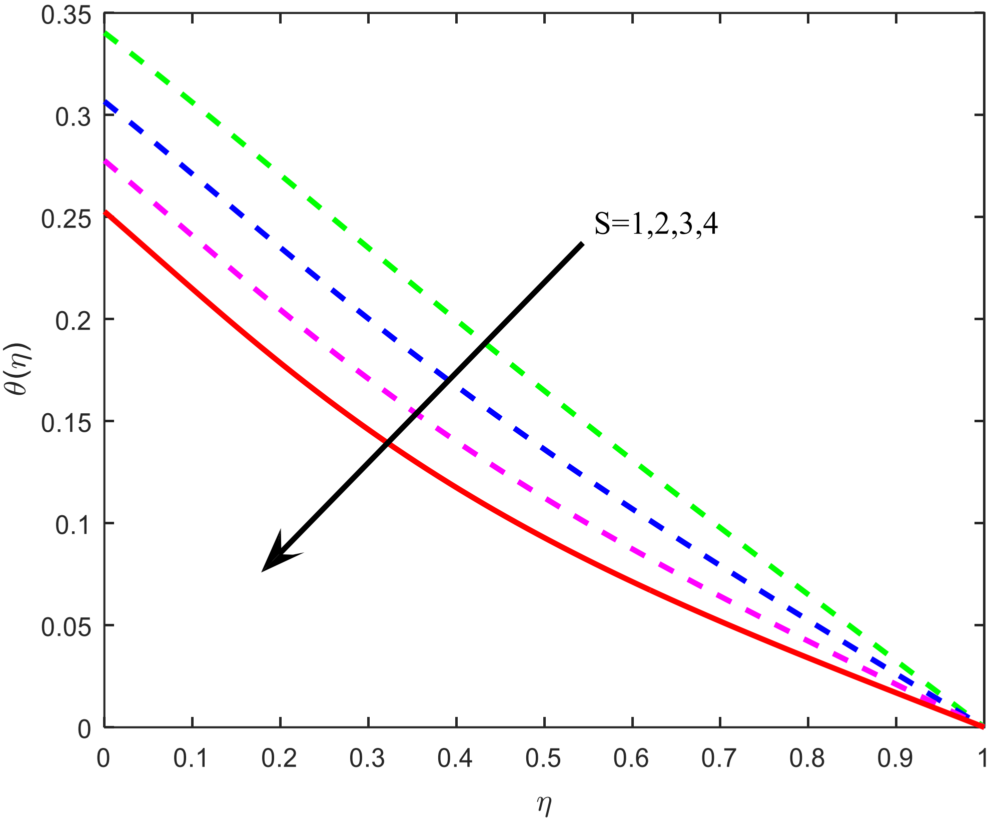

Figure 6. Temperature Profile for Different Values of S

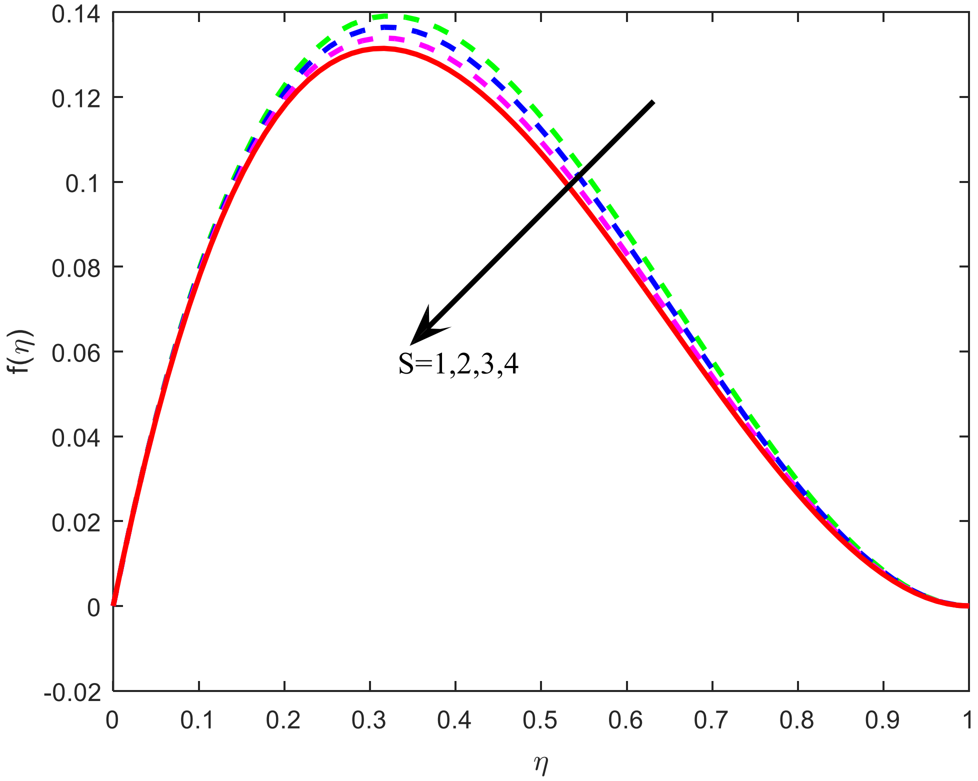

Figure 7. Velocity Profile for Different Values of S

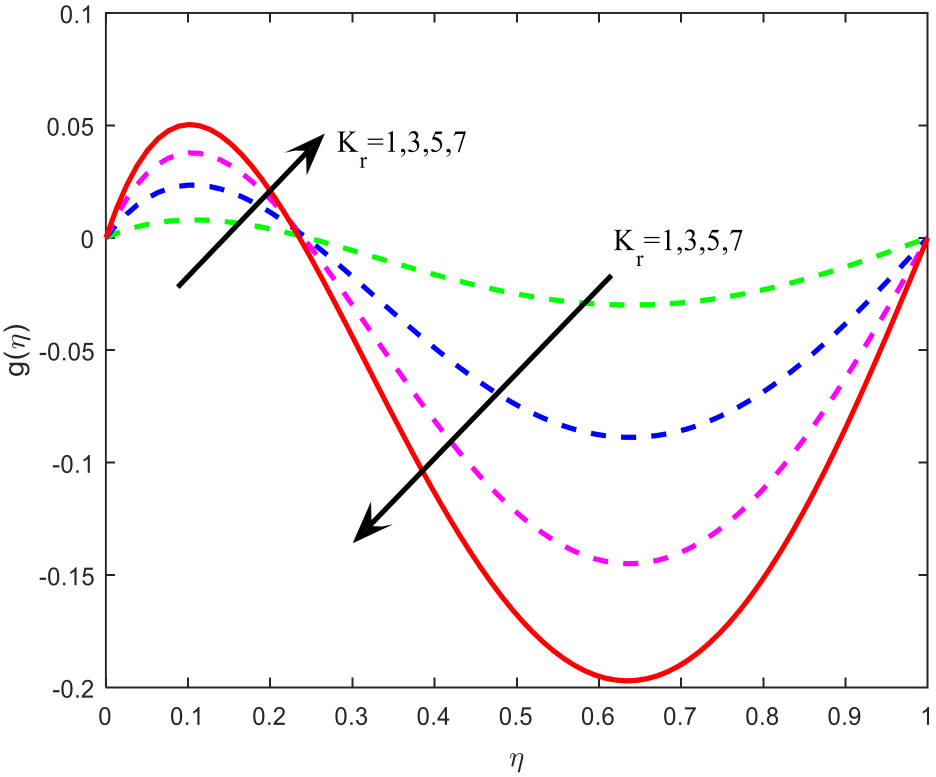

Figure 8. Velocity Profile for Different Values of K_r

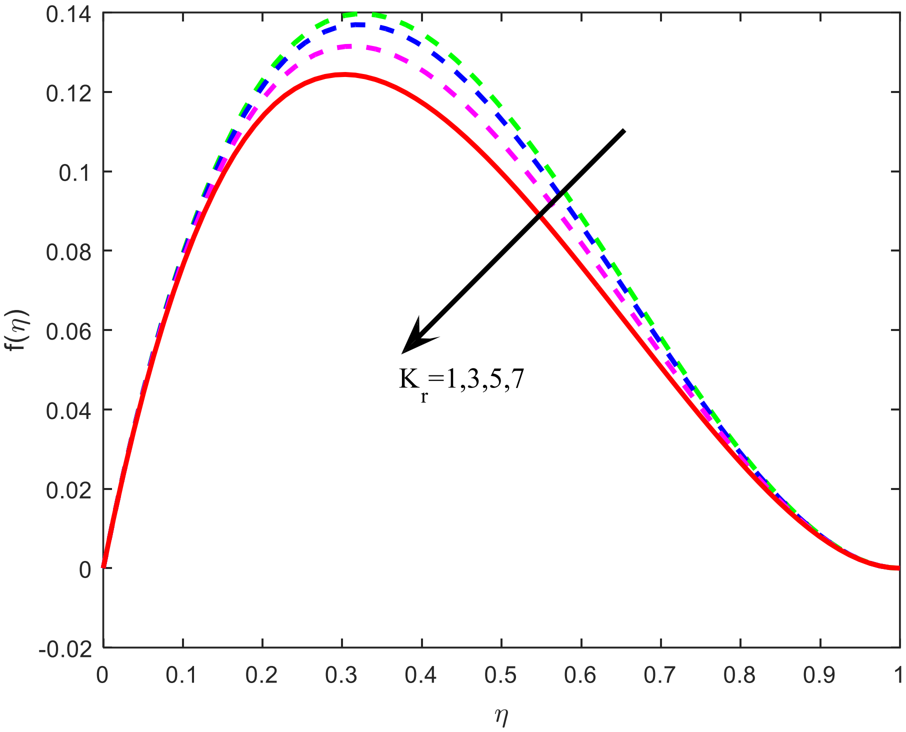

Figure 9. Velocity Profile for Different Values of K_r

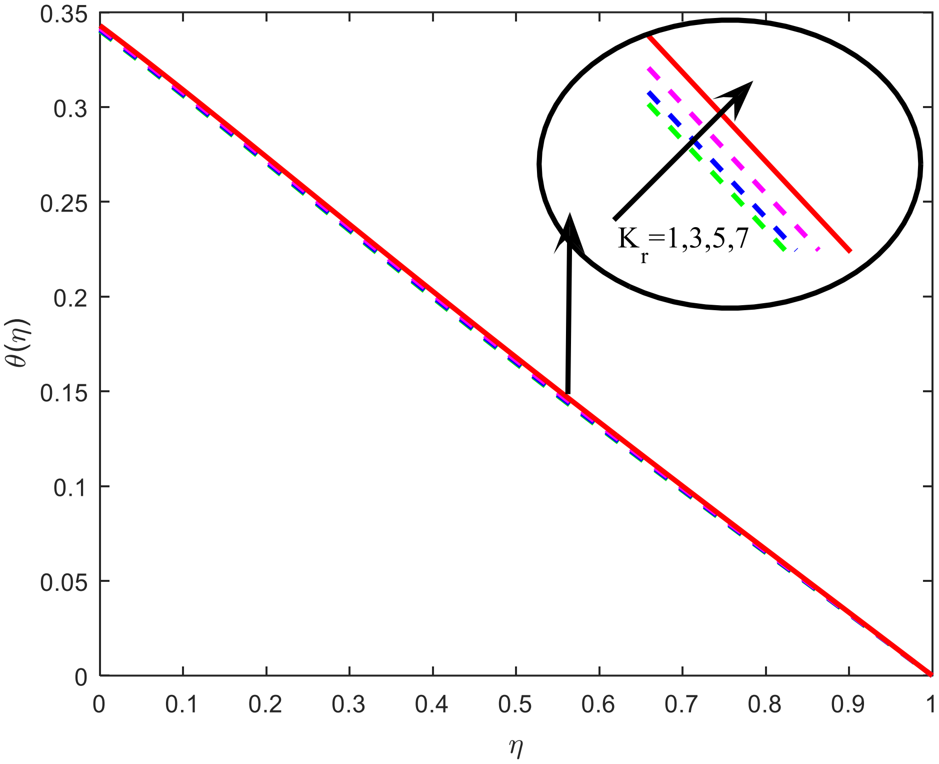

Figure 10. Temperature Profile for Different Values of K_r

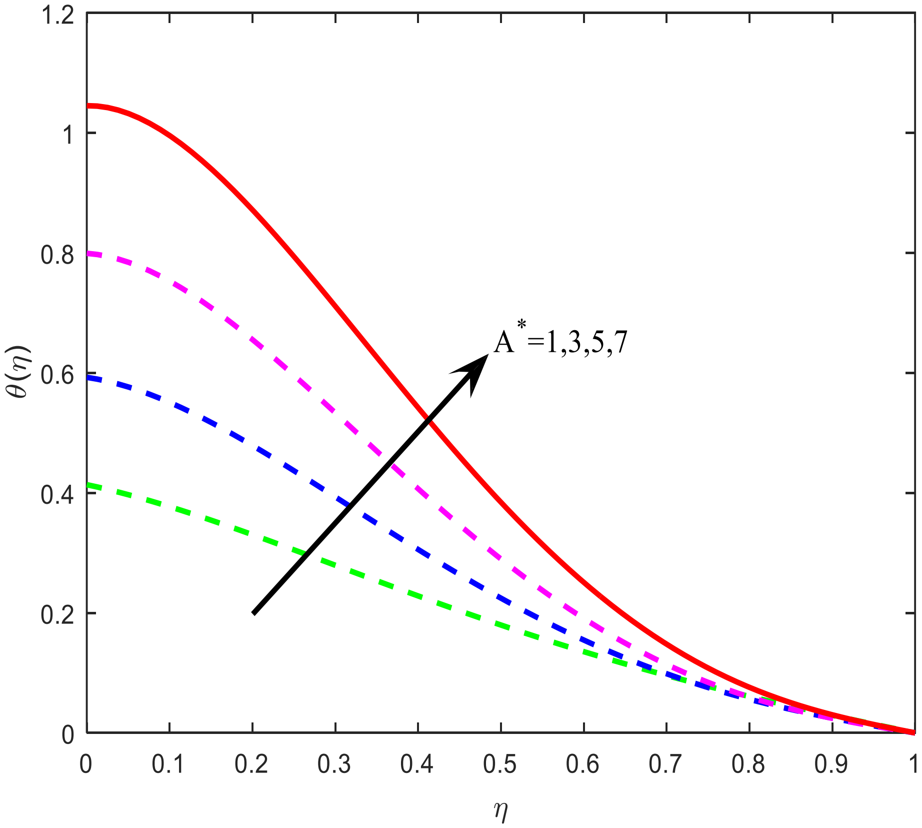

Figure 11. Temperature Profile for Different Values of A^*

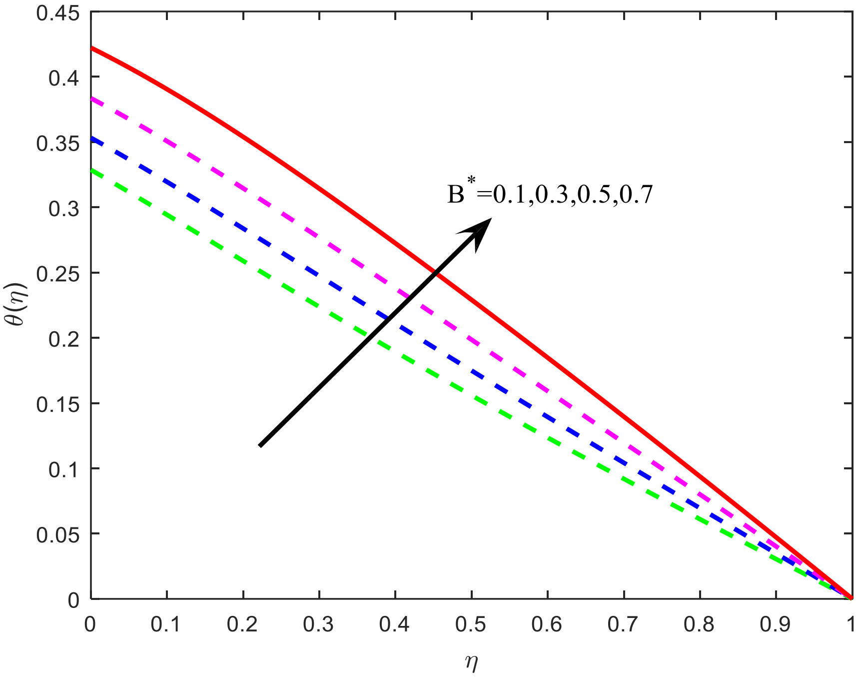

Figure 12. Temperature Profile for Different Values of B^*

Table 1. Physical Parameter Values at Different Non-Dimensional Parameters

R	θ_w	Bi	S	k_r	A^*	B^*	M	$f''(0)$	$-\theta'(0)$
0.3								-4.499393	0.332034
0.6								-4.499393	0.328463
0.9								-4.499393	0.323145
	0.3							-4.499394	0.353596
	0.6							-4.499393	0.347918
	0.9							-4.499393	0.339055
		0.3						-4.499393	0.229076
		0.6						-4.499393	0.370243
		0.9						-4.499393	0.464244
			1					-4.499393	0.329828
			2					-4.580666	0.346651
			3					-4.661546	0.361148
				1				-4.474960	0.329878
				3				-4.539640	0.329746
				5				-4.664612	0.329494
					1			-4.499393	0.293169
					3			-4.499394	0.203715
					5			-4.499394	0.100368
						0.1		-4.499393	0.335658
						0.3		-4.499393	0.323391
						0.5		-4.499393	0.308233
							1	-4.253791	0.330213
							3	-4.499393	0.329828
							5	-4.734748	0.329471

4. Conclusions

Nonlinear thermal radiation and non-uniform heat source/sink on MHD fluid flow between rotating horizontal plates with convective boundary conditions are studied. By making use of the appropriate self-similarity transformations, the equations which govern the boundary layer equations are reduced into a set of nonlinear ordinary differential equations. Further, the resultant equations are solved numerically using the Runge-Kutta based shooting technique. Conclusions of the present study are as follows:

- Increasing values of the Rotation parameter decreases the skin friction coefficient and enhances the local Nusselt number.
- Rise in the strength of magnetic field decreases both the skin friction and the local Nusselt number.
- An increase in the Biot number enhances the temperature profiles of the flow.
- Viscosity parameter have tendency to enhance the heat transfer rate.

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