

## Heat and Mass Transfer in Unsteady MHD Casson Fluid Flow with Convective Boundary Conditions

K. Pushpalatha<sup>1</sup>, V.Sugunamma<sup>2\*</sup>, J. V. Ramana Reddy<sup>3</sup> and N. Sandeep<sup>4</sup>

<sup>1,2,3</sup>Department of Mathematics, S.V.University, Tirupati-517502, India

<sup>4</sup>Department of Mathematics, VIT University, Vellore-632014, India

<sup>2</sup>vsugunar@gmail.com

### Abstract

Through this paper, we investigated the unsteady free convection flow of a Casson fluid bounded by a moving vertical flat plate in a rotating system with convective boundary conditions. The governing equations of the flow have been solved analytically using perturbation technique. The effects of various non-dimensional governing parameters like Casson parameter, magnetic field parameter, thermal diffusion parameter, chemical reaction parameter and thermal radiation parameter on velocity, temperature and concentration profiles are discussed and presented through graphs. We also evaluated the friction factor, Nusselt and Sherwood numbers and presented numerically. Through this study it is found that the Casson parameter controls the velocity profiles and Soret number have tendency to enhance the both velocity and concentration fields.

**Keywords:** Casson fluid, Soret effect, Rotation, Radiation, MHD, Chemical reaction

### Nomenclature:

$u, v, w$  : Velocity components of the fluid in  $x, y$  and  $z$  directions respectively

$U_r$  : Velocity characteristic

$x, y, z$  : Cartesian coordinates

$t$  : Time

$\Omega$  : Rotating velocity of the system

$\rho$  : Density of the fluid

$\mu$  : Dynamic viscosity of the fluid

$\beta$  : Casson parameter

$\nu$  : Kinematic viscosity of the fluid

$\beta$  : Coefficient of thermal expansion of the fluid due to temperature difference

$g$  : Acceleration due to gravity

$T$  : Temperature of the fluid

$T_\infty$  : Ambient temperature of the fluid

$T_w$  : Temperature of the fluid near the plate

$\beta_T$  : Coefficient of thermal expansion of the fluid due to temperature difference

$\beta_c$  : Coefficient of thermal expansion of the fluid due to concentration difference

$C$  : Concentration of the fluid

$C_\infty$  : Ambient Concentration of the fluid

$C_w$  : Concentration of the fluid near the plate

$B_0$	: Uniform magnetic field
$\sigma$	: Electrical conductivity of the fluid
$c_p$	: Specific heat capacity of the fluid at constant pressure
$q_r$	: The radiative heat term
$k$	: Thermal conductivity of nanofluid
$D_B$	: Chemical molecular diffusivity
$k_l$	: Dimensioned chemical reaction parameter
$N_c$	: Convective parameter
$N_d$	: Diffusive parameter
$S$	: Suction/injection parameter
$M$	: Magnetic field parameter
$R$	: Rotational parameter
$N$	: Thermal radiation parameter
$K_r$	: Dimensionless chemical reaction parameter
$Pr$	: Prandtl number
$Sc$	: Schmidt number
$C_f$	: Skin friction coefficient
$Nu$	: Local Nusselt number
$Sh_x$	: Sherwood number

## 1. Introduction

Non-Newtonian fluid flow arises in many branches of chemical and material processing engineering. There are different types of non-Newton fluids like Viscoelastic fluid, couple stress fluid, micropolar fluid and power-law fluid etc. In addition with these, there is another non-Newtonian fluid model is known as the Casson fluid model. In the published literature, it is sometimes claimed that for many materials, the Casson model is better than the general visco plastic models in fitting the rheological data. So, it becomes the preferred rheological model for blood and chocolate. The influence of thermal radiation and chemical reaction on micro polar fluid flow in a rotating frame was discussed by Das [1] and concluded that an increase in the volume fraction of nano particles enhances the velocity profiles. Hayat *et al.*, [2] discussed the cross diffusion effects on MHD Casson fluid flow.

Rashidi *et al.*, [3] analytically discussed the steady flow over a rotating disk in a porous medium by using homotopy analysis method. The effects of radiation on unsteady free convection flow of a nanofluid past an infinite plate was discussed by Sandeep *et al.*, [4]. Further Sandeep and Sugunamma [5] studied the effect of inclined magnetic field on dusty viscous fluid between two infinite flat plates. Nandy [6] studied the heat transfer characteristics of MHD Casson fluid flow over a stretching sheet and found that an increase in the value of dimensionless thermal slip parameter reduces the velocity profiles. Sandeep and Sugunamma [7] discussed the effects of radiation and inclined magnetic field on natural convection flow over an impulsively moving vertical plate. The influence of chemical reaction on MHD flow past a stretching sheet with heat generation has been investigated by Mohan krishna *et al.*, [8]. Das [9] analyzed the effects of magnetic field and volume fraction of nano particles on nanofluid flow in a rotating frame by considering convective boundary conditions. The effect of thermal diffusion on unsteady MHD dusty fluid flow was studied by Ramana Reddy *et al.*, [10]. Sandeep *et al.*,

[11] investigated the unsteady MHD free convection flow past an impulsively moving vertical plate with radiation and rotation effects.

Ramana Reddy *et al.*, [12] studied the influence of nonlinear thermal radiation on MHD flow between rotating plates with homogeneous and heterogeneous reactions. Heat transfer characteristics on Casson fluid flow between parallel plates were analyzed by Khan *et al.*, [13]. Ramana Reddy *et al.*, [14] discussed the effects of radiation and chemical reaction on the flow of nanofluid by taking into consideration of electrical conductivity of the nanofluid. Further, the impact of thermal diffusion and hall current on nanofluids under the influence of inclined magnetic field was studied in detail by Ramana Reddy *et al.*, [15]. The effects of hall current and thermal diffusion on unsteady micropolar fluid flow past an infinite vertical plate were discussed by Anika *et al.*, [16]. Sulochana *et al.*, [17] investigated the effects of Dufour and Soret on nanofluid over an exponentially stretching sheet with aligned magnetic field. Through this paper, it is found that an increase in the aligned angle decreases the velocity profiles.

Raju *et al.*, [18] studied the effects of heat and mass transfer on MHD Casson fluid flow past an exponentially permeable stretching sheet. Recently, Sulochana *et al.* [19] discussed the influence of non linear thermal radiation on MHD 3D Casson fluid flow with viscous dissipation. A comparative study has been done by Sandeep *et al.* [20] to study the heat and mass transfer characteristics in non-Newtonian nanofluid past a permeable stretching surface. Raju *et al.*, [21] discussed the effects of thermal diffusion and diffusion thermo on the flow over a stretching surface with inclined magnetic field. This paper concludes that an increase in the Soret number increases the friction factor but reduces the heat transfer rate. Very recently, the researchers [22-31] investigated the heat and mass transfer characteristics of non-Newtonian and Newtonian flows by considering various channels. They concluded that heat and mass transfer is not uniform in all channels. It varies according to the channel.

By making use of all the above cited articles, we investigated the effects of thermal diffusion and radiation on Casson fluid flow with convective boundary conditions. The governing equations of the flow are first converted into dimensionless form and then solved analytically. Finally the influence of various physical parameters involved in the flow has been discussed in detail.

## 2. Mathematical Analysis:

Consider an unsteady free convection flow of an electrically conducting, incompressible Casson fluid of an ambient temperature  $T_\infty$  past a semi-infinite vertical moving plate with convective boundary conditions. Fig.1 describes the physical model and co-ordinate system. The flow is assumed to be in the  $x$ -direction, which is taken along the plate in the upward direction and  $z$ -axis is normal to it. Also it is assumed that the whole system is rotating with a constant velocity  $\Omega$  about  $z$ -axis. A uniform external magnetic field  $B_0$  is taken to be acting along  $z$ -axis.

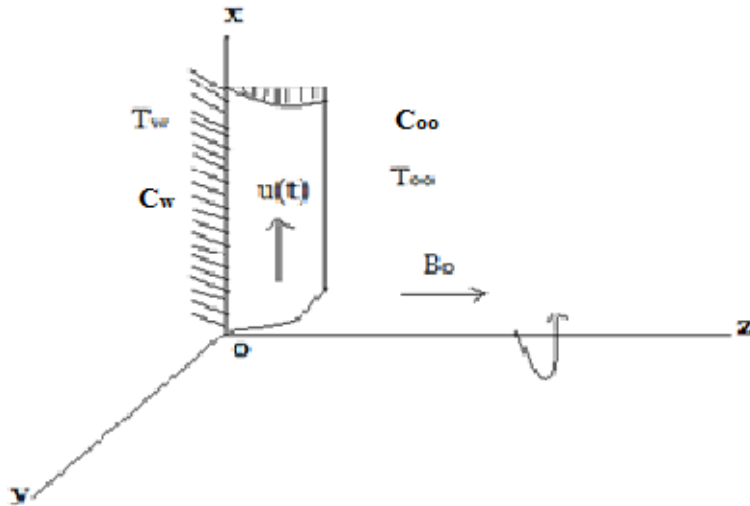
The rheological equation of state for the Cauchy stress tensor of Casson fluid can be written as,

$$\tau = \tau_0 + \mu \dot{\gamma}$$

or

$$\tau_{ij} = \begin{cases} 2 \left( \mu_B + \frac{p_y}{\sqrt{2\pi}} \right) e_{ij} & ; \pi > \pi_c \\ 2 \left( \mu_B + \frac{p_y}{\sqrt{2\pi_c}} \right) e_{ij} & ; \pi < \pi_c \end{cases}$$

where  $\pi = e_{ij} e_{ij}$  and  $e_{ij}$  is the  $(i, j)^{th}$  component of the deformation rate with itself, is the critical value of this product based on the non-Newtonian model,  $\mu_B$  is the plastic dynamic viscosity of the non-Newtonian fluid and  $p_y$  is yield stress of the fluid.



**Figure 1. Physical Model of the Problem**

The governing equations of the flow are given by,

$$\frac{\partial w}{\partial z} = 0, \quad (1)$$

$$\rho \left( \frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} + 2\Omega v \right) = \mu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial z^2} + \rho \beta_T g (T - T_\infty) + \rho \beta_C g (C - C_\infty) - \sigma B_0^2 u, \quad (2)$$

$$\rho \left( \frac{\partial v}{\partial t} + w \frac{\partial v}{\partial z} - 2\Omega u \right) = \left[ \mu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 v}{\partial z^2} - \sigma B_0^2 v \right], \quad (3)$$

$$\rho C_p \left( \frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z} \right) = k \frac{\partial^2 T}{\partial z^2} - \frac{\partial q_r}{\partial z}, \quad (4)$$

$$\frac{\partial C}{\partial t} + w \frac{\partial C}{\partial z} = D_m \frac{\partial^2 C}{\partial z^2} - k_l (C - C_\infty) + \frac{D_m K_l}{T_m} \frac{\partial^2 T}{\partial z^2}, \quad (5)$$

Where  $u, v, w$  are velocity components along  $x, y, z$ -axis directions respectively.  $\rho$  is the density of the fluid,  $\mu$  is the viscosity of the fluid,  $\beta$  is the Casson parameter,  $\beta_T$  is the coefficient of thermal expansion of the fluid due to temperature difference,  $\beta_C$  is the coefficient of thermal expansion of the fluid due to concentration difference,  $g$  is the acceleration due to gravity,  $\sigma$  is the electrical conductivity of the fluid,  $\rho c_p$  is the heat capacitance of the fluid,  $q_r$  is the radiative heat term,  $D_m$  is the diffusion parameter and,  $k_l$  is the dimensioned chemical reaction parameter. Further, we assumed that the plate surface temperature is maintained by convective heat transfer at a certain value  $T_w$ . So the boundary conditions for this problem are given by

$$u(z, t) = 0, v(z, t) = 0, T(z, t) = T_\infty, C(z, t) = C_\infty, \quad \text{for } t \leq 0 \text{ and any } z, \quad (6)$$

$$u(z, t) = U_r \left[ 1 + \frac{\varepsilon}{2} (e^{\text{int}} + e^{-\text{int}}) \right], v(z, t) = 0, \quad \text{for } t > 0 \text{ and } z = 0, \quad (7)$$

$$-k \frac{\partial T}{\partial z} = h_f (T_w - T), -D_m \frac{\partial C}{\partial z} = h_s (C_w - C),$$

$$u(z, t) \rightarrow 0, v(z, t) \rightarrow 0, T(z, t) \rightarrow T_\infty, C(z, t) \rightarrow C_\infty, \quad \text{for } t > 0 \text{ and } z \rightarrow \infty, \quad (8)$$

where  $U_r$  is the uniform velocity and  $\varepsilon$  is the small constant quantity. The Oscillatory plate velocity in Eq. (7) is taken.

The radiative heat term by using the Rosseland approximation is given by

$$\frac{\partial q_r}{\partial z} = -16 \frac{T_\infty^3 \sigma^*}{3k^*} \frac{\partial^2 T}{\partial z^2}, \quad (9)$$

We introduce the following non dimension variables into Eqs. (1)-(5).

$$u' = \frac{u}{U_r}, v' = \frac{v}{U_r}, z' = \frac{zU_r}{\nu}, t' = \frac{tU_r^2}{\nu}, n' = \frac{n\nu}{U_r^2}, \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \psi = \frac{(C - C_\infty)}{(C_w - C_\infty)}, \quad (10)$$

here,  $\nu$  is the kinematic viscosity of the fluid.

Using the equations (9) and (10) in equations (2)-(5), yields the following dimensionless equations. (after dropping the primes)

$$\frac{\partial u}{\partial t} - S \frac{\partial u}{\partial z} + Rv = A_1 \frac{\partial^2 u}{\partial z^2} + Gr\theta + Gc\psi - Mu, \quad (11)$$

$$\frac{\partial V}{\partial t} - S \frac{\partial u}{\partial z} - Ru = A_1 \frac{\partial^2 v}{\partial z^2} - Mv, \quad (12)$$

$$\text{Pr} \left( \frac{\partial \theta}{\partial t} - S \frac{\partial \theta}{\partial z} \right) = (1 + N) \frac{\partial^2 \theta}{\partial z^2}, \quad (13)$$

$$\frac{\partial \psi}{\partial t} - S \frac{\partial \psi}{\partial z} = \frac{1}{Sc} \frac{\partial^2 \psi}{\partial z^2} - Kr\psi + Sr \frac{\partial^2 \theta}{\partial z^2}, \quad (14)$$

Here  $R = \frac{2\Omega\nu}{U_r^2}$  is the rotational parameter,  $s = \frac{w_0}{U_r}$  is the suction ( $s > 0$ ) or injection

( $s < 0$ ) parameter,  $M = \frac{\sigma B_0^2}{\rho U_r^2}$  is the magnetic field parameter,  $\text{Pr} = \frac{\mu C_p}{k}$  is the Prandtl

number,  $Kr = \frac{k_r\nu}{U_r^2}$  is the chemical reaction parameter,  $Sc = \frac{\nu}{D_B}$  is the Schmidt number,

$N = \frac{16\sigma^* T_\infty^3}{kk^*}$  is the thermal radiation parameter,  $Sr = \frac{D_m K_T (T_w - T_\infty)}{T_m \nu (C_w - C_\infty)}$  is the Soret

number,  $Gr = \frac{\nu \beta_T g (T_w - T_\infty)}{U_r^3}$  is the thermal Garshof number and  $Gc = \frac{\nu \beta_C g (C_w - C_\infty)}{U_r^3}$  is

the mass Garshof number.

Also the boundary conditions (6)-(8) become

$$u = 0, v = 0, \theta = 0, \psi = 0 \text{ for } t \leq 0 \text{ and for any } z. \quad (15)$$

$$\left. \begin{aligned} u &= \left\{ 1 + \frac{\varepsilon}{2} (e^{\text{int}} + e^{-\text{int}}) \right\}, v = 0, \\ \theta'(z) &= -N_c (1 - \theta(z)), \psi'(z) = -N_d (1 - \psi(z)) \end{aligned} \right\} \text{for } t > 0 \text{ and } z = 0, \quad (16)$$

$$u \rightarrow 0, v \rightarrow 0, \theta \rightarrow 0, \psi \rightarrow 0, \text{ for } t > 0 \text{ as } z \rightarrow \infty, \quad (17)$$

where  $N_c = \frac{h_f v}{k U_r}$  is the convective parameter and  $N_d = \frac{h_s v}{D_m U_r}$  is the diffusive Parameter.

We now simplify equations (11) and (12) by putting the fluid velocity in the complex form as  $V = u + iv$  and we get

$$\frac{\partial V}{\partial t} - S \frac{\partial V}{\partial z} - iRv = A_1 \frac{\partial^2 v}{\partial z^2} + Gr\theta + Gr\psi - Mv, \quad (18)$$

The corresponding boundary conditions become

$$V = 0, \theta = 0, \psi = 0, \quad \text{for } t \leq 0, \quad (19)$$

$$\left. \begin{aligned} V(z) &= \left\{ 1 + \frac{\varepsilon}{2}(e^{int} + e^{-int}) \right\}, \\ \theta'(z) &= -N_c(1 - \theta(z)), \psi'(z) = -N_d(1 - \psi(z)), \end{aligned} \right\} \text{ at } z = 0 \text{ and } t > 0, \quad (20)$$

$$V \rightarrow 0, \theta \rightarrow 0, \psi \rightarrow 0, \quad \text{for } t > 0 \text{ as } z \rightarrow \infty, \quad (21)$$

### 3. Solution of the Problem:

To obtain the solution of the system of partial differential equations (13), (14) and (18) under the boundary conditions given by equations (19)-(21), we express  $v$ ,  $\theta$  and  $\psi$  as

$$V(z, t) = V_0 + \frac{\varepsilon}{2} [e^{int} V_1(z) + e^{-int} V_2(z)], \quad (22)$$

$$\theta(z, t) = \theta_0 + \frac{\varepsilon}{2} [e^{int} \theta_1(z) + e^{-int} \theta_2(z)], \quad (23)$$

$$\psi(z, t) = \psi_0 + \frac{\varepsilon}{2} [e^{int} \psi_1(z) + e^{-int} \psi_2(z)], \quad (24)$$

Substituting the above equations (22)-(24) in the equations (13), (14) and (18), and equating the harmonic and non-harmonic terms and neglecting the higher order terms of  $\varepsilon^2$ , we get following equations.

The zeroth order equations are:

$$A_1 \frac{\partial^2 V_0}{\partial z^2} + S \frac{\partial V_0}{\partial z} + V_0(iR - M) + Gr\theta_0 + Gc\psi_0 = 0, \quad (25)$$

$$A_2 \frac{\partial^2 \theta_0}{\partial z^2} + S \frac{\partial \theta_0}{\partial z} = 0, \quad (26)$$

$$A_3 \frac{\partial^2 \psi_0}{\partial z^2} + S \frac{\partial \psi_0}{\partial z} - Kr\psi_0 + Sr \frac{\partial^2 \theta_0}{\partial z^2} = 0, \quad (27)$$

The first order equations are:

$$A_1 \frac{\partial^2 V_1}{\partial z^2} + S \frac{\partial V_1}{\partial z} + (i(R - n) - M)V_1 + Gr\theta_1 + Gc\psi_1 = 0, \quad (28)$$

$$A_2 \frac{\partial^2 \theta_1}{\partial z^2} + S \frac{\partial \theta_1}{\partial z} - in\theta_1 = 0, \quad (29)$$

$$A_3 \frac{\partial^2 \psi_1}{\partial z^2} + S \frac{\partial \psi_1}{\partial z} - (in + Kr)\psi_1 + Sr \frac{\partial^2 \theta_1}{\partial z^2} = 0, \quad (30)$$

The second order equations are:

$$A_1 \frac{\partial^2 V_2}{\partial z^2} + S \frac{\partial V_2}{\partial z} + (i(R + n) - M)V_2 + Gr\theta_2 + Gc\psi_2 = 0, \quad (31)$$

$$A_2 \frac{\partial^2 \theta_2}{\partial z^2} + S \frac{\partial \theta_2}{\partial z} + in\theta_2 = 0, \quad (32)$$

$$A_3 \frac{\partial^2 \psi_2}{\partial z^2} + S \frac{\partial \psi_2}{\partial z} + (in - Kr)\psi_2 + Sr \frac{\partial^2 \theta_2}{\partial z^2} = 0, \quad (33)$$

Where  $v_0, \theta_0, \psi_0, v_1, \theta_1, \psi_1, v_2, \theta_2$  and  $\psi_2$  are functions of  $z$  only and prime denotes the differentiation with respect to  $z$ .

The corresponding boundary conditions are given by

$$\left. \begin{aligned} V_0 = V_1 = V_2 = 1, \\ \theta_0' = -N_c(1 - \theta_0), \theta_1' = N_c\theta_1, \theta_2' = N_c\theta_2, \\ \psi_0' = -N_d(1 - \psi_0), \psi_1' = N_d\psi_1, \psi_2' = N_d\psi_2, \end{aligned} \right\} \text{ at } z = 0, \quad (34)$$

$$\left. \begin{aligned} V_0 \rightarrow 0, V_1 \rightarrow 0, V_2 \rightarrow 0, \\ \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, \theta_2 \rightarrow 0, \\ \psi_0 \rightarrow 0, \psi_1 \rightarrow 0, \psi_2 \rightarrow 0, \end{aligned} \right\} \text{ as } z \rightarrow \infty, \quad (35)$$

Solving the equations from (25) - (33) under the boundary conditions (34) and (35), we will obtain the expressions for  $v_0, \theta_0, \psi_0, v_1, \theta_1, \psi_1, v_2, \theta_2$  and  $\psi_2$ . Now substitution of this in (23) - (24), gives the expressions for velocity, temperature and concentration as follows.

$$V = (1 + A_{16} + A_{17})e^{-A_{15}z} - A_{16}e^{-A_7z} - A_{17}e^{-A_{11}z} + \frac{\varepsilon}{2}(e^{-A_{18}z}e^{\text{int}} + e^{-A_{19}z}e^{-\text{int}}), \quad (36)$$

$$\theta = A_8e^{-A_7z}, \quad (37)$$

$$\psi = A_{12}e^{-A_7z} + A_{13}e^{-A_{11}z}, \quad (38)$$

For engineering interest the local skin friction coefficient  $C_f$ , local Nusselt number  $Nu_x$  and local Sherwood number  $Sh_x$  are defined by

$$C_f = \frac{\tau_w}{\rho U_w^2}, Nu = \frac{xq_w}{k(T_w - T_\infty)} \text{ and } Sh_x = \frac{xq_m}{D_B(T_w - T_\infty)}, \quad (39)$$

$$C_f = -A_{15}(1 + A_{16} + A_{17}) + A_7A_{16} + A_{11}A_{17} + \left(\frac{\varepsilon}{2}\right)(-A_{18}e^{\text{int}} - A_{19}e^{-\text{int}}), \quad (40)$$

$$\frac{Nu}{Re} = A_7A_8, \quad (41)$$

$$Sh_x = A_7A_{12} + A_{11}A_{13}, \quad (42)$$

## 4. Results and Discussion

The results obtained shows the influence of the non-dimensional governing parameters, namely Magnetic field parameter  $M$ , Thermal diffusion parameter (Soret number)  $sr$ , Rotation parameter  $R$ , Convective parameter  $N_c$ , Diffusive parameter  $N_d$ , Suction parameter  $S$ , Radiation parameter  $N$  and Chemical reaction parameter  $Kr$  on velocity, temperature, concentration, skin friction coefficient, local Nusselt and Sherwood

numbers. For graphical results we considered  $n = 10, \beta = 0.3, Gr = Gc = 3, N = M = 2, Kr = 0.5, Pr = 6.72, Sr = 0.5, Sc = 0.6, t = 0.5, S = 2, R = 0.8, N_c = 0.2, N_d = 0.3$ . These values are kept as common in entire study except the varied values as shown in respective figures and tables.

Figure 2 represents the velocity profiles for different values of Casson parameter  $\beta$ . It is observed that an increase in the Casson parameter causes for a depreciation in the velocity profiles. Figures 3-4 display the influence of Soret number  $S_r$  on velocity and concentration profiles respectively. From these figures, we may conclude that both the velocity and concentration profiles enhances with an increase in the Soret number. This is due to the fact that an increase in Soret number causes for thicker momentum and concentration boundary layers.

Figures 5-7 depict the velocity, temperature and concentration profiles for different values of Radiation parameter  $N$ . It found that an increase in the Radiation parameter develops the momentum boundary layer thickness. Also temperature and concentration profiles rises with an increase in  $N$ . This is due to the fact that increasing values of Radiation parameter generates the heat energy to the flow. From Figure 8, it is found that an increase in the Magnetic field parameter  $M$ , slowdowns the motion of the fluid. The reason behind this is an increase in the magnetic field leads to a force called Lorentz force. This Lorentz force works in opposite direction of fluid flow.

The effect of Suction parameter  $s$  on velocity, temperature and concentration profiles is presented through Figures 9-11, respectively. It is observed that an increase in the values of suction parameter reduces the velocity, temperature and concentration fields. This may be due to the reason that the positive values of Suction parameter leads to thinner the momentum, thermal and concentration boundary layers. It is also observed that the concentration profiles are significantly affected by Suction parameter. Increasing values of the chemical reaction parameter causes to decrease in the velocity and concentration profiles; this is displayed in Figures 12-13. From Figures 14 and 15 we observed that an increase in the convection parameter  $N_c$  enhances the both velocity as well as temperature of the fluid. Also, rise in the diffusion parameter increases the concentration profiles of the flow, which we can conclude from Figure 16.

Tables 1-3 respectively display the effect of different physical parameters on Skin friction coefficient, Nusselt and Sherwood numbers. It is evident that an increase in Soret number ( $S_r$ ) decreases the friction factor ( $C_f$ ) and Sherwood number ( $Sh_x$ ). Rise in Casson parameter enhances the friction factor ( $C_f$ ). Convection parameter ( $N_c$ ) have the tendency to increase the heat transfer rate. Suction parameter is capable to enhance the heat and mass transfer rate. Rise in chemical reaction parameter increases the mass transfer rate. Prandtl number have tendency to enhance the heat transfer rate.



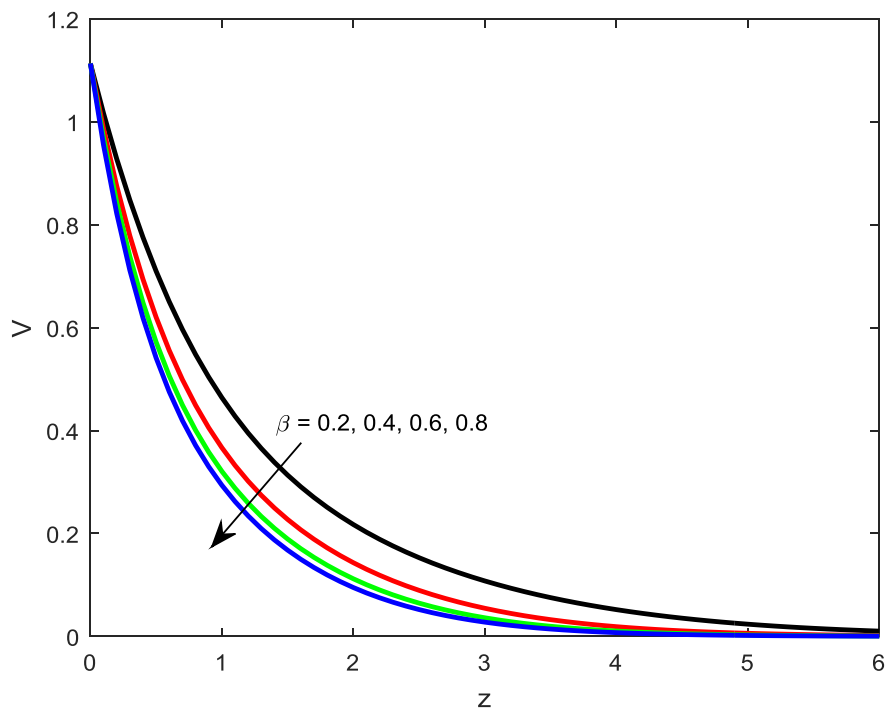


Figure 2. Velocity Profiles for Various Values of  $\beta$

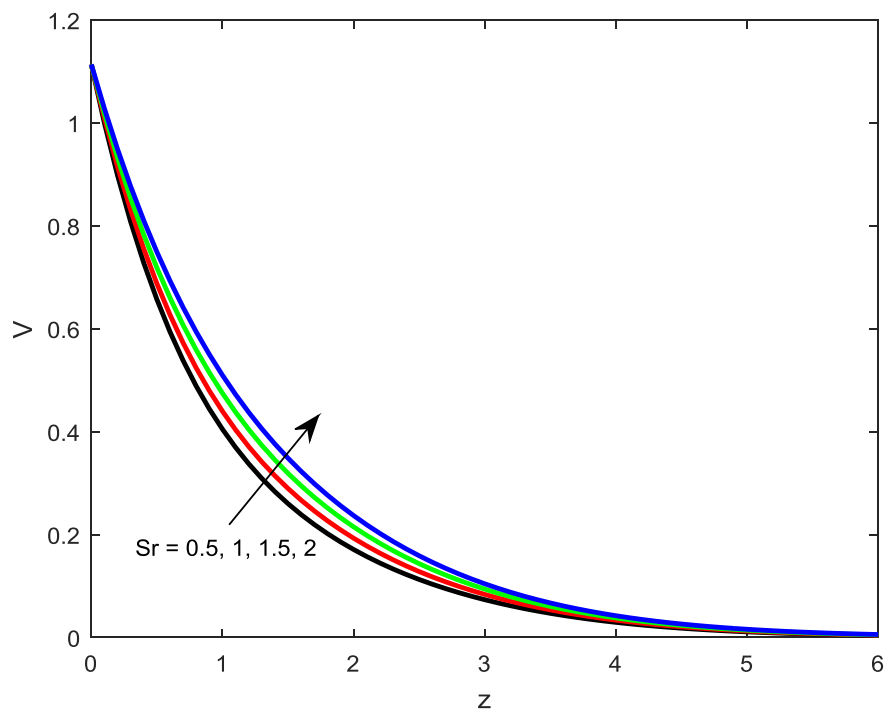


Figure 3. Velocity Profiles for Various Values of  $Sr$

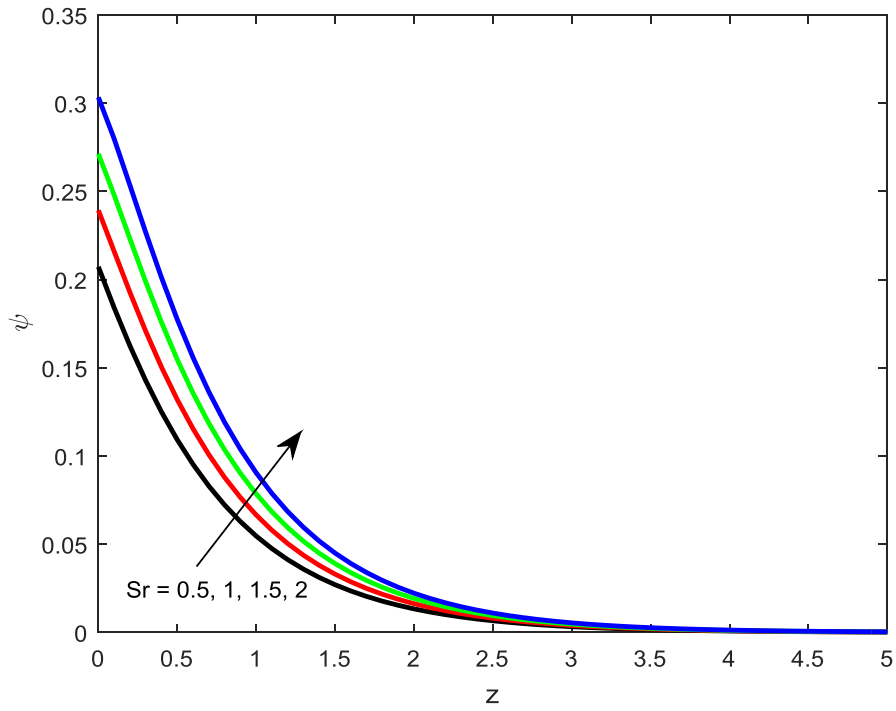


Figure 4. Concentration Profiles for Various Values of  $Sr$

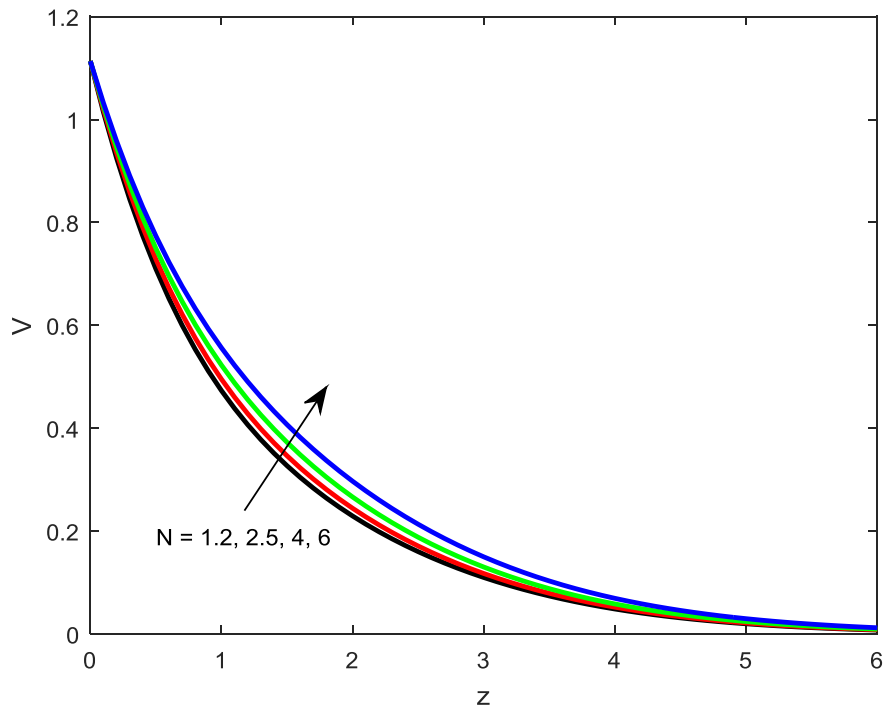


Figure 5. Velocity Profiles for Various Values of  $N$

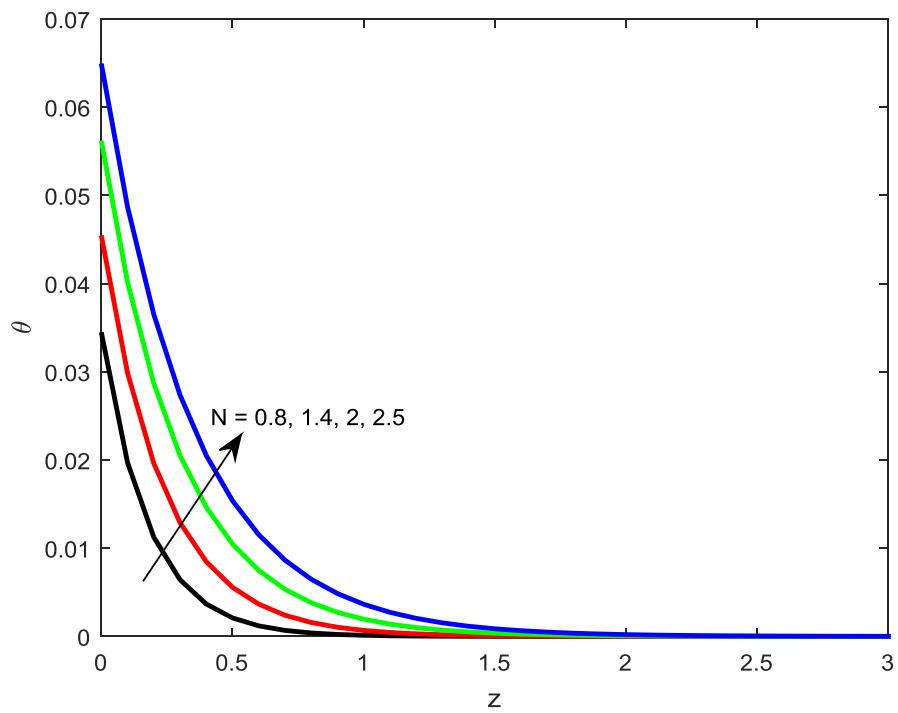


Figure 6. Temperature Profiles for Various Values of  $N$

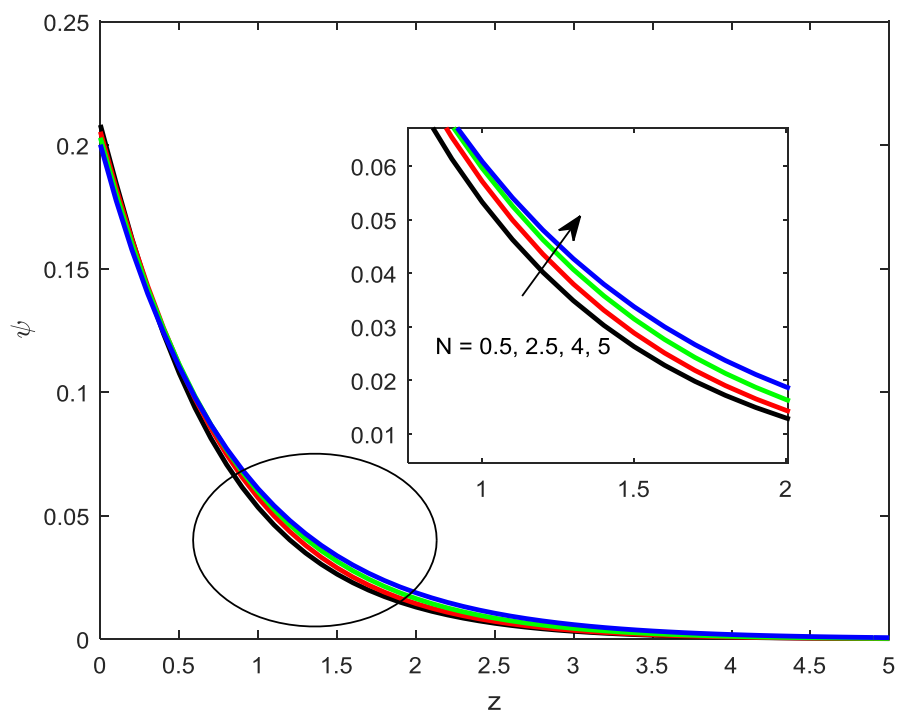


Figure 7. Concentration Profiles for Various Values of  $N$

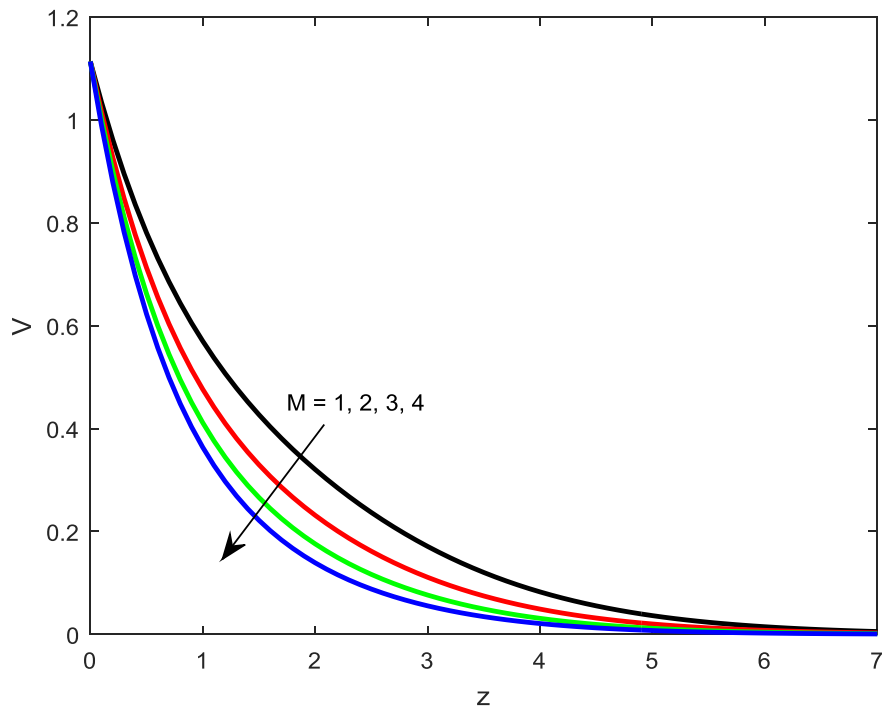


Figure 8. Velocity Profiles for Various Values of  $M$

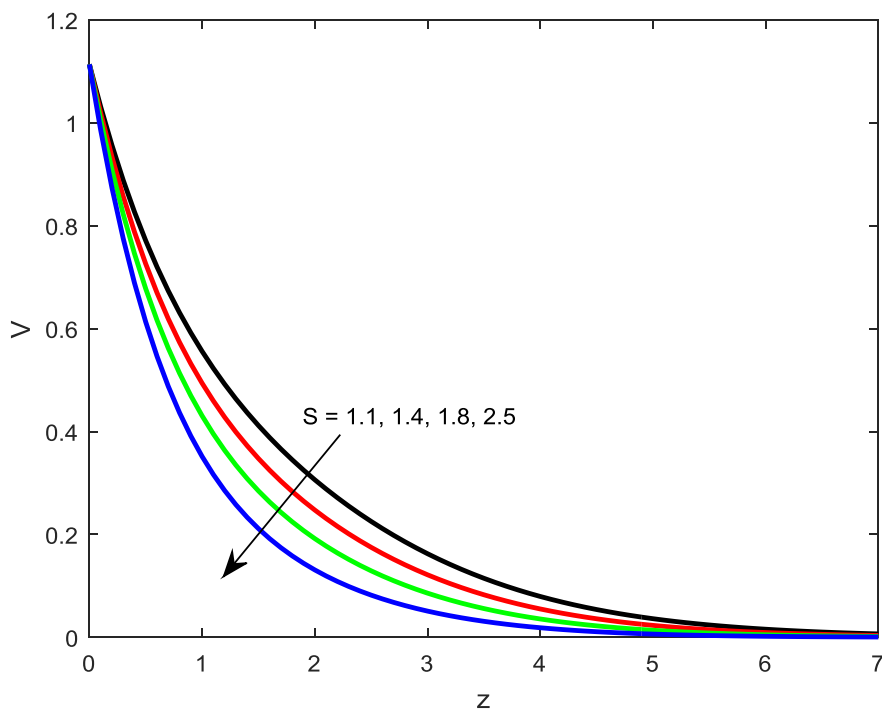


Figure 9. Velocity Profiles for Various Values of  $s$

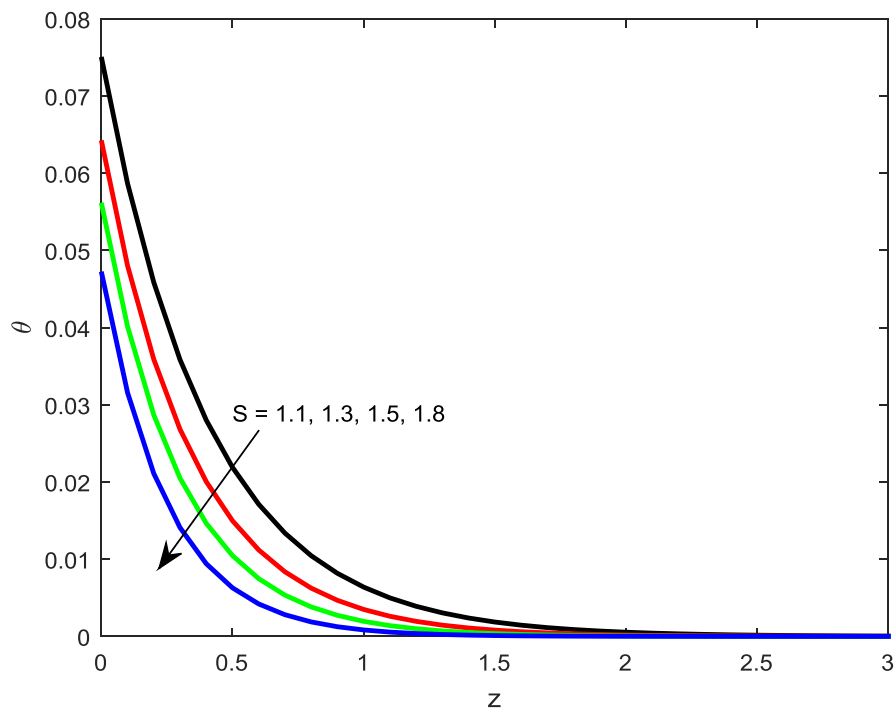


Figure 10. Temperature Profiles for Various Values of  $s$

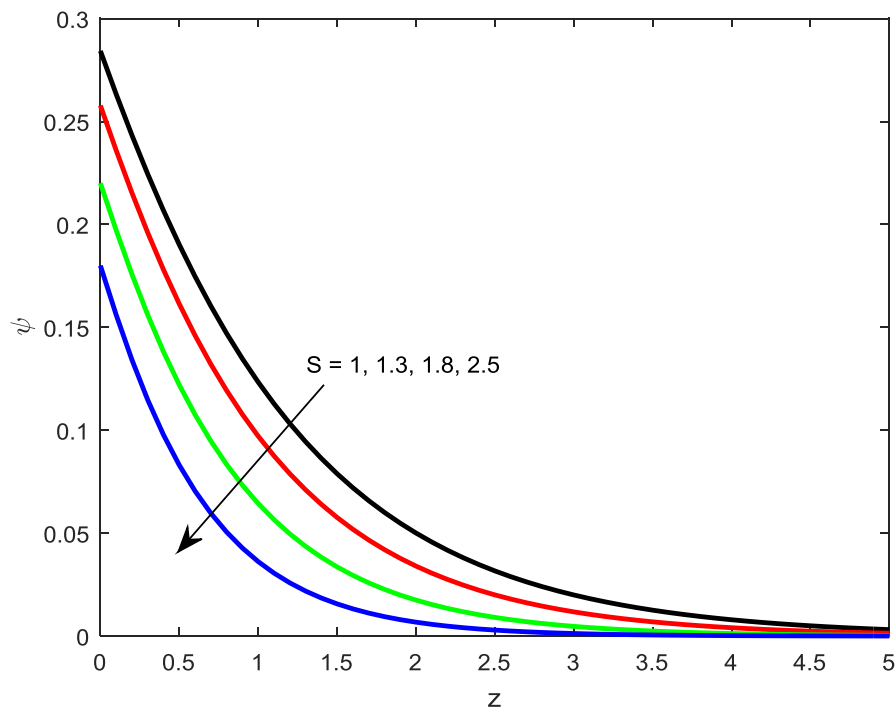


Figure 11. Concentration Profiles for Various Values of  $s$

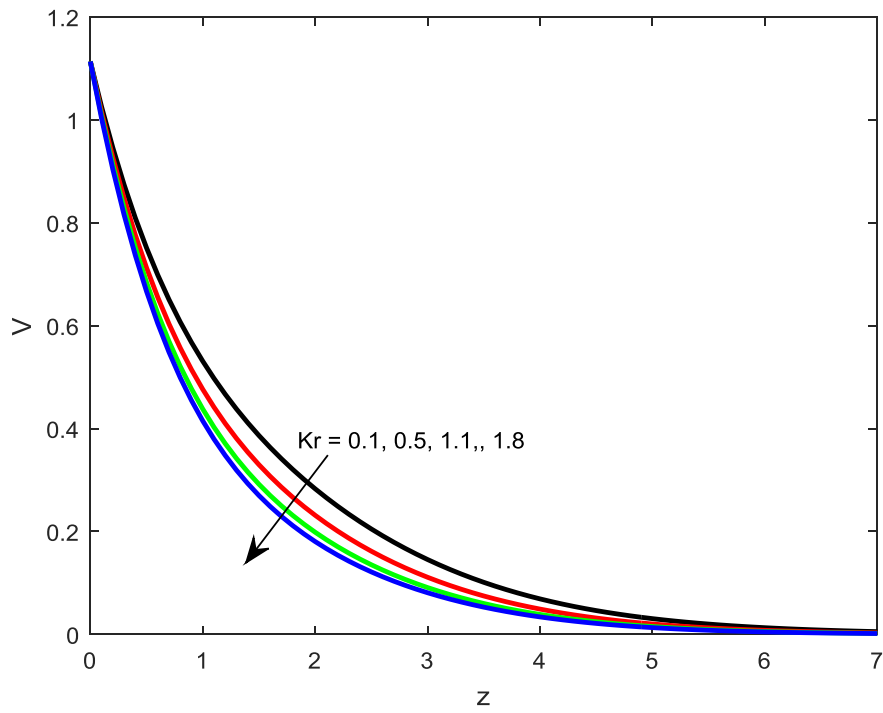


Figure 12. Velocity Profiles for Various Values of  $Kr$

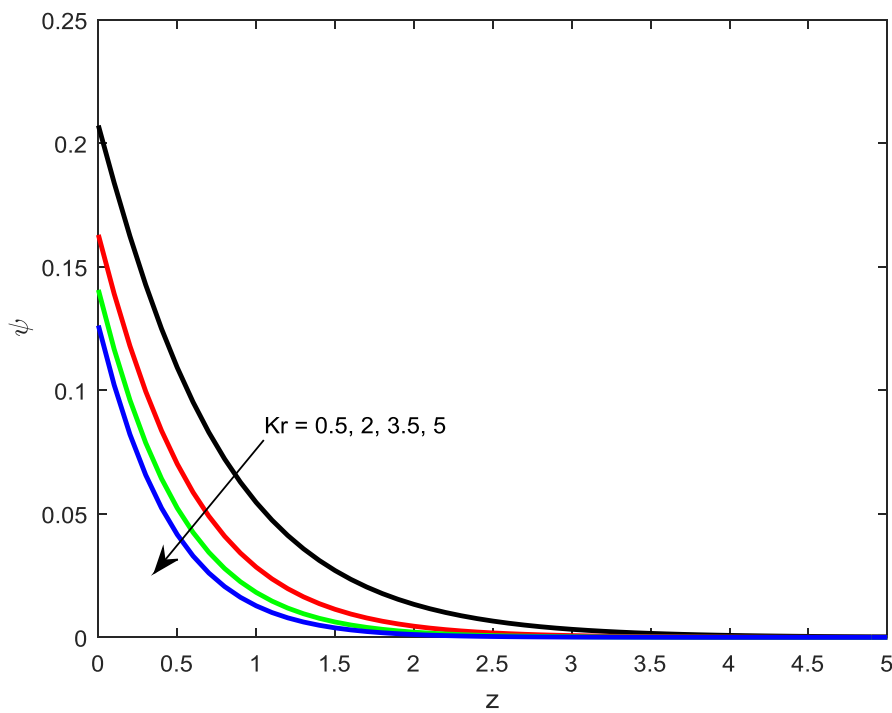
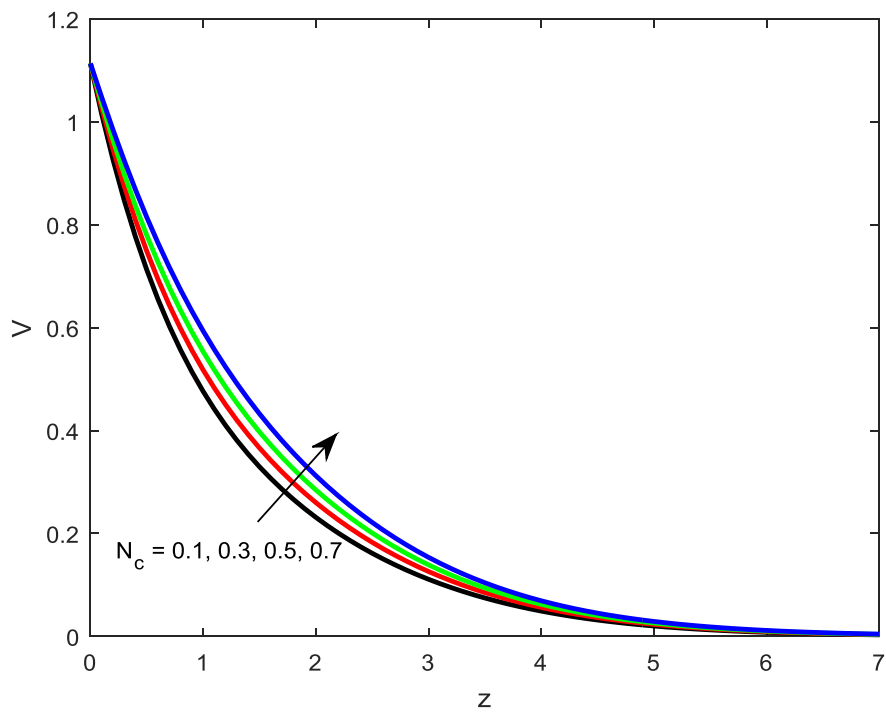
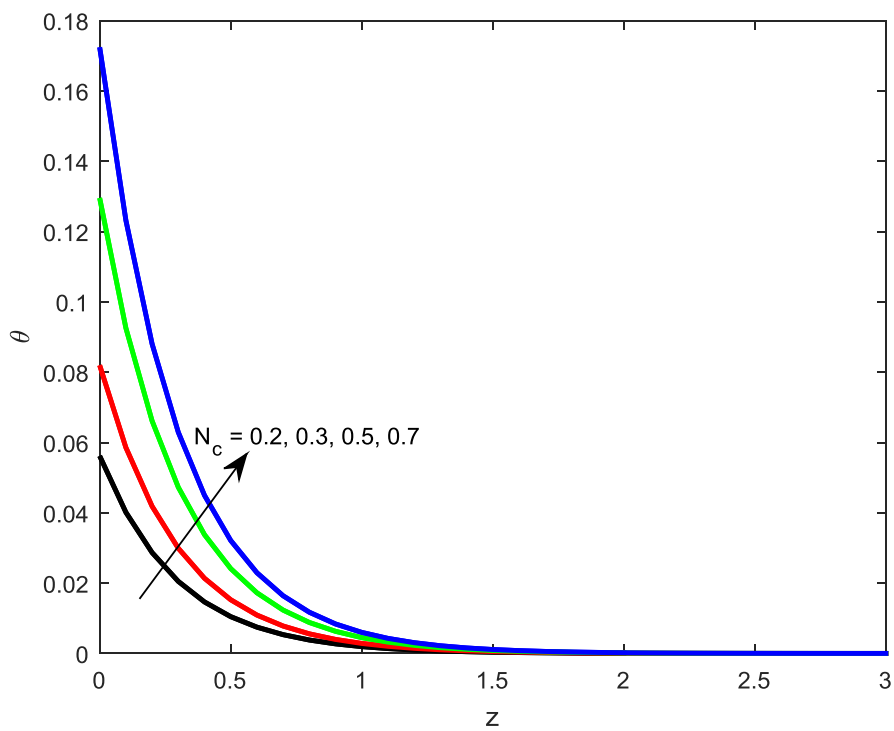


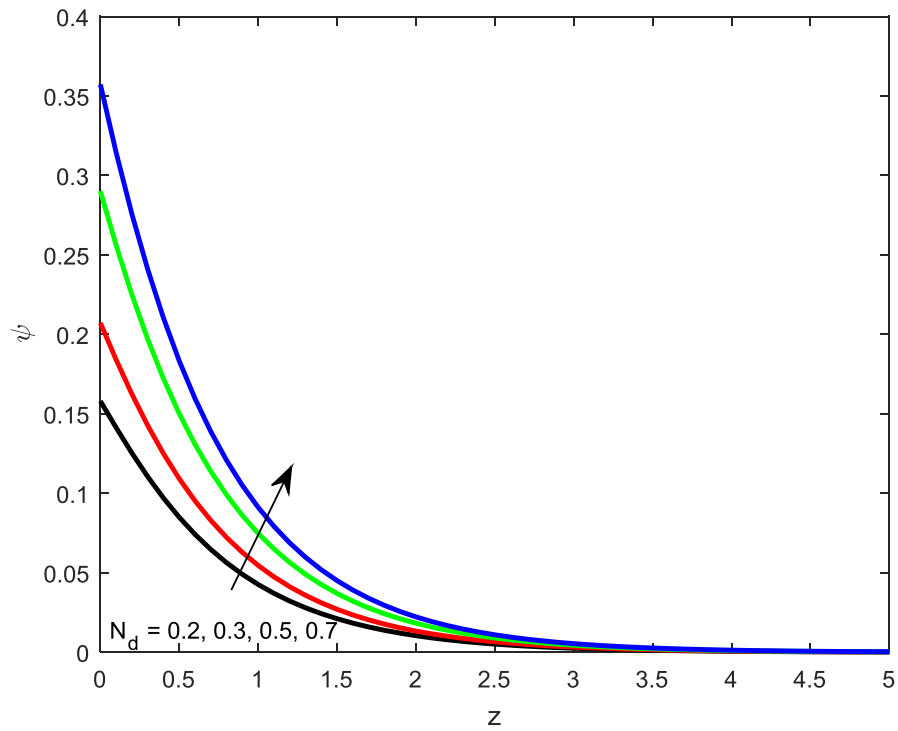
Figure 13. Concentration Profiles for Various Values of  $Kr$



**Figure 14. Velocity Profiles for Various Values of  $N_c$**



**Figure 15. Temperature Profiles for Various Values of  $N_c$**



**Figure 16. Concentration Profiles for Various Values of  $N_d$**

**Table 1. Skin Friction Coefficient ( $C_f$ ) for Different Values of Physical Parameters**

$\beta$	$N$	$Sr$	$M$	$S$	$Kr$	$C_f$
0.2						1.1675
0.5						1.7584
0.8						2.1006
	0.5					1.4163
	1.0					1.4147
	1.5					1.4129
		1.0				1.3954
		1.5				1.3801
		2.0				1.3647
			0.5			1.1061
			0.8			1.1630
			1.2			1.2355
				1.2		1.1673
				1.5		1.2451
				2.0		1.3647
					0.5	1.3647
					1.0	1.3853
					1.5	1.3991



**Table 2. Nusselt Number ( $Nu_x$ ) at Different Physical Parameters**

$N_c$	$Pr$	$N$	$S$	$Nu_x$
0.3				0.2812
0.5				0.4498
0.7				0.6054
	0.7			0.1400
	1.0			0.1538
	2.0			0.1739
		0.5		0.1956
		1.0		0.1942
		1.5		0.1928
			1.2	0.1861
			1.5	0.1888
			2.0	0.1915

**Table 3. Sherwood Number ( $Sh_x$ ) at Different Physical Parameters**

$Sr$	$Kr$	$N_d$	$S$	$Sh_x$
1.0				0.2282
1.5				0.2186
2.0				0.2090
	0.5			0.2378
	1.0			0.2437
	1.5			0.2479
		0.3		0.2378
		0.5		0.3549
		0.7		0.4499
			1.2	0.2201
			1.5	0.2275
			2.0	0.2378

## 5. Conclusions

This paper presents the effect of thermal diffusion, chemical reaction, radiation and rotation on unsteady free convection flow of a Casson fluid bounded by a moving vertical flat plate through porous medium in a rotating system with convective boundary conditions. The conclusions of the present study are as follows.

- Thermal diffusion parameter ( $Sr$ ) have tendency to enhance the velocity and concentration profiles.
- Rise in Casson parameter ( $\beta$ ) reduces the velocity profiles of the flow.
- Convection parameter ( $N_c$ ) has the tendency to enhance the heat transfer rate.
- Increasing values of Suction parameter enhances the heat and mass transfer rate.
- Rotation causes to reduce the momentum boundary layer thickness.
- Magnetic field parameter helps to control the flow field.

## References

- [1] K. Das, "Effect of Chemical Reaction and Thermal Radiation on Heat and Mass Transfer Flow of MHD Micropolar Fluid in a Rotating Frame of Reference", International Journal of Heat and Mass Transfer., vol. 54, no. 15-16, (2011), pp. 3505-3513.
- [2] T. Hayat, S. A. Shehzad and A. Alsaedi, "Soret and Dufour Effects on Magneto Hydrodynamic (MHD) Flow of Casson Fluid", Applied Mathematics and Mechanics, vol. 33, no. 10, (2012), pp. 1301-1312.
- [3] M. M. Rashidi, S. A. M. Pour, T. Hayat and S. Obaidat, "Analytic Approximate Solutions for Steady Flow over a Rotating Disk in Porous Medium with Heat Transfer by Homotopy Analysis Method", Computers & Fluids., vol. 54, (2012), pp. 1-9.
- [4] N. Sandeep, V. Sugunamma and P. Mohankrishna, "Effects of Radiation on an Unsteady Natural Convective Flow of a EG-Nimonic 80a Nanofluid Past an Infinite Vertical Plate", Advances in Physics Theories and Applications, vol. 23, no. 7, (2014), pp. 36-43.
- [5] N. Sandeep and V. Sugunamma, "Effect of Inclined Magnetic Field on Unsteady Free Convection Flow of a Dusty Viscous Fluid between Two Infinite Flat Plates Filled by Porous Medium", International Journal of Applied Mathematics and Modeling, vol. 1, no. 1, (2013), pp. 16-33.
- [6] S. K. Nandy, "Analytical Solution of MHD Stagnation-point Flow and Heat Transfer of Casson Fluid over a Stretching Sheet with Partial Slip", ISNR Thermodynamics., vol. 2013, (2013), Article Id:108264.
- [7] N. Sandeep and V. Sugunamma, "Radiation and Inclined Magnetic Field Effects on Unsteady Hydrodynamic Free Convection Flow Past an Impulsively Moving Vertical Plate in a Porous Medium", Journal of Applied Fluid Mechanics., vol. 7, no. 2, (2014), pp. 275-286.

- [8] P. Mohan Krishna, N. Sandeep and V. Sugunamma, "Effects of Radiation and Chemical Reaction on MHD Convective Flow over a Permeable Stretching Surface with Suction and Heat Generation", *Walailak Journal of Science and Technology*, vol. 12, no. 9, (2014), pp. 831-847.
- [9] K. Das, "Flow and Heat Transfer Characteristics of Nanofluids in a Rotating Frame", *Alexandria Engineering Journal*, vol. 53, no. 3, (2014), pp. 757-766.
- [10] J. V. Ramana Reddy, V. Sugunamma, N. Sandeep and P. Mohan Krishna, "Thermal Diffusion and Chemical Reaction Effects on Unsteady MHD Dusty Viscous Flow", *Advances in Physics Theories and Applications*, vol. 38, (2014), pp. 7-21.
- [11] N. Sandeep, V. Sugunamma and P. Mohan Krishna, "Aligned Magnetic Field, Radiation and Rotation Effects on Unsteady Hydromagnetic Free Convection Flow Past an Impulsively Moving Vertical Plate in a Porous Medium", *International Journal of Engineering Mathematics*, vol. 2014, (2014), Article Id: 565162.
- [12] J. V. Ramana Reddy, V. Sugunamma and N. Sandeep, "Effect of Nonlinear Thermal Radiation on MHD Flow between Rotating Plates with Homogeneous-Heterogeneous Reactions", *International Journal of Engineering Research in Africa*, vol. 20, (2015), pp. 130-143.
- [13] U. Khan, S. I. Khan, N. Ahmad, S. Bano and S. T. Mohyud-Din, "Heat Transfer Analysis for Squeezing Flow of a Casson Fluid between Parallel Plates", *Ain Shams Engineering Journal*, doi:10.1016/j.asej.201502.009, In press, (2015).
- [14] J. V. Ramana Reddy, V. Sugunamma, N. Sandeep and C. Sulochana, "Influence of Chemical Reaction and Rotation on MHD Nanofluid Flow Past a Permeable Flat Plate in Porous Medium", *Journal of the Nigerian Mathematical Society*, doi:10.1016/j.jnnms.2015.08.004, In press, (2015).
- [15] J. V. Ramana Reddy, V. Sugunamma and N. Sandeep, "Thermo Diffusion and Hall Current Effects on an Unsteady Flow of a Nanofluid under the Influence of Inclined Magnetic Field", *International Journal of Engineering Research in Africa*, vol. 20, (2015), pp. 61-79.
- [16] N. N. Anika, M. N. Hoque, S. I. Hossain and M. M. Alam, "Thermal Diffusion Effect on Unsteady Viscous MHD Micropolar Fluid Flow through an Infinite Vertical Plate with Hall and Ion-Slip Current", *Procedia Engineering*, vol. 105, (2015), pp. 160-166.
- [17] C. Sulochana, N. Sandeep, V. Sugunamma and B. Rushi Kumar, "Aligned Magnetic Field and Cross Diffusion Effects of a Nanofluid over an Exponentially Stretching Surface in Porous Medium", *Appl Nanosci.*, doi:10.1007/s13204-015-0475-x, In press, (2015).
- [18] C. S. K. Raju, N. Sandeep, V. Sugunamma, M. Jayachandrababu and J. V. Ramana Reddy, "Heat and Mass Transfer in Magneto Hydrodynamic Casson Fluid over an Exponentially Permeable Stretching Surface", *Engineering Science and Technology, an International Journal*, vol. 19, no. 1, (2015), pp. 45-52.
- [19] C. Sulochana, M. K. Kishor Kumar and N. Sandeep, "Nonlinear Thermal Radiation and Chemical Reaction Effects on MHD 3D Casson Fluid Flow in Porous Medium", *Chemical and Process Engineering Research*, vol. 37, (2015), pp. 24-36.
- [20] N. Sandeep, B. Rushi Kumar and M. S. Jagadeesh Kumar, "A Comparative Study of Convective Heat and Mass transfer in Non-Newtonian Nanofluid flow Past a Permeable Stretching Sheet", *Journal of Molecular Liquids*, vol. 212, (2015), pp. 585-591.
- [21] C. S. K. Raju, N. Sandeep, C. Sulochana, V. Sugunamma and M. Jayachandrababu, "Radiation, Inclined Magnetic Field and Cross-Diffusion Effects on Flow over a Stretching Surface", *Journal of the Nigerian Mathematical Society*, vol. 34, no. 2, (2015), pp. 169-180.
- [22] C. Sulochana and N. Sandeep, "Dual Solutions for Radiative MHD Forced Convective Flow of a Nanofluid over a Slandering Stretching Sheet in Porous Medium", *Journal of Naval Architecture and Marine Engineering*, vol. 12, (2015), pp. 115-124.
- [23] C. S. K. Raju and N. Sandeep, "Heat and Mass Transfer in MHD Non-Newtonian Bio-convection Flow over a Rotating Cone/Plate with Cross Diffusion", *Journal of Molecular Liquids*, vol. 215, (2016), pp. 115-126.
- [24] M. Satish Kumar, N. Sandeep and B. Rushi Kumar, "Dual Solutions for Heat and Mass Transfer in MHD Bio-convective Flow over a Stretching/Shrinking Surface with Suction/Injection", *International Journal of Engineering Research in Africa*, vol. 21, (2015), pp. 84-101.
- [25] N. Sandeep, C. Sulochana and I. L. Animasaun, "Stagnation-point Flow of a Jeffrey Nanofluid over a Stretching Surface with Induced Magnetic Field and Chemical Reaction", *International Journal of Engineering Research in Africa*, vol. 20, (2016), pp. 93-111.
- [26] N. Sandeep, A. Vijaya Bhaskar Reddy and V. Sugunamma, "Effect of Radiation and Chemical Reaction on Transient MHD Free Convective flow over a Vertical Plate through Porous Media", *Chemical and Process Engineering Research*, vol. 2, (2012), pp. 1-9.
- [27] C. Sulochana, S. P. Samrat and N. Sandeep, "Non-uniform Heat Source or Sink Effect on the Flow of 3D Casson Fluid in the Presence of Soret and Thermal radiation", *International Journal of Engineering Research in Africa*, vol. 20, (2016), pp. 112-129.
- [28] J. V. Ramana Reddy, V. Sugunamma, N. Sandeep and K. Anantha Kumar, "Influence of non-uniform heat source/sink on MHD nanofluid flow past a slandering stretching sheet with slip effects", *Global Journal of Pure and Applied Mathematics*, vol. 12, (2016), pp. 247-254.

- [29] M. Sathish Kumar, N. Sandeep and B. Rushi Kumar, "Effect of nonlinear thermal radiation on unsteady MHD flow between parallel plates", Global Journal of Pure and Applied Mathematics, vol. 12, (2016), pp. 61-65.
- [30] C. S. K. Raju and N. Sandeep, "The effect of thermal radiation on MHD ferrofluid flow over a truncated cone in the presence of non-uniform heat source/sink", Global Journal of Pure and Applied Mathematics, vol. 12, no. 1, (2016), pp. 9-15.
- [31] A. Veerasuneela Rani, V. Sugunamma and N. Sandeep, "Hall Current effects on convective heat and mass transfer flow of viscous fluid in a vertical wavy channel", International Journal of Emerging trends in Engineering and Development, vol. 4, no. 2, (2012), pp. 252-278.

## Authors



**K. Pushpalatha** was born in India in the year 1982. She has completed her B.Sc in 2002 and M. Sc., Mathematics in 2004 from Sri Venkateswara University, Tirupati, India. Presently she is a Research scholar (Ph.D) in the department of Mathematics, Sri Venkateswara University, India. His research interests include the fluid dynamics.



**Dr. V. Sugunamma** obtained her M.Sc., M.Phil and Ph.D degrees from Sri Krishna Devaraya University, Anantapuram in 1991, 1995 and 1997 respectively. She published more than 60 papers in reputed scientific journals and presented several papers in national and international conferences and produced 6 Ph.D's under her guidance. At present she is working as an Associate Professor in Mathematics Department, Sri Venkateswara University, Tirupati. Her area of interest is Fluid Dynamics.



**J. V. Ramana Reddy** was born in India in 1990. He received his B. Sc degree from Acharya Nagarjuna University, and M.Sc degree in Mathematics from Sri Venkateswara University, India. Presently he is a Ph.D scholar in department of Mathematics of Sri Venkateswara University, India. His research interests include fluid dynamics, magneto hydrodynamics and nano fluids. He attended a few national and International conferences, seminars and workshops. He also presented research articles in several national and International conferences.



**Dr. N. Sandeep** obtained his M.Sc degree from Sri Venkateswara University, Tirupati, India in 2006, M.Phil degree from Madurai Kamaraj University, Madurai in 2008 and Ph.D degree from Sri Venkateswara University, Tirupati in 2013. He published more than 60 papers in reputed scientific journals and presented several papers in national and international conferences. His area of interest is Fluid Mechanics.

