

## Experimental and Mathematical Study of Brass Casting During Solidification

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### Abstract

*In the present study numerical and experimental study of the brass casting has been conducted. Cooling of the brass casting has been studied both experimentally and numerically. Two-dimensional heat conduction equation in Cartesian co-ordinate governs the physical problem. At the interface convective boundary conditions have been considered. Heat conduction equation in unsteady state and transient state will be solved using. Finite difference software has been used to study the temperature flow and cooling rate. The pure brass and green sand mould will be used as a molten material and mould cavity material respectively.*

**Keywords:** Brass, Numerical, Experimental, solidification

### 1. Introduction

Casting which also known as founding is one of the earliest metal shaping methods. It means the pouring of molten metal into the refractory mould then allowing it to solidify. After solidification removing of part from the mould either by breaking the mould or ejecting the mould part. Casting has been most often selected over other manufacturing methods, because of the following reasons. Any complex shape with internal cavities or hollow sections can be cast using casting. Big parts can be formed in one piece. Materials which are difficult to process can be utilised by casting.

Vijayaram et.al, studied numerical simulation of casting solidification in permanent metallic moulds. They concluded that casting solidification simulation procedure is utilised to distinguish the damaged areas in the castings from the produced time-temperature contours. It is inferred that casting solidification simulation is utilised to dispose of deformities like shrinkage, porosity and to find the hot spot regions which serves to outline the components adequately. In addition, it is utilised to determine the solidification time and behaviour of diverse materials accurately. Henceforth, it is utilised to find the cooling rate impacted by the grain structure of castings. Solidification simulation of castings gives time-temperature information, temperature contours, hot spot locations, and latent heat of fusion and solidification time.

Masoud Jabbari and Azin Hosseinzadeh Numerical modelling of coupled heat transfer and phase transformation for solidification of the gray cast iron. The nucleation and growth hypothesis was utilised, and cooling rate was executed as a key consider in the solidification stages. A FDM model was developed for measure of graphite, cementite, and aggregate austenite on the premise of the cooling rate and level principle. As the cooling rate builds, the unstable stages increment. It is found that expanding cooling rate brings about the increment of cementite stage and the abatement of graphite and austenite stages. Hardness of gray cast iron reductions as the volume portion of graphite goes up. Increasing the cementite, results in the improvements of the hardness of the gray cast iron. The proposed equations developed by numerical modelling find in decent concurrence with the experiments results.

Choudhari et.al, conducted Modelling and Simulation with Experimental Validation of Temperature Distribution during Solidification Process in Sand Casting. They studied heat flow within the casting, as well as from the casting to the mould, and finally obtains the temperature history of all points inside the casting. They found that that most important instant of time is when the hottest region inside the casting is solidifying. They performed Transient thermal analysis using ANSYS software to obtain the temperature distribution in the casting process. They obtained results by simulation software and compared that with the experimental reading of temperature. They also studied the significance of filling pattern and appropriate orientation of gating system. In the end they concluded that the simulation of casting process helps in finding temperature distribution of different parts of the mould. They also concluded that simulation helps in reducing the cost of development and material utilization (yield).

Choudhari et.al, studied Optimum Design and Analysis of Riser for Sand Casting using ANSYS. They find that simulation of the solidification process allows visualization of solidification inside a casting and helps in finding last freezing regions or hot spots. They noticed that simulation also helps in optimizing the placement and design of feeders and feeding aid. They concluded that solidification defect can be minimized after finding optimum location of riser. They determined that simulation can help in optimizing dimensions of riser and casting feeding efficiency. They also validated their results of optimized riser dimensions based on simulation by carrying out actual trials in a foundry.

## 2. Physical Domain

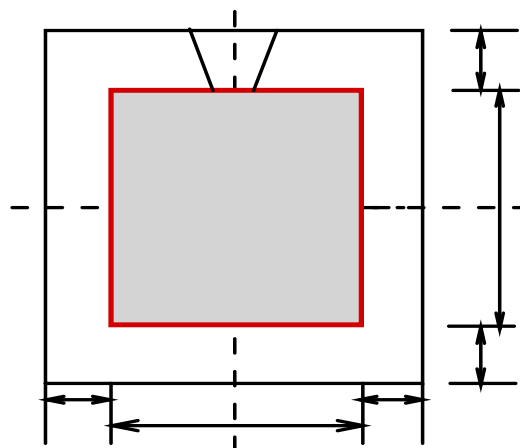


Figure 1. Schematic of Mould Cavity and Casting Part

## 3. Governing Equation

When the temperature of the body changes heat transfer take place. Heat can be transferred either by conduction, convection and radiation. Which mode of heat transfer will dominate that depends upon the temperature value. As in the solidification process of hot liquid metal no movement of particle will occur. So the conduction will dominate over other two modes of heat transfer convection and radiation. As the two-dimensional casting part study will be done in the present study, Two-dimensional heat conduction equation governs the physical flow problem.

$$\text{Unsteady state} \quad \alpha \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] = \frac{\partial T}{\partial t} \quad (1)$$

$$\text{Steady State} \quad \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] = 0 \quad (2)$$

## 4. Boundary Conditions

$$x=0 \text{ (Left vertical wall)} \quad k \frac{\partial T}{\partial x} \Big|_{x=0} = h_f [T_{x=0} - T_{ref}] \quad (3)$$

$$x=L \text{ (Right vertical wall)} \quad -k \frac{\partial T}{\partial x} \Big|_{x=L} = h_f [T_{x=L} - T_{ref}] \quad (4)$$

$$y=0 \text{ (Bottom wall)} \quad k \frac{\partial T}{\partial y} \Big|_{y=0} = h_f [T_{y=0} - T_{ref}] \quad (5)$$

$$y=H \text{ (Top wall)} \quad -k \frac{\partial T}{\partial y} \Big|_{y=H} = h_f [T_{y=H} - T_{ref}] \quad (6)$$

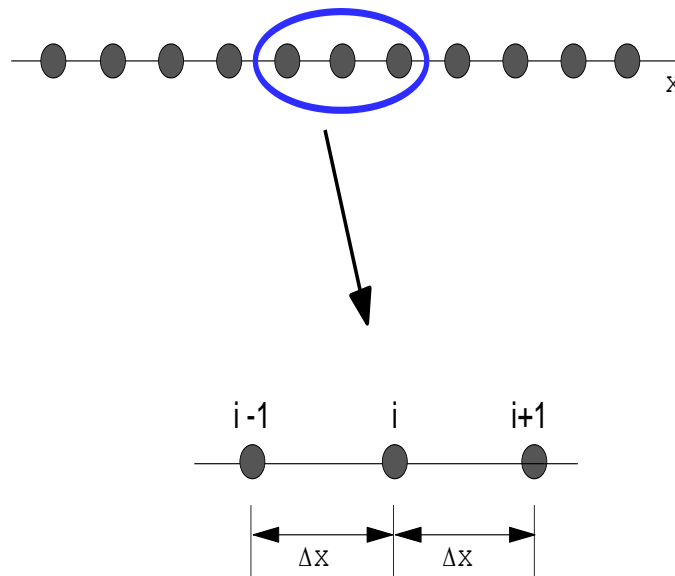
## 5. Assumptions

1. Convection and Radiation have been neglected
2. Molten metal and green sand mould properties are considered to be isotropic
3. Temperature independent properties
4. Mass and momentum change has been neglected.

## 6. Numerical Technique

To solve the two-dimensional heat conduction equation first we will have to convert this equation from partial differential form to a discrete form. The simplest procedure used to drive the discrete form of a differential equation, which we also call as finite difference equations consists of approximating the derivatives in the differential equation using a truncated Taylor series.

The finite difference method is the simplest method to apply, particularly on uniform grids. However it requires high degree of mesh regularity. Let us consider a one dimensional situation where the independent variable T is a function of space coordinates x. We will discretize the spatial domain using equal space interval  $\Delta x$  shown in Figure 2.



**Figure 2. Spatial Domain Discretization**

The difference approximation can be done using Taylor-series expansion or Taylor's formula. Taylor-series expansion for  $T[x+\Delta x]$  and  $T[x-\Delta x]$  about  $[x]$  can be represented as,

$$T[x + \Delta x] = T[x] + \frac{\partial T[x]}{\partial x} \frac{\Delta x}{1!} + \frac{\partial^2 T[x]}{\partial x^2} \frac{[\Delta x]^2}{2!} + \frac{\partial^3 T[x]}{\partial x^3} \frac{[\Delta x]^3}{3!} + O[\Delta x]^4 \quad (7)$$

$$T[x - \Delta x] = T[x] - \frac{\partial T[x]}{\partial x} \frac{\Delta x}{1!} + \frac{\partial^2 T[x]}{\partial x^2} \frac{[\Delta x]^2}{2!} - \frac{\partial^3 T[x]}{\partial x^3} \frac{[\Delta x]^3}{3!} + O[\Delta x]^4 \quad (8)$$

Using equations 7 and 8 we can find the discrete form of space derivative term  $\frac{\partial^2 T}{\partial x^2}$  in the two-dimensional heat conduction equation as,

$$\frac{\partial^2 T[x,y]}{\partial x^2} = \frac{T[x+\Delta x,y] - 2T[x,y] + T[x-\Delta x,y]}{[\Delta x]^2} + O[\Delta x]^2 \quad (9)$$

Taylor-series expansion for  $T[y+\Delta y]$  and  $T[y+2\Delta y]$  about  $[y]$  can be represented as,

$$T[y + \Delta y] = T[y] + \frac{\partial T[y]}{\partial y} \frac{\Delta y}{1!} + \frac{\partial^2 T[y]}{\partial y^2} \frac{[\Delta y]^2}{2!} + \frac{\partial^3 T[y]}{\partial y^3} \frac{[\Delta y]^3}{3!} + O[\Delta y]^3 \quad (10)$$

$$T[y - \Delta y] = T[y] - \frac{\partial T[y]}{\partial y} \frac{\Delta y}{1!} + \frac{\partial^2 T[y]}{\partial y^2} \frac{[\Delta y]^2}{2!} - \frac{\partial^3 T[y]}{\partial y^3} \frac{[\Delta y]^3}{3!} + O[\Delta y]^3 \quad (11)$$

Similarly using equation 10 and 11 we can now write the discrete form of space derivative  $\frac{\partial^2 T}{\partial y^2}$  in the two-dimensional heat conduction equation as,

$$\frac{\partial^2 T[x,y]}{\partial y^2} = \frac{T[x,y+\Delta y] - 2T[x,y] + T[x,y-\Delta y]}{[\Delta y]^2} + O[\Delta y]^2 \quad (12)$$

Now we can write the above equations 9 and 12 into vector form using  $i$  and  $j$  notation as,

$$\frac{\partial^2 T[x,y]}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{[\Delta x]^2} \quad (13)$$

$$\frac{\partial^2 T[x,y]}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{[\Delta y]^2} \quad (14)$$

Final form of two-dimensional governing eq.1 and eq.2 can be written as,

For unsteady case,

$$\alpha \left[ \frac{T_{i+1,j,t} - 2T_{i,j,t} + T_{i-1,j,t}}{[\Delta x]^2} + \frac{T_{i,j+1,t} - 2T_{i,j,t} + T_{i,j-1,t}}{[\Delta y]^2} \right] = \left[ \frac{T_{i,j,t+1} - T_{i,j,t}}{\Delta t} \right] \quad (15)$$

For steady case,

$$\frac{T_{i+1,j,t} - 2T_{i,j,t} + T_{i-1,j,t}}{[\Delta x]^2} + \frac{T_{i,j+1,t} - 2T_{i,j,t} + T_{i,j-1,t}}{[\Delta y]^2} = 0 \quad (16)$$

From the above equation we can find out the final form which we will further use for coding,

For unsteady state,

$$T_{i,j,t+1} = T_{i,j,t} + \frac{\alpha}{\Delta t} \left[ \frac{T_{i+1,j,t} - 2T_{i,j,t} + T_{i-1,j,t}}{[\Delta x]^2} + \frac{T_{i,j+1,t} - 2T_{i,j,t} + T_{i,j-1,t}}{[\Delta y]^2} \right] \quad (17)$$

For steady state,

$$T_{i,j,t} = \frac{\left[ \frac{T_{i+1,j,t} + T_{i-1,j,t}}{(\Delta x)^2} + \frac{T_{i,j+1,t} + T_{i,j-1,t}}{[\Delta y]^2} \right]}{\left[ \frac{2}{[\Delta x]^2} + \frac{2}{[\Delta y]^2} \right]} \quad (18)$$

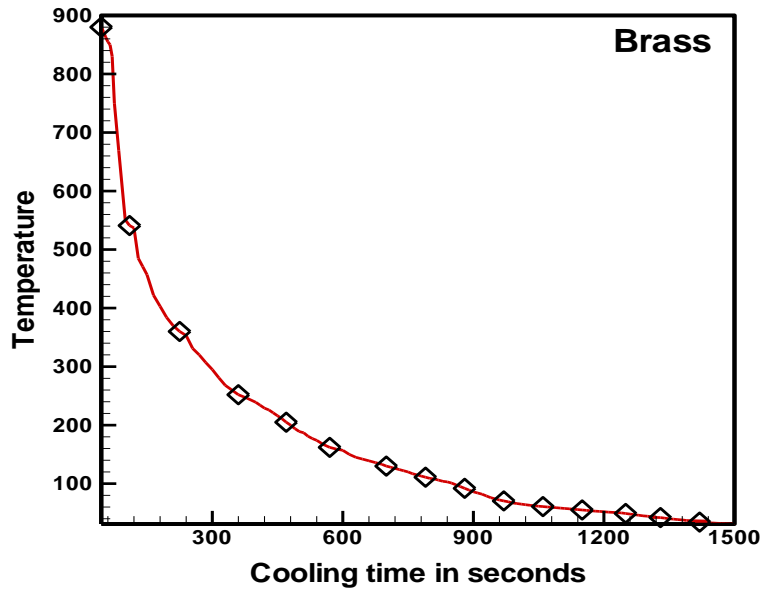
Equation 17 and 18 are the final equations for finding the temperature at the internal nodes, while temperature at the walls of the mould cavity will be calculated using the boundary conditions explained in equations 3-6.

## 7. Experimental Results

**Table 1. Temperature-Time Table with its Location in the Casting**

Time in seconds	Temperature in °C	Place of molten metal	Time in seconds	Temperature in °C	Place of molten metal
0	993	Furnace	830	104	Mould
45	880	Pouring	840	103	Mould
50	870	Mould	850	101	Mould
55	863	Mould	860	98	Mould
60	855	Mould	870	95	Mould
65	850	Mould	880	92	Mould
70	830	Mould	890	89	Mould
75	750	Mould	900	86	Mould
85	669	Mould	910	84	Mould
100	551	Mould	920	82	Mould
110	541	Mould	930	79	Mould
120	537	Mould	940	76	Mould
130	485	Mould	950	74	Mould
140	471	Mould	960	72	Mould
150	457	Mould	970	70.5	Mould
165	422	Mould	980	69	Mould
180	403	Mould	990	67	Mould
195	384	Mould	1000	66	Mould
210	370	Mould	1010	65	Mould
225	360	Mould	1020	64	Mould
240	353	Mould	1030	63	Mould
255	331	Mould	1040	62.2	Mould
270	320	Mould	1050	61.5	Mould
285	307	Mould	1060	60.8	Mould
315	281	Mould	1080	59.6	Mould
330	268	Mould	1090	58.9	Mould
345	260	Mould	1100	58.2	Mould
360	252	Mould	1110	57.6	Mould
380	246	Mould	1120	57	Mould
400	239	Mould	1130	56.2	Mould
410	234	Mould	1140	55.7	Mould
420	229	Mould	1150	55	Mould
430	226	Mould	1160	54.3	Mould
440	221	Mould	1170	53.7	Mould
450	216	Mould	1180	53	Mould
460	211	Mould	1190	52.6	Mould
470	205	Mould	1200	52	Mould
480	200	Mould	1210	51.3	Mould

490	194	Mould	1230	50.6	Mould
500	189	Mould	1240	50	Mould
510	187	Mould	1250	49	Mould
520	181	Mould	1260	48	Mould
530	177	Mould	1270	47	Mould
540	174	Mould	1280	46	Mould
555	167	Mould	1290	45	Mould
570	162	Mould	1300	44.3	Mould
585	159	Mould	1310	43.6	Mould
600	157	Mould	1320	43	Mould
615	150	Mould	1310	42.5	Mould
630	145	Mould	1330	42	Mould
645	142	Mould	1340	41	Mould
660	139	Mould	1350	40	Mould
675	136	Mould	1360	39.3	Mould
690	133	Mould	1370	38.6	Mould
700	130	Mould	1380	38	Mould
710	128	Mould	1390	37	Mould
720	126	Mould	1440	36	Mould
730	124	Mould	1410	35	Mould
740	122	Mould	1420	34.3	Mould
750	120	Mould	1430	33.7	Mould
760	117	Mould	1440	33	Mould
770	115	Mould	1450	32.75	Mould
780	113	Mould	1460	32.35	Mould
790	111	Mould	1470	32	Mould
800	109	Mould	1480	31.75	Mould
810	108	Mould	1490	31.35	Mould
820	106	Mould	1500	31	Mould



**Figure 3. Cooling Rate of Brass in Mould Cavity**

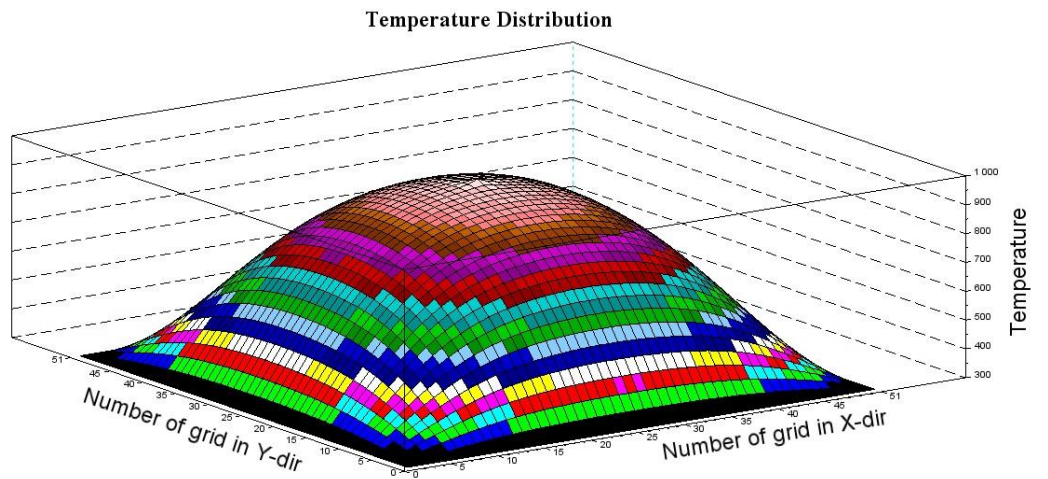
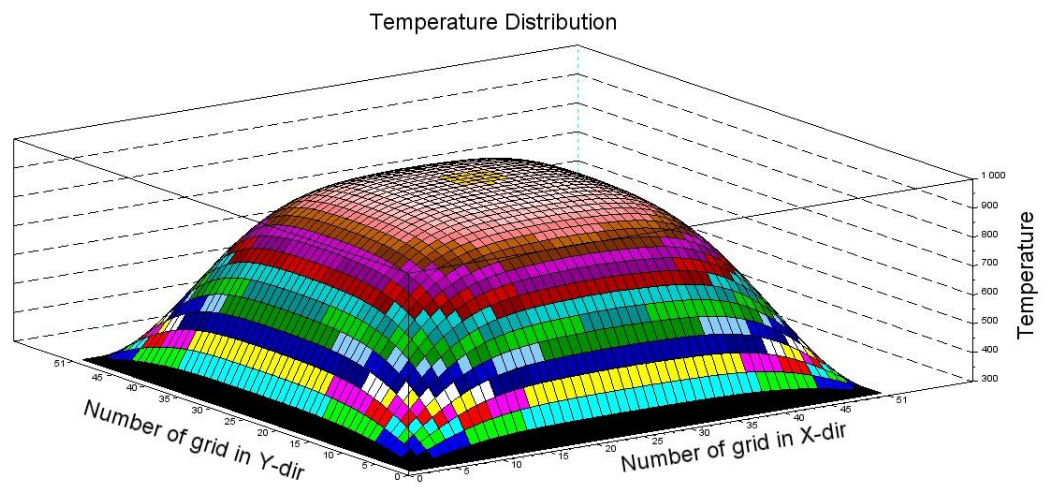
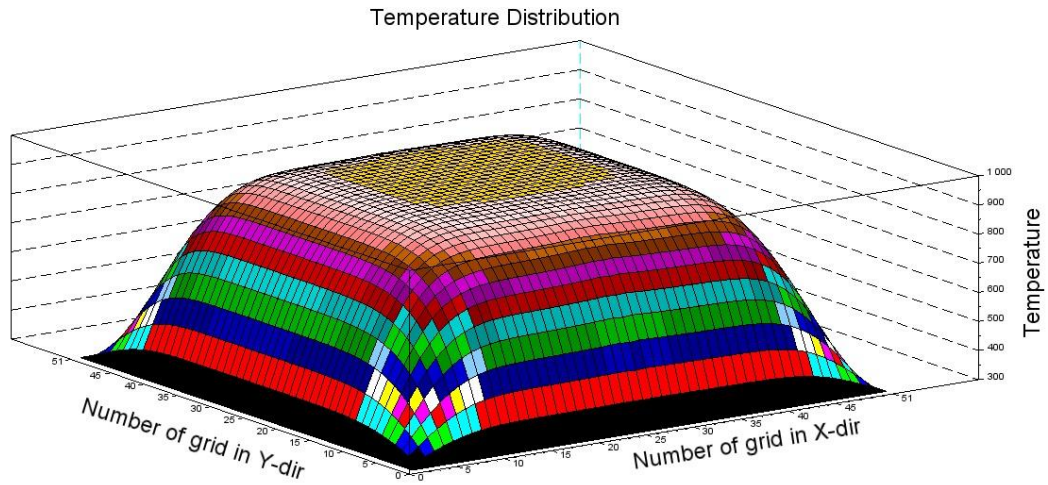
Figure 3 represents the cooling rate of brass. It can be observed that from the figure that pouring temperature is around  $880^{\circ}\text{C}$  and with time it reaches to the atmospheric temperature around  $31^{\circ}\text{C}$  in 1500 seconds. Initially the data has been taken in thermoelectric voltage (mV) with time (in seconds) then the data from mv have been converted into degree centigrade ( $^{\circ}\text{C}$ ) using ITS-90 table for K type thermocouple.

Figure 4 represents the figures taken during experiments. Brass has been melted first at a temperature of  $993^{\circ}\text{C}$  around. Then it has been poured into the casting part. Temperature has been obtained using a K-type thermocouple.



**Figure 4. (a) Melting of Molten Metal (b) Pouring of Molten Metal (c) Final Casting Piece**

Figure 5 represents the solidification curve or temperature distribution inside the brass casting. These curves have been plotted for steady case, where represents no time variation. The curves have been plotted for different number of iterations. One can notice that for small number of iteration temperature in most part of the casting is equal to the melting point temperature while with increment in number of iteration temperature gradually get distributed in whole part of the casting.



**Figure 5. Temperature Distribution or Solidification Curve during Brass Casting**



## 8. Conclusion

1. A finite difference method can be used to formulate the casting problem.
2. Brass casting cooling rate has been plotted by experimental data.
3. Open source software like Scilab can be used to plot the solidification curve.
4. With number of iteration for steady case temperature gradually gets distributed in the whole casting.
5. A grid size of  $51 \times 51$  has been used which represents the actual flow inside the casting.

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