Find Out the Optimum Condition for a Pump Storage Hydro-Power Plant Using AHP & PSO

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Abstract

The year round insufficiency of water in rivers of Tripura (state of North-East India) is the main problem of hydro-power generation of this state. The only hydro-power station of the state (Dombor Hydro-Power Plant) is presently in deadly condition. So a plant which can operate with fixed volume of water, is very much acceptable to pursue this condition. So, pump storage hydro power plant is this type of plant which can operate according to the expected condition. But consumption of power during the pumping phase reduces the net output of PSP hydro-plant. So parametric estimation has been adopted to find the most beneficial & non beneficial parameters for PSP and how the efficiency changes with the variation of these parameters it is also observed here. Finally the optimum condition for maximum power generation has been developed from these estimations.

Keywords: Parametric estimation, finding value of PSP constant, objective equation, optimum condition

1. Introduction

A pump storage plant contains higher reservoir, lower reservoir, penstock, turbine / pump & power house. Various losses are taken place in these parts of the plant which affects the power output as well as the plant efficiency.

2. Parametric Estimation

Efficiency of pump storage plant is given by this equation

\[ \eta = \frac{1 - K}{1 + K \eta_t \eta_p} \]

Here, \( \eta \) = Overall efficiency, \( \eta_t \) = Turbine efficiency, \( \eta_p \) = Pump efficiency & \( K \) = Constant, which depends on gross head & frictional head loss.

\[ K = \frac{h_f}{H} \]

Here, \( h_f \) = head loss due to friction & \( H \) = gross head of the plant.

For the constant values of turbine & plant efficiencies the overall efficiency \( \eta = f(K) \) i.e., \( \eta \propto 1/K \). If the value of ‘K’ decreases the overall efficiency will increase for same pump & turbine efficiencies i.e., for same gross head if head loss due to frication is reduced it will increase the plant efficiency.
2.1. Estimation for Head Loss

Various head losses like head loss due to friction in penstock, head loss due to sudden contraction in the entrance of penstock from higher reservoir, sudden enlargement in the turbine entrance during generating phase etc. various head losses has been listed here.

Head losses during generating phase:

<table>
<thead>
<tr>
<th>Head losses</th>
<th>Calculation of losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sudden contraction in the entrance of penstock</td>
<td>$h = \frac{0.5V^2}{2g}$</td>
</tr>
<tr>
<td>Head loss due to friction inside penstock</td>
<td>$h = \frac{4fV^2}{2gd}$</td>
</tr>
<tr>
<td>Head loss due to sudden enlargement in turbine</td>
<td>$h = \frac{(\frac{V}{d})^2}{2g}$</td>
</tr>
</tbody>
</table>

Head losses during pumping phase:

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</tr>
<tr>
<td>Head loss due to sudden enlargement in turbine</td>
<td>$h = \frac{(\frac{V}{d})^2}{2g}$</td>
</tr>
</tbody>
</table>

Except frictional loss, other losses are only function of flow velocity ($hav^2$). So the other losses can minimize by maintaining the optimum flow velocity corresponding to max output with minimum losses.

But the loss due to friction is not depends only flow velocity only. It depends on other parameters also & directly affecting the plant efficiency. Head loss due to friction is given by

$$h_f = \frac{4fV^2}{2gd}$$

Here, $h_f \propto f$ (co-efficient of friction).

$h_f \propto L$ (length of the penstock).

$h_f \propto V^2$ (velocity of flow).

$h_f \propto 1/d$ (diameter of penstock).

2.1.1. Parametric Analysis of Co-efficient of Friction

Head loss is directly proportional to the co-efficient of friction ($f$). So if the value of ‘$f$’ is reduced the loss due to friction will reduce. But the ‘$f$‘= $f(Re)$ i.e., function of Reynolds Number.

$$f = \frac{0.079}{4\sqrt{Re}}$$

So, the $f\propto 1/Re$. If the value of ‘Ra’ is maximize the ‘$f$’ will minimize.

$$Re = \frac{\rho Vd}{\mu}$$

Now, ‘Re’ is proportional to velocity ‘V’. So if ‘V’ increases the value of ‘$f$’ will decrease. But the value of ‘V’ is directly responsible for frictional loss.
‘Re’ will increase with increased value of diameter ‘d’. So the ‘f’ will decrease. Again the increment of ‘d’ is directly responsible for reduction of ‘h_f’.

‘Re’ is proportional to the density ‘ρ’. In ideal cases density of water is considered as a constant (1 or 0.99) but in practical or experimental expect the value of density will vary in between 0.983-0.997 gm/cm^3 for the temperature variation 60-25°C.

‘μ’ is the dynamic viscosity of water. The value of ‘Re’ is inversely proportional to the viscosity so ‘f’ will increase with the increase of viscosity. The value of viscosity varies 8.684-7.005 Kg/ms for the temperature variation of 25-35°C.

2.1.2 Parametric Analysis of Length of Penstock

Penstock length depends on the slope or inclination of plant alignment with horizontal plane. If the alignment is less, penstock length will more. For more inclination panstrok length will comparatively less.

![Figure 1.](image)

Hear, for same operating ‘H’ head L_1 > L_2. So in case of length ‘L_2’ head loss (h_f) is comparatively less so the net head (H_n = H - h_f) of operation as well available kinetic energy at the turbine will be more than ‘L_1’. So the inclination should maintain as more as possible to reduce the length of penstock.

2.1.3 Parametric Analysis of Velocity

Velocity of water is function of head mainly. It depends on the height of any particular position. But the variation of other parameters like length, surface roughness, diameter of penstock & viscosity of water affects the instantaneous value of velocity slightly.

\[ V = \sqrt{2gH} \]

2.1.3 Parametric Analysis of Penstock Diameter

Diameter of penstock is a design parameter. It can vary according to necessity. Again the head loss due to friction also decrease with the increased value of diameter ‘d’. So the optimum value of ‘d’ should find correspondence to minimum head loss & maximum efficiency.

3. Finding the Most Beneficial & Non Beneficial Parameter with AHP

AHP (Analytic Hierarchy Process) is a process or approach to find the most important event. Here with the help of this process one most important parameter will justify (beneficial parameter) & the parameter which has least importance will consider as non beneficial parameter.

‘AHP’ is a multi criteria based determination technique. So the criteria’s are:

1. Cost required to regulate these variables(C).
2. Effects of parameters on Efficiency (\( \eta \))
3. Effects of parameters onVelocity (V)

Velocity has been considered as a criteria because change or fluctuation of parameters changes the velocity slightly (though it is a major function of operating head). Decrement in velocity is required to reduce the frictional & other losses but on the other hand it is solely essential for runner power at turbine inlet. That’s why change in velocity due to the variation of other parameter is listed among criteria’s not in events.

\[
\begin{bmatrix}
C & \eta & V \\
2/1 & 1 & 3/1 \\
\eta & 1/3 & 1/4 & 1 \\
V & 1/3 & 1/4 & 1
\end{bmatrix}
= \begin{bmatrix}
1.5 \\
2.33 \\
0.526
\end{bmatrix}
\]

Now, the total value= \((1.5+2.33+0.526) = 4.35\)

\[
\begin{bmatrix}
1.5/4.35 \\
2.33/4.35 \\
0.526/4.35
\end{bmatrix}
= \begin{bmatrix}
0.344 \\
0.53 \\
0.120
\end{bmatrix}
\]

So, the maximum wattage is given to efficiency & minimum to the velocity.

The events or alternatives are listed here:

<table>
<thead>
<tr>
<th>Events</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of penstock</td>
<td>D</td>
</tr>
<tr>
<td>Length of penstock</td>
<td>L</td>
</tr>
<tr>
<td>Viscosity of water</td>
<td>M</td>
</tr>
</tbody>
</table>

Now w.r.t cost:

\[
\begin{bmatrix}
d & L & \mu \\
1 & 4/1 & 2/1 \\
L & 1/4 & 1 & 1/3 \\
\mu & 2 & 3 & 1
\end{bmatrix}
= \begin{bmatrix}
2.33 \\
0.526
\end{bmatrix}
\]

Total value = \((0.61+1.167+2) = 4.856\)

\[
\begin{bmatrix}
2.33/4.856 \\
0.526/4.856 \\
2/4.856
\end{bmatrix}
= \begin{bmatrix}
0.479 \\
0.108 \\
0.411
\end{bmatrix}
\]

Now, w.r.t efficiency:

\[
\begin{bmatrix}
1/2 & 2/1 & 4/1 \\
1/4 & 1 & 3/1 \\
1/3 & 1
\end{bmatrix}
= \begin{bmatrix}
2.33 \\
1.50 \\
0.526
\end{bmatrix}
\]

Total value = \((2.33+1.5+0.526) = 4.356\)

\[
= \begin{bmatrix}
0.53 \\
0.344 \\
0.120
\end{bmatrix}
\]

Similarly, w.r.t velocity:

\[
= \begin{bmatrix}
0.53 \\
0.344 \\
0.120
\end{bmatrix}
\]
The results of ‘AHP’ indicate that diameter ‘d’ is most important event or beneficial event & viscosity is second important event. Length ‘L’ is third important event.

4. Variation of ‘hₙ’ & ‘η’ with ‘d’

![Figure 2. (Variation of ‘hₙ’ with ‘d’)](image1)

Figure 2. (Variation of ‘hₙ’ with ‘d’)
(Penstock length = 100 m)

![Figure 3. (Variation of ‘η’ with ‘d’)](image2)

Figure 3. (Variation of ‘η’ with ‘d’)
(Turbine efficiency = 66% & pump efficiency = 65%)

It is clear from ‘AHP’ that diameter is most important parameter for efficiency & the variation of head loss & efficiency with penstock diameter have been represented in Figures 2 & 3.

5. Variation of ‘hₙ’ & ‘η’ with ‘L’

![Figure 4. (Variation of ‘hₙ’ w.r.t ‘L’)](image3)

Figure 4. (Variation of ‘hₙ’ w.r.t ‘L’)
(Penstock diameter = 2.4 m)

![Figure 5. (Variation of ‘η’ w.r.t ‘L’)](image4)

Figure 5. (Variation of ‘η’ w.r.t ‘L’)
(Constant Turbine Head is Maintained with ηᵣ=0.66 & ηₚ=0.65)

The parametric analysis, AHP & graphs implies the following correlations:

\[ \eta \propto d, \eta \propto 1/L, \eta \propto 1/\mu \]
Now, $\eta \propto \frac{d}{\mu L}$ or, $\eta = S \frac{d}{\mu L}$

Here, $S = $ Constant. This constant is first time introduced in the formula of PSP. That’s why, it is termed as PSP constant. This formula can use to calculate the efficiency of PSP if the value of constant is known.

6. Finding the Value of ‘S’ with the Help of Particle Swarm Optimization

PSO is a robust stochastic optimization technique based on the movement and intelligence of swarms. PSO applies the concept of social interaction to problem solving. It was developed in 1995 by James Kennedy (social-psychologist) and Russell Eberhart (electrical engineer). It uses a number of agents (particles) that constitute a swarm moving around in the search space looking for the best solution. Each particle is treated as a point in a N-dimensional space which adjusts its “flying” according to its own flying experience as well as the flying experience of other particles.

\[
V_{i}^{k+1} = wV_{i}^{k} + c_{1} \text{rand}_{1}(\ldots)(p_{best} - s_{i}^{k}) + c_{2} \text{rand}_{2}(\ldots) x (g_{best} - s_{i}^{k}) \ldots \tag{1}
\]

![Flow chart of PSO](image)
Penstock for PSP is within the range of 80-150 meter. So here the optimization is done using the boundary conditions of the parameters. The total optimization is divided in two steps. The first step is for the value of ‘S’ within 100 m limit of penstock length & in the next part for penstock length more than 100 m but not exceeding 150 m. 

The objective equation for optimization is

\[ S = \frac{\eta L \mu}{d} \]

Parameters with their boundary limit have been listed here:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Min Values</th>
<th>Max Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency((\eta))</td>
<td>55%</td>
<td>70%</td>
</tr>
<tr>
<td>Viscosity ((\mu))</td>
<td>7.005</td>
<td>8.684</td>
</tr>
<tr>
<td>Penstock Length (L)</td>
<td>80 m</td>
<td>100 m</td>
</tr>
<tr>
<td>Penstock diameter (d)</td>
<td>0.30 m</td>
<td>0.38 m</td>
</tr>
</tbody>
</table>

After optimization optimum values of different parameters has been listed

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimum Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency((\eta))</td>
<td>0.61</td>
</tr>
<tr>
<td>Viscosity ((\mu))</td>
<td>8.19</td>
</tr>
<tr>
<td>Penstock Length (L)</td>
<td>0.34</td>
</tr>
<tr>
<td>Penstock diameter (d)</td>
<td>82.10</td>
</tr>
</tbody>
</table>

The optimization is done to maximize the value of ‘S’. Now, the value of ‘S’ corresponding to the optimum condition is

\[ S = \frac{0.61 \times 8.19 \times 82.10}{0.34} = 1206.36 \approx 1.206 \times 10^3 \]

Now, for penstock of length more than 100 m but less than 150 m.

<table>
<thead>
<tr>
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<tbody>
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</tr>
<tr>
<td>Penstock Length (L)</td>
<td>100 m</td>
<td>150 m</td>
</tr>
<tr>
<td>Penstock diameter (d)</td>
<td>0.30 m</td>
<td>0.38 m</td>
</tr>
</tbody>
</table>

After optimization parameters are:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimum Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency((\eta))</td>
<td>0.69</td>
</tr>
<tr>
<td>Viscosity ((\mu))</td>
<td>8.64</td>
</tr>
<tr>
<td>Penstock Length (L)</td>
<td>0.31</td>
</tr>
<tr>
<td>Penstock diameter (d)</td>
<td>108</td>
</tr>
</tbody>
</table>

Now, the value of ‘S’ for penstock length within 100-150 m is
Therefore, the standard value of ‘S’ for ‘PSP’ within the range $80 \leq L \leq 100$ is

$$S = \frac{0.69 \times 8.64 \times 10^8}{0.31} = 2076.94 = 2.076 \times 10^3$$

The equation of efficiency for ‘PSP’ is

$$\eta = 1.641 \times 10^3 \frac{d}{\mu L} \text{ for } 80 \leq L \leq 150.$$ 

7. Comparing the Objective Equation with Standard Equation

The standard equation for pump storage plant efficiency is

$$\eta = \frac{1 - K}{1 + K \eta_t \eta_f}$$

The new objective equation is

$$\eta = 1.641 \times 10^3 \frac{d}{\mu L}$$

To optimized efficiency with standard formula, value of ‘K’, turbine efficiency, plant efficiency etc are required to calculate. But in case of new formula, direct values of penstock length & diameter are required. So, it is a easier process of efficiency optimization. The value of efficiency, calculated by new formula is much closer to practical efficiency because it is depends on some independent design variables. But the parameters like ‘K’, turbine efficiency & pump efficiency of standard formula, are not independent variables because these parameters are also functions of other parameters.

The value of turbine efficiency & pump efficiency depends on length, diameter & other designed parameters. Here, the values of length & diameter are directly used to find the efficiency. So this equation can also use as the best objective equation to find optimum condition correspondence to maximum efficiency.

8. Finding the Optimum Operating Condition for a Micro-Hydro PSP

A micro-hydro power station has generation limit <5-10 MW. The penstock length is mostly near about 100 m. Now, the optimum condition for power generation is

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Max Values</th>
<th>Min Values</th>
<th>Optimized Values</th>
<th>Wattage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity</td>
<td>8.684</td>
<td>7.005</td>
<td>7.739</td>
<td>0.21</td>
</tr>
<tr>
<td>Penstock diameter</td>
<td>0.38</td>
<td>0.30</td>
<td>0.335</td>
<td>0.22</td>
</tr>
<tr>
<td>Penstock length</td>
<td>100</td>
<td>80</td>
<td>83.37</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Penstock length & diameter are design parameters. So these two parameters can regulate according to optimum value but practically it is very tough to maintain a constant & optimum value of viscosity. For the constant values of design parameters if viscosity changes, it will affects the efficiency. The variation of efficiency with the change of viscosity keeping penstock length & diameter constant, has been represented here
Figure 6. Variation of Efficiency with Viscosity (Penstock length = 83.37 m & Penstock Diameter = 0.335 m)

Graph-6, represents the variation of efficiency with viscosity by maintaining the optimum value of design parameters (penstock diameter & length). From graph-6, it is clear that if the value of viscosity changes (either increase or decrease) from optimum value, maintaining design parameter constant the efficiency will decrease.

9. Conclusion

The effects of various designed & non designed parameters on the output & efficiency of pump storage hydro power plant has been analyzed & discussed here. Mitigation of frictional head loss by maximizing plant alignment has first time introduced here. The more accurate objective equation of PSP with finite value of constant has developed here. Lastly the optimum condition of design parameters has found out for a micro-hydro pump storage plant.

References
