

## **Design of Multivariable Adaptive Generalized Predictive Control for the Part Turbine/Generator of Micro-Hydro Power Plant**

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### **Abstract**

*This paper provides the design steps of a multivariable Adaptive Generalized Predictive Controller AGPC whose duty is to drive a micro-hydro power system that comprises a hydraulic turbine driving a synchronous generator which is connected to an infinite bus via a step-up transformer and a transmission line. The simulation model of the part turbine/generator of micro-hydropower plant was constructed based on mathematical equations that summarize the behavior of the micro-hydro power plant. Multivariable AGPC is considered here because of its wide use in the industry and also at universities, showing good performance and a certain degree of robustness. In this study, the standard multivariable (GPC) algorithm is presented. The model parameters are estimated using an identification algorithm based on Recursive Least Squares (RLS) method. In order to validate the effectiveness of AGPC, simulation studies for the part turbine/generator of micro hydropower plant are used. Encouraging results are obtained that motivate for further investigations.*

**Keywords:** *Generalized Predictive Control, Adaptive Control, Modeling, Multivariable Systems, Micro-hydropower Plant*

### **1. Introduction**

The most important elements of hydropower plant are synchronous generator and turbine, because it is the source of electrical energy. In generator, mechanical energy (usually from a turbine) is transformed into electrical energy. Energy transformation is possible only if generator excitation exists. Excitation of generator also defines generator output values: voltage and real power. This means that generator excitation regulation is actually regulation of generator output energy and also impacts the stability of entire electric power system [1-4]. The objective of the control strategy is to generate and deliver power in an interconnected system as economically and reliably as possible while maintaining the voltage and frequency within permissible limits. Micro-hydropower plant is equipped with hydraulic turbine governor and excitation control. The errors in the terminal voltage and in the output active power, with respect to their respective references, represent the controller inputs and the generator-exciter voltage and governor-valve position represent the controller outputs. The control of real power output and the terminal voltage keeps the system in the steady state [5-10].

This paper presents the application of multivariable GPC control to achieve sets points tracking of the outputs of the plant. The GPC control is one of the most favorite predictive control methods; it has obtained great success in process industries. It is applicable [11-13] to the systems with non-minimal phase, unstable systems in open loop, systems with unknown or varying dead time, systems with unknown order and nonlinear systems approximated by linear models. This approach can be easily extended to deal with multivariable plants which results in the multivariable GPC algorithm.

The basic idea behind multivariable GPC scheme [14], [15] like many other predictive control approaches is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a control horizon. The index to be optimized is the expectation of a quadratic function measuring the distance between the predicted system output and some reference sequence over the horizon plus a quadratic function measuring the control effort. The predictive model is carried out based on the solving Diophantine equations.

Multivariable GPC depends on a dynamic predictive model of the system obtained by employing time series analysis approaches. These models can quickly become obsolete and require maintenance when the operating conditions become significantly different from the design conditions. In such cases, the stability and performance of a multivariable GPC approach cannot be guaranteed. The need to generate good predictions in the face of changing operating conditions and / or plant characteristics can be fulfilled through updating the linear model parameters online, and then generate optimal controllers at each time step assuming that the estimated model at that time step is the correct one. This work is aimed at the development of adaptive GPC (AGPC) scheme based on ARIMAX models. In addition, adaptive control is one of the most well studied areas in control systems theory. In adaptive control algorithms and techniques are developed for dealing with modeling uncertainties and disturbances [16].

The paper is organized as follows. Section II presents the system modeling. Section III describes the designed multivariable GPC Controller. Section IV is devoted the description of adaptive algorithm. In section V, the effectiveness and superiority of the proposed algorithm, is demonstrated by simulation studies for the part turbine/generator of micro hydropower plant. Some concluding remarks end the paper.

## 2. System Modeling

The block diagram of the sample controlled power system is shown in figure 1 that comprises a hydraulic turbine driving a synchronous generator which is connected to an infinite bus via a step-up transformer and a transmission line. The output real power  $P_t$  and terminal voltage  $V_t$  at the generator terminals are measured and fed to the controller. The outputs of the controller (system control inputs) are fed into the generator-exciter and governor-valve. In the simulation studies described here, the nonlinear equations of the synchronous generator are represented by a third-order nonlinear model based on park's equations. The hydraulic turbine, governor valve and exciter are each represented by a first order model. The model equations are as follows [17-28]:

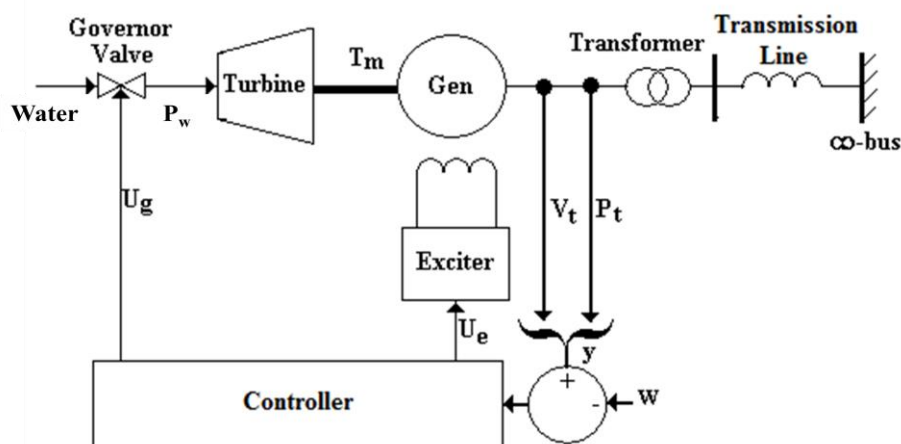


Figure 1. Controlled Sample Hydropower System

The Mechanical equations

The rotor speed of the generator is given by:

$$\dot{\delta}(t) = \omega(t) \quad (1)$$

The mechanical equation of the motion is as follows:

$$\frac{H}{\pi f_0} \frac{d\omega(t)}{dt} + D\omega = P_m - P_t$$

*i.e.,*

$$M \frac{d\omega(t)}{dt} + D\omega = P_m - P_t \quad (2)$$

$$\text{Where, } M = \frac{H}{\pi f_0} \text{ and } f_0 = \frac{\omega_0}{2\pi}$$

The electrical generator dynamics equations

$$\frac{dE'_q(t)}{dt} = \frac{1}{T'_{do}} (E_{fd}(t) - E_q(t)) \quad (3)$$

The electrical equations (assumed  $x'_d = x_q$ )

$$E_q(t) = E'_q(t) + (x_d - x_q)I_d(t) \quad (4)$$

$$P_t(t) = E_q(t)I_q(t) \quad (5)$$

$$I_d(t) = \frac{E'_q(t) - V_s \cos\delta(t)}{x'_{ds}} \quad (6)$$

$$I_q(t) = \frac{V_s \sin\delta(t)}{x'_{ds}} \quad (7)$$

$$E'_q(t) = x_{ad}I_f(t) \quad (8)$$

$$V_t(t) = \left[ (V_d(t))^2 + (V_q(t))^2 \right]^{\frac{1}{2}} \quad (9)$$

$$V_d(t) = E'_q(t) - x'_d I_d(t) \quad (10)$$

$$V_q(t) = x'_d I_q(t) \quad (11)$$

Where,  $x_{ds} = x_d + x_T + x_L$   $x'_{ds} = x'_d + x_T + x_L$   $x_s = x_T + x_L$

More details about power system modeling can be seen in [20-23]

Using the above equations, we can express  $P_t(t)$  as

$$P_t(t) = \frac{V_s x_{ds}}{(x'_{ds})^2} E'_q(t) \sin\delta(t) - \frac{(x_d - x_q)(V_s)^2}{(x'_{ds})^2} \sin\delta(t) \cos\delta(t) \quad (12)$$

In terms of the state variables  $\delta$  and  $\omega(t) = \dot{\delta}(t)$ , the equation (2) becomes

$$\frac{d\omega(t)}{dt} = (P_m - \frac{V_s x_{ds}}{(x'_{ds})^2} E'_q(t) \sin\delta(t) + \frac{(x_d - x_q)(V_s)^2}{(x'_{ds})^2} \sin\delta(t) \cos\delta(t) - D\omega(t)) \frac{\omega_0}{2H} \quad (13)$$

In terms of the state variables  $E'_q(t)$  and  $E_{fd}(t)$  the equation (3) becomes

$$\frac{dE'_q(t)}{dt} = \frac{\omega_0 r_{fd}}{x_{ad}} E_{fd}(t) + \frac{x_{ds}}{x'_{ds} T'_{do}} E'_q(t) + \frac{(x_d - x_q) V_s}{x'_{ds} T'_{do}} \quad (14)$$

$$\text{Where, } T'_{do} = \frac{x_{ad}}{\omega_0 r_{fd}}$$

The governor valve equation is given by

$$\frac{P_w}{U_g} = \frac{K_v}{1 + \tau_g s} \quad (15)$$

The exciter equation defined by

$$\frac{E_{fd}}{U_e} = \frac{1}{1 + \tau_e s} \quad (16)$$

The turbine equation

$$\frac{P_m}{P_w} = \frac{1}{1 + \tau_b s} \quad (17)$$

In terms of the state variables  $E_{fd}$ ,  $P_w$  and  $P_m$  the equations (15)-(17) written as follow:

$$P_w \frac{dP_w(t)}{dt} = \frac{-P_w}{\tau_g} + \frac{K_v}{\tau_g} U_g \quad (18)$$

$$\frac{dE_{fd}(t)}{dt} = \frac{-E_{fd}(t)}{\tau_e} + \frac{1}{\tau_e} U_e \quad (19)$$

$$\frac{dP_m}{dt} = \frac{-P_m}{\tau_b} + \frac{1}{\tau_b} P_w \quad (20)$$

Defining  $x = [\delta \ \dot{\delta} \ E'_q \ E_{fd} \ P_w \ P_m]^T$  the state variables vector then the equations above can be written in the key form:

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = (x_6 - K_1 x_3 \sin x_1 - K_2 \sin x_1 \cos x_1 - D x_2) \frac{\omega_0}{2H} \\ x_3 = \frac{\omega_0 r_{fd}}{x_{ad}} x_4 + K_3 x_3 - K_4 \cos x_1 \\ \dot{x}_4 = \frac{-x_4}{\tau_e} + \frac{1}{\tau_e} U_e \\ \dot{x}_5 = \frac{-x_5}{\tau_g} + \frac{1}{\tau_g} U_g \\ \dot{x}_6 = \frac{-x_6}{\tau_b} + \frac{x_5}{\tau_b} \end{array} \right. \quad (21)$$

The output,  $y_1$  and  $y_2$  may be expressed in terms of these state variable by

$$y_1 = P_t = K_1 x_3 \sin x_1 + K_2 \sin x_1 \cos x_1 \quad (22)$$

$$y_2 = V_t = (V_d^2 + V_q^2)^{1/2} \quad (23)$$

Where,

$$V_d = K_5 \sin x_1 \quad (24)$$

$$V_q = K_6 x_3 + K_7 \cos x_1 \quad (25)$$

### 2.1. Linear Model of Synchronous Generator

A linear Multi-Input Multi-output (MIMO) model of the generator system is required to design a controller for such system. It is derived from the system nonlinear model by linearizing the nonlinear equations (13) and (14) around a specific operating point. The linear state-space model is derived next where the variables shown represent small displacements around the selected operating point.

$$\begin{cases} \dot{x}(t) = F_X x(t) + F_U u(t) \\ y(t) = G_X x(t) + G_U u(t) \end{cases} \quad (26)$$

$$\text{Where, } F_X = \left. \frac{\partial f}{\partial x^T} \right|_{\bar{X}, \bar{U}}; F_U = \left. \frac{\partial f}{\partial U^T} \right|_{\bar{X}, \bar{U}}; G_X = \left. \frac{\partial g}{\partial x^T} \right|_{\bar{X}, \bar{U}}; G_U = \left. \frac{\partial g}{\partial U^T} \right|_{\bar{X}, \bar{U}}$$

$F_X, F_U, G_X$  et  $G_U$  are the Jacobian matrices of partial derivatives of  $f$  and  $g$  respectively to  $X$  and  $U$  evaluated at the point  $(\bar{X}, \bar{U})$ .

The linear state-space model defined by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad (27)$$

$$\text{Where, } A = F_X \quad B = F_U \quad C = G_X \quad D = G_U$$

The matrices  $A, B, C$  and  $D$  have the form:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ K_8 & \frac{-D\omega_0}{2H} & K_9 & 0 & 0 & \frac{\omega_0}{2H} \\ K_{10} & 0 & K_3 & \frac{\omega_0 r_{fd}}{x_{ad}} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_e} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{\tau_g} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\tau_b} & \frac{-1}{\tau_b} \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\tau_e} & 0 \\ 0 & \frac{K_g}{\tau_g} \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} K_{11} & 0 & K_{12} & 0 & 0 & 0 \\ K_{13} & 0 & K_{14} & 0 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Where,

$x = [\delta \quad \dot{\delta} \quad E'_q \quad E_{fd} \quad P_w \quad P_m]^T$  State variables vector

$u = [U_e \quad U_g]^T$  Control input vector

$y = [P_t \quad V_t]^T$  Measurement vector  
 $P_t = K_{11}x_1 + K_{12}x_3$  Output Power  
 $V_t = K_{13}x_1 + K_{14}x_3$  Terminal voltage  
 Expressions for parameters  $K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, K_{13}$  and  $K_{14}$  are given in Appendix 2.

### 2.2. State Space to Transform Function Conversion

Consider the state equation (27). We may take its Laplace transform and rearrange it as follows:

$$sX(s) = AX(s) + BU(s) \rightarrow (sI - A)X(s) = BU(s) \tag{28}$$

If we combine this with the transform of the output equation:  $Y(s) = CX(s) + DU(s)$ , we get  $Y(s) = C(sI - A)^{-1}BU(s) + DU(s)$   
 Or, equivalently

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D \tag{29}$$

In the Control Systems Toolbox, the command  $[\text{num}, \text{den}] = \text{ss2tf}(A, B, C, D, i)$  converts the state equation to a transfer function.

### 3. Multivariable Generalized Predictive Control Algorithm

Let us, consider the following  $m \times m$  process model [28]:

$$A(q^{-1})y(t) = B(q^{-1})u(t - 1) + C(q^{-1})\xi(t) \tag{30}$$

Where,

$$A(q^{-1}) = I_m + A_1q^{-1} \dots \dots \dots + A_{na}q^{-na} \quad A_j \in \mathbb{R}^{m,m}$$

$$B(q^{-1}) = B_0 + B_1q^{-1} \dots \dots \dots + B_{nb}q^{-nb} \quad B_j \in \mathbb{R}^{m,m}$$

$$C(q^{-1}) = C_0 + C_1q^{-1} \dots \dots \dots + C_{nc}q^{-nc} \quad C \in \mathbb{R}^{m,m}$$

$y(t) \in \mathbb{R}^m$  is the output vector

$u(t) \in \mathbb{R}^m$  is the input vector

$\xi(t) \in \mathbb{R}^m$  is a sequence of independent random vectors with zero mean value and finite covariance matrix

$q^{-1}$  is the backward shift operator such that  $q^{-1}f(t) = f(t - 1)$

The objective of the GPC control is the output  $y(t)$  to follow some reference signal  $y^*(t)$  taking into account the control effort. This can be expressed in the following cost function:

$$J(h_i, h_p, h_c, t) = E \left\{ \sum_{h_i}^{h_p} [y(t + j) - y^*(t + j)]^T R [y(t + j) - y^*(t + j)] + \sum_{h_i}^{h_c} \Delta u^T(t + j - 1) Q \Delta u(t + j - 1) \right\} \tag{31}$$

$$\Delta u(t + j - 1) = 0 \text{ for } j > h_c$$

Where:

$h_p$  is the prediction horizon.

$h_i$  is the initial horizon.

$h_c$  is the control horizon.

$y^*(t)$  is the output reference.

$R$  is the output weighting factor.

$Q$  is the control weighting factor.

The control objective is to compute at each time  $t$ , control inputs that minimize the quadratic criterion  $J(h_i, h_p, h_c, t)$  for this there are two cases:

Let us first build  $j$ -step ahead predictors with following Diophantine equation:

$$1_m = E^j(q^{-1})A(q^{-1})\Delta(q^{-1}) + q^{-j}F^j(q^{-1}) \quad (32)$$

$$j = 1 \dots h_p$$

Where:

$$E^j(q^{-1}) = 1_m + E_1q^{-1} \dots \dots \dots + E_{j-1}q^{-(j-1)} \quad E_j \in R^{m,m}$$

$$F^j(q^{-1}) = F_0^j + F_1^jq^{-1} \dots \dots \dots + F_{na}^jq^{-na} \quad F_j^j \in R^{m,m}$$

The polynomial matrices  $E^j(q^{-1})$  and  $F^j(q^{-1})$  are uniquely defined by:  $A(q^{-1}), \Delta(q^{-1})$  and  $j$ .

Using equation (30) and (32) we obtain:

$$y(t+j) = E^j(q^{-1})B(q^{-1})\Delta u(t+j-1) + F^j(q^{-1})y(t) + E^j(q^{-1})\xi(t+j) \quad (33)$$

The optimal predictor  $y(t+j)$  at time  $t$  is given by:

$$\hat{y}(t+j/t) = G^j(q^{-1})B(q^{-1})\Delta u(t+j-1) + F^j(q^{-1})y(t) \quad (34)$$

Where,  $G^j(q^{-1}) = E^j(q^{-1})B(q^{-1})$

Defining  $G^j(q^{-1}) = g_0^j + g_1^jq^{-1} \dots \dots \dots + g_{j-1}^jq^{-(j-1)}$  then the equation above can be written in the key vector form:

$$\hat{Y} = G\Delta U_t + Y_0 \quad (35)$$

Where the vectors are all  $h_p \times 1$ :

$$\hat{Y} = [\hat{y}(t+1/t) \ \hat{y}(t+2/t) \ \dots \ \hat{y}(t+h_p/t)]^T$$

$$\Delta U = [\Delta u(t) \ \Delta u(t+1) \ \dots \ \Delta u(t+h_c-1)]^T$$

$$Y_0 = [Y_0(t+1) \ Y_0(t+2) \ \dots \ Y_0(t+h_p)]^T$$

Note that:  $G^j(q^{-1}) = \frac{B(q^{-1})[1-q^{-j}F^j(q^{-1})]}{A(q^{-1})}$  so that one way to computing  $G^j$  is simply to consider the Z-transform plant's step-response and to take the first  $j$  terms and therefore  $g_j^j = g_j$  for  $j=0, 1, 2 \dots < i$  independent of the particular  $G$  polynomial [11].

The matrix  $G$  is then lower-triangular of dimension  $mh_p \times mh_c$ :

$$G = \begin{bmatrix} g_0 & 0 & \dots & \dots & 0 \\ g_1 & g_0 & \dots & \dots & 0 \\ \vdots & \vdots & \dots & \dots & g_0 \\ \vdots & \vdots & \dots & \dots & \vdots \\ g_{h_p-1} & g_{h_p-2} & \dots & \dots & g_{h_p-h_c} \end{bmatrix}$$

From the definitions above of the vectors and with:

$$Y^* = [Y^*(t+1) \ Y^*(t+2) \ \dots \ Y^*(t+h_p)]^T \quad (36)$$

The expectation of the cost-function of (4) can be written as follow:

$$J(h_i, h_p, h_c, t) = (G\Delta U_t + Y_0 - Y^*)^T \bar{R}(G\Delta U_t + Y_0 - Y^*) + \Delta U_t \bar{Q} \Delta U_t^T \quad (37)$$

The solution,  $\Delta U_t$  minimizing the criterion can be explicitly found, using:

$$\frac{\partial J}{\partial \Delta U_t} = 0 \quad (38)$$

it follows that:

$$\Delta U_t^* = (G^T G + Q)^{-1} G^T R (Y_0 - Y^*) \quad (39)$$

Note that the first element  $\Delta U_t^*$  of is  $\Delta u(t)$  so that the current control  $u(t)$  is given by:

$$u(t) = u(t-1) + (G^T G + Q)^{-1} G^T R (Y_0 - Y^*) \quad (40)$$

#### 4. Adaptive Control Algorithm

In order to derive an adaptive version of the control law described previously, it is necessary to add a parameter estimation procedure.

Hence, at each sampling time the process model parameters (the  $A_i$  and  $B_i$  matrices appearing in  $\theta$ ) are estimated using standard recursive identification algorithms.

The estimated parameters are obtained, via the following parameter estimation algorithm:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\varepsilon(t) \Delta \Phi(t-1) F(t-1)}{1 + \Delta \Phi^T(t-1) F(t-1) \Delta \Phi(t-1)} \quad (41)$$

$$\varepsilon(t) = \Delta y(t-1) - \theta(t-1) \Delta \Phi(t-1) \quad (42)$$

$$F(t) = \frac{1}{\lambda_1(t)} \left[ F(t-1) - \frac{\Delta \Phi(t-1) \Delta \Phi^T(t-1) F(t-1)}{\lambda_1(t) \Delta \Phi^T(t-1) F(t-1) \Delta \Phi(t-1)} \right] \quad (43)$$

$$0 < \lambda \leq 1; \ 0 \leq \lambda < 2; \ F(0) > 0$$

The adaptive control law is then computed, starting from the  $\hat{A}_i$  and  $\hat{B}_i$  matrices instead of the  $A_i$  and  $B_i$  ones.

#### 5. Simulation and Discussion

This section presents simulation results to validate the theoretical developments and to demonstrate the performance of the proposed adaptive generalized predictive control scheme in multi-inputs multi-outputs systems. In the simulation to test the reference tracking performance, parameters convergence, and disturbance rejection capacity, the reference input is changed with time and a load disturbance is applied. The simulation results are obtained by using Matlab Toolbox.



Initial condition (operating point) for the non linear system:

$$x = [0.775 \quad 0 \quad 1.434 \quad -0.0016 \quad 0.8 \quad 0.8]^T$$

The micro-hydropower plant model is as follow:

$$A(q^{-1})y(t) = B(q^{-1})u(t - 1)$$

Where,

$$A_1 = \begin{bmatrix} -1.826 & 0 \\ 0 & -1.54 \end{bmatrix}; A_2 = \begin{bmatrix} 1.21 & 0 \\ 0 & 0.7693 \end{bmatrix}; A_3 = \begin{bmatrix} -0.3479 & 0 \\ 0 & -0.1275 \end{bmatrix}; A_4 = \begin{bmatrix} 0.03653 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 1.367 & -0.03534 \\ -0.07828 & 1.218 \end{bmatrix}; B_1 = \begin{bmatrix} -2.107 & -0.06115 \\ 0.1818 & -0.8947 \end{bmatrix};$$

$$B_2 = \begin{bmatrix} 1.043 & 0.06876 \\ -0.1276 & 0.1182 \end{bmatrix}; B_3 = \begin{bmatrix} -0.1629 & -0.01589 \\ 0.02571 & 0.02204 \end{bmatrix}; B_4 = \begin{bmatrix} 0.003059 & 0 \\ 0 & 0 \end{bmatrix}.$$

The objective of the micro-hydropower plant control is to track a reference. The prediction controller parameters ( $h_p$ ,  $h_c$ ,  $h_i$ ,  $Q$  and  $R$ ) for GPC controller are chosen in order to get an acceptable tracking.

$$h_p = 10; h_c = 5; h_i = 1; Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}; R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The reference is chosen as a square wave.

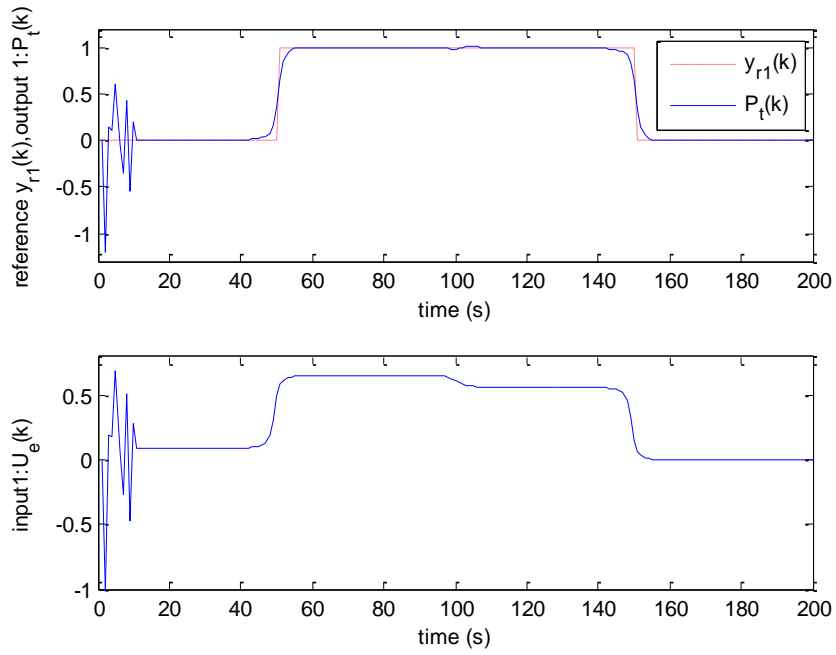
Simulations were carried out to verify the advantages of using multivariable AGPC control in this application.

From the results shown in Figures 2, 3, 5 and 6 it can be seen that the proposed controller is able to adequately (attain and) control the system output at the desired reference.

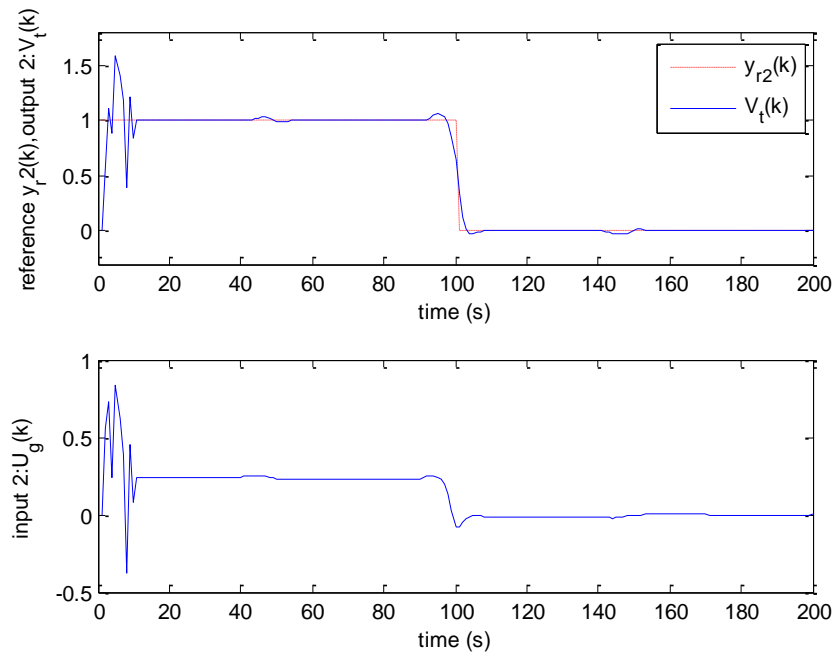
In terms of initial response of the controller, it can be observed that the courses of the control variables oscillate in the initial control interval. When model parameter estimates are converged, the quality of the control process is very good.

When the load disturbance is applied there is an oscillation and overshoots in system response in the initial control interval. As can be seen in Figures 5 and 6 the controller eliminates this disturbance.

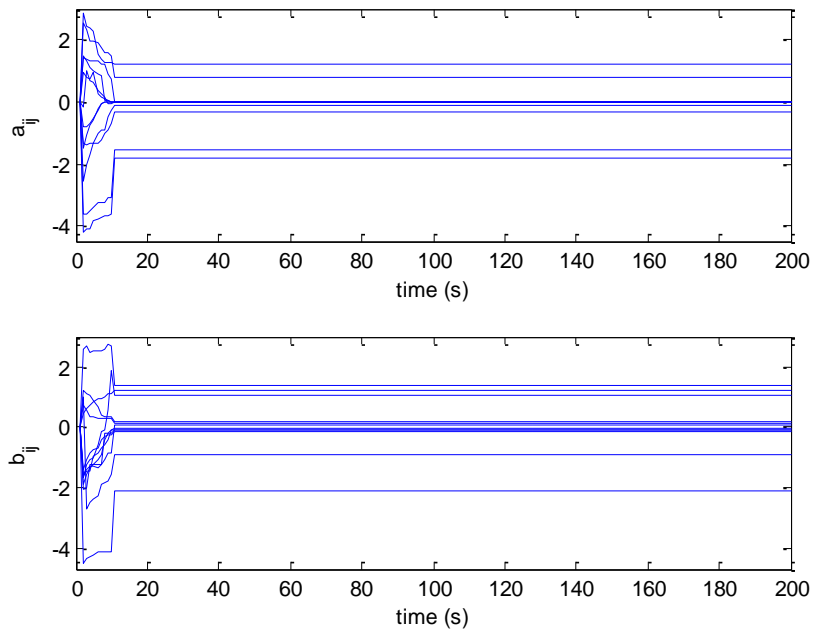
The temporal evolution of the adjustable parameters of the controller is show in Figures 4 and 7. The model parameters are initialized with values near zero, but then they are adjusted taking in account the desired response. When the load disturbance is applied, the parameters are again adjusted taking into account the corresponding changes in the system.



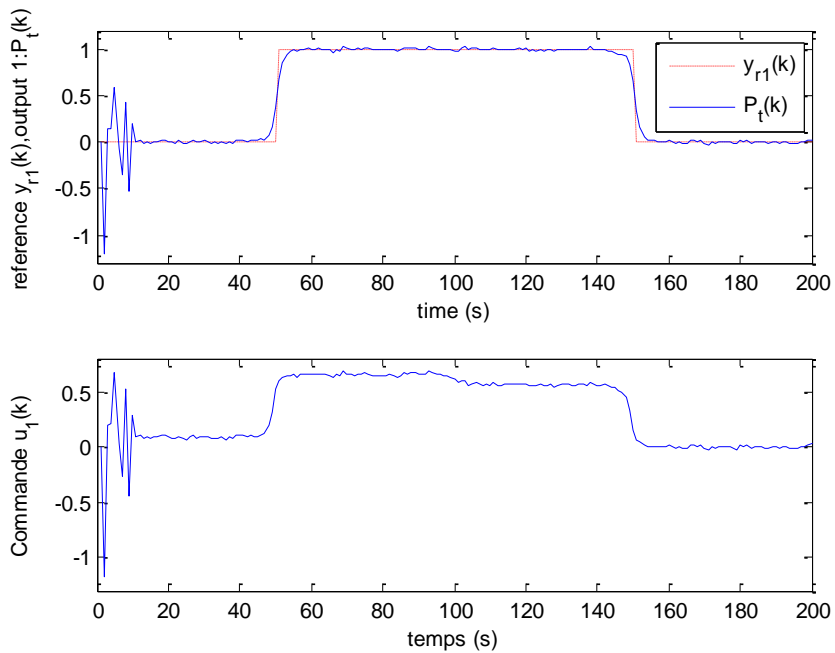
**Figure 2. Power Output  $P_t$  and Exciter Input  $U_e$  in Noise Absence Conditions**



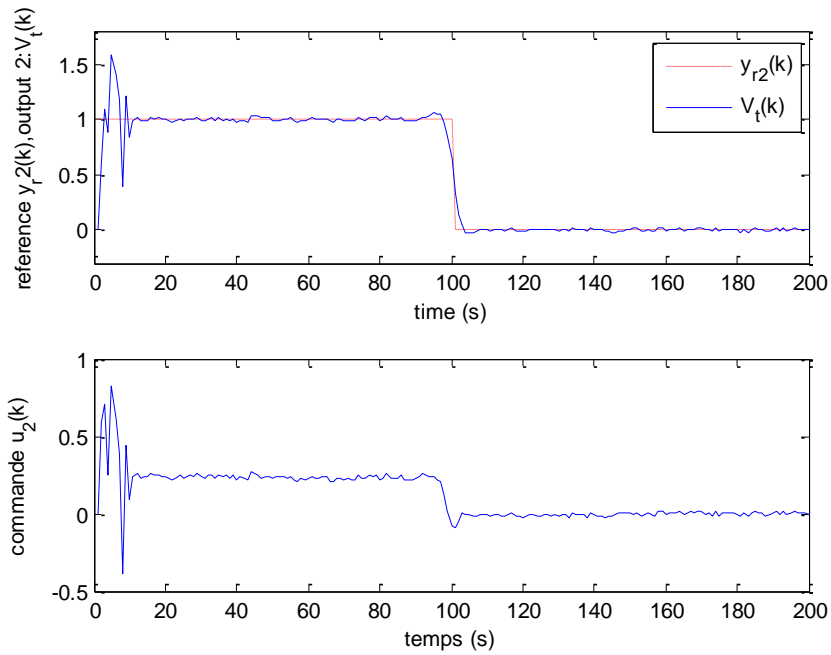
**Figure 3. Terminal Voltage  $V_t$  and Governor Input  $U_g$  in Noise Absence Conditions**



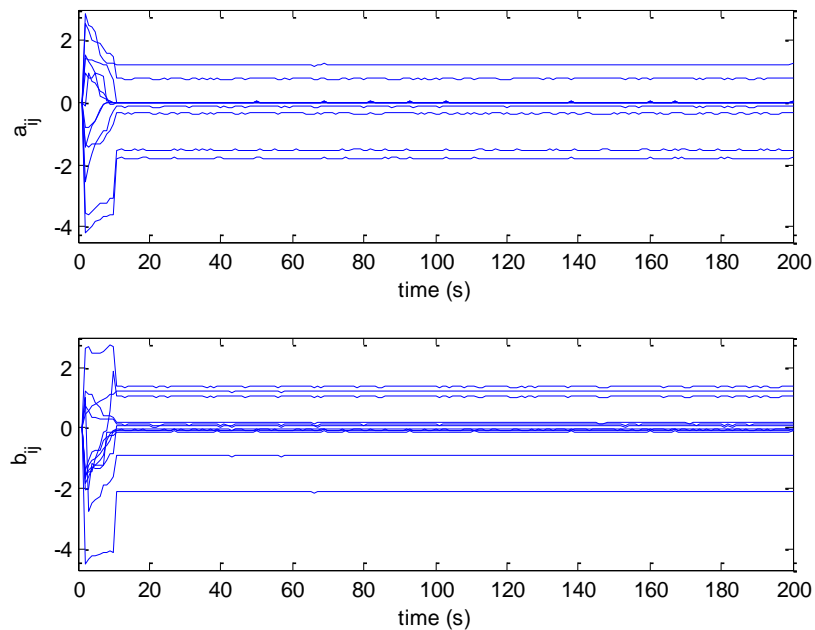
**Figure 4. Temporal Evolution of the Adjustable Parameters  $a_{ij}$  and  $b_{ij}$  in Noise Absence Conditions**



**Figure 5. Power Output  $P_t$  and Exciter Input  $U_e$  Affected by a Random Disturbance**



**Figure 6. Terminal Voltage  $V_t$  and Governor Input  $U_g$  Affected by a Random Disturbance**



**Figure 7. Temporal Evolution of the Adjustable Parameters  $a_{ij}$  and  $b_{ij}$  in Presence of Random Disturbance**

## 6. Conclusion

This paper has proposed an adaptive predictive controller for a class of multivariable process was designed for a micro-hydropower plant comprising a water turbine driving a synchronous generator. The model of micro-hydropower plant was constructed based on mathematical equations that summarize the behavior of the hydropower plant. The

proposed controller is based on GPC algorithm and identification on-line of the parameters of the process using RLS algorithm.

The simulation results show that the proposed method is able to adequately control the plant, and has good tracking performance and disturbance rejection capacity. This evidence suggests that the proposed controller could be a good option for industrial process control. As can be seen in the simulations, the adjustable parameters are adjusted for control of the unknown plant and taking into account changes in the system.

## Appendix 1

### List of Symbols

$V_d, V_q$	Stator voltage in d-axis and q-axis circuit
$V_t$	Terminal voltage
$E'_q$	Transient EMF in the quadratic axis of the generator
$x_{ad}$	Stator – rotor mutual reactance
$E_{fd}$	Field voltage
$r_{fd}$	Field resistance
$X_{fd}$	Self reactance of field winding
$U_e$	Exciter input
$\delta$	Rotor angle
$P_m$	Mechanical power
$P_w$	Water power
$H$	Inertia constant
$\omega(t)$	Rotor speed of the generator
$\omega_0$	Angular frequency of the infinite bus bar
$K_d$	Mechanical damping torque coefficient
$T_d$	Damping torque coefficient due to damper windings
$P_t$	Real power output at the generator terminals
$\tau_e$	Exciter time constant
$\tau_g$	Governor valve time constant
$\tau_b$	Turbine time constant
$U_g$	Governor input
$G_v$	Governor valve position
$K_v$	Valve constant
$x_d$	Total d-axis synchronous reactance between the generator and the infinite busbar
$x_q$	Total q-axis synchronous reactance between the generator and the infinite busbar
$x'_d$	Total d-axis transient reactance including the generator and the infinite busbar
$T'_{do}$	d-axis transient open-circuit time constant
$x_T$	Reactance of the transformer
$x_L$	Reactance of the transmission line
$x_s$	Reactance of the system

## Appendix 2

Expressions for parameters  $K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, K_{13}$  and  $K_{14}$  in the system model are:

$$K_1 = \frac{V_s x_{ds}}{(x'_{ds})^2}, K_2 = -\frac{(x_d - x_q) V_s^2}{(x'_{ds})^2}, K_3 = -\frac{x_{ds}}{x'_{ds} T'_{do}}, K_4 = -\frac{(x_d - x_q) V_s}{x'_{ds} T'_{do}}, K_5 = \frac{x_q V_s}{x'_{ds}}, K_6 = \frac{x_t + x_l}{x'_{ds}}$$

$$K_7 = \frac{x'_d V_s}{x'_{ds}} \quad , \quad K_8 = -K_1 x_{30} \cos(x_{10}) - K_2 \cos(2x_{10}) \quad , \quad K_9 = -K_1 \sin(x_{10}) \quad , \quad K_{10} = -K_4 \sin(x_{10})$$

$$K_{11} = -K_1 x_{30} \cos(x_{10}) + K_2 \cos(2x_{10}), \quad K_{12} = K_1 \sin(x_{10})$$

$$K_{13} = ((K_5 - K_7^2) \sin(x_{10}) \cos(x_{10}) - K_6 K_7 x_{30} \sin(x_{10})) ((K_5 \sin(x_{10}))^2 + (K_6 x_{30} + K_7 \cos(x_{10}))^2)^{-1/2}$$

$$K_{14} = 2K_6 (K_6 x_{30} + K_7 \cos(x_{10})) ((K_5 \sin(x_{10}))^2 + (K_6 x_{30} + K_7 \cos(x_{10}))^2)^{-1/2}$$

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