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Densely Distribution Pores and Coriolis Force on Thermohaline Convection in a Ferrofluid with Soret and Anisotropy Effects

R. Sekar[1*], D. Murugan[2] and K. Raju[3]

[1,2,3]Department of Mathematics, Pondicherry Engineering College, Pillaichavady, Puducherry-605 014, India.
**rsekar@pec.edu*

Abstract

Thermal convection in multi-component fluid has wide applications in industrial, ionospheric and geothermal systems. The effect of Coriolis force on Soret driven ferrothermohaline convection in densely packed anisotropic porous medium has been studied. A linear stability analysis is carried out using normal mode technique. It is found that stationary mode is favorable for Darcy model and oscillatory instability is studied. The porous medium is assumed to be variable and the effect of permeable parameter and vertical anisotropy are to destabilize the system. The non-buoyancy magnetization and Soret effects are found to stabilize the system in consideration with anisotropy of the system. The results are depicted graphically.

Keywords: *Darcy model; Soret effect; ferrofluid; thermohaline convection; anisotropy porous medium*

1. Introduction

The last millennium has seen many fascinating materials that possess promising physical properties and which are technologically useful. The ferrofluid is one such material. The magnetic materials play an important role in the overall development of many scientific applications. The ferrofluid has to be synthesized and it has widespread applications in various fields ranging from physics, chemistry, electrical engineering, biomedicine and instrumentation to computer technology. Its commercial usage includes novel- zero leakage, rotary-shaft seals used in computer disc drives [1], liquid cooled loudspeakers [2] and energy conversion devices [3].

The study of thermoconvective instability of ferrofluids has been the subject of investigation for the past four decades due to its remarkable applications. The magnetization of ferrofluids depends on the magnetic field, the temperature and density of the fluid. The variation of any one of these causes a change in the body force. This induces convection in ferromagnetic fluids in the presence of a magnetic field gradient. This mechanism, known as ferroconvection, is similar to the Rayleigh-Benard convection in ordinary fluids [4].

The thermohaline (double diffusive) convection in ferrofluid in a rotating porous medium has a very wide application in geothermal mineral fluid motion causing a deposit of ferric oxide in shale rock layers. In India these deposits are found very close to Eastern part of the Western Ghats and forestry region of Bihar. The same deposits are found very close to the rocky region in various part of the world. The analysis of double diffusive convection becomes complicated in case the diffusivity of one property is much greater than the other. Further, when two transport processes take place simultaneously, they

[*] Corresponding Author

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interfere with each other and produce cross-diffusion effect. The flux of mass caused by temperature gradient is known as Soret coefficients.

In astrophysical system, the ion layers are affected by the magnetic field surrounding the earth, which in turn rotates with constant angular velocity. This causes the ionspheric drift and storm. The suspension of the dust particle may be visualized as a porous medium. Finlayson [5] was the first to study the linear stability of ferroconvection in a horizontal layer of ferrofluid heated from below in the presence of uniform vertical magnetic field. Schwab *et al.*, [6] have conducted experiments and their results are found to be in good agreement with Finlayson [5]. Lalas and Carmi [7] have analyzed the same problem using the energy method. A similar analysis but with the fluid confined between ferromagnetic plates has been carried out by Gotoh and Yamada [8] using the linear stability analysis. Stiles and Kagan [9] have extended the problem to allow for the dependence of effective shear viscosity on temperature and colloid concentration.

Odenbach [10] has focused on recent developments in the field of rheological investigations of ferrofluids and their importance for the general treatment of ferrofluids. The nonlinear stability analysis for a magnetized ferrofluid layer heated from below has been performed by Sunil and Amit Mahajan [11] for stress-free boundaries. Nanjundappa and Shivakumara [12] have considered variety of velocity and temperature boundary conditions on the onset of ferroconvection in an initially quiescent ferrofluid layer. Thermal convection of ferrofluids in the presence of a uniform vertical magnetic field with the boundary temperatures modulated sinusoidally about some reference values is investigated by Singh and Bajaj [13]. Vaidyanathan *et al*., [14] have the convective instability of ferromagnetic fluid through porous medium of large permeability and mentioned that stationary convection can occur and oscillatory convection cannot occur by use of Brinkman number. This work has been extended to anisotropic porous medium by Sekar *et al*., [15] and Vaidyanathan *et al*., [16] modified the above work with use of Darcy model.

The effect of magnetic field along the vertical axis on thermoconvective instability in a ferromagnetic fluid saturating a rotating porous medium with Darcy model has been studied by Sekar *et al.,* [17]. The same with Brinkman model was also studied by Sekar *et al.,* [18]. Initially the effects of rotation and anisotropy of a porous medium on ferroconvection was analyzed by Vaidyanathan *et al*., [19]. This was extended to a study on the effect of rotation on ferrothermohaline convection saturating a porous medium was carried out by Sekar *et al*., [20]. The effect of rotation on ferrothermohaline convection has been analyzed and linear theory is used by Sekar *et al.*, [21]. It was observed that stationary mode is favored when compared to oscillatory mode for optimum heat transfer.

Kaloni and Lou [22] have studied convective instability in a horizontal layer of a magnetic fluid by considering the relaxation time and the rotational viscosity effects. Ryskin *et al.,* [23] analyzed the Soret -driven convection in ferrofluids using a non-linear analysis. Vaidyanathan *et al.,* [24] analyzed Soret-driven ferro thermohaline convection. Effect of Coriolis force on a Soret driven ferrothermohaline convective system was studied by Sekar *et al.,* [25]. Following this, the same analysis in a medium of sparse particle suspension was analyzed by Vaidyanathan *et al.,* [26]. The effect of Coriolis force on thermal convection in a couple stress fluid saturated rotating rigid porous layer was studied by Shivakumara *et al.,* [27].

More recently, the presence and absence of an anisotropy porous medium on Soret driven ferrothermohaline convection have been investigated by Sekar *et al*. [28-30] using Brinkman and Darcy models. Also, with and without of MFD viscosity on Soret driven ferrothermohaline convection in an anisotropic porous medium have been studied by Sekar and Raju [31-32] and the temperature dependent viscosity and coriolis force are studied in Soret driven ferrothermohaline convection in a porous medium and anisotropy effect have been studied by Sekar *et al*. [33] and Sekar and Raju [34].

Keeping in mind the importance of densely distributed porous medium on the onset of ferroconvection. In the present investigation, the convection of Soret-driven ferro thermohaline instability of multi-component fluid heated from below and salted from above is investigated in an anisotropic porous medium with coriolis force. Linear stability analysis is used. The conditions for the onset of stationary and oscillatory instabilities have been obtained.

2. Formulation of Problem

A horizontal layer of an incompressible Boussinesq ferromagnetic fluid of thickness '*d*' saturating a densely packed anisotropic porous medium with coriolis force in the presence of transverse applied magnetic field heated from below and salted from above is considered. The temperature and salinity at the bottom and top surfaces $z = \pm d/2$ are $T_0 \pm \Delta T/2$ and $S_0 \pm \Delta S/2$, respectively. Both the boundaries are taken to be free and perfect conductors of heat and solute. Consider the Soret effect on the temperature gradient. Further the whole system is assumed to rotate with uniform constant angular velocity Ω and anisotropy along the vertical direction taken as z axis (Figure 1). The mathematical equations governing the above investigation are as follows.

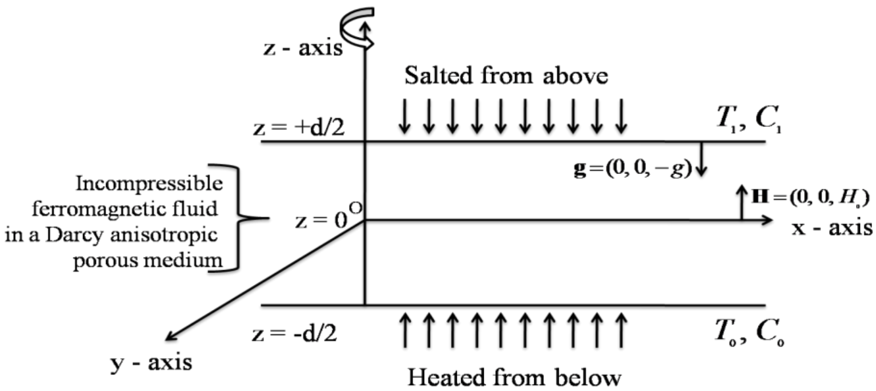

Figure 1. Geometrical Configuration

The continuity equation for an incompressible fluid is

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

The corresponding momentum equation is

$$\rho_o \frac{D\mathbf{q}}{Dt} = -\nabla p + \rho \mathbf{g} + \nabla \cdot (\mathbf{H}\mathbf{B}) + 2\rho_o (\mathbf{q} \times \mathbf{\Omega}) + \frac{\rho_o}{2} \nabla \left(\left| \mathbf{\Omega} \times \mathbf{r} \right|^2 \right) - \frac{\eta}{k} \mathbf{q} \tag{2}$$

The temperature equation for an incompressible ferrofluid is

$$\left[\rho_o C_{V,H} - \mu_o \mathbf{H} \cdot \left(\frac{\partial \mathbf{M}}{\partial T} \right)_{V,H} \right] \frac{dT}{dt} + \mu_o T \left(\frac{\partial \mathbf{M}}{\partial T} \right)_{V,H} \cdot \frac{d\mathbf{H}}{dt} = K_1 \nabla^2 T + \phi \tag{3}$$

The mass flux equation is given by

$$\frac{DS}{Dt} = K_s \nabla^2 S + S_T \nabla^2 T \tag{4}$$

where ρ_o, $\mathbf{q} = (u, v, w)$, $\mathbf{g} = (0,0,-g)$, k, t, p, η, $\mathbf{H}$, $\mathbf{B}$, $C_{V,H}$, T, $\mathbf{M}$, K_1, S, K_S, $\Omega = (0, 0, \Omega)$, S_T and ϕ are the fluid density, velocity, acceleration due to gravity, permeability of the porous medium, time, pressure, coefficient of viscosity, magnetic field, magnetic induction, heat capacity at constant volume and magnetic field, temperature, magnetization, thermal conductivity, salinity, concentration diffusivity, angular velocity, Soret coefficient and viscous dissipation factor containing second-order terms in velocity, respectively.

Using Maxwell's equation for non-conducting fluids, one can assume that the magnetization is aligned with the magnetic field and depends on the magnitude of the magnetic field, temperature and salinity, so that

$$\mathbf{M} = \frac{\mathbf{H}}{H} M\,(H, T, S) \tag{5}$$

The magnetic equation of state is linearized about the magnetic field H_0, the average temperature T_0 and the average salinity S_0 and so

$$M = M_0 + \chi (H - H_0) - K (T - T_0) + K_2 (S - S_0) \tag{6}$$

where $\chi = (\partial M / \partial H)_{H_0, T_0}$ is the susceptibility, $K = -(\partial M / \partial T)_{H_0, T_0}$ is the pyromagnetic coefficient and $K_2 = (\partial M / \partial S)_{H_0, S_0}$ is the salinity magnetic coefficient.

The density equation of state for a Boussinesq two-component fluid is

$$\rho = \rho_0 \left[1 - \alpha_t (T - T_0) + \alpha_s (S - S_0) \right] \tag{7}$$

where α_t is the thermal expansion coefficient and α_s is the solute analog of α_t.

Basic state is assumed to be quiescent state and basic state quantities are obtained by substituting the velocity of quiescent state in the governing Equations (1)-(4). The techniques of linearization and normal mode method are used in finding the solutions of the Equations (1)-(7). This can be written as

$$f(x, y, z, t) = f(z, t) e^{i(k_x x + k_y y)} \tag{8}$$

where $f(z,t)$ represents perturbed variables $w(z,t)$, $\theta(z,t)$, $\phi(z,t)$ and $S(z,t)$ and the wave number k_0 is given by $k_0^2 = k_x^2 + k_y^2$.

The vertical component of momentum equation can be calculated as

$$\left.\begin{aligned}
&\rho_0 \frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial z^2} - k_0^2\right) w \\
&= \frac{\mu_0 K \beta_t}{1+\chi}\left[(1+\chi)\frac{\partial \phi}{\partial z} - K\theta(1 - S_T)\right] k_0^2 - \rho_0 g \alpha_t k_0^2 \theta \\
&\quad + \frac{\mu_0 K_2 \beta_s}{1+\chi}\left[(1+\chi)\frac{\partial \phi}{\partial z} + K_2 S\right] k_0^2 + \rho_0 g \alpha_s k_0^2 S \\
&\quad - \frac{\mu_0 K K_2}{1+\chi}\left[\beta_s (1 - S_T)\theta - \beta_t S\right] k_0^2 - 2\rho_0 \Omega \frac{\partial \xi}{\partial z} + \frac{\eta}{k'} w k_0^2 - \frac{\eta}{k}\frac{\partial^2 w}{\partial z^2}
\end{aligned}\right\} \tag{9}$$

$$\left(\rho_0 \frac{\partial}{\partial t} + \frac{\eta}{k}\right)\xi = 2\rho_0 \Omega \frac{\partial w}{\partial z} \tag{10}$$

where ξ is the z- component of vorticity given by $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

The modified Fourier heat conduction equation is

$$\rho_0 C_{V,H}\frac{\partial\theta}{\partial t}-\mu_0 K T_0\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right)=K_1\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)\theta+\left[\rho_0 C\beta_t-\frac{\mu_0 K^2 T_0^2\beta_t}{1+\chi}+\frac{\mu_0 K K_2 T_0\beta_s}{1+\chi}\right]w \tag{11}$$

where $\rho_0 C=\rho_0 C_{V,H}+\mu_0 K H_0$

The salinity equation is

$$\frac{\partial S}{\partial t}+\beta_s w=K_s\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)S+S_T\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)\theta \tag{12}$$

Using the analysis similar to Finlayson [5], one gets

$$(1+\chi)\frac{\partial^2\phi}{\partial z^2}-\left(1+\frac{M_0}{H_0}\right)k_0^2\phi-K\frac{\partial\theta}{\partial z}(1-S_T)+K_2\frac{\partial S}{\partial z}=0 \tag{13}$$

The non-dimensional numbers can be written using

$$t^*=\frac{\nu t}{d^2},\quad w^*=\frac{wd}{\nu},\quad T^*=\left(\frac{K_1 a R^{1/2}}{\rho_0 C_{V,H}\beta_t\nu d}\right)\theta,\quad \phi^*=\left(\frac{(1+\chi)K_1 a R^{1/2}}{K\rho_0 C_{V,H}\beta_t\nu d^2}\right)\phi,\quad z^*=\frac{z}{d},\quad a=k_0 d$$

$$D=\frac{\partial}{\partial z^*},\quad S^*=\left(\frac{K_s a R_s^{1/2}}{\rho_0 C_{V,H}\beta_s\nu d}\right)S,\quad \nu=\frac{\eta}{\rho_0},\quad \xi^*=\frac{\xi d^2}{\nu},\quad k'^*=\frac{k'}{d^2},\quad k^*=\frac{k}{d^2}$$

Then the Equations (9) – (13) become

$$\left.\begin{aligned}\frac{\partial}{\partial t^*}(D^2-a^2)w^*&=aR^{1/2}\left[M_1 D\phi^*-(1+M_1(1-S_T)T^*)\right]+aR^{1/2}M_1M_5D\phi^*\\&\quad-aR^{1/2}M_1M_5(1-S_T)T^*+aR_S^{1/2}\left[1+M_4+\frac{M_4}{M_5}\right]S^*\\&\quad-(T_a)^{1/2}D\xi^*+\frac{a^2}{k'^*}-\frac{1}{k^*}D^2w^*\end{aligned}\right\} \tag{14}$$

$$\left(\frac{\partial}{\partial t^*}+\frac{1}{k^*}\right)\xi^*=(T_a)^{1/2}Dw^* \tag{15}$$

$$P_r\left[\frac{\partial T^*}{\partial t^*}-M_2\frac{\partial}{\partial t^*}(D\phi^*)\right]=(D^2-a^2)T^*+aR^{1/2}(1-M_2-M_2M_5)w^*, \tag{16}$$

$$P_r\frac{\partial S^*}{\partial t^*}=\tau(D^2-a^2)S^*-aR_S^{1/2}M_6w^*+S_T\left(\frac{M_5}{M_6}\right)\left(\frac{R_S}{R}\right)^{1/2}(D^2-a^2)T^*, \tag{17}$$

$$D^2\phi^*-M_3a^2\phi^*-(1-S_T)DT^*+\frac{M_5}{M_6}\left(\frac{R}{R_S}\right)^{1/2}DS^*=0, \tag{18}$$

where the non-dimensional parameters used are

$$\left.\begin{aligned}&M_1=\frac{\mu_0K^2\beta_t}{(1+\chi)\rho_0 g\alpha_t},\quad M_2=\frac{\mu_0K^2T_0}{(1+\chi)\rho_0C_{v,H}},\quad M_3=\frac{1+M_0/H_0}{(1+\chi)},M_4=\frac{\mu_0K^2\beta_s}{(1+\chi)\rho_0 g\alpha_s},\\&M_5=\frac{K_2\beta_s}{K\beta_t},\quad M_6=\frac{K_S}{K_1},\quad \tau=\rho_0C_{v,H}\left(\frac{K_S}{K_1}\right),P_r=\frac{\eta C_{v,H}}{K_1},\quad R_S=\frac{\rho_0C_{v,H}\beta_s\alpha_s g d^4}{\nu K_S},\\&R=\frac{\rho_0C_{v,H}\beta_t\alpha_t g d^4}{\nu K_1},\end{aligned}\right\} \tag{19}$$

where R_S is the salinity Rayleigh number, R is the thermal Rayleigh number, P_r is the Prandtl number and other parameters represent non-dimensional parameters used appropriately.

3. Analysis of Solution at Free Boundaries

The boundary conditions on velocity, temperature and salinity are

$$w^* = D^2 w^* = T^* = D\phi^* = S^* = \xi^* = D\xi^* = 0 \quad \text{at} \quad z^* = \pm 1/2. \tag{20}$$

The exact solutions satisfying Equation (20) are

$$w^* = Ae^{\sigma t^*}\cos\pi z^*,\ T^* = Be^{\sigma t^*}\cos\pi z^*, S^* = Ce^{\sigma t^*}\cos\pi z^*,$$
$$D\phi^* = Fe^{\sigma t^*}\cos\pi z^*,\ \phi^* = \frac{F}{\pi}\sin\pi z^* \tag{21}$$

where *A, B, C* and *F* are constants. These functions substituted in the set of Equations (14) – (18) gives the following four linear homogeneous algebraic equations in the constant *A, B, C* and *F* are obtained upon $k'^* = \varepsilon k^*$ and removing the asterisks for our convenience, where ε is non-dimensional parameter, which is the ratio of vertical to the horizontal plane permeability, governing the anisotropy leads to

$$\left\{ \left[\sigma(\pi^2 + a^2) + \left(\frac{\pi^2\varepsilon + a^2}{k\varepsilon} \right) + \frac{T_a\pi^2}{\left(\sigma + \frac{1}{k}\right)} \right] A - aR^{1/2}\left[1 + M_1(1 - S_T)(1 + M_5)\right]B + aR_S^{1/2}(1 + M_4 + M_4 M_5^{-1})C + aR^{1/2}\ M_1(1 + M_5)F = 0, \right\} \tag{22}$$

$$aR^{1/2}(1 - M_2 - M_2M_5)A - (\pi^2 + a^2 + \sigma P_r)B + P_r\sigma M_2 F = 0, \tag{23}$$

$$aR_S^{1/2}M_6 A + S_T\left(\frac{M_5}{M_6}\right)\left(\frac{R_S}{R}\right)^{1/2}(\pi^2 + a^2)B + \left[\tau(\pi^2 + a^2) + \sigma P_r\right]C = 0, \tag{24}$$

$$-R_S^{1/2}\pi^2(1 - S_T)B + R^{1/2}\pi^2 M_5 M_6^{-1} C + R_S^{1/2}(\pi^2 + a^2 M_3)F = 0, \tag{25}$$

For the existence of non-trivial Eigen functions, the determinant of the co-efficient of *A*, *B*, *C* and *F* in Equations (22) – (25) must vanish. Following the techniques and analysis of Sekar *et al*. [25], Equations (22) – (25) lead to

$$U\sigma^4 + V\sigma^3 + W\sigma^2 + X\sigma + Y = 0 \tag{26}$$

$$U = \left(\pi^2 + a^2\right)\left(\pi^2 + a^2 M_3\right)P_r^2$$

$$V = \left(\pi^2 + a^2 M_3\right)\left[\left(\pi^2 + a^2\right)^2(1 + \tau) + P_r\left\{\frac{1}{k}\left(\pi^2 + a^2\right) + \left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right)\right\}\right]P_r$$

$$W = \left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\left[\tau\left(\pi^2 + a^2\right)^2 + \mathrm{P}_r(1 + \tau)\left\{\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right) + \frac{1}{k}\left(\pi^2 + a^2\right)\right\}\right]$$
$$+ a^2 R\mathrm{P}_r M_1(1 + M_5)\left[\left(1 - S_\mathrm{T}\right)\left(\pi^2 + a^2 M_3\right) - \pi^2(1 - S_\mathrm{T} + M_5)\right]$$
$$+ a^2 R_s P_r\left(\pi^2 + a^2 M_3\right)\left(1 + M_4 + M_4 M_5^{-1}\right)M_6 + a^2 R\mathrm{P}_r\left(\pi^2 + a^2 M_3\right)$$
$$+ \left(\pi^2 + a^2 M_3\right)\left[T_a\pi^2 + \frac{1}{k}\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right)\right]P_r^2$$

$$X = \left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\begin{bmatrix} \frac{1}{k}(1+\tau)\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right)P_r + \pi^2 T_a \mathrm{P}_r \\ +\tau\left(\pi^2 + a^2\right)\left\{\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right) + \frac{1}{k}\left(\pi^2 + a^2\right)\right\} \end{bmatrix}$$

$$+ a^2 R\tau\left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\left[1 + (1 - S_{\mathrm{T}})M_1(1 + M_5)\right]$$

$$+ \frac{1}{k}a^2 R\mathrm{P}_r\left[\left(\pi^2 + a^2 M_3\right)\left\{1 + M_1(1 + M_5)(1 - S_{\mathrm{T}})\right\} - \pi^2 M_1(1 + M_5)\left\{(1 - S_{\mathrm{T}}) + M_5\right\}\right]$$

$$- a^2 R\left(\pi^2 + a^2\right)M_1(1 + M_5)\pi^2\left[S_{\mathrm{T}}\left(\frac{M_5}{M_6}\right)^2 + \tau(1 - S_{\mathrm{T}}) + M_5\right]$$

$$+ a^2 R_s\left(\pi^2 + a^2 M_3\right)\left(1 + M_4 + M_4 M_5^{-1}\right)\left[\frac{1}{k}M_6 P_r + \left(\pi^2 + a^2\right)\left\{S_{\mathrm{T}}\left(\frac{M_5}{M_6}\right) + M_6\right\}\right]$$

$$Y = \tau\left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)^2\left[T_a\pi^2 + \frac{1}{k}\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right)\right]$$

$$+ \frac{1}{k}a^2 R\tau\left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\left[1 + (1 - S_{\mathrm{T}})M_1(1 + M_5)\right]$$

$$- \frac{1}{k}a^2 R\left(\pi^2 + a^2\right)M_1(1 + M_5)\pi^2\left[S_{\mathrm{T}}\left(\frac{M_5}{M_6}\right)^2 + \tau(1 - S_{\mathrm{T}}) + M_5\right]$$

$$+ \frac{1}{k}a^2 R_s\left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\left(1 + M_4 + M_4 M_5^{-1}\right)\left[S_{\mathrm{T}}\left(\frac{M_5}{M_6}\right) + M_6\right]$$

For obtaining stationary instability, the time-independent term Y=0. Equation (26) helps one to obtain Eigen value R^{sc} for which a solution exists;

$$R^{sc} = \frac{N_r}{D_r},$$

where

$$N_r = \left(\pi^2 + a^2\right)\left[T_a\pi^2 k + \left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right)\right] - a^2 R_s\tau^{-1}\left(1 + M_4 + M_4 M_5^{-1}\right)\left[S_{\mathrm{T}}\left(\frac{M_5}{M_6}\right) + M_6\right]$$

and

$$D_r = a^2\left[1 + (1 - S_{\mathrm{T}})M_1(1 + M_5)\right] - \pi^2\left[\frac{a^2 M_1(1 + M_5)}{\pi^2 + a^2 M_3}\right]\left[S_{\mathrm{T}}\left(\frac{M_5}{M_6}\right)^2\tau^{-1} + (1 - S_{\mathrm{T}}) + M_5\tau^{-1}\right]$$

For M_1 very large, one gets the results for the magnetic mechanism, and the critical thermo magnetic Rayleigh number for stationary mode is obtained using

$$N^{sc} = R^{sc}M_1 = \frac{N_r}{D_r},$$

where

$$\mathrm{N}_r = \left(\pi^2 + a^2\right)\left[T_a\pi^2 k + \left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right)\right] - a^2 R_s\tau^{-1}\left(1 + M_4 + M_4 M_5^{-1}\right)\left[S_{\mathrm{T}}\left(\frac{M_5}{M_6}\right) + M_6\right]$$

and

$$D_r = a^2\left[(1 - S_{\mathrm{T}})(1 + M_5)\right] - \pi^2\left[\frac{a^2(1 + M_5)}{\pi^2 + a^2 M_3}\right]\left[S_{\mathrm{T}}\left(\frac{M_5}{M_6}\right)^2\tau^{-1} + (1 - S_{\mathrm{T}}) + M_5\tau^{-1}\right]$$

The conditions for the onset of oscillatory stabilities are obtained as follows. Taking $\sigma = i\sigma_1$ and $\sigma_1^2 > 0$, following the analysis and techniques of Sekar *et al*. [25], the critical Rayleigh number for oscillatory mode has been calculated using

$$R^{oc} = \frac{C_2 A_2 + B_2 D_2}{A_2^{\,2} + B_2^{\,2}}$$

where

$$A_2 = -U_1\sigma_1^2 + V_1,\; B_2 = W_1\sigma_1,\; C_2 = -U_2\sigma_1^4 + W_2\sigma_1^2 - Y_1$$

$$D_2 = V_2\sigma_1^3 - X_1\sigma_1,\quad \sigma_1^2 = \frac{-B_1 \pm sqrt(B_1^2 - 4A_1C_1)}{2A_1}$$

where

$$A_1 = U_2W_1 - U_1V_2,\; B_1 = V_1V_2 + U_1X_1 - W_1W_2,\; C_1 = W_1Y_1 - V_1X_1$$

$$U_1 = a^2P_rM_1(1+M_5)\left[(1-S_T)\left(\pi^2 + a^2M_3\right) - \pi^2(1 - S_T + M_5)\right] + a^2RP_r\left(\pi^2 + a^2M_3\right)$$

$$V_1 = \frac{1}{k}a^2\tau\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)\left[1 + (1-S_T)M_1(1+M_5)\right]$$
$$-\frac{1}{k}a^2\left(\pi^2 + a^2\right)M_1(1+M_5)\pi^2\left[S_T\left(\frac{M_5}{M_6}\right)^2 + \tau(1-S_T) + M_5\right]$$

$$W_1 = a^2\tau\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)\left[1 + (1-S_T)M_1(1+M_5)\right]$$
$$+\frac{1}{k}a^2P_r\left[\left(\pi^2 + a^2M_3\right)\left\{1 + M_1(1+M_5)(1-S_T)\right\} - \pi^2M_1(1+M_5)\left\{(1-S_T) + M_5\right\}\right]$$
$$-a^2\left(\pi^2 + a^2\right)M_1(1+M_5)\pi^2\left[S_T\left(\frac{M_5}{M_6}\right)^2 + \tau(1-S_T) + M_5\right]$$

$$X_1 = \left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)\left[\begin{array}{l}\frac{1}{k}(1+\tau)\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right)P_r + \pi^2T_aP_r \\ +\tau\left(\pi^2 + a^2\right)\left\{\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right) + \frac{1}{k}\left(\pi^2 + a^2\right)\right\}\end{array}\right]$$
$$+a^2R_s\left(\pi^2 + a^2M_3\right)\left(1 + M_4 + M_4M_5^{-1}\right)\left[\frac{1}{k}M_6P_r + \left(\pi^2 + a^2\right)\left\{S_T\left(\frac{M_5}{M_6}\right) + M_6\right\}\right]$$

$$Y_1 = \tau\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)^2\left[T_a\pi^2 + \frac{1}{k}\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right)\right]$$
$$+\frac{1}{k}a^2R_s\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)\left(1 + M_4 + M_4M_5^{-1}\right)\left[S_T\left(\frac{M_5}{M_6}\right) + M_6\right]$$

$$U_2 = \left(\pi^2 + a^2\right)\left(\pi^2 + a^2M_3\right)P_r^2$$

$$V_2 = \left(\pi^2 + a^2 M_3\right)\left[\left(\pi^2 + a^2\right)^2 (1+\tau) + P_r\left\{\frac{1}{k}\left(\pi^2 + a^2\right) + \left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right)\right\}P_r\right]$$

$$W_2 = \left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\left[\tau\left(\pi^2 + a^2\right)^2 + P_r(1+\tau)\left\{\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right) + \frac{1}{k}\left(\pi^2 + a^2\right)\right\}\right]$$

$$+ a^2 R_s P_r\left(\pi^2 + a^2 M_3\right)\left(1 + M_4 + M_4 M_5^{-1}\right)M_6 + \left(\pi^2 + a^2 M_3\right)\left[T_a\pi^2 + \frac{1}{k}\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k}\right)\right]P_r^2$$

4. Results and Discussions

The Soret-driven thermoconvective instability of ferromagnetic fluid heated from below and salted from above rotating a densely packed anisotropic porous medium has been analyzed using Darcy model. The effect of anisotropy is studied by the anisotropic parameter ε, which is the ratio of vertical to the horizontal plane permeability, which takes the values from 10 to 70. The Prandtl number P_r is assumed to be 0.01. The Taylor number T_a is assumed to vary from 10 to 10^5. The Soret parameter S_T is assumed to take values from -0.002 to 0.002, the salinity Rayleigh number R_s is varied from -500 to 500 and the non-buoyancy magnetization parameter M_3 is allowed to take values from 5 to 25. The values of ratio of the mass transport to heat transport τ is assumed to be 0.03, 0.05, 0.07, 0.09 and 0.11 (Sekar *et al.*, [28]). The buoyancy magnetization parameter M_1 is assumed to be 1000 (Finlayson [7]). For these fluids, M_2 will have a negligible value and hence is taken as zero. M_6 is taken to be 0.1 and M_4 is the effect on magnetization due to salinity. This is allowed to vary from 0.1 to 0.5 taking values less than the non-buoyancy magnetization parameter M_3. M_5 represents the ratio of the salinity effect on magnetic field and pyromagnetic coefficient. This is varied between 0.1 and 0.5. The permeability of porous medium k is assumed to take the values from 0.001, 0.003, 0.005, 0.007 and 0.009 (Darcy number).

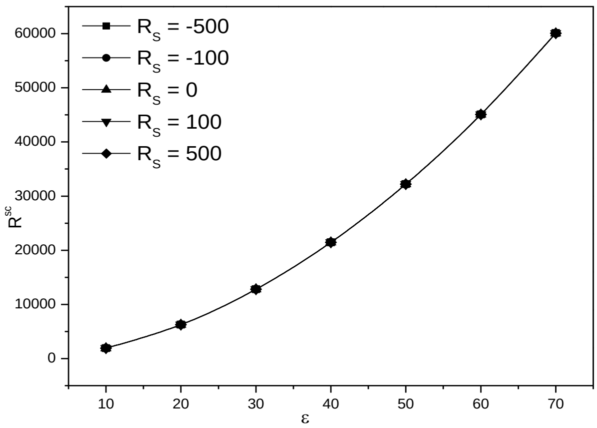

Figure 2. Variation of R^{sc} versus ε for Different R_S with S_T = -0.002, M_3 = 5, k = 0.001, T_a = 10 and $\tau = 0.03$.

Figure 2 shows variation of R^{sc} with ε (anisotropic ratio) for different R_S (-500, -100, 0, 100, 500) and keeping the values of S_T = -0.002, M_3 = 5, k = 0.001, T_a = 10 and $\tau = 0.03$ are fixed. It has been observed that as R_S increases there is no notable variation in the curves. That is, there is no any variation in convection for different effect of salt. Therefore, the effect of salinity Rayleigh number is negligible. Anyhow, stability pattern is observed.

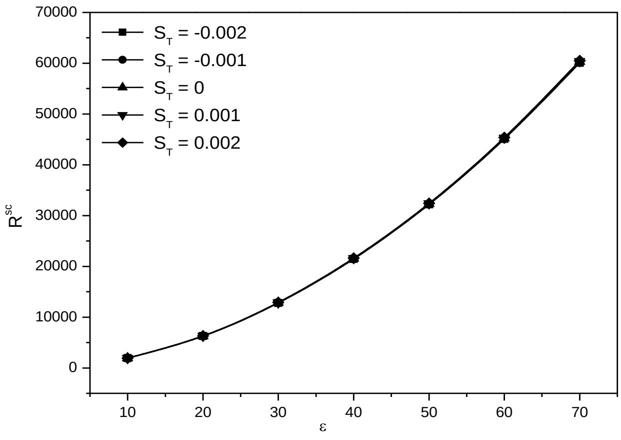

Figure 3. Variation of R^{sc} versus ε for Different S_T with R_S = -500, M_3 = 5, k = 0.001, T_a = 10 and $\tau = 0.03$.

Figure 3 represents variation of R^{sc} versus ε drawn for the different S_T as shown in the figure. It has been observed that as S_T increases, R^{sc} increases, the convective system leads to stabilization. But, in the physical situation, critical Rayleigh number R^{sc} gets the same value for various Soret parameter S_T from -0.002 to 0.002 which is also studied in Figure 2.

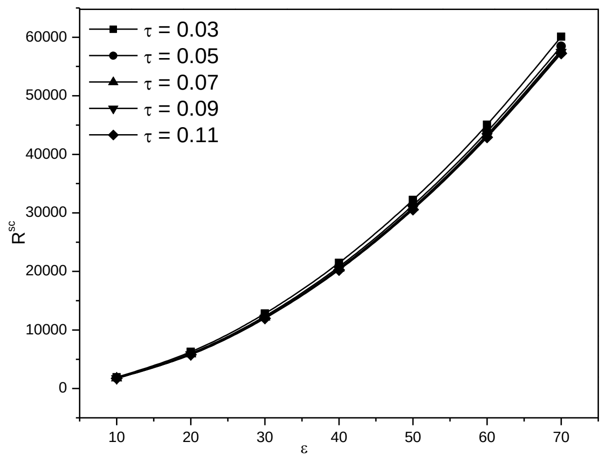

Figure 4. Variation of R^{sc} versus ε for Different τ with R_S = -500, M_3 = 5, k = 0.001, T_a = 10 and S_T = -0.002

In Figure 4 the stability curves for different values of τ (the ratio of the mass transport to heat transport) is analyzed. When $\varepsilon = 10$ the critical Rayleigh number R^{sc} gets the unique value. But in the increasing of ε from 20 to 70 there is a variation in the convective system, τ is decreased from 0.03 to 0.11. This is because the increase in mass transport to heat transport leads the system to be top heavy. The same trend of stabilization is seen in Figure 5. Also, in which variation of R^{sc} versus ε for different M_3 and k = 0.001. The application of magnetic field makes the magnetic fluid acquire larger magnetization M_3. This on interacting with the applied magnetic field once again releases large energy.

Figure 6 shows the plot of critical thermal Rayleigh number R^{sc} versus anisotropic ratio ε for various values of Taylor number T_a. When ε increases from 10 to 70, there is an increase in R^{sc}. It is clear that the system gets stabilized through oscillatory mode.

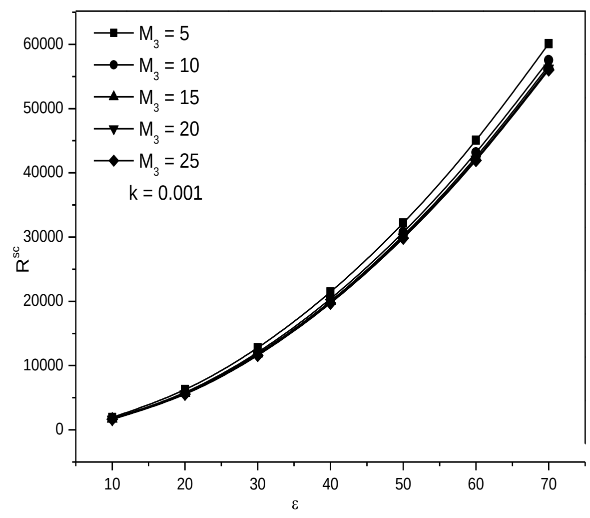

Figure 5. Variation of R^{sc} versus ε for Different M_3 with S_T = -0.002, R_S = -500, T_a = 10 and $\tau = 0.03$.

Figure 7 obtains the stabilization of the system is not much pronounced because of coriolis force for various ε. In other words, as Taylor number T_a increases, the critical thermal Rayleigh number R^{sc} is rather increased which is studied in Figure 8 also for destabilization because of various non-buoyancy magnetization parameter M_3. But increase in anisotropic ratio ε increases the critical thermal Rayleigh number R^{sc} increases. Therefore anisotropy effect leads to stability of the system.

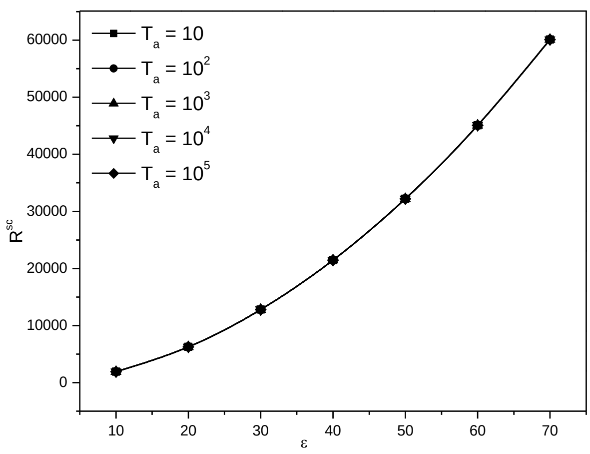

Figure 6. Variation of R^{sc} versus ε for different T_a with R_S = -500, M_3 = 5, k = 0.001 $\tau = 0.03$ and S_T = -0.002

Figure 8 analyze the plot of critical thermal Rayleigh number R^{sc} versus Taylor number T_a for various values of non-buoyancy magnetization parameter M_3. As Taylor number T_a increases, the critical thermal Rayleigh number R^{sc} is almost constant and the system gets equilibrium state due to the rotation. But, when the values of non-buoyancy magnetization parameter M_3 are increased, the critical thermal Rayleigh number R^{sc} gets decreased. Therefore for larger rotation, magnetization leads to destabilization of the system.

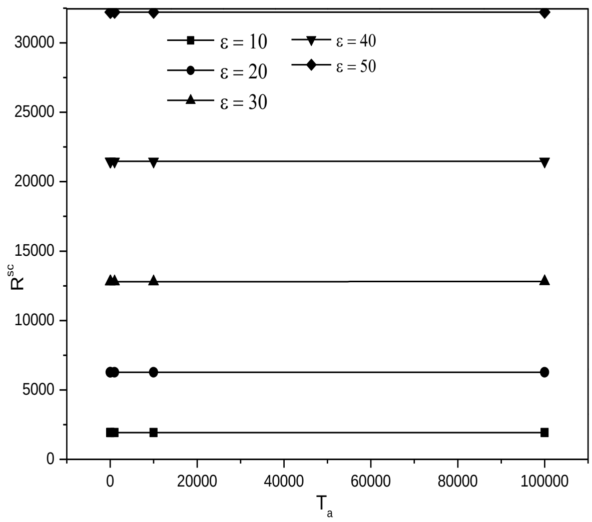

Figure 7. Variation of R^{sc} versus T_a for Different ε with S_T = -0.002, R_S = -500, k = 0.001, M_3 = 5 and $\tau = 0.03$.

Figure 9 gives the variation of the critical thermal Rayleigh number R^{sc} versus the ratio of the mass transport to heat transport τ (varied from 0.03 to 0.11). It has been observed that as the Taylor number T_a is increased from 10 to 10^5, there is no notable variation in rotation. The effect of Taylor number T_a is negligible as noticed from the curves. It is clear that as the ratio of the mass transport to heat transport τ increases from 0.03 to 0.11, the critical thermal Rayleigh number R^{sc} values tend to decrease leading to destabilization. This is because the increase in mass transport leads the system to be top heavy.

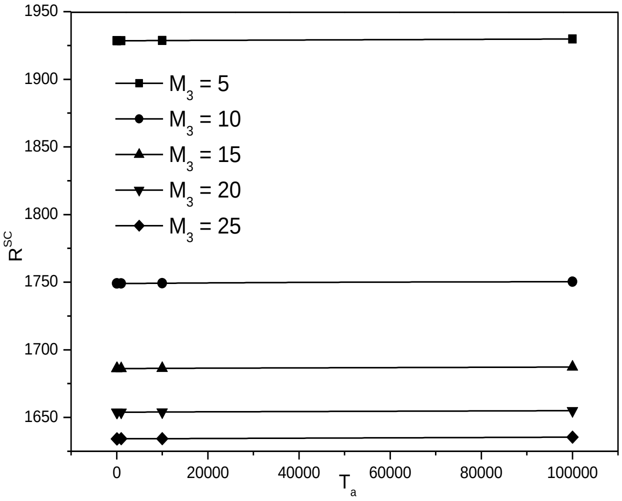

Figure 8. Variation of R^{sc} versus T_a for Different M_3 with S_T = -0.002, R_S = -500, k = 0.001, $\varepsilon = 10$ and $\tau = 0.03$.

Figure 10 investigates the variation of the critical thermal Rayleigh number R^{sc} versus non-buoyancy magnetization parameter M_3 for T_a = 10. When the values of ratio of the mass transport to heat transport τ is varied from 0.03 to 0.11, it is seen that, when M_3 increases from 5 to 25, R^{sc} decreases indicating the onset of instability. This is because high magnetization tends to release large energy to the system causing instability to set in earlier. Also as the ratio of the mass transport to heat transport τ increases from 0.03 to

0.11, there is a fall in the values of R^{sc}. Thus larger values of τ leads to destabilization of the system. The magnetization of the fluid is found to destabilize the system through oscillatory mode, which was discussed by Sekar *et al*. [25] in the absence of anisotropy effect.

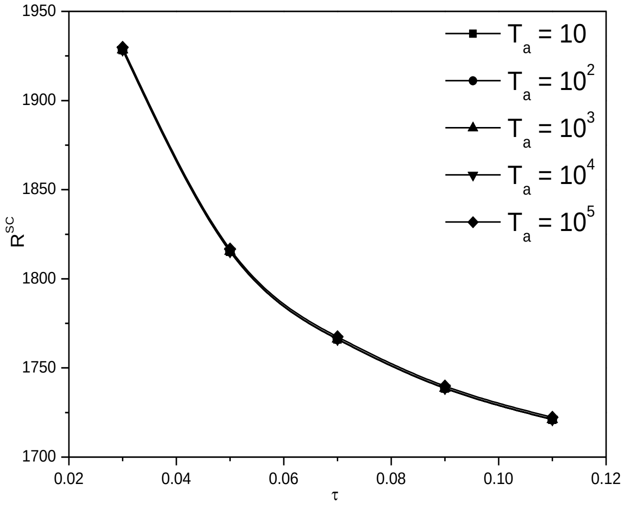

Figure 9. Variation of R^{sc} versus τ for Different T_a with S_T = -0.002, R_S = -500, M_3 = 5, $\varepsilon = 10$ and k = 0.001

Figure 11 represents the variation of the critical thermal Rayleigh number R^{sc} versus the permeability of porous medium k. It has been observed that as salinity Rayleigh number R_S increases from -500 to 500, there is no notable variation. The effect of salinity Rayleigh number is negligible. For different R_S, no appreciable change in the curves are noticed. It is clear that as the permeability k increases from 0.001 to 0.009, the critical thermal Rayleigh number R^{sc} values tend to decrease leading to destabilization. This is due to the fact that increase in pore size makes the flow of the fluid easier causing instability to set in earlier.

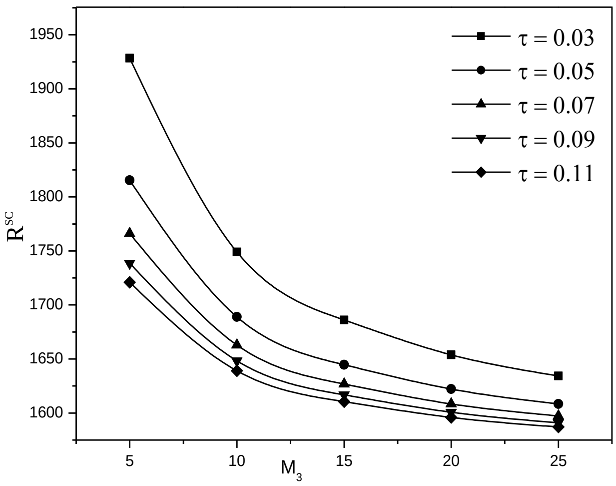

Figure 10. Variation of R^{sc} versus M_3 for different τ with S_T = -0.002, R_S = -500, T_a = 10, $\varepsilon = 10$ and k = 0.001

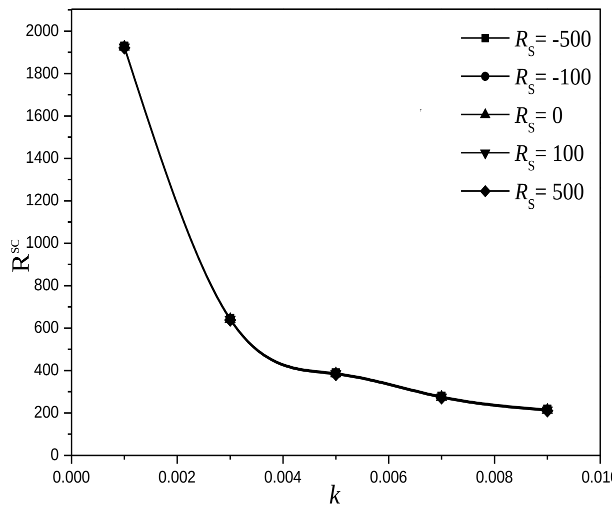

Figure 11. Variation of R^{sc} versus k for Different R_S with S_T = -0.002, M_3 = 5, T_a = 10, $\tau = 0.03$ and $\varepsilon = 10$.

Figure 12 gives the variation of the critical thermal Rayleigh number R^{sc} versus the permeability of porous medium k. It is seen that as Soret coefficient S_T increases from -0.002 to 0.002, there is no notable variation which analyzed in Figure 11. But for different S_T, no appreciable changes in the curves are noticed. This is due to the effect of Soret parameter S_T, which provides additional temperature gradient by cross diffusion of salinity on temperature. It is clear that as the permeability of a porous medium k increases from 0.001 to 0.009, it leads to decrease in the values of the critical thermal Rayleigh number R^{sc} indicating destabilization. This is due to the fact that increase in pore size makes the flow of the fluid easier causing instability to set in earlier.

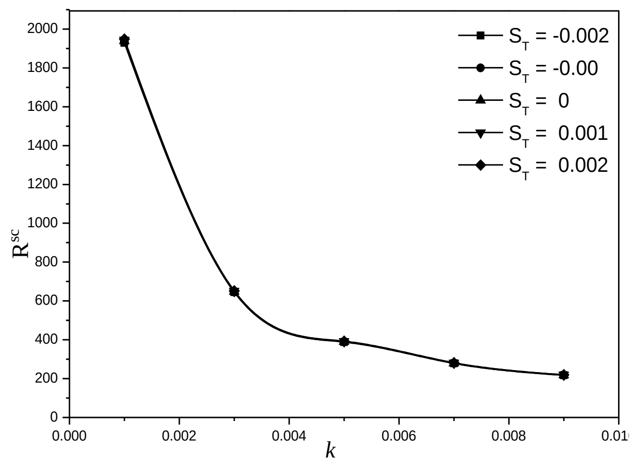

Figure 12. Variation of R^{sc} versus k for Different S_T, R_S = -500, M_3 = 5, T_a = 10, $\tau = 0.03$ and $\varepsilon = 10$.

5. Conclusions

The linear stability of thermohaline convection in a ferrofluid layer heated from below and salted from above saturating an anisotropic porous medium subjected to a transverse uniform magnetic field has been considered with effect of rotation using Darcy model. In this investigation, the effect of various parameters like permeability of the porous medium, anisotropic parameter, non – buoyancy magnetization, buoyancy magnetization, Prandtl number, ratio of mass transport to heat transport, Rayleigh number and salinity Rayleigh number on the onset of convection have been calculated. The thermal critical magnetic Rayleigh numbers for the onset of instability are also determined numerically for sufficient large values of buoyancy magnetization parameter M_1 and results depicted graphically. Furthermore, the principle of exchange of instability is applied to find out the mode of attaining instability.

In conclusion, we see that convection can encourage in a ferromagnetic fluid by means of spatial variation in magnetization, which is induced when the magnetization of the ferrofluid depends on temperature and salinity. For the stationary convection, the anisotropy effect ε has a stabilizing behavior for various values of R_S, S_T, τ, M_3 and T_a which are studied in Figures 2 – 6. But, for the value of $\varepsilon = 0.03$, the convective system has a destabilizing effect which is analyzed in Figures 9 – 12 and Figures 7 and 8 showed a stabilizing effect which is not much pronounced. Also, when increasing value of anisotropic porous medium, there is an increasing convection process in the system. Therefore, the anisotropy effect dominates the system due to high energy.

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