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PI Controller Performance Analysis Using Lambert W Function Approach for First Order Systems with Time Delay

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Abstract

Time delays are inherent in process industry. The presence of time delays limits and degrades the possible performance of the system and also leads to instability. The states of the delay system not only depend on the present state, they also depend on the previous states. With the help of a suitable controller design, the delay systems can be controlled for getting fruitful results. In all industrial feedback control applications most commonly and practically used controllers are Proportional Integral Derivative (PID) controllers. With proper parameter tuning of the controller, required performance can be achieved from the system. In general, the first order processes with time delay can be easily controlled with PI controllers. By choosing the proper controller gains the effect of time delays can be avoided. In this paper, PI controller design with Lambert W function analysis in smith configuration has been proposed. After the selection of dominant poles, the PI controller gains were chosen accordingly to shift the desired eigen values to required positions using the Lambert W function based analysis. A first order process with time delay has been considered in this paper and the comparative analysis of proposed tuning algorithm with Smith predictor (SP) and Zeigler-Nichols (ZN) methods has been done. By using the time response performance specifications and errors, the performance of different tuning algorithms has been analyzed. From the simulation and different performance indices results, it has been observed that proposed smith configured Lambert W based controller tuning approach gives improved results compared to remaining methods in terms of the measures like overshoot, peak time, settling time, rise time, errors etc.

Keywords: *Lambert W Function, PI Controller, Smith Predictor, Time Delay, Zeigler-Nichols*

1. Introduction

Time delays arise in numerous natural and engineered systems, such as processes in industry, biological systems, tele-operation, etc. These delays are mainly because of the time required to transport mass, energy, information or they may be inherent in the plant also. The presences of delay produces decrease in phase and give rise to non-rational transfer function of the system. The existence of delays disturbs the desired performance of the system and sometimes may leads to instability of the system also. Because of these characteristics, the delay system problems have been attracted by many researchers. Many articles and books devoted to this active area of research came since two decades [1-6]. The processes with dead times are difficult to control since, the effect of the disturbances is not felt until significant time has elapsed and the control action which is applied based on actual error tries to rectify a state that originated a few time before. Also characteristic equation of delay system has infinite number of eigen values and makes difficult to analyze and

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to design controller. By using approximation to delay term the systems were analyzed using Pade's approximation in past [7]. But these methods have drawbacks of limited accuracy and higher order Pade's approximation produces transfer function with cluster poles.

O.J. Smith invented a first predictive controller for delay systems in 1957. Smith predictor uses an internal model to predict the delay free response. It consists of two loops in its structure which results in elimination of the delay term in the characteristic equation [8]. By modifying the basic predictor structure several contributions have been reported in literature. For controlling the processes with long dead time an approach, which is the combination of programmable logic controller (PLC) and smith predictor was designed by Asim V. in [9]. With the benefit of advanced controller PLC, combined with Smith predictor significant improvement in results was reported. Improved tuning of classical PI controller in Smith predictor configuration was proposed for systems with time delay by S.Shokri *et al.*, in [10]. For controlling unstable processes with dead time D.G.Padhan and S.Majhi, proposed a modified Smith predictor based on cascaded control in [11].

For the analysis and control of time delay systems and to overcome the difficulty of infinite dimensionality Lambert proposed a theory based on Lambert W function. The concept is identical to the ordinary differential equation solution. With the help of Lambert W function approach, the delay differential equations can be solved in an analogous way as ordinary differential equations solution [12]. Later, PI controller design based on Lambert solution for delay systems has been proposed by Sun Yi *et al.*, [13]. By selecting the dominant poles, controller gains were tuned accordingly. The same controller and PV controller was applied to DC motor control as an application to Lambert W approach by Sun Yi *et al.*, [14]. The same work was extended by R. K. Rao *et al.*, [15] advanced the same by combining proportional, integral and velocity controllers for the same DC motor control problem with time delays. An extensive review on Lambert's approach has been given in [16] by citing various applications of its use.

In process control most of the control loops are combination of proportional integral and derivative. This is due to robust nature, simple structure and easy implementation of PID controllers. Several works have reported in literature by applying different tuning algorithms starting with Ziegler Nichols method [17]. For the first order processes with time delay PI controllers are most suitable since the PI controller can give off-set free response. This article gives the relative study of proposed smith configured Lambert W method of PI controller design performance with respect to ZN method of tuning and Smith predictor for first order plus time delay system. For illustration, a benchmark problem of first order system with delay has been considered. Simulation exercise has been done which gives the efficacy of various methods in terms of different measures.

The remaining paper is prepared as: Section 2 gives ZN method based PI controller design and controller tuning with Smith predictor approach. Controller design using Lambert W function is given in Section 3. Simulation results with illustration have been explained in Section 4. Conclusions with observations presented in Section 5.

2. Proportional Integral Controller Design

PI controller is a particular case of PID controller, where derivative of the error is not used. Many tuning methods exist to tune the controller gains of the PI controller. Ziegler Nichols method is a basic tuning method. This section presents the ZN method of tuning controller gains and PI controller design in Smith predictor configuration.

2.1. Controller tuning by Ziegler Nichols Method

A heuristic method for tuning PID controller has been given by Ziegler and Nichols. There exist two methods of tuning by ZN method. First one is known as reaction curve method or open loop ZN method, which considers the open loop step response of the plant. For the processes of first order plus delay, Ziegler-Nichols proposed a damped oscillation method relations for calculating the controller parameters [17].

The second method of ZN tuning is known as closed loop ZN method. This method uses only a closed loop experiment by considering proportional gain. The gain is increased till the response reaches a nondamped oscillation. When this occurs that gain is known as ultimate gain and the time period at that instant is said to be ultimate time period [18]. After getting these two quantities, the analyst can calculate the remaining controller parameters easily by using the well established formulae [18]. Figure 1 gives the block diagram representing delay system with controller.

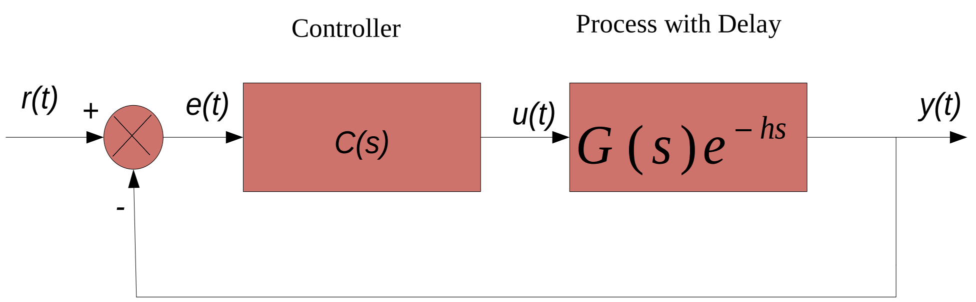

Figure 1. Controller Structure for Process with Time Delay

2.2. PI Controller Design in Smith Predictor Configuration

From the advantages like good reference tracking, disturbance rejection and robust performance, Smith-predictor technique acquired considerable attention from the researchers for dead time compensation. The fundamental idea is to put up a parallel model that cancels the delay. So, by eliminating the delay term from the characteristic equation with the help of the structure of SP, delay systems can be easily analyzed. The controller design is with the pseudo-complementary sensitivity function [18]. By using controller D(S) in Smith configuration the structure of the process is given in Figure 2.

Suppose, a first order process with time delay is considered as in eq.(1)

$$G(s) = G_p(s)e^{-sh} = \frac{K_N}{\tau_N s + 1}e^{-sh} \quad (1)$$

where K_N is the steady-state gain, h is the time delay and τ_N is the time constant. D(s) is a PI controller as in eq.(2)

$$D(s) = K_p + \frac{K_I}{s} \quad (2)$$

For a preferred natural frequency ω_n and damping ratio ζ, the eigen values can be determined using eq.(3) and the controller gains can be selected by using eq.(4)

$$\lambda_d = -\omega_n\zeta \pm \omega_n\sqrt{1-\zeta^2}i = -\sigma \pm \omega_d i \quad (3)$$

$$K_P = \frac{2\zeta\omega_n\tau_M - 1}{K_M} \text{ and } K_I = \frac{\omega_n^2\tau_M}{K_M} \tag{4}$$

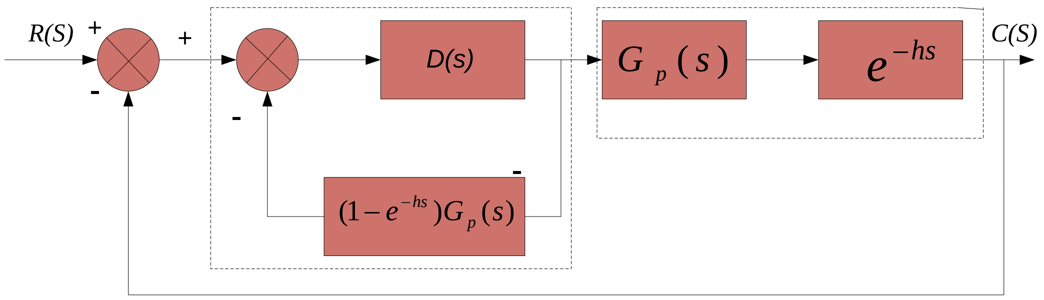

Figure 2. Controller Structure in Smith Predictor Configuration

3. PI Controller Design using Lambert W Function Analysis

A technique for analysis and control of linier time-invariant time delay systems with single delay using Lambert W function based approach is discussed in this section. By choosing the dominant eigen values and assigning them to desired positions with proper tuning of PI controller gains using Lambert W approach time delay systems can be controlled. The PI controller design in terms of delay differential equation is derived as given below.

Considering PI controller for an open loop system of first order is

$$G(s) = \frac{K_N}{\tau_N s + 1} e^{-sh}\left(K_P + \frac{K_I}{s}\right) = \frac{y}{e} \tag{5}$$

here, $e = -y + r$.

By converting closed-loop system into time domain becomes

$$\ddot{y} = -\frac{1}{\tau_N}\dot{y} - \frac{K_N K_P}{\tau_N}\dot{y}(t-h) - \frac{K_N K_I}{\tau_N} y(t-h) + \{K_N K_P s + K_N K_I\} r(t-h) \tag{6}$$

The state space representation of eq.(6) by assuming $x_1 \equiv y, x_2 \equiv \dot{y}$ is given in eq.(7)

$$\dot{x}(t) = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & \frac{-1}{\tau_N} \end{bmatrix}}_{A} x(t) - \underbrace{\begin{bmatrix} 0 & 0 \\ \frac{K_N K_I}{\tau_N} & \frac{K_N K_P}{\tau_N} \end{bmatrix}}_{A_d} x(t-h) \tag{7}$$

The eq.(7) is in the form of delay differential equation. From the Lambert W approach, this equation can be solved by calculating the solution matrix S_0. The eigen values of the solution matrix represent the eigen values of the system. By choosing appropriate controller gains the dominant eigenvalues can be assigned to the required positions. With the help of the direct commands available in Lambert W DDE toolbox of MATLAB, and with known system parameters like A and A_d the Lambert solution matrix S_0 can be evaluated from eq.(8)

$$S_0 = \frac{1}{h} W_0(A_d h Q_0) + A \tag{8}$$

W_0 represents the Lambert W matrix and Q_0 is unknown matrix which can be found from

$$W_0(A_d hQ_0)e^{W_0(A_d hQ_0)+Ah} = A_d h \tag{9}$$

Since, the Lambert solution involves all the system parameters i.e A, A_d and h one can determine how the parameters variation affects the system response. By changing the controller gains the eigen values can be assigned to required positions [13].

4. Results and Discussion

This section presents the simulation analysis of different methods discussed. Simulation exercise was carried out by considering a benchmark case of first order system with delay. Tuning of controller parameters with different methods has been done on the considered system. The system considered is

$$G(s) = \frac{e^{-0.2S}}{0.6s+1} \tag{10}$$

The delay differential equation of the considered system with PI controller can be expressed as

$$\dot{x}(t) = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -1.66 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ -1.66K_I & -1.66K_P \end{bmatrix}}_{A_d} x(t-0.2) \tag{11}$$

For several desired natural frequencies and damping ratios the PI controller gains are adjusted by Smith predictor approach and also by Lambert W function based approach. The gains obtained for both methods were tabulated in Table 1.

Table 1. PI Controller Parameters Attained by Lambert W Function Approach and Smith Predictor

ω_n	ζ	Dominant Pole	Smith Predictor		Lambert W Function	
			K_I	K_P	K_I	K_P
1.01	0.28	$-0.2828 \pm 0.9696i$	0.6120	-0.6606	1.1485	-0.3425
1.07	0.38	$-0.4066 \pm 0.9897i$	0.6869	-0.5120	1.2576	0.1546
1.22	0.38	$-0.4636 \pm 1.1284i$	0.8930	-0.4436	2.0354	0.3421
1.77	0.44	$-0.7788 \pm 1.5894i$	1.8794	-0.0654	2.6348	0.6426

For the analysis of results, the response at $\omega_n = 1.77$ and $\zeta = 0.44$ has been considered. Figure 3 shows the simulated response for Smith Predictor, ZN and Lambert W based PI controller tuning methods.

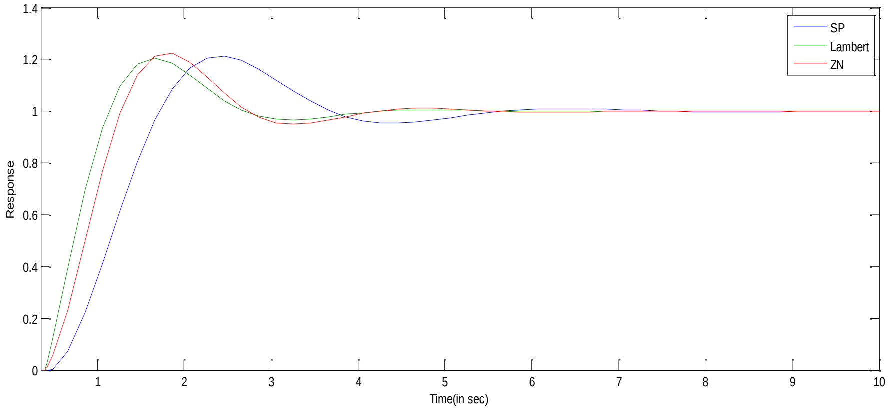

Figure 3. Response of Lambert W function based Approach, Smith Predictor and Zeigler-Nichols Method

From simulation result shown in Figure 3, it can be observed that the control action is very smooth in case of Lambert W method. A system is considered an optimum control system, when the system parameters are adjusted so that the index reaches required value, *i.e.*, minimum or maximum value. In order to analyze the performance of the system with respect to various methods of tuning, different time domain performance measures like overshoot, settling time and peak time has been measured and given in Table 2. The performance in terms of errors like integral absolute error (IAE), integral time absolute error (ITAE), integral square error (ISE) and mean square error (MSE) were evaluated and tabularized in Table 3.

Table 2. Transient Response Characteristics

Transient Response Characteristics	Smith Predictor(SP)	Ziegler Nichols(ZN)	Lambert W Function(LWF)
Rise Time(Sec)	0.86	0.627	0.81
Settling Time(Sec)	4.76	3.47	2.57
Maximum Overshoot (%)	21.5	22.6	6.48
Peak Time(Sec)	2.01	1.42	1.66

Table 3. Error Indices Results

Method	IAE	IATE	ISE	MSE
Zeigler-Nichols	0.7873	0.8701	0.3658	0.2017
Lambert W Function	0.7662	0.7409	0.4576	0.2428
Smith Predictor	1.267	1.681	0.8906	0.5924

From the performance indices results, it can be seen that most of the performance specifications have been enhanced greatly with Lambert W approach. The rise time is less in case of ZN method but the settling time is more where as in case of Lambert W method the settling time is very less. Error indices results also favor the Lambert W approach. Except the ISE and MSE all the errors gives improved results for Lambert W approach compared to other methods.

5. Conclusion

Many system dynamics can be represented adequately by first order system with time-delay transfer function. Optimal controller design plays vital role in control of delay systems for getting fruitful results. PI controllers can sufficiently give better results for first order plus time delay systems. The tuning of controller parameters is an essential task. This paper presents the comparison of ZN, Smith Predictor and proposed Smith configured Lambert W based PI controller tuning algorithms. The solution based on Lambert W method is dominant in terms of right most eigen values. By assigning the dominant eigen values to desired positions, the controller can be designed using Lambert W analysis. As the Lambert W method does not include the pole-zero cancellation, the controller tuned by this method gives efficient results. One example of first order plus delay system has been considered for illustration of performance of different methods. Simulation results clearly demonstrate that the Lambert W function based tuning of PI controller in smith configuration gives improved results in terms various performance measures considered.

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