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Verification of Dynamic Relaxation (DR) Method in Isotropic, Orthotropic and Laminated Plates using Small Deflection Theory

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Abstract

Dynamic Relaxation (DR) method is presented for the analysis of geometrically linear laterally loaded, rectangular laminated plates. The analysis uses the Mindlin plate theory which accounts for transverse shear deformations. A computer program has been compiled. The convergence and accuracy of the DR solutions of isotropic, orthotropic, and laminated plates for elastic small deflection response are established by comparison with different exact and approximate solutions. The present Dynamic Relaxation (DR) method shows a good agreement with other analytical and numerical methods used in the verification scheme.

It was found that: The convergence and accuracy of the DR solution is dependent on several factors which include boundary conditions, mesh size and type, fictitious densities, damping coefficients, time increment and applied load. Also, the DR small deflection program using uniform meshes can be employed in the analysis of different thicknesses for isotropic, orthotropic or laminated plates under uniform loads in a fairly good accuracy.

Keywords: *Dynamic Relaxation (DR) method, Dynamic Relaxation Solution, Verification of the Dynamic Relaxation, numerical comparisons*

1. Introduction

There are many situations in engineering applications where no single material will be suitable to meet a particular design requirement. However, two materials in combination may possess the desired properties and provide a feasible solution to the materials selection problem. A composite can be defined as a material that is composed of two or more distinct phases. It is usually a reinforced material that supported in a compatible matrix, assembled in prescribed amounts to give specific physical, mechanical and chemical properties.

Many composites used today are at the leading edge of materials technology, with their performance and cost appropriate to overwhelming applications such as that in space industries. Nevertheless, heterogeneous materials combining the best aspect of dissimilar constituents have been used by nature for millions of years ago. Ancient societies, imitating nature, used this approach as well: The book of exodus explains the usage of straw to reinforce mud in brick making without which the bricks would have almost no strength. Here in Sudan, the population from ancient ages dated back to Meroe civilization, and up to now used zibala (*i.e.*, animal dung) mixed with mud as a strong building material.

Composites possess two desirable features: the first one is their high strength to weight ratio, and the second is their properties that can be tailored through the variation of the fiber orientation and the stacking sequence which give the designer a wide choice of a suitable composite material. The incorporation of high strength, high modulus and low density fibers in a low strength and a low modulus matrix material result in a structural composite material

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which owns a high strength to weight ratio. Thus, the potential of a composite structure for use in aerospace, under – water, and automotive applications has stimulated considerable research activities in the theoretical prediction of the behavior of these materials. Usually a composite structure consists of many layers bonded on top of one another to form a high strength and rigid laminated composite plate. Each lamina is fiber reinforced along a single direction, with adjacent layers usually having different fiber orientations. For these reasons, composites are continuing to replace other materials used in structures such as steels, Aluminum alloys… *etc*. In fact composites are classified as the potential structural materials of the future as their cost continues to decrease due to the continuous improvements in production techniques and expanding rate of sales.

Three – dimensional theories of laminated plates in which each layer is treated as homogeneous anisotropic medium (Reddy [1]) are intractable as the number of layers becomes moderately large. Thus, Reddy [1] concluded that a simple two dimensional theory of plates that accurately describes the global behavior of laminated plates seems to be a compromise between accuracy and ease of analysis. Numerical results obtained using refined finite element analysis (D.J. vuksanovic [2], and [3]) and their comparison with exact three dimensional analysis pointed out that the higher order theory provides results which are accurate and acceptable for all ranges of thickness and modular ratio.

Putcha and Reddy [4] classified the two dimensional analyses of laminated composite plates into two categories: (1) the classical lamination theory, and (2) shear deformation theories (including first and higher order theories). In both theories the laminates are assumed in a state of plane stress, the individual lamina is linearly elastic, and there is perfect bonding between layers. The classical laminated plate theory (CLPT), which is an extension of the classical plate theory (CPT) applied to laminated plates was the first theory formulated for the analysis of laminated plates by Reissner and Stavsky [5] in 1961, in which they utilized the Kirchhoff – love assumption that normals to the middle surface before deformation remain straight and normal to the middle surface after deformation, but it is not adequate for the flexural analysis of moderately thick laminates. However, it gives reasonably accurate results for many engineering problems specially those related to thin composite plates, as proved by Srinivas and Rao [6], Reissner and Stavsky [5], Hui – Shen Shen [7], and Ji – Fan He, and Shuang – Wang Zheng [8]. This theory ignores the transverse shear stress components and models a laminate as an equivalent single layer. The high values of modular ratios classify classical laminate theory as inadequate for the analysis of composite plates as verified by Turvey and Osman [9-11], Reddy [1], Pagano [12], and Taner Timarci and Metin Aydogdu [13].

The theory used in the present work comes under the class of displacement based theories which are classified according to Phan and Reddy [14]. In this theory, which is called first order shear deformation theory (FSDT), the transverse planes, which are originally normal and straight to the middle plane of the plate, are assumed to remain straight but not necessarily normal after deformation, and consequently shear correction factors are employed in this theory to adjust the transverse shear stress, which is constant through thickness. Numerous studies involving the application of the first order theory to bending and buckling analyses can be found in the works of Reddy [15], Reddy and Chao [16] Prabhu Madabhusi – Raman and Julio F. Davalo [17], and J. Wang, K.M. Liew, M.J. Tan, S. Rajendran [18].

2. Small Deflection Theory

The equilibrium, strain, constitutive equations and boundary conditions are introduced below without derivation.

2.1. Equilibrium Equations

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$
$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0$$
$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0 \qquad (1)$$
$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0$$
$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = 0$$

2.2. Strain Equations

The small deflection strains of the mid – plane of the plate are as given below:

$$\varepsilon_x^o = \frac{\partial u^o}{\partial x} + z\frac{\partial \phi}{\partial x}$$
$$y_y^o = \frac{\partial v^o}{\partial y} + z\frac{\partial \psi}{\partial y} \qquad (2)$$
$$\varepsilon_{xy}^o = \frac{\partial u^o}{\partial y} + \frac{\partial v}{\partial x} + z\left\{\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}\right\}$$

$$\varepsilon_{xz}^\circ = \frac{\partial w}{\partial y} + \psi$$
$$\varepsilon_{yz}^\circ = \frac{\partial w}{\partial x} + \varphi$$

2.3. The Constitutive Equations

The laminate constitutive equations can be represented in the following form:

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j^\circ \\ \chi_j^\circ \end{Bmatrix} \qquad (3)$$
$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{yz}^\circ \\ \varepsilon_{xz}^\circ \end{Bmatrix}$$

Where N_i denotes N_x , N_y and N_{xy} and M_i denotes M_x , M_y and M_{xy} . A_{ij} , B_{ij} and D_{ij} , $(i, j = 1, 2, 6)$ are respectively the membrane rigidities, coupling rigidities and flexural rigidities of the plate.

χ_j° Denotes $\frac{\partial \varphi}{\partial x}$, $\frac{\partial \psi}{\partial y}$ and $\frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x}$. A_{44}, A_{45} and A_{55} denote the stiffness coefficients, and are calculated as follows:

$$A_{ij} = \sum_{k=1}^{n} K_i K_j \int_{z_k}^{z_{k+1}} c_{ij} \, dz, \; (i, j = 4, 5)$$

Where c_{ij} are the stiffness of a lamina referred to the plate principal axes and K_i, K_j are the shear correction factors.

2.4. Boundary Conditions

All of the analyses described in this paper have been undertaken assuming the plates to be subjected to identical support conditions in the flexural and extensional senses along all edges. The three sets of edge conditions used here are designated as SS1, SS2 and SS3 and are shown in Figure (1) below.

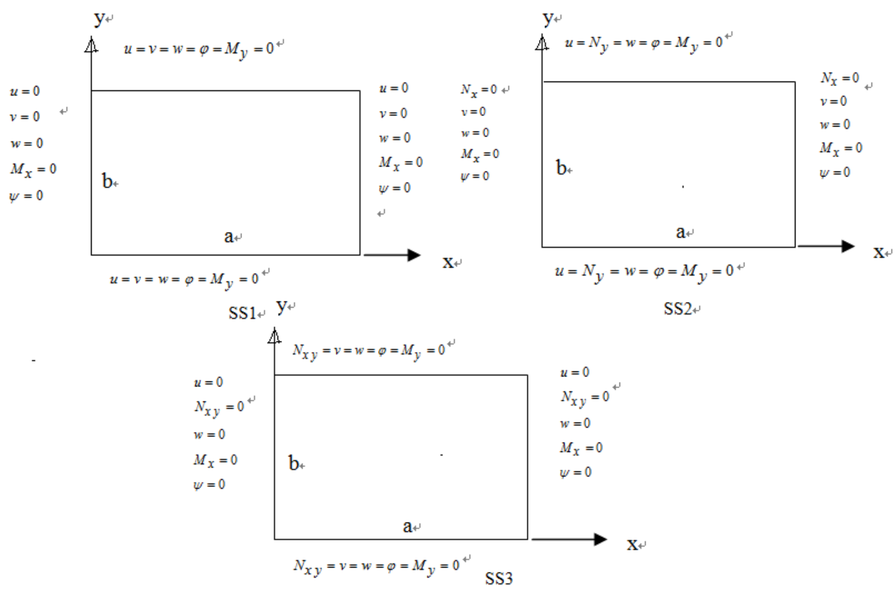

Figure 1. Simply Supported Boundary Conditions

3. Dynamic Relaxation Solution of the Plate Equations

In the present work, a numerical method known as Dynamic Relaxation (DR) coupled with finite differences is used. The DR method was first proposed in 1960s and then passed through a series of studies to verify its validity by Turvey and Osman Refs. [9, 10] and [11] and Rushton [19], Cassel and Hobbs [20], and Day [21]. In this method, the equations of equilibrium are converted to dynamic equations by adding damping and inertia terms. These are then expressed in finite difference form and the solution is obtained through iterations.

Numerical techniques other than the DR include finite element method, which is widely used in several studies *i.e.*, of Damodar R. Ambur *et al.*, [22], ying Qing Huang *et al.*, [23], Onsy L. Roufaeil *et al.*, [3]… *etc*. In a comparison between the DR and the finite element method, Aalami [24] found that the computer time required for finite element method is eight times greater than for DR analysis, whereas the storage capacity for finite element analysis is ten times or more than that for DR analysis. This fact is supported by Putcha and Reddy [4] who they noted that some of the finite element formulation requires large storage capacity and computer time. Hence due to less computations and computer time involved in the present study, the DR method is considered more suitable than the finite element method.

The plate equations are written in dimensionless forms. Damping and inertia terms are added to Eqation. (1). Then the following approximations are introduced for the velocity and acceleration terms:

$$\frac{\partial \alpha}{\partial t} = \frac{1}{2}\left[\frac{\partial \alpha^a}{\partial t} + \frac{\partial \alpha^b}{\partial t}\right]$$
$$\frac{\partial^2 \alpha}{\partial t^2} = \left(\frac{\partial \alpha^a}{\partial t} - \frac{\partial \alpha^b}{\partial t}\right) / \partial t \tag{4}$$

In which $\alpha \equiv u, v, w, \varphi, \psi$. Hence Eqns (1) becomes :

$$\frac{\partial u^a}{\partial t} = \frac{1}{1+k_u^*}\left[\left(1-k_u^*\right)\frac{\partial u^b}{\partial t} + \frac{\delta t}{\ell_u}\left(\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y}\right)\right]$$

$$\frac{\partial v^a}{\partial t} = \frac{1}{1+k_u^*}\left[\left(1-k_u^*\right)\frac{\partial v^b}{\partial t} + \frac{\delta t}{\ell_v}\left(\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y}\right)\right]$$

$$\frac{\partial w^a}{\partial t} = \frac{1}{1+k_w^*}\left[\left(1-k_w^*\right)\frac{\partial w^b}{\partial t} + \frac{\delta t}{\ell_w}\left(\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q\right)\right] \tag{5}$$

$$\frac{\partial \varphi^a}{\partial t} = \frac{1}{1+k_\varphi^*}\left[\left(1-k_\varphi^*\right)\frac{\partial \varphi^b}{\partial t} + \frac{\delta t}{\ell_\varphi}\left(\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x\right)\right]$$

$$\frac{\partial \psi^a}{\partial t} = \frac{1}{1+k_\psi^*}\left[\left(1-k_\psi^*\right)\frac{\partial \psi^b}{\partial t} + \frac{\delta t}{\ell_\psi}\left(\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y\right)\right]$$

In equations. (4) and (5) the superscripts (a) and (b) refer respectively to the values of velocities after and before the time step δt , and $k_\alpha^* = \frac{1}{2}k_\alpha \delta t \ell_\alpha^{-1}$. The displacements at the end of each time increment, δt , are evaluated using the following simple integration procedure:

$$\alpha^a = \alpha^b + \delta t \frac{\partial \alpha^b}{\partial t} \tag{6}$$

The complete equation system is represented by eqns. (5), (6),(2) and (3). The DR algorithm operates on these equations is as follows:
Step 1: set initial conditions (usually all variables are zero).
Step 2: compute velocities from equation (5).
Step 3: compute displacements from equation (6).
Step 4: Apply displacement boundary conditions.
Step 5: compute strains from equation (2).
Step 6: compute stress resultants, etc from equation (3).
Step 7: Apply stress resultants … etc, boundary conditions.

Step 8: check if velocities are acceptably small (say < 10-6).
Step 9: If Step 8 is satisfied print out results, otherwise repeat steps. (2 to 8).

4. Verification of the Dynamic Relaxation (DR) method

The present DR results are compared with similar results generated by other DR and / or alternative techniques including approximate analytical and exact solutions so as to validate the DR program. In the following discussion a wide range of small deflections are dealt with including isotropic, orthotropic, and laminated plates subjected to static uniformly distributed loading scheme.

Table (1) shows the variations in the central deflections of a moderately thick isotropic plate $(h/a=0.1)$ with simply supported condition (SS1). These results suggest that a 5×5 mesh over one quarter of the plate is quite enough for the present work (*i.e.*, less than 0.3% error compared to the finest mesh available). In table (2) the comparison of the present DR deflections and stresses with that generated by Turvey and Osman [9] and Reddy [25] is presented for a uniformly loaded plate of thin $(i.e.\ \ h/a=0.01)$, moderately thick $\left(i.e.\ \ h/a=0.1\right)$, and thick laminates $(i.e.\ \ h/a=0.2)$ using simply supported condition (SS1). The present DR results of central deflections and stresses showed good agreement with the other results even though the plate is square or rectangle. Another comparison analysis for small deformations of thin and moderately thick square simply supported isotropic plates (SS1) between the present DR method, and Roufaeil [26] two and three node strip method is shown in table (3). Again, these results provide further confirmation that a DR analysis based on a 5×5 quarter – plate mesh produces results of acceptable accuracy.

In the following analyses, several orthotropic materials were employed; their properties are given in table (4). Exact FSDT solutions are available for plates simply supported on all four edges (SS2). By imposing only a small load on the plate, the DR program may be made to simulate these small deflections. In table (5), the computations were made for uniform loads and for thickness / side ratios ranging from 0.2 to 0.01 of square simply supported in – plane free plates made of material I with $(\bar{q}=1.0)$. In this case the central deflections of the present DR method are close to those of Turvey and Osman [10], and Reddy [25]. Another small deflection analysis is shown on table (6), and it was made for uniformly loaded plates with simply supported in – plane fixed condition (SS1) of material II and subjected to uniform loading $\left(\bar{q}=1.0\right)$. In this analysis, the four sets of results are the same for the central deflections and stresses at the upper and lower surfaces of the plate and also the same for the mid – plane stresses. Nevertheless, the exact solution of Srinivas and Rao [6] is not in a good agreement with the others as far as stresses are concerned. These differences may be attributed to the exact solution theory adopted in Ref. [6].

Most of the published literature on laminated plates are devoted to linear analysis and in particular to the development of higher order shear deformation theories. Comparatively, there are few studies on the nonlinear behavior of laminated plates and even fewer are those which include shear deformations. The elastic properties of the material used in the analyses are given in table (4). The shear correction factors are $k_4^2=k_5^2=5/6$, unless otherwise stated.

In table (7) which shows a comparison between the present DR method and finite element results of Ref. [6] for a simply supported condition (SS3) plate. There are four antisymmetric angle ply laminates of material III which are subjected to a small uniform load $(\bar{q}=1.0)$. The central deflections and stresses are recorded for different thickness ratios including thick, moderately thick, and thin laminates. These results are compared with Reddy's finite element results [6] and are found in a good agreement despite the different theory adopted in the latter case.

Another comparison analysis of central deflections between the present DR method, Zenkour *et al.*, [27] using third order shear deformation theory and Librescu and Khdeir [28] which are made of material IV are illustrated in table (8). The three results showed a good agreement especially as the length to thickness ratio increases.

Table 1. DR Solution Convergence Results for a Simply Supported (SS1) Square Plate Subjected to Uniform Pressure $\bar{q} = 1.0, h/a = 0.1$ and v= 0.3

Mesh size	$\bar{w}_c$
2 × 2	0.04437
3 × 3	0.04592
4 × 4	0.04601
5 × 5	0.04627
6 × 6	0.04629
7 × 7	0.04638
8 × 8	0.04640

Table 2. Comparison of Present DR, Turvey and Osman [1], and Exact Values of Reddy [25] Small Deflection Results for Uniformly Loaded Simply Supported (SS1) Square and Rectangular Plates of Various Thickness Ratios $\bar{q} = 1.0$, v= 0.3

a/b	h/a	s	$\bar{w}_c$	$\bar{\sigma}_x(1)$	$\bar{\sigma}_y(1)$	$\bar{\sigma}_{xy}(2)$	$\bar{\sigma}_{xz}(3)$	$\bar{\sigma}_{yz}(4)$
1	0.20	1	0.0529	0.2879	0.2879	- 0.2035	0.3983	0.3983
		2	0.0529	0.2879	0.2879	- 0.2035	0.3984	0.3984
		3	0.0536	0.2873	0.2873	- 0.1946	0.3928	0.3928
	0.10	1	0.0463	0.2866	0.2866	- 0.2038	0.3983	0.3960
		2	0.0463	0.2865	0.2865	- 0.2038	0.3990	0.3990
		3	0.0467	0.2873	0.2873	- 0.1946	0.3928	0.3928
	0.01	1	0.0440	0.2853	0.2853	- 0.2033	0.3960	0.3960
		2	0.0441	0.2860	0.2860	- 0.2039	0.3990	0.3990
		3	0.0444	0.2873	0.2873	- 0.1946	0.3928	0.3928
2	0.20	1	0.1204	0.2825	0.6165	- 0.2952	0.4230	0.5400
		2	0.1216	0.2840	0.6225	- 0.2829	0.4341	0.5410
		3	0.1248	0.2779	0.6100	- 0.2769	0.4192	0.5451
	0.10	1	0.1111	0.2819	0.6148	- 0.2964	0.4200	0.5412
		2	0.1122	0.2838	0.6209	- 0.2843	0.4358	0.5447
		3	0.1142	0.2779	0.6100	- 0.2769	0.4192	0.5451
	0.01	1	0.1080	0.2823	0.6141	- 0.2970	0.4200	0.5400
		2	0.1109	0.2842	0.6212	- 0.2857	0.4377	0.5472
		3	0.1106	0.2779	0.6100	- 0.2769	0.4192	0.5451

S (1): present DR results
S (2): DR results of Ref. [9]
S (3): Exact results of Ref. [25]

$$(1)\ x=\frac{1}{2}a,\ y=\frac{1}{2}b,\ z=\frac{1}{2}h;\ (2)\ x=y=0,\ z=\frac{1}{2}h;\ (3)\ x=0,\ y=\frac{1}{2}b,\ z=0;\ (4)\ x=\frac{1}{2}a,\ y=z=0$$

Table 3. Dimensionless Central Deflection of a Square Simply Supported Isotropic Plate (SS1)

$\left(\bar{q}=1.0,\ v=0.3,\ k^{2}=0.0.833\right)$

a/h	Present DR Results	3 – node strip Ref. [31]	2 – node strip Ref. [31]
100	0.00403	0.00406	0.00406
10	0.00424	0.00427	0.00426

Table 4. Material Properties used in the Orthotropic and Laminated Plate Comparison Analysis

Material	E_1/E_2	G_{12}/E_2	G_{13}/E_2	G_{23}/E_2	v_{12}	$SCF\left(k_4^2=k_5^2\right)$
I	25.0	0.5	0.5	0.2	0.25	5/6
II	1.904	0.558	0.339	0.566	0.44	5/6
III	40.0	0.5	0.5	0.5	0.25	5/6
IV	12.308	0.526	0.526	0.335	0.24	5/6

Table 5. Comparison of Present DR, Turvey and Osman [10], and Ref [25] Center Deflections of a Simply Supported (SS2) Square Orthotropic Plate Made of Material I for Different Thickness Ratios when Subjected to Uniform Loading

$\left(\bar{q}=1.0\right)$

Thickness ratio h/a	Uniform Loading		
	$\bar{w}_c\,(DR)$ present	$\bar{w}_c\,(DR)$ Ref. [4]	$\bar{w}_c\,(exact)$ Ref. [2]
0.2	0.017914	0.017912	0.018159
0.1	0.009444	0.009441	0.009519
0.08	0.008393	0.008385	0.008442
0.05	0.007245	0.007230	0.007262
0.02	0.006617	0.006602	0.006620
0.01	0.006512	0.006512	0.006528

Table 6. Comparison of Present DR, Ref. [10], Ref. [25], and Exact Solutions Ref [6] for a Uniformly Loaded Simply Supported (SS1) Orthotropic Plate Made of Material II when Subjected to Uniform Loading $\left(\bar{q}=1.0\right)$

b/a	h/a	s	$\bar{w}_c$	$\bar{\sigma}_x(1)$	$\bar{\sigma}_{xz}(2)$
1	0.05	1	0.0306	0.3563	0.4387
		2	0.0306	0.3562	0.4410
		3	0.0308	0. 3598	0.4351
		4	0.0308	0.3608	0.5437
	0.10	1	0.0323	0.3533	0.4393
		2	0.0323	0.3534	0.4395

		3	0.0326	0.3562	0.4338
		4	0.0325	0.3602	0.5341
	0.14	1	0.0344	0.3498	0.4367
		2	0.0344	0.3498	0.4374
		3	0.0347	0.3516	0.5328
		4	0.0346	0.3596	0.5223
2	0.05	1	0.0629	0.6569	0.6506
		2	0.0629	0.6568	0.5637
		3	0.0636	0.6550	0.5600
		4	0.0636	0.6567	0.7024
	0.10	1	0.0657	0.6566	0.5623
		2	0.0657	0.6566	0.5628
		3	0.0665	0.6538	0.5599
		4	0.0664	0.6598	0.6927
	0.14	1	0.0692	0.6564	0.5613
		2	0.0692	0.6564	0.5613
		3	0.0703	0.6521	0.5597
		4	0.0701	0.6637	0.6829

S (1): present DR results
S (2): DR results of Ref [10]
S (3): Finite element solution Ref [25]
S (4): Exact solution Ref [6]

Table 7. Comparison of Present DR, and Reddy Finite Element Results Ref. [15] for $[45^\circ/-45^\circ/45^\circ/-45^\circ]$ Simply Supported (SS3) Square Laminate Made of Material III and Subjected to Uniform Loads and for Different Thickness Ratios $\left(\bar{q}=1.0\right)$

h/a	s	$\bar{w}_c \times 10^3$	$\bar{\sigma}_x\,(1)$
0.20	1	9.0809	0.2022
	2	9.0000	0.1951
0.10	1	4.3769	0.2062
	2	4.2000	0.1949
0.05	1	3.2007	0.2081
	2	3.0000	0.1938
0.04	1	3.0574	0.2090
	2	2.9000	0.1933
0.02	1	2.8371	0.2063
	2	2.8000	0.1912

S (1): present DR results.

S (2): Reddy [15] as read from graph. $x=y=\frac{1}{2}a,\ z=\frac{1}{2}h$

Table 8. Non – Dimensionalized Deflections in Three Layers Cross – ply $[0^\circ/90^\circ/0^\circ]$ Simply Supported (SS1) Square Laminates of Material IV Under Uniform Load $(\bar{q} = 1.0)$

a/h	s	$\bar{w}_c$
2	1	0.0693
	2	0.0726
	3	0.0716
5	1	0.0224
	2	0.0232
	3	0.0235
10	1	0.0147
	2	0.0150
	3	0.0151
20	1	0.0127
	2	0.0128
	3	0.0128

S (1): present DR results linear analysis
S (2): Librescu L and Khdeir A.A [28]
S (3): A.M. Zenkour, and M.E.Fares [27] results.

5. Conclusions

A Dynamic relaxation (DR) program based on finite differences has been developed for small deflection analysis of rectangular laminated plates using first order shear deformation theory (FSDT). The displacements are assumed linear through the thickness of the plate. A series of new results for uniformly loaded thin, moderately thick, and thick plates with simply supported edges have been presented. Finally a series of numerical comparisons have been undertaken to demonstrate the accuracy of the DR program. These comparisons show the following:-

1. The convergence of the DR solution depends on several factors including boundary conditions, mesh size, fictitious densities and load.

2. The type of mesh used (*i.e.*, uniform or graded mesh) may be responsible for the considerable differences in the mid – side and corner stress resultants.

3. For simply supported (SS1) edge conditions, all the comparison results confirmed that deflection depends on the direction of the applied load or the arrangement of the layers.

4. The DR small deflection program using uniform finite difference meshes can be employed with less accuracy in the analysis of moderately thick and flat isotropic, orthotropic or laminated plates under uniform loads.

5. Time increment is a very important factor for speeding convergence and controlling numerical computations. When the increment is too small, the convergence becomes tediously slow, and when it is too large, the solution becomes unstable. The proper time increment in the present study is taken as 0.8 for all boundary conditions.

6. The optimum damping coefficient is that which produces critical motion. When the damping coefficients are large, the motion is over – damped and the convergence becomes

very slow. And when the coefficients are small, the motion is under – damped and can cause numerical instability. Therefore, the damping coefficients must be selected carefully to eliminate under – damping and over – damping.

7. Finer meshes reduce the discretization errors, but increase the round – off errors due to the large number of calculations involved.

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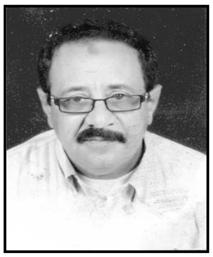

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