

Statistical Modeling of Head Loss Components in Ventilation Ducts

John I. Sodiki

*Department of Mechanical Engineering
Rivers State University of Science and Technology
P. M. B. 5080
Port Harcourt, Nigeria
jisodiki_partners@yahoo.com*

Abstract

The results of an earlier study are utilized to obtain regression model equations for predicting the variation of the fraction of the total pressure loss which is due to duct fittings for varying numbers of air inlets and lengths of extract ventilation ductwork, in a set of ventilation system configurations. The obtained second order equations give a general increase of the fraction from 0.590 to 0.667 for an increase in number of air inlets (and number of ventilated rooms) from 2 to 12 and an increase of the fraction from 0.598 to 0.667 for an increase of duct length from 2.8m to 22.4m. The correlation coefficients for a 90% confidence level are, respectively, 0.707 and 0.713 for the variations of the fitting loss fraction with number of air inlets and length of index duct run. The obtained fractions are useful for quick estimate of total pressure losses in ventilation duct systems, which are needed in determining the fan system pressure requirements.

Keywords: *Head loss fractions, ventilation systems, regression analysis*

1. Introduction

The determination of the fan pressure in a ducted ventilation system requires the calculation of the pressure losses in the index run of ductwork; these losses being the frictional loss and the loss through duct fittings such as elbows, tees and enlargements. The latter loss is referred to as separation loss. Usually, increased index duct lengths result in increased frictional loss while a multiplicity of fittings is associated with increased separation loss. In general, the ratio between the two loss components in an air extraction system may vary with varying length of index run (hence with number of air intake terminals).

In an earlier study [1] a relationship had been drawn between the fraction of the total head loss which represents the separation loss, on one hand; and the number of intake terminals and length of index duct run, on the other hand. The resulting graphs showed a second order variation for the relationship.

In the study, total pressure loss components were calculated for six configurations of toilet rooms (namely 2, 4, 6, 8, 10, and 12 rooms). The 4-room configuration is illustrated in plan and as an isometric sketch in Figures 1 and 2, respectively; while the 6-room configuration is illustrated in plan in Figure 3. In Figure 2 the duct sections in the index run are labeled using boxes which touch the sections. In each box, the number on the left is the duct section number, that on the top right is the length of the section (in m), while the number on the bottom right is the air quantity (in l/s) flowing through the duct section.

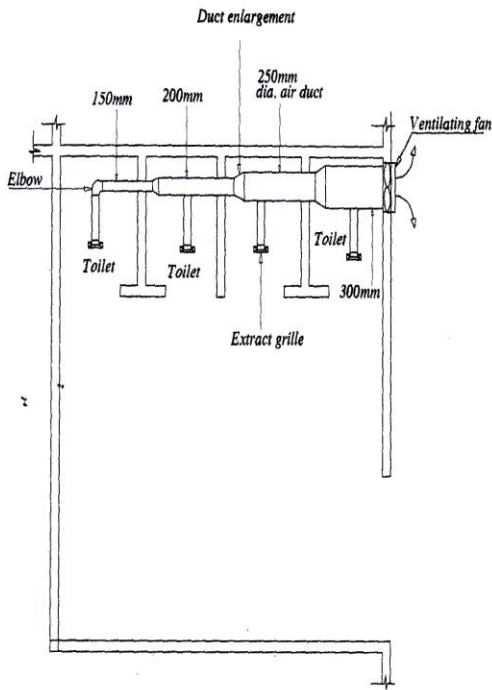


Figure 1: Plan of Ventilation Duct System for Four Toilet Rooms

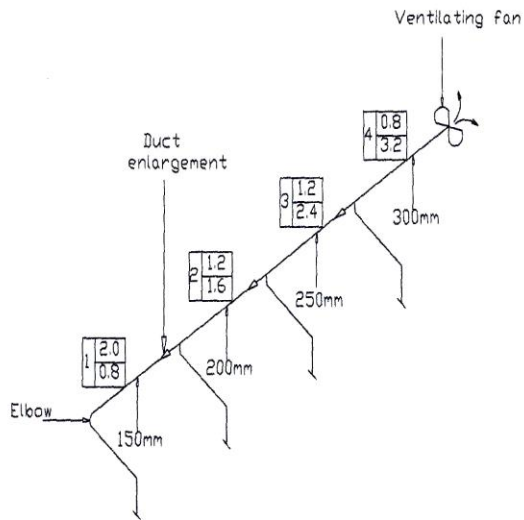


Figure 2: Isometric Line Sketch of Ventilation Ductwork for Four Toilet Rooms

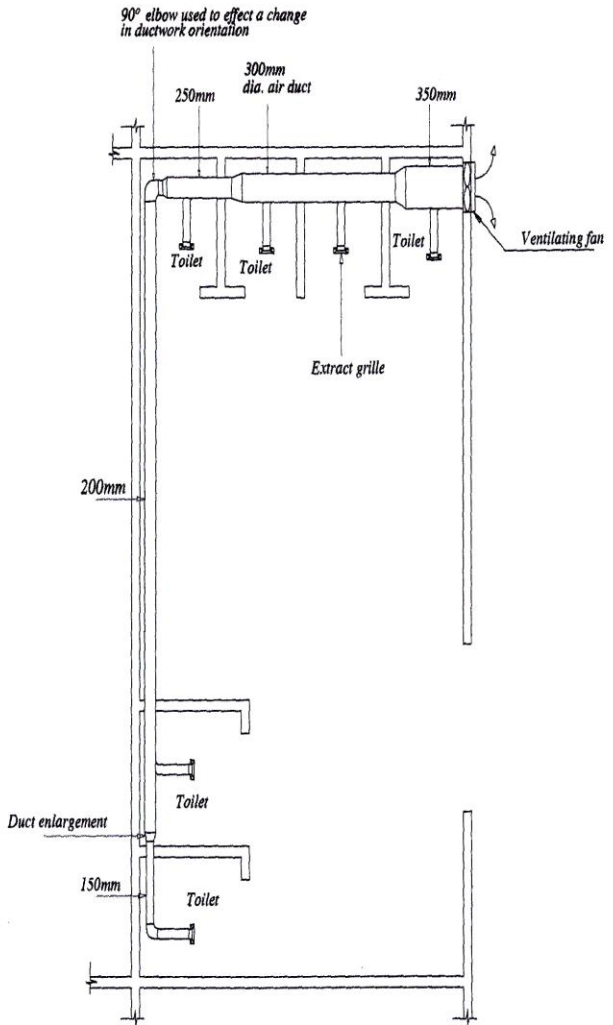


Figure 3: Plan of Ventilation Duct System for Six Toilet Rooms

In the present study, the data generated in the earlier study are utilized to obtain regression equations which are useful in predicting the fraction of total head loss due to duct fittings in index duct runs.

2. Calculation of Pressure Loss Components

For each ductwork configuration, the frictional head loss was calculated by the D'Arcy-Weisbach formula expressed as [1]

$$h_{friction} = 0.3304 \sum_{i=1}^{i=n} \frac{f_i l_i q_i^2}{d_i^5} \quad \text{----- (1)}$$

where i denotes the i^{th} duct section and n is the number of sections in the composite index run.

- f = duct section friction factor
- l = duct section length (in m)
- q = flow rate in duct section (in m³/s)
- d = duct section diameter (in m)

f is a function of the flow Reynolds number which is given as [2]

$$Re = \frac{\rho v d}{\mu} \quad \text{----- (2)}$$

- where ρ = air density
- v = flow velocity in the particular duct section
- u = air dynamic velocity

Now, for Re up to 200000 (which includes the range of Re realized in the ventilation duct configurations being considered) the Blasius equation [3]

$$f = 0.079 Re^{0.25} \quad \text{----- (3)}$$

was utilized in the analysis.

The separation loss component was calculated in terms of the loss coefficient k [4] of each fitting obtained from the literature as [5]

$$h_{fitting} = \sum_{j=1}^{j=m} k_j q_j^2 d_j^{-4} \quad \text{----- (4)}$$

where j denotes the j^{th} fitting and m the number of fittings in the composite index duct run.

Typically, the estimation of head loss components for the 4 – room toilet ventilation configuration is shown in Table 1, while in Table 2 the summary of the estimated loss components for all six configurations are presented. The ‘ Excel ‘ plots of Figs 4 and 5 depict the respective variations of the fraction of the total head loss which represents the loss through duct fittings with varying number of ventilation air inlet terminals and lengths of index duct run.

Table 1. Parameters of Ventilation Index Duct Run for 4-Room Toilet Configuration

Duct Section	Flow Rate, Q (m^3/s)	Fractional Flow with Respect to Total Fan Discharge Q	Length L (m)	Diameter, d (mm)	Reynolds Number, Re	Friction Factor, f	Frictional Head Loss (m)	Type of Fitting	Number of Particular Type of Fitting in Section	Head Loss Coefficient of Fitting, k^*	Head Loss Through Fitting (m)
1	0.8	0.25 Q	2.0	130	45413	0.0054	2.937 Q^2	150mm radius elbow ($R/D=1.0$)	1	0.16	2 x 1.631 Q^2
								150mm x 200mm enlargement ($d_2/d_1 = 1.3$)	1	0.13	0.254 Q^2
								200mm x 150mm radius tee	1	0.20	2.039 Q^2
2	1.6	0.50 Q	1.2	200	68120	0.0049	1.518 Q^2	250mm x 300mm enlargement ($d_2/d_1 = 1.3$)	1	0.13	0.217 Q^2
								250mm x 150mm radius tee	1	0.20	2.039 Q^2
3	2.4	0.75 Q	1.2	250	81744	0.0047	1.073 Q^2	250mm x 300mm enlargement ($d_2/d_1 = 1.3$)	1	0.08	0.089 Q^2
								300mm x 150mm radius tee	1	0.20	2.039 Q^2
4	3.2	Q	0.8	300	90827	0.0046	0.500 Q^2

*Source: J. J. Barton (1964).

Table 2. Summary of Head Loss Estimates

No. of Air Inlet Terminals (or Rooms)	Length of Index Duct Run (m)	Frictional Head Loss (m)	Head Loss Due to Fittings (m)	Total Head Loss (m)	Fraction of Total Loss Due of Fittings
2	2.8	15.795Q ²	22.215Q ²	38.010Q ²	0.584
4	5.2	6.028Q ²	9.939Q ²	15.967Q ²	0.622
6	11.4	5.630Q ₂	7.767Q ²	13.397Q ²	0.580
8	13.8	2.963Q ²	4.962Q ²	7.925Q ²	0.626
10	20.0	2.872Q ²	5.535Q ²	8.407Q ²	0.658
12	22.4	1.931Q ²	3.744Q ²	5.675Q ²	0.660

3. Derivation of Regression Models

The second order variation of the fraction of total loss due to separation loss, denoted as the dependent variable y , which is regressed, in turn, on the independent variables of number of inlet air terminals (denoted as x_1) and length of index duct run (denoted as x_2), may be expressed as

$$y = a_0 + a_1x + a_2x^2 \quad \text{---- (5)}$$

where the regression parameters a_0 , a_1 and a_2 would be obtained for each of the independent variables x_1 and x_2 by the solution of the simultaneous equations [6]

$$\sum y = na_0 + a_1 \sum x + a_2 \sum x^2 \quad \text{---- (6)}$$

$$\sum xy = a_0 \sum x + a_1 \sum x^2 + a_2 \sum x^3 \quad \text{---- (7)}$$

$$\sum yx^2 = a_0 \sum x^2 + a_1 \sum x^3 + a_2 \sum x^4 \quad \text{---- (8)}$$

where n is the number of data points, which is 6 in this case.

Utilizing the values presented in Table 2, the statistical variables are computed in Table 3. Substitution of values from Table 3 into Eqns. 6, 7 and 8 and simultaneous solution results in

$$y = 0.5876 + 3.7943 \times 10^{-4}x_1 + 5.178 \times 10^{-4}x_1^2 \quad \text{---- (9)}$$

and $y = 0.6019 - 2.1816 \times 10^{-3}x_2 + 2.255 \times 10^{-4}x_2^2 \quad \text{---- (10)}$

as the respective relationships between the depend variable y and the independent variables x_1 and x_2 .

Taking the derivation of Eqn. 10 as an example, substitution of values from Table 3 yields the simultaneous equations

$$3.73 = 6a_0 + 75.6a_1 + 1257.04a_2 \quad \text{--- (a)}$$

$$48.064 = 75.6a_0 + 1257.04a_1 + 23511.6a_2 \quad \text{--- (b)}$$

$$810.352 = 1257.04a_0 + 23511.6a_1 + 465712.722a_2 \quad \text{--- (c)}$$

Solving for a_0 , a_1 and a_2 gives $a_0 = 0.6019$, $a_1 = -2.1816 \times 10^{-3}$ and $a_2 = 2.255 \times 10^{-4}$

Table 3. Compilation of Statistical Variables and Terms for x_1 and x_2

n_1 No. of Air inlet terminals	n_2 Length of Index Duct Run (m)	Frictional Head Loss (m)	Head Loss Due to Fittings (m)	Total Head Loss (m)	$\frac{y}{\text{Total Loss}}$ Loss thru Fittings	yx_1	x_1^2	yx_1^2	x_1^3	x_1^4	yx_2	x_2^2	yx_2^2	x_2^3	x_2^4
2	2.8	15.795Q ²	22.215Q ²	38.010Q ²	0.584	1.168	4	2.336	8	16	1.635	7.84	4.579	21.952	61.466
4	5.2	6.028Q ²	9.939Q ²	15.967Q ²	0.622	2.488	16	9.952	64	256	3.234	27.04	16.819	140.608	731.162
6	11.4	5.630Q ²	7.767Q ²	13.397Q ²	0.580	3.480	36	20.880	216	1296	6.612	129.96	75.377	1481.544	16889.602
8	13.8	2.963Q ²	4.962Q ²	7.925Q ²	0.626	5.008	64	40.064	512	4096	8.639	190.44	119.215	2628.072	36267.394
10	20.0	2.872Q ²	5.535Q ²	8.407Q ²	0.658	6.580	100	65.800	1000	10000	13.160	400.00	263.200	8000.000	160000.000
12	22.4	1.931Q ²	3.744Q ²	5.675Q ²	0.660	7.920	144	95.040	1728	20736	14.784	501.76	331.162	11239.424	251763.098
$\Sigma = 42$	$\Sigma = 75.6$			$\Sigma =$	3.730	26.644	364	234.072	3528	36400	48.064	1257.04	810.352	23511.600	465712.722

Table 4. Values for Calculating Coefficient of Correlation for x_1

i	x_i	y_i	$y_i - \bar{y}$	$(y_i - \bar{y})^2 \times 10^4$	y_{ic}	$y_i - y_{ic}$	$(y_i - y_{ic})^2 \times 10^4$
1	2	0.584	-0.038	14.44	0.590	-0.006	0.36
2	4	0.622	0.000	0.00	0.597	0.025	6.25
3	6	0.580	-0.042	17.64	0.609	-0.029	8.41
4	8	0.626	0.004	0.16	0.624	0.002	0.04
5	10	0.658	-0.036	12.96	0.643	0.015	2.25
6	12	0.660	0.038	14.44	0.667	-0.007	0.49
		$\Sigma=3.73$ $\bar{y}=0.622$		$\Sigma=59.64$			$\Sigma=17.80$

Table 5. Values for Calculating Coefficient of Correlation for x_2

i	x_i	y_i	$y_i - \bar{y}$	$(y_i - \bar{y})^2 \times 10^4$	y_{ic}	$y_i - y_{ic}$	$(y_i - y_{ic})^2 \times 10^4$
1	2.8	0.584	-0.038	14.44	0.598	-0.014	1.96
2	5.2	0.622	0.000	0.00	0.597	0.025	6.25
3	11.4	0.580	-0.042	17.64	0.606	-0.026	6.76
4	13.8	0.626	0.004	0.16	0.615	0.011	1.21
5	20.0	0.658	-0.036	12.96	0.648	0.010	1.00
6	22.4	0.660	0.038	14.44	0.666	-0.006	0.36
		$\Sigma=3.73$ $\bar{y}=0.622$		$\Sigma=59.64$			$\Sigma=17.54$

The coefficient of correlation is given as [6]

$$r = \sqrt{1 - \left(\frac{S_{y.x}}{s_y}\right)^2} \quad \text{--- (11)}$$

$$\text{where } S_{y.x} = \sqrt{\frac{\sum_{i=1}^n (y_i - y_{ic})^2}{n-3}} \quad \text{--- (12)}$$

= standard error of estimate, y_i being the actual values of y obtained from

the results of section 2 above, y_{ic} being the values of y computed from the regression equation (Eqns. 9 and 10) and n the number of points (equal to 6).

$n-3$ is the number of degrees of freedom, as the number of regression parameters to be estimated in Eqn. 5 is three: a_0 , a_1 and a_2 .

$$s_y = \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}} \quad \text{--- (13)}$$

= sample standard deviation of y , \bar{y} being the mean of y_i

The calculation of r is facilitated by Tables 4 and 5, respectively, for x_1 and x_2 . For x_2 , for instance, the regression equation is

$$y_{ic} = 0.6019 - 2.1816 \times 10^{-3}x_i + 2.255 \times 10^{-4}x_i^2 \quad \text{---- (14)}$$

Then taking values from Table 5 into Eqn.12

$$s_{y, x} = \sqrt{\frac{17.54 \times 10^{-4}}{6-3}} = 0.0242$$

and from Eqn. 13, $s_y = \sqrt{\frac{59.64 \times 10^{-4}}{6-1}} = 0.0345$

$$\therefore r \text{ (from Eqn. 11)} = \sqrt{1 - \left(\frac{0.0242}{0.0345}\right)^2} = 0.713$$

Following a similar procedure, the correlation coefficient for x_1 is obtained as 0.707. The coefficients of correlation for x_1 and x_2 (0.707 and 0.713, respectively), thus, fall within the 90% confidence interval of $0.629 \leq r \leq 0.785$, as obtained from statistical data [7].

4. Discussion of Results

From the obtained coefficients of correlation, it can be inferred with 90% confidence that the fraction of total head loss through duct fittings in the studied ventilation system configurations can be obtained from the derived regression equations (with the number of ventilation air intakes, or rooms; and length of index ductwork as independent variables).

The fractions of head loss through fittings obtained from the regression equations show second order increases from 0.590 to 0.667 for an increase in the number of air intakes (or rooms) from 2 to 12, and from 0.598 to 0.667 for a corresponding increase of 2.8m to 22.4m in index duct length. It is thus observed that the fitting loss fractions fall between 0.590 and 0.667 within the limits of system complexity utilized in the study.

Hence, needed estimates of the fraction of pressure loss due to fittings may be made by interpolating between these limits. Alternatively, the derived regression equations may be applied with reasonable correctness.

It is also observed that, within the limits of system complexity utilized in the study, the fractions of loss due to fittings are greater than those due to friction (*i.e.*, > 0.5). It would, therefore be a misnomer to refer to the head loss through duct fittings as ‘minor loss’.

5. Conclusions

Second order equations have been derived by regression analysis to estimate the head loss fraction due to duct fittings in a set of extract ventilating duct systems. Such regression models facilitate the extract fan selection process since the total system head loss is easily obtained by adding the relevant fraction to the frictional loss. More extensive duct systems can also be analyzed by the same method to obtain regression models which would be useful for wider applications.

References

- [1] J. I. Sodiki, "The Variation of Head Loss through Fittings with Ductwork Complexity in Horizontal Ventilation Systems", *Compendium of Engineering Monographs*, vol. 1, no. 3, (2004), pp. 8-17.
- [2] J. F. Douglas, J. M. Gasiorek and J. A. Swaffield, "Fluid Mechanics", Longman Group Ltd., Essex, (1995).
- [3] J. F. Douglas, "Solutions to Problems in Fluid Mechanics Part 2", Pitman Publishing Ltd., London, (1978).
- [4] J. J. Barton, "Principles and Practice of Heating and Ventilating", George Newnes Ltd., London, (1964).
- [5] J. I. Sodiki, "The Variation of Head Loss through Fittings within Ductwork Complexity in Air Conditioning Systems", *Nigerian Journal of Technological Development*, vol. 4, no. 2, (2004), pp. 145-164.
- [6] C. Lipson and N. J. Seth, "Statistical Design and Analysis of Engineering Experiments", McGraw-Hill Book Co., New York, (1973).
- [7] Statistics Calculators Version 3.0, 2014 [Online]. Available: <http://www.Danielsoper.com>.

Author



J. I. Sodiki received his B.Sc. degree in Mechanical Engineering from University of Lagos, Nigeria in 1981 and M.Sc. degree in Mechanical Engineering from University of Lagos in 1986. He is working as Lecturer in Department of Mechanical Engineering at Rivers State University of Science and Technology, Port Harcourt, Nigeria. He has 19 years of teaching experience at university level. His areas of interest include Thermo-fluids, Building Services, and Corrosion Control.

