Performance Analysis of Different Higher Order Modulations for PAPR Reduction

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Abstract

Next generation wireless communication system uses one of the most competent multi-carrier transmission techniques known as Orthogonal Frequency Division Multiplexing (OFDM). OFDM has several characteristics such as providing greater immunity to multipath fading & impulse noise, eliminating Inter Symbol Interference (ISI) & Inter Carrier Interference (ICI) using a guard interval known as Cyclic Prefix (CP). But, OFDM suffers a serious drawback of high peak to average power ratio (PAPR) which is defined as the ratio of the peak power to the average power of OFDM Signal. A lot of researches are carrying on reducing this high PAPR. In this paper, we investigated the performances of different higher order modulations on our previously designed amplitude clipping & filtering method to reduce PAPR.

Keywords: BER, PAPR, CCDF, OFDM

1. Introduction

The massive expansion in broadband multimedia controlled applications have triggered an insatiable thirst for high data rates and resulted in an increased demand for technologies that support very high speed transmission rates, mobility and efficiently utilize the available spectrum & network resources. OFDM is one of the vital resolutions to achieve this goal and it offers a promising choice for future high speed data rate systems [1].OFDM has been standardized as part of the IEEE802.11a and IEEE 802.11g for high bit rate data transmission over wireless LANs. It is incorporated in other applications and standards such as digital audio broadcasting (DAB), digital video broadcasting (DVB), European HIPERLAN/2 and the Japanese multimedia mobile access communications (MMAC) [2]. In addition, OFDM is also used now as the transmission scheme of choice in the physical layer of the world wide interoperability for microwave access (WiMAX) & long term evolution (LTE) standards. As the data rates and mobility supported by the OFDM system raise, the number of subcarriers also raise, which in turn leads to high PAPR. As future OFDM-based systems may push the number of subcarriers up to meet the higher data rates and mobility demands, there is a need to mitigate the high PAPR [3].

A number of attractive techniques have been proposed & implemented to reduce PAPR with the expense of increase transmit signal power, bit error rate (BER), computational complexity and data rate loss, etc. So, a system trade-off is required. These reduction techniques are basically divided into three types of classes such as signal distortion, multiple signaling & probabilistic and coding. In this paper, amplitude clipping & filtering based design (signal distortion) is used to reduce PAPR with a little compromise of BER. The main
objective of this paper is to investigate the comparative performance analysis of different higher order modulation techniques on one of our earlier PAPR reduction design.

2. System Model

In OFDM modulation scheme, multiple data bits are modulated simultaneously by multiple carriers. This procedure partitions the transmission frequency band into multiple narrower subbands, where each data symbol’s spectrum occupies one of these subbands. As compared to the conventional frequency division multiplexing (FDM), where such subbands are non-overlapping, OFDM increases spectral efficiency by utilizing subbands that overlap (Figure 1). To avoid interference among subbands, the subbands are made orthogonal to each other, which mean that subbands are mutually independent. By breaking the wide transmission band into narrower, multiple subbands, OFDM schemes effectively combat the effect of frequency-selective fading usually encountered in wireless channels. Frequency-selective fading is a consequence of the phenomenon called multipath propagation, where multiple copies of the transmitted signal travelling along different paths combine at the receiver. To overcome the frequency-selective fading, each subband should be narrow enough such that its bandwidth $B$ satisfies [1]

$$B < \frac{1}{2\pi \tau_{av}}$$

(1)

Where, $\tau_{av}$ is the average delay spread defined as the average value of the exponentially distributed random variable used to model the incremental delays of the multiple received rays of the transmitted signal. OFDM converts the frequency selective fading channel into multiple flat-fading subchannels, thereby allows the use of simple frequency-domain equalizers to overcome the problem.

![Figure 1. Comparison of spectral utilization efficiency between FDM and OFDM](image)

2.1. Mathematical Explanation of OFDM Signals

Consider, a data stream with rate $R$ bps where bits are mapped to some constellation points using a digital modulation (QPSK or QAM). Let, $N$ of these constellation points be stored for an interval of $T_s = N/R$, referred to as the OFDM symbol interval. A serial-to-parallel
converter is used to achieve this. Now, each one of the N constellation points is used to modulate one of the subcarriers. Then, all modulated subcarriers are transmitted simultaneously over the symbol interval $T_s$ to get the proper OFDM signal [2]. The OFDM signal $x(t)$ can be expressed as,

$$ x(t) = \sum_{k=0}^{N-1} a_k \exp(j2\pi(f_c + k\Delta f)t) $$

$$ = \exp(j2\pi f_c t) \sum_{k=0}^{N-1} a_k \exp(j2\pi k\Delta f t) $$

$$ = \exp(j2\pi f_c t) a(t) \quad (1) $$

Where, $a_k, \ 0 \leq k \leq N - 1$, are complex-valued constellation points representing data and $f_k = f_c + k\Delta f, \ 0 \leq k \leq N - 1$, is the $k^{th}$ subcarrier, with $f_c$ being the lowest subcarrier frequency. $\Delta f$ is the frequency spacing between adjacent subcarriers, chosen to be $1/T_s$ to ensure that the subscribers are orthogonal. However, OFDM output symbols typically have large dynamic envelope range due to the superposition process performed at the IFFT stage in the transmitter.

3. Synopsis of PAPR

PAPR is extensively used to evaluate this variation of the output envelope. It is also an important factor in the design of both high power amplifier (PA) and digital-to-analog (D/A) converter, for generating error-free (minimum errors) transmitted OFDM symbols. As, there are large number of independently modulated subcarriers are existed in an OFDM system, the peak value of the system can be very large as compared to the average value of the whole system. Coherent addition of $N$ signals of same phase produces a large peak which is $N$ times of the average signal. So, the ratio of peak power to average power is known as PAPR.

$$ PAPR = \frac{\text{Peak Power}}{\text{Average Power}} $$

The PAPR of the transmitted signal is defined as [4],

$$ PAPR = \frac{\max_{0 \leq t \leq NT} |x(t)|^2}{\frac{1}{NT} \int_0^{NT} |x(t)|^2 dt} \quad (2) $$

4. Amplitude Clipping & Filtering

Amplitude Clipping and Filtering is one of the easiest techniques which may be under taken for PAPR reduction for an OFDM system. A threshold value of the amplitude is fixed in this case to limit the peak envelope of the input signal [5].
Figure 2. Clipping Function

The clipping ratio (CR) is defined as,

$$CR = \frac{A}{\sigma}$$

(3)

Where, $A$ is the amplitude and $\sigma$ is the root mean squared value of the unclipped OFDM signal. The clipping function is performed in digital time domain, before the D/A conversion and the process is described by the following expression,

$$x_k^c = \begin{cases} 
  x_k & |x_k| \leq A \\
  A\text{e}^{j\phi(x_k)} & |x_k| > A 
\end{cases} \quad 0 \leq k \leq N - 1$$

(4)

Where, $x_k^c$ is the clipped signal, $x_k$ is the transmitted signal, $A$ is the amplitude and $\phi(x_k)$ is the phase of the transmitted signal, $x_k$.

4.1. Limitations of Conventional Amplitude Clipping & Filtering

- Clipping causes in-band signal distortion, resulting in BER performance degradation [6].
- Clipping also causes out-of-band radiation, which imposes out-of-band interference signals to adjacent channels. Although the out-of-band signals caused by clipping can be reduced by filtering, it may affect high-frequency components of in-band signal (aliasing) when the clipping is performed with the Nyquist sampling rate in the discrete-time domain. However, if clipping is performed for the sufficiently-oversampled OFDM signals (e.g., $L \geq 4$) in the discrete-time domain before a low-pass filter (LPF) and the signal passes through a band-pass filter (BPF), the BER performance will be less degraded [6].
- Filtering the clipped signal can reduce out-of-band radiation at the cost of peak regrowth. The signal after filtering operation may exceed the clipping level specified for the clipping operation [7].

5. Proposed Clipping & Filtering Method

As the main focus is to reduce PAPR, so, in this simulation, we have trade-off between PAPR reduction with BER increment. Very less amount of BER increment is desirable. Pointing out the third limitation in Section 4.1, shows that if clipped signal passes through a Composed filter (FIR based BPF) before passing a LPF to reduce out-of-band radiation, then it causes less BER degradation with medium amount of PAPR reduction. Considering this
concept, we previously designed a scheme for clipping & filtering method where clipped signal passes through a Composed filter (IIR based BPF) before passing a LPF, then it causes a little bit more BER degradation but more amount of PAPR reduction than existing method [8]. The operation was based only QPSK & QAM modulation. Now, we want to compare the performance for different modulation techniques. The proposed method is now shown in the Figure 3. It shows a block diagram of a PAPR reduction scheme using clipping and filtering, where \( L \) is the oversampling factor and \( N \) is the number of subcarriers. The input of the IFFT block is the interpolated signal introducing \( N(L - 1) \) zeros in the middle of the original signal is expressed as,

\[
X'[k] = \begin{cases} 
X[k], & \text{for } 0 \leq k \leq \frac{N}{2} \text{ and } \frac{N}{2} < k < NL \\
0 & \text{Elsewhere}
\end{cases}
\]  

(5)

In this system, the \( L \)-times oversampled discrete-time signal is generated as,

\[
x'[m] = \frac{1}{\sqrt{LN}} \sum_{k=0}^{LN-1} X'[k] \exp \left( \frac{j2\pi n\Delta f k}{LN} \right), \quad m = 0,1,...,NL-1 
\]  

(6)

and is then modulated with carrier frequency \( f_c \) to yield a passband signal \( x''[m] \).

Figure 3. Block Diagram of Proposed Clipping & Filtering Scheme

Let, \( x^p_c[m] \) denote the clipped version of \( x^p[m] \) which is expressed as,

\[
x^p_c[m] = \begin{cases} 
-A & x^p[m] \leq -A \\
x^p[m] & |x^p[m]| < A \\
A & x^p[m] \geq A 
\end{cases}
\]  

(7)

Where, \( A \) is the pre-specified clipping level. After clipping, the signals are passed through the proposed filter (Composed Filter). The filter itself consists on a set of FFT-IFFT operations where filtering takes place in frequency domain after the FFT function. The FFT function transforms the clipped signal \( x^p_c[m] \) to frequency domain yielding \( X^p_c[k] \). The information components of \( X^p_c[k] \) are passed to a IIR based band pass filter (BPF) producing \( \tilde{X}^p_c[k] \).
This filtered signal is passed to the unchanged condition of IFFT block and the out-of-band radiation that fell in the zeros is set back to zero. The IFFT block of the filter transforms the signal to time domain and thus obtain $\tilde{z}^P_c[m]$.

6. Design and Simulation Parameters

In our previous research work, an IIR digital filter (Chebyshev Type I) is used in the composed filtering [8]. Chebyshev Type I filter is equiripple in the passband and monotonic in the stopband. Type I filter rolls off faster than type II filters. Chebyshev filter has the property that it minimizes the error between the idealized & the actual filter characteristic over the range of the filter. Because of the passband equiripple behaviour inherent in Chebyshev Type I filter, it has a smoother response. Using the special type of bandpass filter in the composed filter, significant improvement is observed in the case of PAPR reduction. The observations were actually based on only QPSK & QAM. In this simulation, using this filter, the effects of other higher order modulation techniques (8-PSK, 16-PSK, 32-PSK, 8-QAM, 16-QAM & 32-QAM) will be analyzed. Table 1 shows the values of parameters used in the different modulation systems for analyzing the performance of clipping and filtering technique.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth (BW)</td>
<td>1 MHz</td>
</tr>
<tr>
<td>Over sampling factor (L)</td>
<td>8</td>
</tr>
<tr>
<td>Sampling frequency, $f_s$</td>
<td>$8 MHz$</td>
</tr>
<tr>
<td>Carrier frequency, $f_c$</td>
<td>$2 MHz$</td>
</tr>
<tr>
<td>No. of Subscribers (N)</td>
<td>128</td>
</tr>
<tr>
<td>CP / GI size</td>
<td>32</td>
</tr>
<tr>
<td>Clipping Ratio (CR)</td>
<td>0.8, 1.0, 1.2, 1.4, 1.6</td>
</tr>
<tr>
<td>Modulation Format</td>
<td>QPSK, 8-PSK, 16-PSK, 32-PSK, QAM, 8-QAM, 16-QAM &amp; 32-QAM</td>
</tr>
</tbody>
</table>

6.1. Simulation Results for PAPR Reduction

In this first section, simulation is performed on our design for different higher order modulation techniques and analyzed their performances in case of reducing PAPR. Here, we want to monitor the effect of same number of symbol order (both for QPSK & QAM) step by step. It was analyzed QPSK with QAM previously. Now, other comparative analysis will be discussed in the next section.

6.1.1. Simulation Results: In this section, PAPR distributions for different CR values are shown in the following figures. Clipped & filtered signal are shown in red colors.
Figure 4. PAPR distribution for CR=0.8, 1.0, 1.2, 1.4, 1.6; (a) QPSK and N=128; (b) QAM and N=128; (c) 8-PSK and N=128; (d) 8-QAM and N=128; (e) 16-PSK and N=128; (f) 16-QAM and N=128; (g) 32-PSK and N=128; (h) 32-QAM and N=128

In Table 2, PAPR distribution for the above mentioned data are tabulated. The differences between same order modulations are also shown.

Table 2. PAPR Characteristics comparison of same symbol order modulation

<table>
<thead>
<tr>
<th>CR value</th>
<th>QPSK (dB)</th>
<th>QAM (dB)</th>
<th>Difference between QPSK &amp; QAM (dB)</th>
<th>8-PSK (dB)</th>
<th>8-QAM (dB)</th>
<th>Difference between 8-PSK &amp; 8-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>4.21</td>
<td>4.21</td>
<td>0</td>
<td>4.171</td>
<td>4.184</td>
<td>-0.013</td>
</tr>
<tr>
<td>1.0</td>
<td>4.67</td>
<td>4.65</td>
<td>0.02</td>
<td>4.656</td>
<td>4.673</td>
<td>-0.017</td>
</tr>
<tr>
<td>1.2</td>
<td>5.21</td>
<td>5.11</td>
<td>0.1</td>
<td>5.173</td>
<td>5.211</td>
<td>-0.038</td>
</tr>
<tr>
<td>1.4</td>
<td>5.72</td>
<td>5.71</td>
<td>0.01</td>
<td>5.677</td>
<td>5.72</td>
<td>-0.043</td>
</tr>
<tr>
<td>1.6</td>
<td>6.29</td>
<td>6.27</td>
<td>0.02</td>
<td>6.228</td>
<td>6.241</td>
<td>-0.013</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CR value</th>
<th>16-PSK (dB)</th>
<th>16-QAM (dB)</th>
<th>Difference between 16-PSK &amp; 16-QAM (dB)</th>
<th>32-PSK (dB)</th>
<th>32-QAM (dB)</th>
<th>Difference between 32-PSK &amp; 32-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>4.217</td>
<td>4.192</td>
<td>0.025</td>
<td>4.209</td>
<td>4.185</td>
<td>0.024</td>
</tr>
<tr>
<td>1.0</td>
<td>4.682</td>
<td>4.658</td>
<td>0.024</td>
<td>4.669</td>
<td>4.659</td>
<td>0.01</td>
</tr>
<tr>
<td>1.2</td>
<td>5.163</td>
<td>5.224</td>
<td>-0.061</td>
<td>5.151</td>
<td>5.198</td>
<td>-0.047</td>
</tr>
</tbody>
</table>
6.1.2. Performance Analysis: To begin with the analysis, for the same number of subscribers (N=128) & low CR=0.8, it is observed that there is no difference between using QAM & QPSK. But, with the increasing value of CR (1.0, 1.2, 1.4, 1.6), QAM provides less amount of PAPR than QPSK. So, for high CR (less amount of clipping), QAM is better suited than QPSK for this proposed design.

Secondly, it is found that for the symbol order eight (8), 8-PSK shows the less PAPR than 8-QAM for all values of CR. So, whatever the amount of clipping (more or less), 8-PSK provides the better results.

Thirdly, it is examined that for the symbol order sixteen (16), 16-QAM shows the less PAPR than 16-PSK up to the CR value (1.0). But, at the higher CR values (1.2,1.4,1.6), QPSK gives the less amount of PAPR. The same results are also monitored for the higher symbol order (32).

So, analyzing the simulated results by this designing method, it is clearly observed that in case of symbol order M=16 & M=32, M-QAM provides better PAPR reduction than M-PSK for low CR (more amount of clipping). On the other hand, M-PSK provides less PAPR than M-QAM for high CR (less amount of clipping).

6.2. Simulation Results for BER Performance

The clipped & filtered signal is passed through the AWGN channel and BER are measured for different modulation techniques. It is shown from these figures that the BER performance becomes worse as the CR decreases. That means, for low value of CR, (More amount of clipping), the BER is more.

6.2.1. Simulation Results: In this section, BER Performance for different CR values is shown in the following figures.
Figure 4. BER Performance for CR=0.8, 1.0, 1.2, 1.4, 1.6; (a) QPSK and N=128; (b) QAM and N=128; (c) 8-PSK and N=128; (d) 8-QAM and N=128; (e) 16-PSK and N=128; (f) 16-QAM and N=128; (g) 32-PSK and N=128; (h) 32-QAM and N=128
Table 3. BER Performance comparison of same symbol order modulation

<table>
<thead>
<tr>
<th>CR value</th>
<th>QPSK</th>
<th>QAM</th>
<th>Difference between QPSK &amp; QAM</th>
<th>8-PSK</th>
<th>8-QAM</th>
<th>Difference between 8-PSK &amp; 8-QAM</th>
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</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.10631</td>
<td>0.07535</td>
<td>0.03096</td>
<td>0.2498</td>
<td>0.2095</td>
<td>0.0403</td>
</tr>
<tr>
<td>1.0</td>
<td>0.09012</td>
<td>0.06098</td>
<td>0.02914</td>
<td>0.2376</td>
<td>0.1744</td>
<td>0.0632</td>
</tr>
<tr>
<td>1.2</td>
<td>0.07846</td>
<td>0.05433</td>
<td>0.02413</td>
<td>0.2223</td>
<td>0.1498</td>
<td>0.0725</td>
</tr>
<tr>
<td>1.4</td>
<td>0.06358</td>
<td>0.04631</td>
<td>0.01727</td>
<td>0.2113</td>
<td>0.1411</td>
<td>0.0702</td>
</tr>
<tr>
<td>1.6</td>
<td>0.05748</td>
<td>0.04211</td>
<td>0.01537</td>
<td>0.1955</td>
<td>0.1271</td>
<td>0.0684</td>
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<tbody>
<tr>
<td>0.8</td>
<td>0.3239</td>
<td>0.2827</td>
<td>0.0412</td>
<td>0.369</td>
<td>0.3386</td>
<td>0.0304</td>
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<tr>
<td>1.0</td>
<td>0.3179</td>
<td>0.2488</td>
<td>0.0691</td>
<td>0.3565</td>
<td>0.3211</td>
<td>0.0354</td>
</tr>
<tr>
<td>1.2</td>
<td>0.3089</td>
<td>0.2238</td>
<td>0.0851</td>
<td>0.3511</td>
<td>0.3047</td>
<td>0.0464</td>
</tr>
<tr>
<td>1.4</td>
<td>0.3031</td>
<td>0.2047</td>
<td>0.0984</td>
<td>0.3424</td>
<td>0.2924</td>
<td>0.0500</td>
</tr>
<tr>
<td>1.6</td>
<td>0.2981</td>
<td>0.1915</td>
<td>0.1066</td>
<td>0.339</td>
<td>0.2754</td>
<td>0.0636</td>
</tr>
</tbody>
</table>

6.2.2. Performance Analysis: It is observed from the table 3 that, for all CR values and all types of modulation orders, M-PSK results more BER than M-QAM. As stated earlier, that for low CR means more amount of clipping that consequences more amount of BER, so, it is also monitored that for all cases of modulation.

We have done another performance analysis between different M-PSK and M-QAM in case of PAPR reduction as well as BER increase. It is analyzed that 8-PSK compare with QPSK, results average 4% improvement for PAPR reduction while increasing average 14% BER. 16-PSK provides average 5% PAPR improvement with the increase of 22% BER compare to QPSK. Additionally, 32-PSK provides average 6% PAPR improvement with the increase of 27% BER compare to QPSK. It is also observed that 16-PSK provides almost 1% PAPR reduction with almost 8% increase of BER compare to 8-PSK. Finally, it is found that, 32-PSK shows average 2% PAPR reduction improvement with 4% BER increase compare to 16-PSK.

However, for the M-QAM, we have seen that 8-QAM provides 2% PAPR improvement with average 9% BER increment, 16-QAM provides 2% PAPR improvement with average 17% BER increment, 32-QAM results 1% PAPR improvement with average 23% BER compare to QAM. In addition, 16-QAM shows 7% BER increment without considerable improvement of PAPR reduction compare to 8-QAM. Finally, it is observed that, 32-QAM results average 1% PAPR improvement while average 8% BER increment compare to 16-QAM.
In this paper, the effect of using higher order modulation on a particular PAPR reduction method is analyzed thoroughly.

References


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Mr. Mowla is now working as an Assistant Professor in Electronics & Telecommunication Engineering department of Rajshahi University Engineering & Technology (RUET) since November 2010. He has completed M.Sc Engineering degree in Electrical & Electronic Engineering (EEE) from RUET in May 2013. He has four years telecom job experience in the operators, vendors, ICX etc of Bangladesh telecom market. Mr. Mowla has published several international journals as well as conference papers and three books. He is a member of communication society COMSOC of IEEE, Institutions of Engineers, Bangladesh (IEB) and Bangladesh Electronics Society (BES). His research interest includes advanced wireless communication including LTE, LTE-Advanced, green communication, smart grid communication.