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Comparison of Properties of Transversely Isotropic Lamina Using Method of Cells and Composite Cylinder Assemblage

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Abstract

Composite structures are finding increasing applications because of their high specific stiffness and strength. The behavior characterization of composites under different loading conditions necessitates thermo-mechanical properties of individual layers.

The present work deals with determination of elastic and thermo-mechanical properties of transversely isotropic lamina using Method of Cells and comparison of the results withComposite Cylinder Assemblages .The prerequisite properties of the fibers and matrix used are referred from standard data bank to determine the relevant properties of lamina. Specially orthotropic 3D composites are characterized by nine elastic properties, these properties are determined using Method of Cells, Composite Cylinder Assemblages.

It has been observed that the Method of Cells is useful for determination of the compressive strength of lamina. The results obtained using micromechanical models of Method of Cells are found upper bound for transverse properties of lamina.

Keywords: *Composite structures, Transversely isotropic lamina, Method of Cells, Micromechanics, Elastic properties*

1. Introduction

Today composite is used in various fields, with high end applications as aircraft, wind turbines, *etc.* These components are subjected to cyclic loading resulting in sudden failure, which may cause heavy loss. The properties of very limited composite materials are readily available. The properties of lamina such as elastic properties, thermal properties, and strengths are required to find out laminate properties. To get all these properties a data bank must be available, which contains the properties of fibers, matrix, and laminas. These properties of lamina can be calculated by micromechanics approach easily.
Objective of present work is to study the behavior of composite materials. This investigation deals with lamina composed of polymer matrix and carbon fibers. The aim of this study is to determine following properties.

a) Elastic properties, thermal properties and strength properties of transversely isotropic lamina by all methods of Micromechanics.

b) Verifying the results predicted byMethod of Cellswith the other micromechanics methods like Composite Cylinder Assemblages (CCA) method , Rule of Mixture by Jones [1] & Daniel [2] , Hashin [3], Chamis method [4, 5] and Zing-ming Huang method [6, 11].

Scope of this work is limited to evaluation of properties of unidirectional lamina.

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The word composite signifies that two or more materials are combined on a macroscopic scale to form a useful third material, to get better strength, stiffness, fatigue life, corrosion resistance, weight, thermal conductivity, thermal insulation, and many more. Composite materials are classified as

1. Fibrous composite
2. Laminated composite.
3. Particulate composite
4. Combination of all three.

A lamina or a ply is formed by combination of a large number of fibers in a thin layer of matrix. Fibers in a lamina may be continuous or discontinuous, arranged in a specific direction or in a random orientation. A unidirectional lamina is one where the fibers in a lamina run parallel to one another in a particular direction. The thickness of the lamina ranges from 0.1 – 1 mm. The standard thickness of a unidirectional ply is 0.125 mm where as typical thickness of the wooden ply is 0.25 mm.

The laminate is formed by stacking several laminas. It is made of a desired thickness so as to enable it to support a given load and maintain a given deflection. Fiber orientation of each lamina and stacking sequence of various layers can be varied to obtain a wide range of physical and mechanical properties of composites.

The fiber-reinforced composite materials have been called revolution when we started using them in jet engines. They have major advantages as improved specific strength and specific stiffness on a unit weight basis. For example composite materials can be made that have same strength and stiffness as high-strength steel, yet are 70% lighter. And directional strength can also be achieved.

Micromechanics are used to predict the various properties of orthotropic/transversely isotropic lamina which includes elastic properties, thermal properties, and strengths. The properties are predicted when unidirectional lamina is subjected to unidirectional loading.. Micromechanics deals with the deformation and stress in the basic constituents of a structure. It deals with local failures such as matrix failure, fiber failure and interface failure. As such the constituent materials are examined on a microscopic scale without recourse to their internal structure.

The laminator is engineering software package that analyzes laminated composite plates according to Classical Laminated Plate Theory. A Micromechanics Calculator is also included for estimating lamina properties.

2. Literature Survey

Micromechanics is the study of composite material behavior wherein the material is assumed homogeneous and the effects of the constituent materials are detected only as averaged apparent properties of the composite materials. Fiber reinforced composites are often selected for weight-critical structural applications because of their high specific stiffness and strength. For determining the properties of transversely isotropic lamina following methods are used.

2.1 Mechanics of Material Approach (Rule of Mixture)

The method used is Mechanics of material approach (Rule of mixture). The formulation for prediction of properties of lamina is given by Jones [1] and Daniel [2].

Mechanics of material approach have additional assumption that the strain in the fiber direction of a unidirectional fiber-reinforced composite material are the same in the fiber as in the matrix as shown in Figure 2.1.

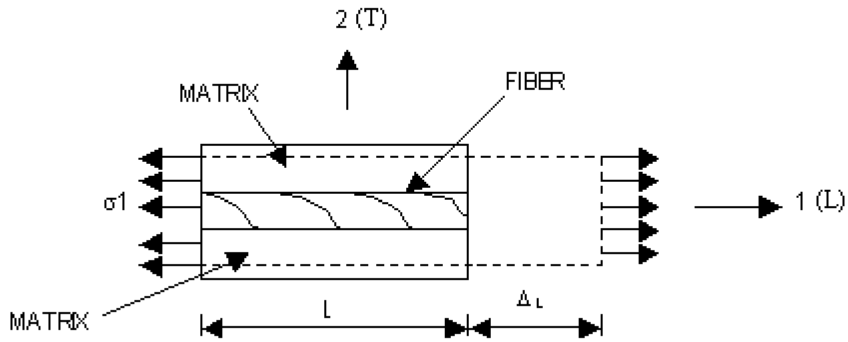

Figure 2.1. Representative Volume Element Loaded in Direction 1

However, deficiencies in micromechanical prediction of properties by mechanics of material approach (rule of mixture) are

1. It considers fiber and matrix as isotropic materials
2. It predicts lower bound transverse properties
3. It predicts only four elastic properties,

The relevant elastic properties are obtained as given below

a) Young's modulus of elasticity (E_l)

The first modulus of the composite material is determined in the 1-direction i.e. in the fiber direction as shown in Figure 2.1, when subjected to loading along fiber direction.

$$E_l = E_f V_f + E_m V_m \quad (2.1.1)$$

b) Young's modulus of elasticity (E_t)

The apparent Young's modulus of the composite material in the direction transverse to the fiber, with assumption the same transverse stress is assumed to be applied to both the fiber and matrix.

$$E_t = \frac{E_f E_m}{V_m E_f + V_f E_m} \quad (2.1.2)$$

c) Poisson's Ratio in L-T plane (v_{lt})

The major Poisson's ratio is obtained by the same approach that used in analysis of E_1

$$v_{lt} = v_f V_f + v_m V_m \quad (2.1.3)$$

d) Shear modulus in L-T plane (G_{lt})

The in plane shear modulus is determined as

$$G_{lt} = \frac{G_m G_f}{V_m G_f + V_f G_m} \quad (2.1.4)$$

2.2 Halpin-Tsai Method

This is an interpolation method which is an approximate representation of more complicated micromechanics results. The relevant elastic properties are obtained as given below are presented by Jones [1], the expressions for axial Young;s modulus (*E1*) and axial Poisson's ratio (v_{lt}) are generally accepted results of rule of mixture. The Halpin-Tsai equations are equally applicable to fiber, ribbon, or particulate composite:

a) Young's modulus of elasticity (E_l)

The first modulus of the composite material is determined in the 1-direction i.e. in the fiber direction when subjected to loading along fiber direction.

$$E_l = E_f V_f + E_m V_m \quad (2.2.1)$$

Young's modulus of elasticity (E_t)

The apparent Young's modulus of the composite material in the direction transverse to the fiber, with assumption the same transverse stress is assumed to be applied to both the fiber and matrix.

$$E_t = \frac{(1+ \xi \times \eta \times V_f) \times E_m}{1 - \eta \times V_f} \quad (2.2.2)$$

b) Poisson's Ratio in L-T plane (v_{lt})

The major Poisson's ratio is obtained by the same approach that used in analysis of E_1

$$v_{lt} = v_f V_f + v_m V_m \quad (2.2.3)$$

a) Shear modulus in L-T plane (G_{lt})

The in plane shear modulus is determined as

$$G_{lt} = \frac{(1 + \xi \times \eta_2 \times V_f) \times G_m}{1 - \eta \times V_f} \quad (2.2.4)$$

2.3 Composite Cylinder Assemblages (CCA)

It is one of the micromechanics based methods given by Hashin [3]. Here a certain semi-random fiber reinforced material (FRM) is considered. In this method each circular cylinder is assumed to be made of a circular cylindrical fiber and concentric matrix shell. This unit is assembled to construct composite.

a) Matrix volume fraction

$$V_m = 1 - V_f \tag{2.3.1}$$

b) Matrix bulk modulus

$$K_{bm} = \frac{E_m}{3\times(1-2\nu_m)} \tag{2.3.2}$$

c) Fiber transverse bulk modulus

$$K_{tbf} = \frac{E_{tf}}{3(1 - 2\nu_{ttf})} \tag{2.3.3}$$

d) Matrix shear modulus

$$G_m = \frac{E_m}{2(1 + \nu_m)} \tag{2.3.4}$$

e) Axial Poisson's ratio

The major Poisson's ratio is obtained by the same approach that used in analysis of E_1

f) $$\nu_{12} = \nu_m V_m + \nu_{ltf} V_f + \frac{\left(\nu_{ltf} - \nu_m\right)\left(\frac{1}{K_{bm}} - \frac{1}{K_{tbf}}\right) V_m V_f}{\left(\frac{V_m}{K_{tbf}} + \frac{V_f}{K_{bm}} + \frac{1}{G_m}\right)} \tag{2.3.5}$$

g) Axial shear modulus of lamina

The in plane shear modulus is determined as

$$G_{12} = G_m \frac{G_m V_m + G_{ltf}(1 + V_f)}{G_m(1 + V_f) + G_{ltf} V_m} \tag{2.3.6}$$

2.4 Chamis Method

Chamis [4, 5] method is used to calculate lamina elastic properties. Fiber properties, matrix properties and fiber volume fraction are input to this method. The elastic properties can be found out as:

a). Calculationaxial Young's modulus of Lamina

$$E_1 = E_{lf} V_f + E_m V_m \tag{2.4.1}$$

b). Calculation of Poisson's ratio of Lamina in one two plane

$$\nu_{12} = \nu_{ltf} V_f + \nu_m V_m \tag{2.4.2}$$

c). Calculation of transverse Young's modulus of lamina

$$E_2 = \frac{E_m}{1 - \sqrt{V_f}\left(1 - {E_m}/{E_{tf}}\right)} \tag{2.4.3}$$

d). Calculation of shear modulus or modulus of rigidity of lamina in l-t plane

$$G_{12} = \frac{G_m}{1 - \sqrt{V_f}\left(1 - {G_m}/{G_{ltf}}\right)} \tag{2.4.4}$$

e). Calculation of shear modulus or modulus of rigidity of lamina in t-t plane

$$G_{23} = \frac{G_m}{1 - \sqrt{V_f}\left(1 - {G_m}/{G_{ttf}}\right)} \tag{2.4.5}$$

f). Calculation of longitudinal tensile strength of lamina

$$X_t = V_f \ X_{lf} \tag{2.4.6}$$

g). Calculation of transverse tensile strength of lamina

$$Y_t = \left[1 - \left(\sqrt{V_f} - V_f\right)\left(1 - {E_m}/{E_{tf}}\right)\right] X_{tm} \tag{2.4.7}$$

h)Calculation of longitudinal compressive strength of lamina

$$X_c = V_f \ X_{cf} \tag{2.4.8}$$

i). Calculation of transverse compressive strength

$$Y_c = \left[1 - \left(\sqrt{V_f} - V_f\right)\left(1 - {E_m}/{E_{tf}}\right)\right] X_{cm} \tag{2.4.9}$$

j). Calculation of axial shear stress in one two plane

$$S_{12} = \left[1 - \left(\sqrt{V_f} - V_f\right)\left(1 - {G_m}/{G_{ltf}}\right)\right] S_m \tag{2.4.10}$$

k) Calculation of coefficient of thermal expansion of lamina in direction one

$$\alpha_1 = \frac{V_f \alpha_{lf} E_{lf} + V_m \alpha_m E_m}{E_1} \tag{2.4.11}$$

l). Calculation of coefficient of thermal expansion of lamina in direction two

$$\alpha_2 = \alpha_{tf} V_f + \alpha_m V_m \left(1 + \frac{V_f \nu_m E_{lf}}{E_1}\right) \tag{2.4.12}$$

2.5 Zheng-ming Huang Method

Zheng-ming Huang is a method which is used to calculate elastic properties of transversely isotropic lamina. Fiber properties, matrix properties and fiber volume fraction are input to this method. The elastic properties can be found out as given by Zheng-ming Huang [6, 11]:

1) Young's modulus of lamina in direction 1

$$E_l = V_f E_{lf} + V_m E_m \tag{2.5.1}$$

2) Poisson's ratio of lamina in 1-2 plane

$$\nu_{lt} = V_f \nu_{ltf} + V_m \nu_m \tag{2.5.2}$$

3) Young's modulus of lamina in direction - 2

$$E_2 = \frac{\left(V_f + V_m a_{11}\right)\left(V_f + V_m a_{22}\right)}{\left(V_f + V_m a_{11}\right)\left(V_f S_{22}^f + a_{22} V_m S_{22}^m\right) + V_f V_m \left(S_{21}^m - S_{21}^f\right) a_{12}} \tag{2.5.3}$$

4) Shear modulus of lamina in 1-2 plane

$$G_{lt} = G_m \times \frac{\left(G_{ltf} + G_m\right) + V_f \left(G_{ltf} - G_m\right)}{\left(G_{ltf} + G_m\right) - V_f \left(G_{ltf} - G_m\right)} \tag{2.5.4}$$

5) Shear modulus of lamina in 2-3 plane

$$G_{tt} = \frac{0.5\left(V_f + V_m a_{22}\right)}{V_f \left(S_{22}^f - S_{23}^f\right) + V_m a_{22} \left(S_{22}^m - S_{23}^m\right)} \tag{2.5.5}$$

2.6 Method of Cells (MOC)

The major contribution to the Method of Cells is by Aboudi [7, 8]. He has developed this theory to predict properties of lamina. The prediction of ultimate stresses of unidirectional fiber composites under complex loading system using micromechanics approach has been presented [9]. Aboudi and Pindera [10] extended this method to generate initial yield surfaces unidirectional and cross-ply metal matrix composites.

2.6.1 Determination of properties of Lamina

To determine the properties of lamina by method of cells a representative cell considered as shown in Figure 2.2. As a result of this periodic arrangement, it is sufficient to analyze a representative cell. The representative cell contains four sub cells, one hatched sub cell represents fiber and remaining three are matrix. The positions of which is defined by $\beta, \gamma = 1, 2.$ Let four local coordinate systems (x_1, $\bar{x}_2^{\beta}$, $\bar{x}_3^{\gamma}$) be introduced, all of which have origins that are located at the center of each sub cell. The cross section of the square fiber is h_1^2 and h_2 represents its spacing in the matrix. Properties of lamina are derived as given below.

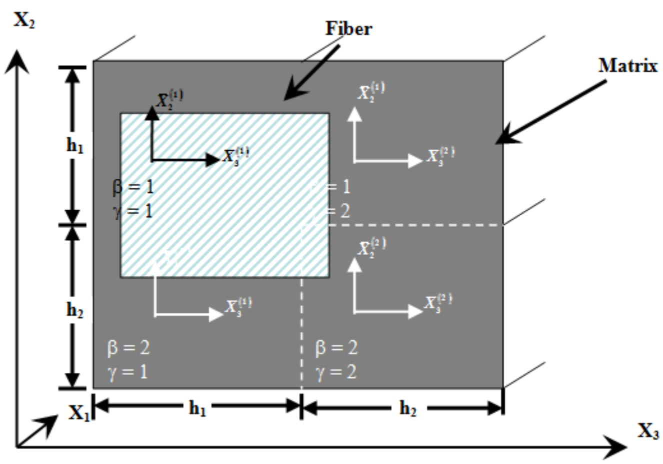

Figure 2.2. A Representative Cell with Four Sub cells of Transversely Isotropic Lamina

$$\boldsymbol{h = h_1 + h_2 \quad and \quad h_4 = h_1 + h_2}$$
$$\boldsymbol{n = 1}$$

2.6.2Elastic properties of lamina(E_1, E_2, G_{12}, v_{12}, v_{21})

The elastic properties of lamina are calculated by obtaining first order displacement expansion in each sub cell then strains are calculated by imposing the displacement interfacial conditions. From stress – strain relation stresses in each sub cells of representative cell are calculated, then effective elastic constants are calculated. From elastic constants the coefficients of stiffness matrix $E = [e_{ij}]$ are calculated. Elastic properties are calculated from this elastic stiffness matrix.

The constants used here given in appendix, so the elastic stiffness matrix B = [b_{ij}]:

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 & 0 & 0 \\ b_{12} & b_{22} & b_{23} & 0 & 0 & 0 \\ b_{13} & b_{23} & b_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{66} \end{bmatrix} \qquad (2.6.1)$$

The composite constitutive relations are of the form

$$\bar{\sigma} = B \,\bar{\in} \qquad (2.6.2)$$

This representation effectively provides an orthotropic material with a square symmetry.

a) The effective elastic constants

The effective elastic constants of transverse isotropic material can be determined from

$$E = \frac{1}{\pi}\int_0^{\pi} B'(\xi)d\xi. \quad (2.6.3)$$

Where $B' = \left[b'_{ij}\right]$ obtain by transformation i.e. by rotating the x_1, x_2, x_3 coordinates around x_1-axis by an angle 'ξ'.

Which provides the elastic stiffness matrix E = [e_{ij}] in the form

$$E = \begin{bmatrix} e_{11} & e_{12} & e_{13} & 0 & 0 & 0 \\ e_{12} & e_{22} & e_{23} & 0 & 0 & 0 \\ e_{13} & e_{23} & e_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & e_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{66} \end{bmatrix} \quad (2.6.4)$$

$$e_{11} = b_{11} \quad (2.6.5)$$

$$e_{12} = e_{13} = b_{12} \quad (2.6.6)$$

$$e_{22} = e_{33} = (3/4)b_{22} + (1/4)b_{23} + (1/2)b_{66} \quad (2.6.7)$$

$$e_{23} = (1/4)b_{22} + (3/4)b_{23} - (1/2)b_{66} \quad (2.6.8)$$

$$e_{44} = e_{55} = b_{44} \quad (2.6.9)$$

$$e_{66} = (1/2)(e_{22} - e_{23}) \quad (2.6.10)$$

Coefficients of compliance matrix

Coefficients of compliance matrix are calculated S_{ij} (when n = 1) are as given below for transversely isotropic lamina

$$S_{ij} = \left(e_{ij}\right)^{-1}$$

The elastic properties of transversely isotropic lamina

$$1)\ E_l = e_{11} - \left(2e_{12}^2/\left(e_{22} + e_{23}\right)\right)$$

$$2)\ E_t = \left[e_{11}\left(e_{22}+e_{23}\right)-2e_{12}^2\right]\left(e_{22}-e_{23}\right)\Big/\left(e_{11}e_{22}-e_{12}^2\right)$$

$$3)\ v_{lt} = e_{12}\Big/\left(e_{22}+e_{23}\right)$$

$$4)\ v_{tt} = \left(e_{11}e_{23}-e_{12}^2\right)\Big/\left(e_{11}e_{22}-e_{12}^2\right)$$

$$5)\ G_{lt} = e_{44} \qquad (2.6.11)$$

b) Coefficient of thermal expansion (α_i)

The effective coefficient of thermal expansion of a unidirectional composite in the axial and transverse directions can be readily obtained given by Aboudi [1]

c) Thermal conductivity of lamina

a) Free expansion due to temperature difference is calculated as shown

$$\Delta\theta^{(\beta\gamma)} = \Delta T + \bar{x}_2^{(\beta)}\xi_2^{(\beta\gamma)} + \bar{x}_3^{(\gamma)}\xi_3^{(\beta\gamma)} \qquad (2.6.12)$$

b) The average heat flux in the composite is given asthe average heat flux is calculated by using Fourier law for anisotropic material

$$\bar{q}_i = \frac{1}{A'}\sum_{i=1}^{n} a'_{\beta\gamma}\bar{q}_i^{(\beta\gamma)} \qquad (2.6.12)$$

$$q_i = -k_{ij}\left(\partial t\big/\partial x_j\right) \qquad (2.6.14)$$

Wher $\boldsymbol{k_{ij}}$ is thermal conductivity tensor

The continuity condition of the heat flux at the interface is given as

$$\bar{q}_i^{(1\gamma)} = \bar{q}_i^{(2\gamma)} \qquad (2.6.15)$$

Where $\boldsymbol{i = 2, 3}$ and by eliminating the micro-variables

$$\bar{q}_j^{(\beta\gamma)} = -k_i^{(\beta\gamma)}\,\partial T\big/\partial x_j \quad i = l, t\ \&\ j = 1, 2, 3 \qquad (2.7.16)$$

3. Results and Discussion

3.1 Analytical Methods for Prediction of Properties of Transversely Isotropic Lamina

The properties of transversely isotropic lamina calculated by varying fiber volume fraction from 0.1% to 0.9% are predicted by different methods of micromechanics by a program. In the present work Polymer matrix and carbon fibers are used as input.. Then by varying the volume fraction of fiber, different properties of lamina are calculated by different methods and compared with experimental results and results by software package Laminator.

3.2 Comparison between analytical and experimental results

The analytical and experimental results are compared graphically as shown in Figures below. The properties for which experimental data was not available are compared with results by software package Laminator. From the results obtained by analytical work and experimental results it is concluded that results of analytical method gives excellent agreement with experimental results. The values of axial Young's modulus, transverse Young's modulus, Poisson's ratio, coefficient of thermal expansion, and strength of lamina are close to the experimental results.

Comparison of all methods for axial Young's modulus with experimental results is shown in Figure 3.1. Comparison of all methods for transverse Young's modulus with experimental results is shown in Figure 3.2. Comparison of all methods for axial Poisson's ratio with results obtained by software Laminator is shown in Figure 3.3. Values of axial shear are plotted against fiber volume fractions for all methods and compared with experimental results as shown in Figure 3.4. Comparison of all methods for transverse shear modulus with experimental results is shown in Figure 3.5. Comparison of all methods for axial strength with results obtained by software laminator is shown in Figure 3.6.

Values of axial coefficient of thermal expansion, transverse coefficient of thermal expansion and axial compressive strength are plotted against fiber volume fraction by CCA method and MOC and compared with results obtained by software package laminator and it is shown in Figure 3.7.

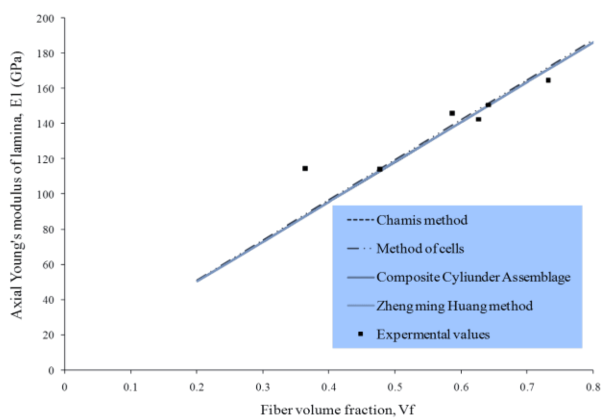

Figure 3.1. Axial Youngs modulus Vs Fiber Volume Fraction

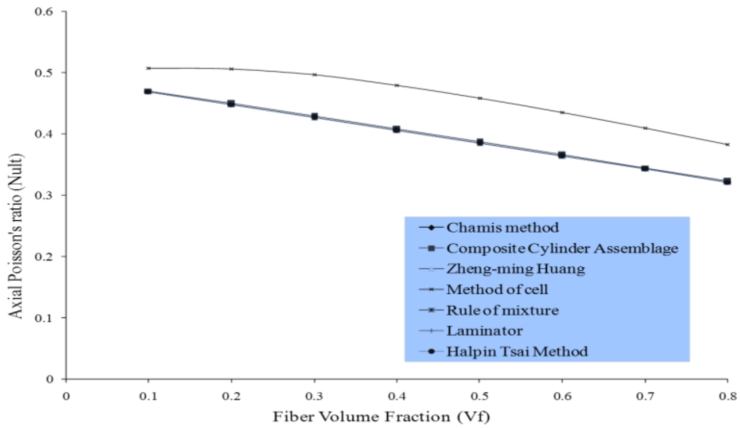

Figure 3.2. Modulus of elasticity (Transverse) Vs fiber volume fraction

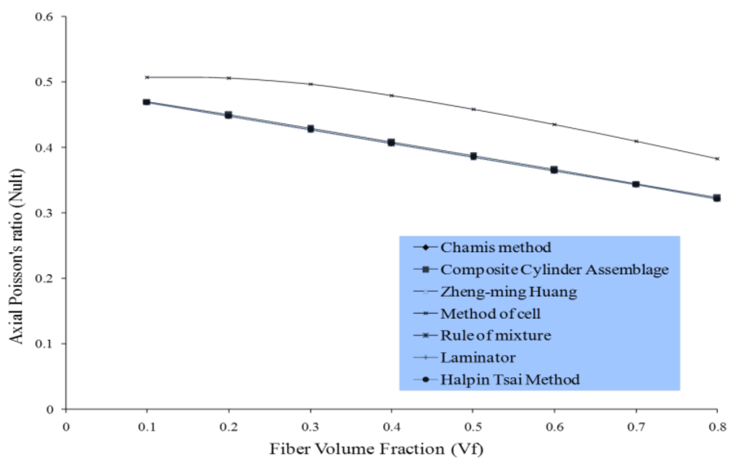

Figure 3.3. Poisson's ratio (Longitudinal) Vs fiber volume fraction

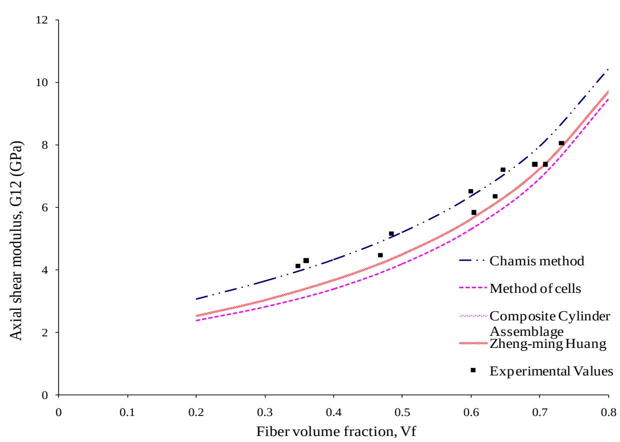

Figure 3.4. Shear Modulus of lamina Vs Fiber volume Fraction

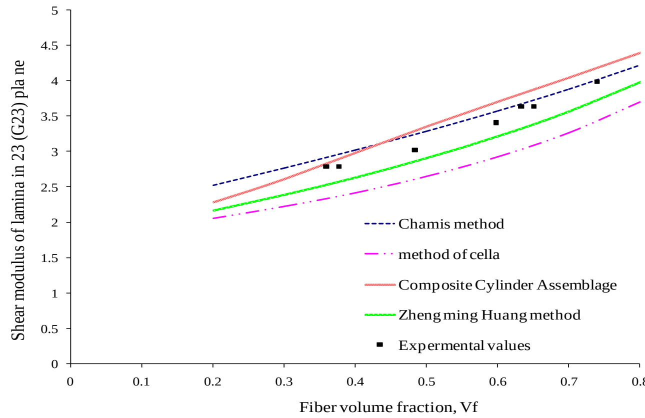

Figure 3.5. Shear Modulus of lamina Vs Fiber volume Fraction

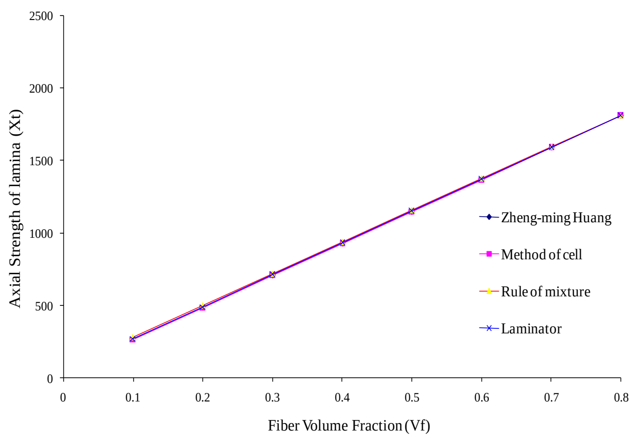

Figure 3.6. Axial strength Vs fiber volume fraction

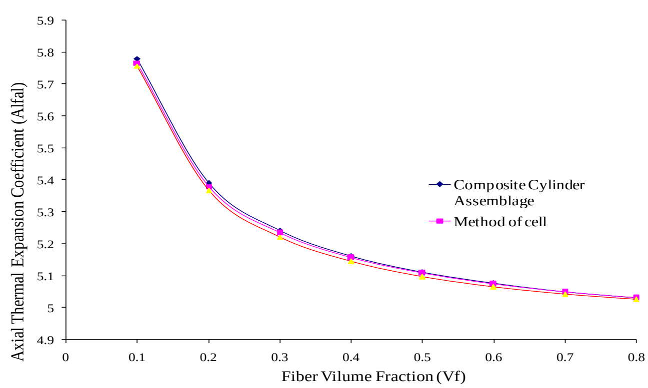

Figure 3.7. Coefficient of Thermal expansion (Axial) Vs fiber volume Fraction

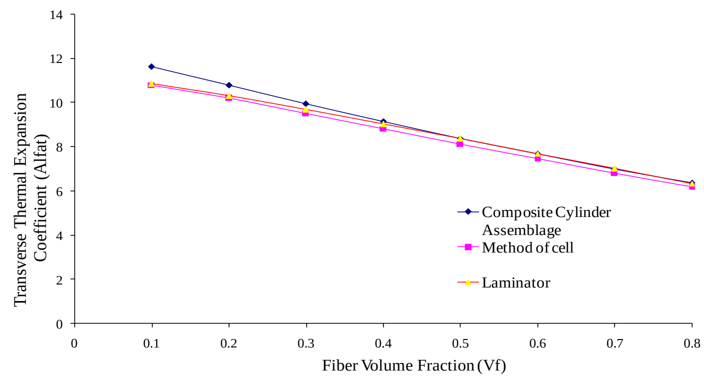

Figure 3.8. Coefficient of Thermal expansion (Transverse) Vs fiber volume fraction

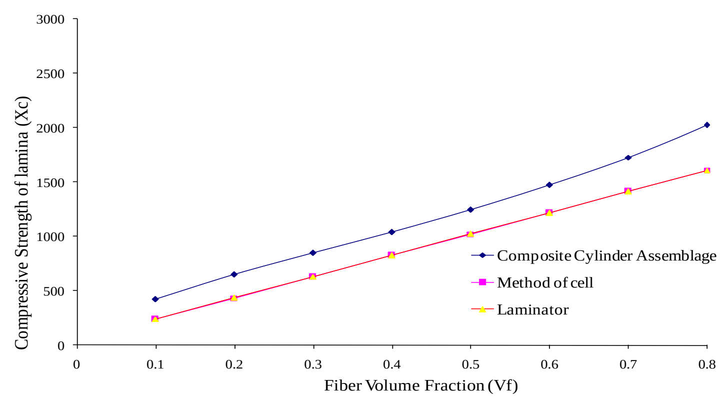

Figure 3.9. Axial Compression Strength of Lamina Vs fiber volume fraction

4. Conclusions

This paper presents a summary of present study, the major conclusions and future scope of the investigation.

1. Axial Young's modulus, transverse Young's modulus, shear modulus and strength of lamina goes on increasing as the percentage of fiber volume fraction increases.
2. Poisson's ratio and coefficient of thermal expansion goes on decreasing as percentage of fiber volume fraction increases.
3. The results obtained using micromechanical models of Method of Cells are found upper bound for transverse properties.

4. This method of cells, composite cylinder assemblage method are useful for prediction of all properties of lamina like elastic, thermal and Strength properties where as other methods cannot predict all properties.
5. Results obtained for all properties of lamina by analytical methods are in excellent agreement with experimental results and results by software package the Laminator.
6. Empirical expressions are developed to predict properties of orthotropic lamina by method of cells. Method of cells can be effectively applied for transversely isotropic as well as orthotropic lamina where as other methods used only for transversely isotropic lamina.

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