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Unsteady MHD Free Convective Visco-Elastic Fluid Flow Bounded by an Infinite Inclined Porous Plate in the Presence of Heat Source, Viscous Dissipation and Ohmic Heating

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Abstract

The present study deals with an unsteady magneto hydrodynamic free convective, Visco-elastic, dissipative fluid flow embedded in porous medium bounded by an infinite inclined porous plate in the presence of heat source, and Ohmic heating under the influence of transversely applied magnetic field of uniform strength. The equations governing the fluid flow are solved using a multiple parameter perturbation technique, subject to the relevant boundary conditions. Expressions for velocity and temperature distributions are obtained. Non dimensional skin friction coefficient and the rate of heat transfer in the form of Nusselt number are also derived and illustrated using graphs and tables. The effects of various physical parameters on the above flow quantities are discussed.

Keywords: *MHD, Unsteady, free convection, Visco-elastic fluid, Ohmic heating, viscous dissipation, heat source, Sink and Porous inclined plate*

1. Introduction

The study of MHD visco-elastic fluid flows with heat transfer past a porous plate is attracting the attention of any researchers for its wide applications in various fields like soil physics, Aerodynamics, Aeronautics, Geo physics, Nuclear power reactors and so on. Palani *et al.*, [1] studied free convection MHD flow with a thermal radiation from an impulsively-started vertical plate. Takhar and Ram [2] have studied MHD Free convection flow of water at through a porous medium. MHD free convection near a moving vertical plate in the presence of thermal radiation is studied by Das and Das [3]. Deka and Neog [4] considered unsteady MHD flow past a vertical oscillating plate with thermal Radiation and variable mass diffusion. Unsteady MHD free convection flow and mass transfer near a moving vertical plate in the presence of thermal radiation is investigated by Abzal *et al.*, [5]. Rapits and Singh [6] have analyzed MHD free convection flow past an accelerated vertical plate. Kythe and Puri [7] have considered the unsteady MHD free convection flows on a porous plate with time-dependent heating in a rotating medium. Abel and Veena [8] studied the Visco-elasticity on the flow and heat transfer in a porous medium over a stretching sheet. Sajid *et al.*, [9] have obtained analytical solution for the problem of fully developed mixed convection flow of viscoelastic fluid between two permeable parallel vertical walls. Dash *et al.*, [10] have considered free convective MHD flow of a visco-elastic fluid past an infinite vertical porous plate in rotating frame of reference in the presence of chemical reaction. Sivraj and Rushi Kumar [11] have considered MHD flow of visco-elastic fluid with short memory. Kandasamy

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et al., [12] have studied the heat and mass transfer under a chemical reaction with a heat source. Srinivas and Muthuraj [13] considered a MHD flow with slip effects and temperature-dependent heat source in a vertical wavy porous space. The effects of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet is studied by Sharma and Singh [14].Variable heat and mass transfer, radiation and heat source or sink are considered by Chamka and Ahmed [15].MHD free convection and mass transfer flow through a porous medium with heat source is studied by Jha and Prasad [16]. Israle-coobey *et al.*, [17] investigated the influence of viscous dissipation and radiation on unsteady MHD free convective flow past a heated vertical plate with time dependent suction in an optically thin environment. Aydin and Kaya [18] studied MHD mixed convection of a viscous dissipating fluid about permeable vertical flat plate. Aydin and Kaya [19] have also analyzed the non-darcian forced convection flow of viscous dissipating over a flat plate embedded in a porous medium. Babu and Reddy [20] have studied the mass transfer effects on MHD mixed convective flow from a vertical surface with ohmic heating and viscous dissipation. Chien-Hsin-Chen [21] studied combined heat and mass transfer in MHD free convection from a vertical surface with ohmic heating and viscous dissipation. Hossain [22] studied the effect of ohmic heating on the MHD free convection heat transfer has been examined for a Newtonian fluid. Chen [23] considered the problem of combined heat and mass transfer of electrically conducting fluid in MHD natural convection adjacent to a vertical surface with ohmic heating. Recently Reddy *et al.*, [24] studied an unsteady free convective MHD non Newtonian flow through a porous medium bounded by an infinite inclined porous plate. Unsteady MHD free convection oscillatory couette flow through a porous medium with periodic wall temperature was considered by Raju and Varma [25]. Ravikumar *et al.*, [26-27] investigated Magnetic field and radiation effects on a double diffusive free convective flow bounded by two infinite impermeable plates in the presence of chemical reaction. Very recently Reddy *et al.*, [28] investigated Chemical reaction and radiation effects on MHD free convection flow through a porous medium bounded by a vertical surface with constant heat and mass flux. In all the above studied the combined effects of viscous dissipation along with Joule heating in the presence of Visco-elastic fluid was not considered. Motivated by the above studies, in this paper we have investigated an unsteady magneto hydrodynamic free convective, Visco-elastic, dissipative fluid flow embedded in porous medium bounded by an infinite inclined porous plate in the presence of heat source, and Ohmic heating under the influence of transversely applied magnetic field of uniform strength. The results of this study are found to be in good agreement with the results of Reddy *et al.*, [24], in all aspects.

2. Mathematical Formulation

We have considered an unsteady MHD free convection viscous incompressible fluid flow past an infinite inclined porous plate in the presence of constant suction, viscous dissipation and ohmic heating. Let x*-axis is taken in the direction of the flow along the infinite inclined plate and y*-axis is taken perpendicular to the fluid flow. Let u* be the velocity of the fluid along the x* direction. The inclined plate makes an angle Ø ϕ with the x*-axis. In the analysis of flow, the following assumptions are made:

i. All the fluid properties are constant except the density in the buoyancy force term.

ii. The influence of the density variation in terms of momentum and energy equations, and the variation of the expansion coefficient with temperature, is negligible.

iii.The Eckert number Ec and the magnetic Reynolds number Rm are small, so that the induced magnetic field can be neglected.

iv. Viscous dissipation cannot be neglected.

v. Since the magnetic Reynolds number is very small, the induced magnetic field is negligible when comparison with the applied magnetic field.

Using Boussineq's approximation with the above assumptions, the basic flow equations through porous medium are:

Equation of continuity

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -v_0 \tag{1}$$

Equation of Motion

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} + B_1 \left(\frac{\partial^3 u^*}{\partial t^* \partial y^{*2}} + \upsilon \frac{\partial^3 u^*}{\partial y^{*3}}\right) - \left(\frac{\sigma B_0^2}{\rho} + \frac{\upsilon}{K}\right) u^* + g \sin\phi \beta (T^* - T_\infty) \tag{2}$$

Equation of Energy

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k_T}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} + Q^*(T^* - T_\infty) + \frac{\upsilon}{C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 - \frac{k_0}{\rho C_p}\left(v^* \frac{\partial u^*}{\partial y^*} \frac{\partial^2 u^*}{\partial y^{*2}}\right) + \frac{\sigma B_0^2 u^{*2}}{\rho C_p} \tag{3}$$

The relevant boundary conditions are given by

$$y^* = 0; u^* = 0, v^* = -v_0, T^* = T_W + \varepsilon (T_W - T_\infty) e^{iwt}$$
$$y^* = \infty; u^* \to 0, T^* \to T_\infty$$

(4)

On introduce the following non-dimensional quantities and variables

$$y = \frac{y^* v_0}{\upsilon}, t = \frac{t^* v_0^2}{4\upsilon}, \omega = \frac{4 v \omega^*}{v_0^2}, u = \frac{u^*}{v_0}, \upsilon = \frac{\mu}{\rho}, \Pr = \frac{\upsilon}{k^*}, Q = \frac{4 Q^* \upsilon}{V_0^2}, Ec = \frac{v_0^2}{C_p (T_W - T_\infty)},$$
$$k^* = \frac{k_T}{\rho C_p}, T = \frac{T^* - T_\infty}{T_W - T_\infty}, k = \frac{K v_0^2}{\upsilon^2}, Gr = \frac{\upsilon g \beta (T_W - T_\infty)}{v_0^3}, M = \frac{\sigma B_0^2 \upsilon}{\rho v_0^2}, R_m = \frac{B_1 v_0^2}{\upsilon^2} \tag{5}$$

By substituting equations (5) into set of equations (2)-(4), we get

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = Gr \sin\phi T + \frac{\partial^2 u}{\partial y^2} + R_m \left(\frac{1}{4}\frac{\partial^3 u}{\partial t \partial y^2} - \frac{\partial^3 u}{\partial y^3}\right) - \left(M + \frac{1}{K}\right) u \tag{6}$$

$$\frac{\Pr}{4}\frac{\partial T}{\partial t} - \Pr \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{\Pr}{4} QT + \Pr Ec \left(\frac{\partial u}{\partial y}\right)^2 + k_1 \Pr Ec \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + \Pr M Ec u^2 \tag{7}$$

The corresponding boundary conditions in non-dimensional form are:

$$\left.\begin{array}{l} y = 0; u = 0, T = 1 + \varepsilon e^{iwt} \\ y \to \infty; u \to \infty, T \to 0 \end{array}\right\} \tag{8}$$

3. Method of Solution

To solve the Eqs. (6) and (7), subject to the boundary conditions (8), the velocity u and temperature T in the neighborhood of the plate is assumed to be of the form,

$$\left.\begin{aligned} u(y,t) &= u_0(y) + \varepsilon e^{iwt} u_1(y) \\ T(y,t) &= T_0(Y) + \varepsilon e^{iwt} T_1(y) \end{aligned}\right\} \tag{9}$$

Substituting eq. (9) in the equations (6) and (7), and equating harmonic and non-harmonic terms for velocity And temperature, after neglecting coefficients of ε^2, the set of following equations are obtained:

$$R_m u_0^{111} - u_0^{11} - u_0^{1} + (M + \frac{1}{K}) u_0 = Gr \sin\phi T_0 \tag{10}$$

$$R_m u_1^{111} - \left(1 + \frac{iR_m\omega}{4}\right) u_1^{11} - u_1^{1} + \left(M + \frac{1}{K} + \frac{i\omega}{4}\right) u_1 = Gr \sin\phi T_1 \tag{11}$$

$$T_0^{11} + \Pr T_0^{1} + \frac{\Pr}{4} Q T_0 = -\Pr Ec\left(u_0^{1}\right)^2 - k_1 \Pr Ec u_0^{1} u_0^{11} - \Pr M Ec u_0^{2} \tag{12}$$

$$T_1^{11} + \Pr T_1^{1} + \frac{\Pr}{4}(Q - i\omega) T_1 = -2\Pr Ec u_0^{1} u_1^{1} - k_1 \Pr Ec\left(u_0^{1} u_1^{11} + u_0^{11} u_1^{1}\right) - 2\Pr M Ec u_0 u_1 \tag{13}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} & y = 0; u_0 = u_1 = 0, T_0 = T_1 = 1 \\ & y \to \infty; u_0 \to 0, u_1 \to 0, T_0 \to 0, T_1 \to 0 \end{aligned}\right\} \tag{14}$$

In equations (10) and (11), due to presence of elasticity, we get third order differential equations.

To solve these equations, we need three boundary conditions but we have two. So, following

Beard and Walters [11], we assume the solutions as

$$u_0 = u_{00} + R_m u_{01} + o\left(R_m^{2}\right) \tag{15}$$

$$u_1 = u_{10} + R_m u_{11} + o\left(R_m^{2}\right) \tag{16}$$

$$T_0 = T_{00} + R_m T_{01} + o\left(R_m^{2}\right) \tag{17}$$

$$T_1 = T_{10} + R_m T_{11} + o\left(R_m^{2}\right) \tag{18}$$

Zero order of R_m:

$$u_{00}{}^{11} + u_{00}{}^{1} - \left(M + \frac{1}{K} \right) u_{00} = -Gr \sin\phi T_{00} \tag{19}$$

$$u_{10}{}^{11} + u_{10}{}^{1} - \left(M + \frac{1}{K} + \frac{i\omega}{4} \right) u_{10} = -Gr \sin\phi T_{10} \tag{20}$$

$$T_{00}{}^{11} + \Pr T_{00}{}^{1} + \frac{\Pr}{4} Q T_{00} = -\Pr Ec \left(u_{00}{}^{1} \right)^2 - k_1 \Pr Ec u_{00}{}^{1} u_{00}{}^{11} - \Pr MEc u_{00}{}^{2} \tag{21}$$

$$T_{10}{}^{11} + \Pr T_{10}{}^{1} + \frac{\Pr}{4} (Q - i\omega) T_{10} = -2 \Pr Ec u_{00}{}^{1} u_{10}{}^{1} - k_1 \Pr Ec \left(u_{00}{}^{1} u_{10}{}^{11} + u_{00}{}^{11} u_{10}{}^{1} \right) - 2 \Pr MEc u_{00} u_{10} \tag{22}$$

First order of R_m:

$$u_{01}{}^{11} + u_{01}{}^{1} - \left(M + \frac{1}{K} \right) u_{01} = u_{00}{}^{111} - Gr \sin\phi T_{01} \tag{23}$$

$$u_{11}{}^{11} + u_{11}{}^{1} - \left(M + \frac{1}{K} + \frac{i\omega}{4} \right) u_{11} = u_{10}{}^{111} - Gr \sin\phi T_{11} - \frac{i\omega}{4} u_{10}{}^{11} \tag{24}$$

$$T_{01}{}^{11} + \Pr T_{01}{}^{1} + \frac{\Pr}{4} Q T_{01} = -2 \Pr Ec u_{00}{}^{1} u_{01}{}^{1} - k_1 \Pr Ec \left(u_{00}{}^{1} u_{01}{}^{11} + u_{00}{}^{11} u_{01}{}^{1} \right) - 2 \Pr MEc u_{00} u_{01} \tag{25}$$

$$T_{11}{}^{11} + \Pr T_{11}^{1} + \frac{\Pr}{4} (Q - i\omega) T_{11} = -2 \Pr Ec \left(u_{00}{}^{1} u_{11}{}^{1} + u_{01}{}^{1} u_{10}{}^{1} \right) - k_1 \Pr Ec \left(u_{00}{}^{1} u_{11}{}^{11} + u_{01}{}^{1} u_{10}{}^{11} + u_{00}{}^{11} u_{11}{}^{1} + u_{01}{}^{11} u_{10}{}^{1} \right) - 2 \Pr MEc \left(u_{00} u_{11} + u_{01} u_{10} \right) \tag{26}$$

In order to obtain solutions for the above coupled nonlinear system of equations (19) to (26) expand u_{00}, u_{01}, u_{10}, u_{11}, T_{00}, T_{01}, T_{10} and T_{11} in powers of Eckert number Ec. This is valid as Ec is very small (Ec<<1) for all incompressible fluid. So, we assumed that

$$\begin{aligned}
u_{00} &= u_{000} + Ec u_{001} + o(Ec^2) \\
u_{01} &= u_{010} + Ec u_{011} + o(Ec^2) \\
u_{10} &= u_{100} + Ec u_{101} + o(Ec^2) \\
u_{11} &= u_{110} + Ec u_{111} + o(Ec^2) \\
T_{00} &= T_{000} + Ec T_{001} + o(Ec^2) \\
T_{01} &= T_{010} + Ec T_{011} + o(Ec^2) \\
T_{10} &= T_{100} + Ec T_{101} + o(Ec^2) \\
T_{11} &= T_{110} + Ec T_{111} + o(Ec^2)
\end{aligned}$$

Using these equations in the equations (19) to (26) and equating the coefficient of Ec^0, Ec^1 we get the following sets of differential equations.

Zero order of Ec:

$$u_{000}^{11}+u_{000}^{1}-\left(M+\frac{1}{K}\right)u_{000}=-Gr\sin\phi T_{000} \tag{27}$$

$$u_{100}^{11}+u_{100}^{1}-\left(M+\frac{1}{K}+\frac{i\omega}{4}\right)u_{100}=-Gr\sin\phi T_{100} \tag{28}$$

$$T_{000}^{11}+\Pr T_{000}^{1}+\frac{\Pr}{4}QT_{000}=0 \tag{29}$$

$$T_{100}^{11}+\Pr T_{100}^{1}+\frac{\Pr}{4}(Q-i\omega)T_{100}=0 \tag{30}$$

$$u_{010}^{11}+u_{010}^{1}-\left(M+\frac{1}{K}\right)u_{010}=u_{000}^{111}-Gr\sin\phi T_{010} \tag{31}$$

$$u_{110}^{11}+u_{110}^{1}-\left(M+\frac{1}{K}+\frac{i\omega}{4}\right)u_{110}=u_{100}^{111}-Gr\sin\phi T_{110}-\frac{i\omega}{4}u_{100}^{11} \tag{32}$$

$$T_{010}^{11}+\Pr T_{010}^{1}+\frac{\Pr}{4}QT_{010}=0 \tag{33}$$

$$T_{110}^{11}+\Pr T_{110}^{1}+\frac{\Pr}{4}(Q-i\omega)T_{110}=0 \tag{34}$$

First order of Ec:

$$u_{001}^{11}+u_{001}^{1}-\left(M+\frac{1}{K}\right)u_{001}=-Gr\sin\phi T_{001} \tag{35}$$

$$u_{101}^{11}+u_{101}^{1}-\left(M+\frac{1}{K}+\frac{i\omega}{4}\right)u_{101}=-Gr\sin\phi T_{101} \tag{36}$$

$$T_{001}^{11}+\Pr T_{001}^{1}+\frac{\Pr}{4}QT_{001}=-\Pr(u_{000}^{1})^{2}-k_{1}\Pr u_{000}^{1}u_{000}^{11}-\Pr Mu_{000}^{2} \tag{37}$$

$$T_{101}^{11}+\Pr T_{101}^{1}+\frac{\Pr}{4}(Q-i\omega)T_{101}=-2\Pr u_{000}^{1}u_{100}^{1}-k_{1}\Pr\left(u_{000}^{1}u_{100}^{11}+u_{000}^{11}u_{100}^{1}\right)-2\Pr Mu_{000}u_{100} \tag{38}$$

$$u_{011}^{11}+u_{011}^{1}-\left(M+\frac{1}{K}\right)u_{011}=u_{001}^{111}-Gr\sin\phi T_{011} \tag{39}$$

$$u_{111}^{11}+u_{111}^{1}-\left(M+\frac{1}{K}+\frac{i\omega}{4}\right)u_{111}=u_{101}^{111}-Gr\sin\phi T_{111}-\frac{i\omega}{4}u_{101}^{11} \tag{40}$$

$$T_{011}{}^{11} + \Pr T_{011}{}^{1} + \frac{\Pr}{4} Q T_{011} = -2\Pr u_{000}{}^{1} u_{010}{}^{1} - k_1 \Pr\left(u_{000}{}^{1} u_{010}{}^{11} + u_{000}{}^{11} u_{010}{}^{1}\right) - 2\Pr M u_{000} u_{010}$$

$$T_{111}{}^{11} + \Pr T_{111}{}^{1} + \frac{\Pr}{4}(Q - i\omega) T_{111} = -2\Pr\left(u_{000}{}^{1} u_{110}{}^{1} + u_{010}{}^{1} u_{100}{}^{1}\right) - k_1 \Pr\left(u_{000}{}^{1} u_{110}{}^{11} + u_{000}{}^{11} u_{110}{}^{1} + u_{010}{}^{1} u_{100}{}^{11} + u_{010}{}^{11} u_{100}{}^{1}\right) - \tag{41}$$

$$2\Pr M\left(u_{000} u_{110} + u_{010} u_{100}\right) \tag{42}$$

Along with the boundary conditions:

y=0: $u_{000}=u_{010}=u_{001}=u_{001}=0$, $T_{000}=T_{010}=T_{001}=T_{001}=0$, $u_{100}=u_{110}=u_{101}=u_{111}=0$,

$T_{100}=T_{110}=T_{101}=T_{111}=0$

$y\to\infty$: $u_{000}\to u_{010}\to u_{001}\to u_{001}\to 0$, $T_{000}\to T_{010}\to T_{001}\to T_{001}\to 0$,
$u_{100}\to u_{110}\to u_{101}\to u111\to 0$,

$T_{100}\to T_{110}\to T_{101}\to T_{111}\to 0$ (43)

Solving the set of equations (27)-(42), with the boundary conditions (43), the expressions for the distributions of velocity and temperature are given below.

$$u(y,t) = A_1 e^{-r_3 y} + A_2 e^{-r_1 y} + A_3 e^{-2r_3 y} + A_4 e^{-2r_1 y} + A_5 e^{-(r_1+r_3)y} + A_{12} e^{-r_5 y} + A_{13} e^{-q_1 y} + A_{14} e^{-(r_3+r_5)y} + A_{15} e^{-(r_3+q_1)y} + A_{16} e^{-(r_1+r_5)y} + A_{17} e^{-(r_1+q_1)y} \tag{44}$$

$$T(y,t) = A_{18} e^{-r_1 y} + A_{19} e^{-2r_3 y} + A_{20} e^{-2r_1 y} + A_{21} e^{-(r_1+r_3)y} + A_{22} e^{-q_1 y} + A_{23} e^{-(r_3+r_5)y} + A_{24} e^{-(r_3+q_1)y} + A_{25} e^{-(r_1+r_5)y} + A_{26} e^{-(r_1+q_1)y} \tag{45}$$

Skin friction: The skin friction is obtained from equation (44), which is defined as

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -A_1 r_3 - A_2 r_1 - 2A_3 r_3 - 2A_4 r_4 - A_5(r_1+r_3) - A_{12} r_5 - A_{13} q_1 - A_{14}(r_3+r_5) - A_{15}(r_3+q_1) - A_{16}(r_1+r_5) - A_{17}(r_1+q_1) \tag{46}$$

Rate of heat transfer: The rate of Mass transfer in the form of Nusselt number is given by

$$Nu = \left(\frac{\partial T}{\partial y}\right)_{y=0} = -A_{18} r_1 - 2A_{19} r_3 - 2A_{20} r_1 - A_{21}(r_1+r_3) - A_{22} q_1 - A_{23}(r_3+r_5) - A_{24}(r_3+q_1) - A_{25}(r_1+r_5) - A_{26}(r_1+q_1) \tag{47}$$

5. Results and Discussion

In order to get physical insight into the problem, the velocity, temperature, skin friction and rate of heat transfer have been studied by assigning numerical values for M, Gr, Q, Pr while keeping Rm=0.01,ϕ=π/4,Ec=0.01,w=π/6,K=2,k1=0.2 and ε=0.01 constant. The results obtained are illustrated through the figures 1 to 6 and Tables 1 and 2.

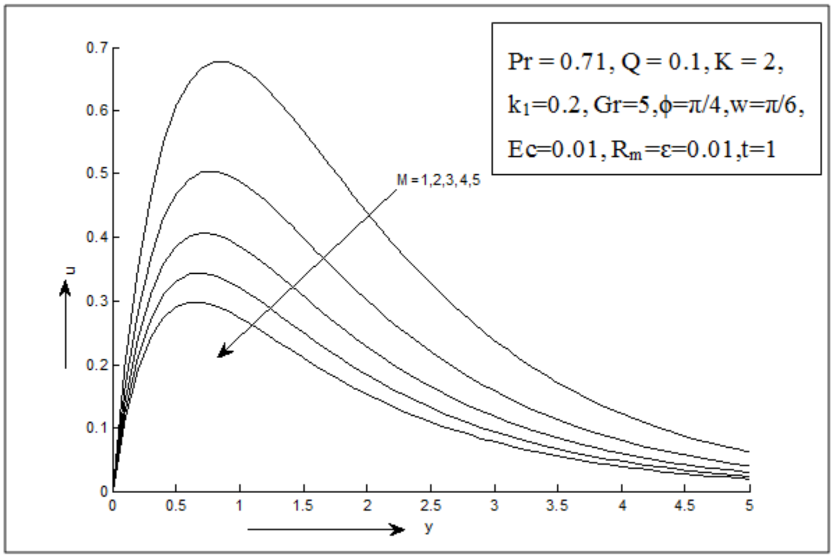

Figure 1. Effect of Magnetic Parameter (M) on Velocity

Figure 1 depicts the velocity profile u against y for different values of M. From this figure, we observe that as M increases, the velocity u decreases. It indicates that magnetic field suppresses the free convection. Figure 2 depicts the velocity profile u against y for different values of heat source parameter Q. From this figure, it is observed that as the heat source parameter Q increases, velocity u increases. In Figure 3, as Gr increases, the velocity u increases. Increase of Gr means Increase of temperature gradients T1-T0, which leads to the increase of velocity distribution. From Figure 4, it is noticed that velocity decreases with the increase in Prandtl number (Pr). From this figure, we observe that velocity (u) is greater for mercury (Pr=0.025) than that of electrolytic solution (Pr=1), *i.e.*, velocity (u) for various viscous fluid is more than the visco-elastic. From Figure 5, it is noticed that the temperature (T) increases as heat source parameter (Q) increase. From Figure 6, it is observed that temperature (T) decreases as Prandtl number (Pr) increases. It is clear that the temperature (T) is more for mercury (Pr=0.025) than that of electrolytic solution (Pr=1).

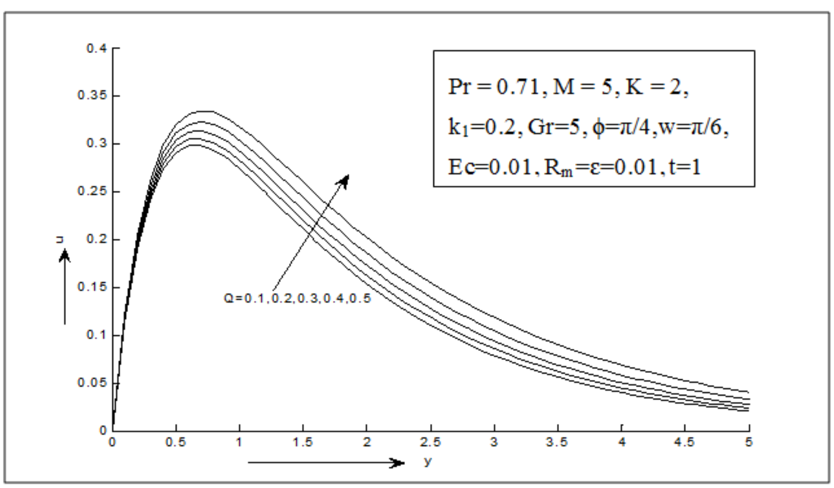

Figure 2. Effect of Heat Source Parameter on Velocity

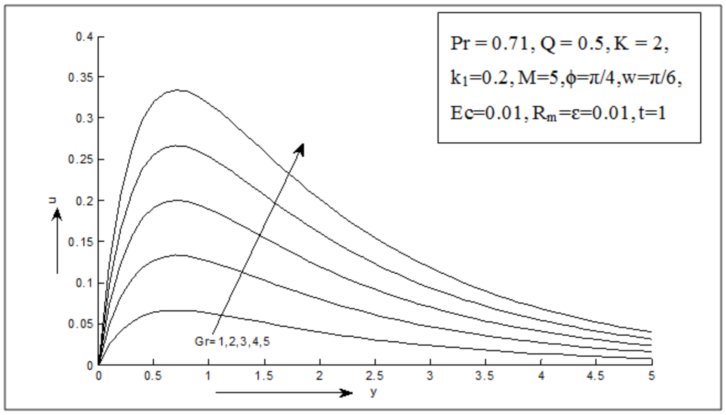

Figure 3. Effect of Grashof Number on Velocity

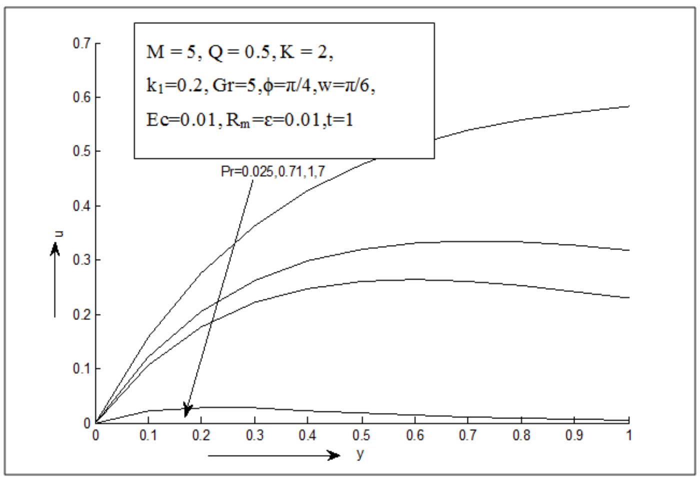

Figure 4. Effect of Prandtl Number on Velocity

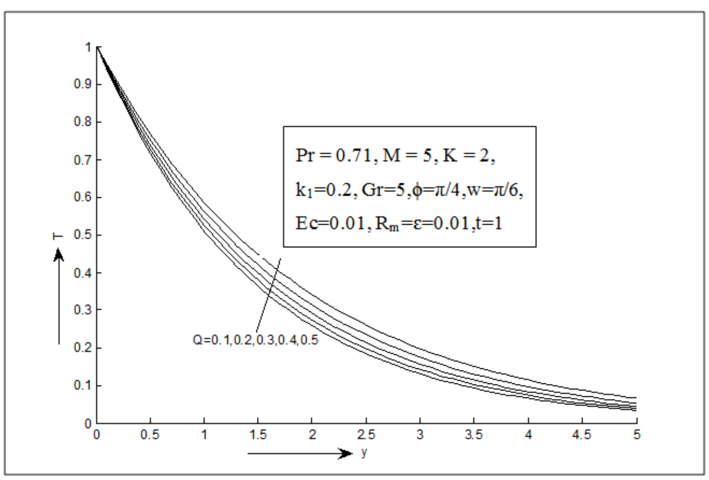

Figure 5. Effect of Heat Source Parameter on Temperature

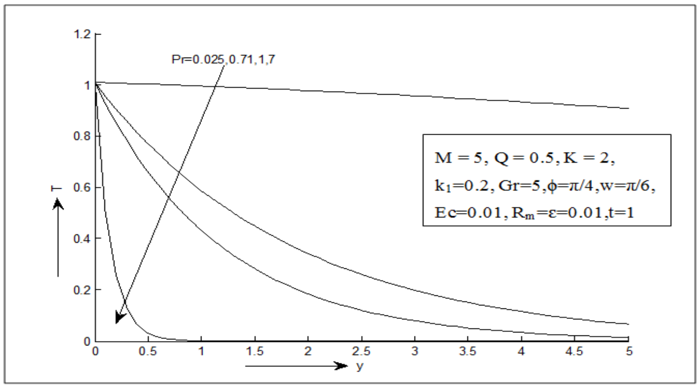

Figure 6. Effect of Prandtl Number on Temperature

From Table 1, we observe that the skin friction and Nusselt number increase with an increase in Grashof number (Gr). From Table 2, we observe that when M Increases the skin friction and Nusselt number decreases. From Table 3, it is clear that Q increases the skin friction and Nusselt number increase.

Table 1. Effect of Gr on Skin friction and Nusselt Number

Gr	$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$	$Nu = \left(\frac{\partial T}{\partial y}\right)_{y=0}$
1	-1.4250	-0.5518
2	-0.9246	-0.5496
3	-0.4157	-0.5459
4	0.1028	-0.5407
5	0.6321	-0.5340

Table 2. Effect of M on Skin Friction and Nusselt Number

M	$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$	$Nu = \left(\frac{\partial T}{\partial y}\right)_{y=0}$
1.2	0.6321	-0.5340
2.2	-0.1455	-0.5377
3.2	-0.6922	-0.5402
4.2	-1.1204	-0.5420
5.2	-1.4766	-0.5433

Table 3. Effect of Q on Skin Friction and Nusselt Number

Q	$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$	$Nu = \left(\frac{\partial T}{\partial y}\right)_{y=0}$
0.1	-1.5858	-0.6842
0.2	-1.5661	-0.6552
0.3	-1.5429	-0.6231
0.4	-1.5144	-0.5866
0.5	-1.4766	-0.5433

6. Conclusion

In this paper, the effect of unsteady MHD free convective visco-elastic fluid flow in the presence of heat source, viscous dissipation and ohmic heating has been studied numerically, the equations governing the velocity and temperature of the fluid are solved by using multi-parameter perturbation technique in terms of dimensionless parameters. In the analysis of the flow the following conclusions are made:

i. Velocity increases with an increase in Q and Gr but it shows the reverse effects in case of M and Pr.

ii. Temperature increases with an increase in Q but it shows the reverse effects in case of Pr.

iii. Skin friction increases with an increase in Gr and Q but it shows the reverse effects in case of M

iv. Nusselt number increases with an increase in Gr and Q but it shows the reverse effects in case of M

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