Multiple Cracks Assessment using Natural Frequency Measurement and Prediction of Crack Properties by Artificial Neural Network

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Abstract

This paper addresses the method of multiple cracks detection in moving parts or beams by monitoring the natural frequency and prediction of crack location and depth using Artificial Neural Networks (ANN). Determination of crack properties like depth and location is vital in the fault diagnosis of rotating machine equipments. For the theoretical analysis, Finite Element Method (FEM) is used wherein the natural frequency of beam is calculated whereas the experimentation is performed using Fast Fourier Transform (FFT) analyzer. In experimentation, simply supported beam with single crack and cantilever beam with two cracks are considered. The experimental results are validated with the results of FEM (ANSYS™) software. This formulation can be extended for various boundary conditions as well as varying cross sectional areas. The database obtained by FEM is used for prediction of crack location and depth using Artificial Neural Network (ANN). To investigate the validity of the proposed method, some predictions by ANN are compared with the results given by FEM. It is found that the method is capable of predicting the crack location and depth for single as well as two cracks. This work may be useful for improving online conditioning and monitoring of machine components and integrity assessment of the structures.

Keywords: *ANN, Crack, FEM, FFT, Modal Analysis, Natural Frequency*

1. Introduction

The presence of crack in structure changes its dynamic characteristics. The change is characterized by change in modal parameters like modal frequencies, modal value and mode shapes associated with each modal frequency. It also alters the structural parameters like mass, damping matrix, stiffness matrix and flexibility matrix of structure. The vibration technique utilizes one or more of these parameters for crack detection [1-2]. The frequency reduction in cracked beam is not due to removal of mass from beam, indeed the reduction in mass would increase natural frequency. But reduction in natural frequency is observed due to removal of material which carries significant stresses when defect is a narrow crack or notch [3]. It reduces the stiffness of structure and natural frequency [4-7]. Due to presence of crack there is local influence which results from reduction and second moment of area of cross section where it is located [8-9]. The system becomes non linear due to crack [10]. This reduction is equivalent to lowering the local bending stiffness of beam and therefore it behaves as two beams connected by means of torsion spring [11-12]. Finite Element Analysis is powerful tool which gives the reasonably accurate results for complicated on line working assemblies in dynamic analysis [13-14]. Non destructive error detection suggests that the variation in monitored signatures is indication of error and it can be located [15]. Beam forming of lamb waves can also be used for structural health monitoring and the results are also promising [16]. To incorporate the non linearity, the crack is simulated by an equivalent linear spring for longitudinal vibration and the torsion spring for transverse vibrations connecting the two segments of beam [17]. The equivalent stiffness may be computed from the crack strain energy function [18]. The expression for the spring stiffness representing a crack depth ratio is presented [19]. As such correct numerical formula is not available hence the use of 2D element in Finite Element Analysis is equally valuable. Till now the effect of crack along the width is considered. It is observed that the crack along the length does not affect the natural frequency up to the considerable mark [20]. This method can be implemented for assessment of multiple cracks also [21-23]. The present study is based on observation of changes in natural frequency. In theoretical analysis, the crack is simulated by a spring connecting the two segments of the beam in the work carried out. For the theoretical analysis, Finite Element Method (FEM) is used wherein the natural frequency of beam is calculated by modal analysis using *ANSYS*™ whereas for experimentation purpose, Fast Fourier Transform (FFT) analyzer is used. In experimentation, simply supported beam with single crack and cantilever beam with two cracks are considered.

2. Analysis of Reduction in Natural Frequencies

A theoretical model based on the receptance technique is presented for analysis. It can be treated as one-dimensional analysis. The crack divides the beam in two sections having receptances β and γ respectively. If K_x is the stiffness of the bar, then the natural frequencies of the cracked bar satisfy the following equation

$$\beta x + \gamma x + \frac{1}{Kx} = 0 \tag{2.1}$$

Decrease in Kx is indication of increase in damage. For a bar with uniform cross section a relationship between the crack stiffness, crack location and natural frequency is given by,

$$EA/Kx = 1/\lambda[Cot\lambda x + Cot\lambda(1-x)] \tag{2.2}$$

Where E, A, and I are the modulus of elasticity, cross-sectional area and length of the beam respectively. $\lambda = \omega \sqrt{1/x}$ is the frequency parameter where ω is natural frequency of an axial vibrations. For torsion springs the relationship is in terms of ratios of two determinants.

$$K = -\lambda \Delta_2/\Delta_1 \tag{2.3}$$

Where Δ is frequency parameter while Δ_1 and Δ_2 are obtained from the characteristic equation of the system [1] Minimum three modes are required for an efficient prediction. For the cantilever beam with multiple cracks (2 cracks), five modes are extracted. Consider Euler Bernoulli beam. To derive the differential equation of motion for the bending vibration of beam, consider an element of beam of length dx where V and M are shear and bending moments. P(x) represents the loading per unit length of beam. Summing the forces in Y direction,

$$(\mathcal{V} + d_v) – \mathcal{V} - P(x)dx = 0, d_v + dx = p(x) \tag{2.4}$$

By assuming moments about any point on right face of elements in limiting case becomes

$$\frac{d_m}{d_x} = V \tag{2.5}$$

The equation (2.4) state that rate of change of shear along the length of the beam is equal to the loading per unit length. Equation (2.5) state that rate of change of the moment along the beam is equal to the shear. From equation (2.4) and equation (2.5)

$$\frac{\partial^2 \mathrm{M}}{\partial \mathrm{x}^2} = \frac{\partial \mathrm{V}}{\partial \mathrm{x}} = \mathrm{p(x)} \tag{2.6}$$

Substituting bending moment $M = EI\frac{\partial^2 y}{\partial x^2}$ in equation (2.6),

$$\frac{\partial^2}{\partial x^2 \left(EI\frac{\partial^2 y}{\partial x^2}\right)} = \mathrm{p(x)} \tag{2.7}$$

For the beam having transverse vibrations, the load per unit length of the beam is the inertia force *i.e.*, mass and acceleration, where M = mass of beam per unit length hence equation (2.7) becomes,

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right) = \mathrm{M}\ \frac{\partial^2 \mathrm{y}}{\partial \mathrm{t}^2}$$

$$EI\ \frac{\partial^4 y}{\partial x^4} = -\rho A\ \frac{\partial^2 y}{\partial t^2} = 0 \tag{2.8}$$

The equation (2.8) is called governing equation of motion of Euler – Bernoulli. The general solution of the equation (2.8) is obtained by method of separation of variable Y (x,t) = Y(x), T(t), Substituting y = Y.T in equation (2.8),

$$EIT\ \frac{\partial^2 Y}{\partial x^4} + \rho A\frac{\partial^2 T}{\partial t^4} = 0$$

$$1/_T\left(\frac{\partial^2 T}{\partial t^2}\right) = \left(\frac{EI}{\rho A}\right)\frac{1}{Y}\left(\frac{\partial^4 Y}{\partial x^4}\right)$$

The left hand side of the above equation is function of ‘T’ while the right hand side is function of ‘Y’ alone. It is possible if each side of this equation is equal to negative constant say - ω^2 where ω is a real number.

$$1/_T\left(\frac{\partial^2 T}{\partial t^2}\right) = \left(\frac{EI}{\rho A}\right)\frac{1}{Y}\left(\frac{\partial^4 Y}{\partial x^4}\right) = -\omega^2$$

$$\left(\frac{EI}{\rho A}\right)\frac{1}{Y}\left(\frac{\partial^4 Y}{\partial x^4}\right) = -\omega^2$$

$$\frac{\partial^4 Y}{\partial x^4} - \left(\frac{\rho A\omega^2}{EI}\right)Y = 0$$

$$\frac{\partial^4 Y}{\partial x^4} - \lambda^4 Y = 0$$

Where $\lambda^4 = \left(\frac{\rho A \omega^2}{EI}\right)$, from theory of linear differential equation, the solution for above equation is,

$$Y(x) = \mathrm{a1} \sin \lambda \mathrm{x} + \mathrm{a2} \cos \lambda \mathrm{x} + \mathrm{a3} \sinh \lambda \mathrm{x} + \mathrm{a4} \cosh \lambda \mathrm{x} \qquad (2.9)$$

This equation represents harmonic motion of the beam. Where a_1, a_2, a_3 and a_4 are constants and can be found by substituting this solution in the boundary condition. Hence, we get different values of y for the range of x = 0 to 1 for each modes and mode shapes are found out.

3. Determination of Crack Location

The equation (2.8) of the motion of Euler-Bernoulli does not satisfy near the crack due to abrupt change in the cross-section. The beam can be treated as two uniform beams connected by a torsional spring at the crack location. The equation (2.8) is then valid for each segment of the beam separately. This kind of modeling for the cracked beam has the advantage of using the exact solution throughout the beam except for a narrow region near the crack where the true stress-strain field is approximated by spring. For two beam segments, we get set of equations from equation (2.9)

$$Y1(x) = \mathrm{a1} \sin \lambda \mathrm{x} + \mathrm{a2} \cos \lambda \mathrm{x} + \mathrm{a3} \sinh \lambda \mathrm{x} + \mathrm{a4} \cosh \lambda \mathrm{x} \qquad (2.10\ \mathrm{a})$$

$$Y2(x) = \mathrm{a5} \sin \lambda \mathrm{x} + \mathrm{a6} \cos \lambda \mathrm{x} + \mathrm{a7} \sinh \lambda \mathrm{x} + \mathrm{a8} \cosh \lambda \mathrm{x} \qquad (2.10\ \mathrm{b})$$

Where the origin of x for both segments is at the support and $\lambda_4 = \rho A\ \omega^2/EI$. The coefficients a_1 can be found by substituting this solution in boundary conditions. In case of stepped beams or shafts, four constants for each step get added. The boundary conditions for simply supported beam are as follows. For the free vibrations of the beam, $Y_{1A} = Y_{2C} = 0$ and $Y''_{1A} = Y''_{2C} = 0$. The continuity conditions at the crack position the displacement, moments and shear forces are $Y_{1B} = Y_{2B}$, $Y''_{1B} = Y''_{2B}$, $Y''_{1B} = Y''_{2B,}$ with the non-dimensional crack section flexibility denoted by 0, the angular displacement between the two beam segments can be related to the moment at this section by, $Y'_{2B} + \Theta L\ Y''_{2B} = Y'_{1B}$. Substituting equation (2.5) in above boundary conditions, a set of eight homogeneous linear algebraic equations for the eight unknown coefficients is formed [24].

1	0	1	0
1	0	-1	0
cosheβ	sinheβ	coseβ	sineβ
sinheβ	cosheβ	sineβ	-coseβ
cosheβ	sinheβ	-coseβ	-sineβ
0	0	0	0
0	0	0	0
(sineβ/θβ)+cosheβ	(cosheβ)/(θβ)+sinheβ	-(sineβ)/(θβ)-coseβ	(coseβ)/(θB)-sineβ

$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\cosh e\beta & -\sinh e\beta & -\cos e\beta & -\sin e\beta \\ -\sinh e\beta & -\cosh e\beta & -\sin e\beta & -\cos e\beta \\ -\cosh e\beta & -\sinh e\beta & \cos e\beta & \sin e\beta \\ \cosh\beta & \sinh\beta & \cos\beta & \sin\beta \\ \cosh\beta & \sinh\beta & -\cos\beta & -\sin\beta \\ -(\sin e\beta/0\beta) & -(\cosh e\beta)/(0\beta) & (\sin e\beta)/(0\beta) & -(\cos e\beta)/(0B) \end{matrix}\Bigg] = 0$$

Where θ - non dimensional flexibility (EI / KfL), β = λL - non dimensional frequency parameter and e = [(L_1 – L/2) / (L/2)] - non dimensional crack location. After solving the above matrix using MATLAB, the set of equation reduces to,

$$4\sin\beta\ \sinh\beta + \beta\theta[\sinh\beta(\cos\beta - \text{cosec}\beta)] + \sin\beta(\cosh\beta - \text{coshe}\beta) = 0$$

For crack location, a partial differentiation with respect to 0 yields,

$$4(cos\beta\sinh\beta + \sin\beta\cosh\beta)\left(\frac{\partial\beta}{\partial\theta}\right) + \beta[\sinh\beta(\cos\beta - \cosh\beta) + \sin\beta(\text{cose}\beta - \text{coshe}\beta)] = 0$$

$$\frac{\partial}{\partial\beta}\{Bsinh\beta(\cos\beta - \text{cose}\beta) + \sinh\beta(\cosh\beta - \text{coshe}\beta)]\}\frac{\partial\beta}{\partial\theta} = 0$$

For uncracked beam with equivalent flexibility, the nominal values of 0 in above equation become zero. Substituting 0 = 0 and β = nπ. By definition of the non-dimensional frequency parameter, 2 Δβ/β = Δf/f. Substituting in above equation and rewriting for the i^{th} mode,

$$2cos\beta i\left(\frac{\Delta f}{fi}\right) = [\text{cosec}\beta i - \cos\beta i]\Delta\theta$$

For the first natural mode $\beta_i = 1\pi$, above equation yields

$$-2\left(\frac{\Delta f1}{fi}\right) = (\cos 2e\pi\ + 1)\Delta\theta \tag{2.11}$$

And for the second mode β2 = 2 π, Above equation yields,

$$2\left(\frac{\Delta f2}{f2}\right) = (\cos 2e\pi\ - 1)\Delta\theta \tag{2.12}$$

Dividing equation (2.11) by equation (2.12),

$$\left(\frac{\Delta f2}{f2}\right)\left(\frac{\Delta f1}{f1}\right) = \frac{(1 - \cos 2e\pi)}{(1 + \cos 2e\pi)}$$

Solving above equation for crack location e,

$$e = \frac{1}{\pi}\cos^{-1}\frac{[1 - \left(\frac{\Delta f2}{f2}\right)/\left(\frac{\Delta f1}{f1}\right)]}{2} \tag{2.13}$$

Where Δfn = fn – $\overline{fn}$, fn and $\overline{fn}$ are the natural frequencies of uncracked and cracked beam. This relation suggests that the ratio of the relative vibrations of two modes depends solely on the location of the crack and is independent on crack geometry or beam properties.

4. Determination of Crack Size

Consider a beam with a discrete crack. Considering the characteristic equations, the frequency change ratio Δfn/fn and the dimensionless stiffness K is given as,

$$\frac{\Delta fn}{fn} = 2gn(x)(\frac{1}{K}) \quad (3.1)$$

Where, x is non dimensional crack location L1/L or (e+1)/2 and K = (K_1L) / EI (3.2)

Δfn is difference between uncracked and cracked beam = fn – $\overline{fn}$, fn and $\overline{fn}$ are the natural frequencies of uncracked and cracked beam. The g_n(x) function for a simply supported beam can be evaluated as,

$$gn(x) = \frac{1}{4}\{\frac{[\varphi n(x^2)]}{\int[\varphi n(x^2)]}dx\} \quad (3.3)$$

From elementary beam theory, for simply supported beam the mode shape is φ_n = sin (n πx). The relationship between the changes in eigen frequencies and the crack location and stiffness of crack based on equation (3.1), (3.2) and (3.3) can be expressed as,

$$\Delta fn/fn = sin^2(n\pi x)EI/\mathrm{K_1L} \quad (3.4)$$

The spring stiffness K_r decreases in the vicinity of the cracked section of a beam having width b, height h and crack depth a. From the crack strain energy function,

$$K_r = EI/[(5.346h)f(a/h)]$$

Substituting the above relation in equation (3.4)

$$\Delta fn/fn = sin^2(n\pi x)\{[(5.346h)f(a/h)]/L\}$$

$$\Delta fn/fn = sin^2[n\pi(e+1)/2]\frac{\mathrm{h}}{\mathrm{L}}\ f(a/h) \quad (3.5)$$

Where, f(a/h) = 1.8624 $(a/h)^2$ – 3.95 $(a/h)^3$ +16.375 $(a/h)^4$ – 37.226 $(a/h)^5$ + 78.81 $(a/h)^6$ – 126.9 $(a/h)^7$ + 172 $(a/h)^8$ – 143.97 $(a/h)^9$ + 66.56 $(a/h)^{10}$. [25]

Substituting above value in equation (3.5) and neglecting higher order values, the equation becomes,

$$(a/h)^2 = (\Delta fn/fn)/\{9.9563sin^2[n\pi(\mathrm{e}+1)/2]\ h\} \quad (3.6)$$

Using equation (3.6) crack depth ratio (a/h) can be found out if the natural frequency of beam is known.

5. Experimental Analysis

The instruments used for experimental analysis *i.e.*, measurement of natural frequencies are Fast Fourier Transform (FFT) analyzer, accelerometer, impact hammer and related accessories. The FFT analyzer used is 4 channel B&K make with measuring range 10-200 dB, amplitude stability $\pm$ 0.1 dB, impedance 10 G Ω, frequency limit 1 Hz to 20 KHz. RT-PRO^{TM} software, compatible with the FFT analyzer is used. The piezoelectric, miniature type unidirectional accelerometer is used to capture the frequency response functions. The accelerometer is mounted on the beam using mounting clips. The accelerometer is mounted near the crack to capture the correct signal. The impact hammer is used to excite the beam whose frequency response function has to be captured. For every test, the location of impact of impact hammer is kept constant. Impact hammer has the range of excitation 1-4000 Hz. The beam is tapped gently with the impact hammer. The experiments are performed on mild steel beams with simply supported boundary conditions having single crack and cantilever boundary conditions with two cracks of different depths at different locations. The properties of mild steel are, Young's modulus (E) 2.0 e11 N/m^2, density (ρ) 7950 N/m^3 and Poisson's ratio 0.3. Specimen beams under consideration have rectangular cross section area. For simply supported beam the cross sectional area is 0.025 x 0.010 m, L = 0.3m and for cantilever beam the cross sectional area is 0.05 x 0.01m, L = 0.5 m. The geometry of beams is as shown in Figure 1. Crack depth is represented in terms of (a/h) ratio where a = depth of crack and h = height of beam and crack location is represented in terms of (e) where e is ratio of location of crack at distance L_1 or L_2 from the support to the length of the beam L. The experimental setup is as shown in Figure 2. The aim of experimental analysis is to verify the practical applicability of the theoretical method developed. For the beam with single or double cracks, transverse and open cracks are considered. Two cracks are parallel to each other as two cracks of same depth with different orientation do not have any effect on natural frequency values [26]. Initially, the natural frequency of uncracked beam is found out. Hairline crack is generated to simulate the actual crack in the working components. Thereafter, the severity *i.e.*, depth of crack is increased. The change in natural frequency due to the crack is monitored. Table 1 shows the natural frequencies of simply supported beam with single crack.

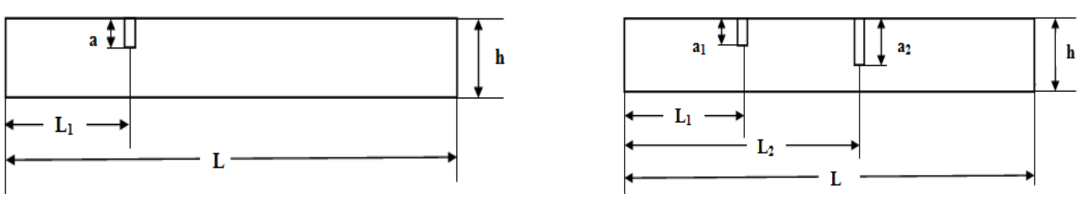

Figure 1. Geometry of Beam with Single Crack and Two Cracks

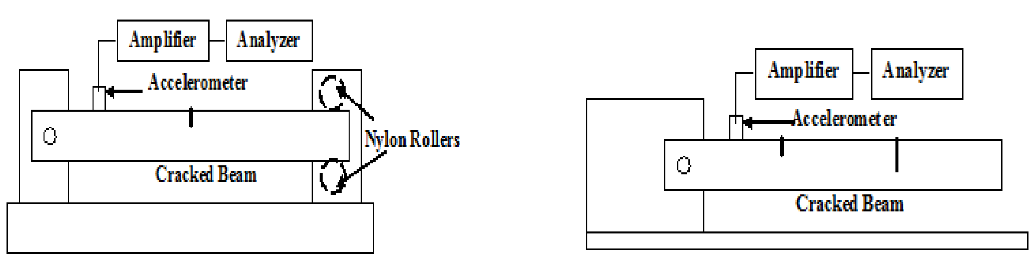

Figure 2. Experimental set-up (Simply Supported and Cantilever Beam)

Table 1. Natural Frequencies and Frequency Ratio for Simply Supported Beam with Single Crack

Sr. No	Crack location & size (mm)		Natural Frequency by FEM (Hz)		Natural Frequency by Experiments (Hz)		Theoretical frequency ratio	Expt. frequency ratio
	e	a/h	ω_1	ω_2	ω_1	ω_2		
1	0	0	624.7	2420.3	581.25	2371.2	-	-
2	0.2	0.1	623.52	2419.0	562.25	2362.5	0.3705	0.3203
3	0.4	0.1	627.85	2416.4	568.75	2325.00	1.2966	1.2172
4	0.6	0.1	624.27	2416.4	568.75	2250.00	2.5565	2.7215
5	0.8	0.1	624.59	2419.1	575.00	2306.00	3.4812	3.1746
6	0.2	0.2	620.96	2415.4	566.75	2356.00	0.3597	0.4036
7	0.4	0.2	620.96	2415.4	568.25	2300.00	1.2343	1.1361
8	0.6	0.2	621.96	2406.7	568.75	2250.00	2.4113	2.678
9	0.8	0.2	624.31	2415.3	575.00	2300.00	3.444	3.4083
10	0.2	0.3	616.93	2409.7	525.00	2331.25	0.347	0.2434
11	0.4	0.3	615.12	2392.6	531.25	2187.5	1.1633	0.9738
12	0.6	0.3	621.74	2392.0	556.25	2181.25	2.2616	2.0085
13	0.8	0.3	623.89	2409.8	562.50	2168.75	3.3976	2.7955
14	0.2	0.4	611.59	2420.3	475.00	2331.25	0.3347	0.358
15	0.4	0.4	615.06	2373.1	493.75	2143.75	1.0626	0.8781
16	0.6	0.4	619.62	2372.6	543.75	2068.75	2.0787	2.0693
17	0.8	0.4	623.27	2401.5	556.25	2087.5	3.3447	2.9214

6. Finite Element Analysis

Finite Element Analysis is performed using *ANSYS*™. The model of beam is generated and used for Finite Element Analysis. The modal analysis is used to determine the natural frequencies and mode shapes of a structure. The element used in Finite Element Analysis is PLANE 82: 2-D8-Node Structural Solid. PLANE82 is a higher order version of the two-dimensional, four-node element (PLANE42). It provides more accurate results for mixed (quadrilateral-triangular) automatic meshes and can tolerate irregular shapes without loss of accuracy [27]. The properties of the material are as mentioned Section 5. Table 1 shows the natural frequencies of simply supported beam with single crack and Table 2 shows the natural frequencies of cantilever beam with two cracks of different depths at different locations determined using *ANSYS*™. The crack location and crack depth of first crack with reference to left support is fixed and the other crack is varied [21].

Table 2. Natural Frequencies of Cantilever Beam with Two Cracks by FEM

Crack location & size (mm)				Natural Frequency (Hz)				
e_1	a_1/h	e_2	a_2/h	ω_1	ω_2	ω_3	ω_4	ω_5
0	0	0	0	143.03	901.83	2143.6	2435.4	4478
0.1	0.1	0.2	0.1	139.07	896.52	2116.9	2417.1	4431.7
0.1	0.1	0.2	0.2	134.78	893.08	2090.7	2381.9	4352.6
0.1	0.1	0.2	0.3	130.25	894.71	2048.2	2350.6	4260
0.1	0.1	0.2	0.4	121.99	890.88	1954.4	2316.9	4156

0.1	0.1	0.2	0.5	111.41	887.6	1802.7	2313.7	4077.5
0.1	0.1	0.3	0.1	145.54	915.35	2210.8	2450	4518.4
0.1	0.1	0.3	0.2	142.85	907.01	2189	2402.7	4503.3
0.1	0.1	0.3	0.3	138.4	893.47	2138.1	2350.3	4481
0.1	0.1	0.3	0.4	132.3	875.37	2021.1	2333.7	4455.7
0.1	0.1	0.3	0.5	123.37	850.35	1876.6	2329.1	4427.8
0.1	0.1	0.4	0.1	1389.7	8865.2	2111.1	2419.2	4442.8
0.1	0.1	0.4	0.2	138.34	868.51	2103.8	2404.7	4404.1
0.1	0.1	0.4	0.3	135.87	836.35	2073.8	2382.9	4341.1
0.1	0.1	0.4	0.4	132.19	793.25	2024.2	2362	4267.6
0.1	0.1	0.4	0.5	126.35	737.48	1945.8	2347	4185.9
0.1	0.1	0.5	0.1	139.11	884.19	2111.2	2425.8	4422
0.1	0.1	0.5	0.2	139.14	861.06	2107.5	2426.5	4333.4
0.1	0.1	0.5	0.3	145.76	847.93	2179	2474.6	4264.6
0.1	0.1	0.5	0.4	141.95	788.23	2122	2463.2	4113.9
0.1	0.1	0.5	0.5	137.36	717.17	2071.7	2458.6	3969.3
0.1	0.1	0.6	0.1	145.95	908.74	2210.2	2456	4518.1
0.1	0.1	0.6	0.2	147.86	891.48	2199.2	2427.2	4505
0.1	0.1	0.6	0.3	147.86	855.95	2173.7	2396.3	4489.5
0.1	0.1	0.6	0.4	143.76	795.21	2099.6	2360.3	4458.2
0.1	0.1	0.6	0.5	144.03	727.79	2015.9	2347.7	4410.6
0.1	0.1	0.7	0.1	146.51	913.31	2212	2445.3	4504
0.1	0.1	0.7	0.2	146.4	899.65	2199	2384.7	4456.2
0.1	0.1	0.7	0.3	145.74	874.61	2148	2321.5	4383.9
0.1	0.1	0.7	0.4	147.96	842.68	2047.6	2288.5	4306.3
0.1	0.1	0.7	0.5	144.72	779.05	1891	2267	4213.9
0.1	0.1	0.8	0.1	146.4	916.34	2213.4	2453.4	4474.9
0.1	0.1	0.8	0.2	146.04	910.82	2206.5	2411	4348.5
0.1	0.1	0.8	0.3	145.96	902.24	2184.2	2343.8	4166.2
0.1	0.1	0.8	0.4	146.2	888.81	2105.2	2291.2	3948.2
0.1	0.1	0.8	0.5	146.1	863.26	1935.4	2261.4	3727.5
0.1	0.1	0.9	0.1	146.5	918.08	2214.3	2468.7	4510.7
0.1	0.1	0.9	0.2	146.52	917.77	2214.5	2461.5	4472
0.1	0.1	0.9	0.3	146.25	916.2	2211.5	2449.2	4389.8
0.1	0.1	0.9	0.4	146.16	914.36	2204.1	2426.2	4229
0.1	0.1	0.9	0.5	146.16	911.85	2185.5	2384.7	3920.5
0.1	0.2	0.2	0.1	137.49	903.06	2154	2441.9	4457.4
0.1	0.2	0.2	0.2	134.33	902.56	2135.7	2411.7	4386.5

0.1	0.2	0.2	0.3	128.49	899.85	2080.8	2376.5	4291.1
0.1	0.2	0.2	0.4	121.73	900.98	2004.8	2348.5	4194.3
0.1	0.2	0.2	0.5	111.01	897.2	1854.1	2339.4	4113.1
0.1	0.2	0.3	0.1	137.53	898.69	2150.4	2431.2	4480.3
0.1	0.2	0.3	0.2	135.98	891.83	2139.7	2386.9	4473.1
0.1	0.2	0.3	0.3	132.24	877.05	2092	2333.1	4454.4
0.1	0.2	0.3	0.4	126.55	855.55	1991.1	2302.4	4431.6
0.1	0.2	0.3	0.5	122.04	862.53	1869.9	2357.7	4389.4
0.1	0.2	0.4	0.1	145.89	910.72	2211.1	2462.7	4508.8
0.1	0.2	0.4	0.2	136.96	875.44	2142	2427.2	4430.6
0.1	0.2	0.4	0.3	134.17	842.01	2107.6	2402.2	4366.5
0.1	0.2	0.4	0.4	130.62	798.28	2054.7	2381.4	4292.2
0.1	0.2	0.4	0.5	125.44	743.66	1976.4	2363.6	4222.2
0.1	0.2	0.5	0.1	138.56	893.69	2156.3	2454.4	4450.8
0.1	0.2	0.5	0.2	145.14	882.26	2196.9	2467.6	4392.8
0.1	0.2	0.5	0.3	145.76	847.93	2179	2474.6	4264.6
0.1	0.2	0.5	0.4	134.34	771.94	2083.3	2445.8	4074.8
0.1	0.2	0.5	0.5	131.03	701.94	2026.8	2440.7	3923
0.1	0.2	0.6	0.1	138.63	894.67	2155.3	2439	4482.4
0.1	0.2	0.6	0.2	138.32	871.9	2144	2404	4471.7
0.1	0.2	0.6	0.3	137.4	834.31	2116	2357.7	4451.8
0.1	0.2	0.6	0.4	135.97	780.42	2060.7	2313.4	4417.5
0.1	0.2	0.6	0.5	135.14	711.28	1977.4	2305.9	4380.1
0.1	0.2	0.7	0.1	138.89	898.95	2158.8	2428.9	4468.7
0.1	0.2	0.7	0.2	138.14	883.51	2143.5	2358.3	4417.2
0.1	0.2	0.7	0.3	138.05	860.55	2110.2	2281.4	4350.2
0.1	0.2	0.7	0.4	138.95	825.55	2012.2	2224	4263.7
0.1	0.2	0.7	0.5	136.96	767.14	1867.4	2215.9	4182.2
0.1	0.2	0.8	0.1	138.84	901.73	2157.9	2437.6	4440.2
0.1	0.2	0.8	0.2	138.13	895.6	2149.5	2388.3	4319.9
0.1	0.2	0.8	0.3	138.36	888.57	2137.5	2321.8	4141.9
0.1	0.2	0.8	0.4	138.55	874.85	2075.1	2246.3	3925.2
0.1	0.2	0.8	0.5	138.05	849.33	1916.7	2205.3	3705.7
0.1	0.2	0.9	0.1	138.86	903.4	2159.7	2452.2	4475.1
0.1	0.2	0.9	0.2	138.23	901.25	2153.5	2441.4	4434.9
0.1	0.2	0.9	0.3	138.93	902.66	2158	2431.9	4362.2
0.1	0.2	0.9	0.4	138.99	901.08	2153	2407.8	4209
0.1	0.2	0.9	0.5	139	898.51	2138.1	2363.1	3905.8

7. Prediction of Crack Properties by Artificial Neural Networks (ANN)

The inverse problem can be converted into forward technique using tools of Artificial Intelligence like Genetic algorithm, Fuzzy Logics, Artificial Neural Network (ANN). These techniques can be used for prediction of life of components or even optimization to minimize the errors in frequencies determined by numerical simulation and experimental measurement. Genetic algorithms are stochastic search algorithms which are based on the mechanics of nature selection and natural genetics. These are designed to search large, non-linear, discrete and poorly understood search space where expert knowledge is difficult to model and traditional optimization techniques may not give accurate results. In the genetic algorithm, this error is used to evaluate the fitness of each individual in the population. Genetic algorithms have been frequently accepted as optimization methods in various fields and have also been proved as an excellent in solving complicated optimization problem. Thus, Genetic Algorithm can be used to solve inverse problem for the crack detection in a shaft [28]. The Artificial Neural Networks (ANN) in a wide sense belongs to the class of evolutionary computing algorithms that try to simulate natural evolution of information handling [29]. The present paper checks the applicability of this tool to predict the crack location and depth depending upon the input. The input to the ANN is the natural frequency of three or more number of modes and output is crack location and crack depth. In case of single crack the output will be prediction of crack location and crack depth *i.e.*, two parameters whereas for two cracks the output will be prediction of four parameters *i.e.*, two predictions for crack depths and two predictions for crack locations. Amongst the available data, 90% data is used to train the network in ANN whereas 10% of the data is used for validation. The back propagation algorithm is used [30]. The network is trained using the data obtained by FEM *i.e.*, Table 1 for simply supported beam with single crack and Table 2 for cantilever beam with two cracks. Thereafter, the network predicts the location and depth of crack. The network can predict the crack location and depth for any intermediate input values of natural frequencies. The network decides the predominant input parameter on its own. The iterations are conducted till the average training error and average validating error is minimized [31]. For simply supported beam with single crack, single layer serves the purpose whereas for cantilever beam with two cracks, three layers give close predictions. For cantilever beam with two cracks, three layers are chosen as average training error and average prediction error is minimum in case of three layers. Less error in both is indication of precise prediction of output. During the routine assessment of the health of component or online conditioning and monitoring, if decrease in natural frequency is observed, these frequency values can be given as input in the form of new query to the network. The network predicts the properties of crack. Any number of queries can be run.

8. Results and Discussions

The crack of known severity is generated at known location in mild steel beam. In case of simply supported beam single crack is generated whereas in case of cantilever beam, two cracks are generated. The changes in natural frequencies for the uncracked and cracked beams are measured. The predicted values are determined by theoretical and experimental technique. Table 1 shows the natural frequency values extracted for simply supported beam with single crack determined by using FEM and experimentation. Non dimensional frequency ratio is also calculated using these values. By the inverse method *i.e.*, by using equation 2.13 and 3.6 the results for crack location (e), crack size (a/h) are computed. Table 3 shows the comparison for crack location (e), crack size (a/h) between actual and determined values for simply supported beam with single crack. It compares between theoretical and experimental frequency ratio

($\Delta f_1/ f_1$) with respect to crack size (a/h)**.** The theoretical results *i.e.*, results obtained by FEM are compared with the experimental results and Figure 3 shows graphical comparison between the theoretical (FEM) and experimental non dimensional frequency ratio. It is observed that experimental results have some deviation from the results obtained by FEM as model of structure generated by Finite Element Analysis differs from actual structure. Hence the response of structure in practice differs. The results are close to the actual for finding the crack locations. These results approach to the actual results found by FEM as compared to the experimental findings. The variation in the results obtained is in the range of 0.2 to 15%. The variation might be the effect of structure prone vibrations. The results of crack depth findings are close to the actual depth for large (a/h) ratio as compared to small (a/h) ratio. It is observed because for small (a/h) ratio, the reduction in the stiffness of beam is less as compared to large (a/h) ratio. Due to the high stiffness, the vibrations are damped and natural frequency does not reduce. The readings obtained are used as database for Neural Networks. ANN can predict the crack location and crack depth by adding new query to ANN grid. The predicted crack location and depth by ANN are verified by using FEM. Table 4 shows the comparison of crack properties like crack depth and crack location predicted by ANN with the results obtained by FEM with the actual. The results are in close agreement. Similar procedure is extended for two cracks at different locations with varying severity in case of cantilever beam. The accuracy in prediction of crack properties is more for single crack than two cracks.

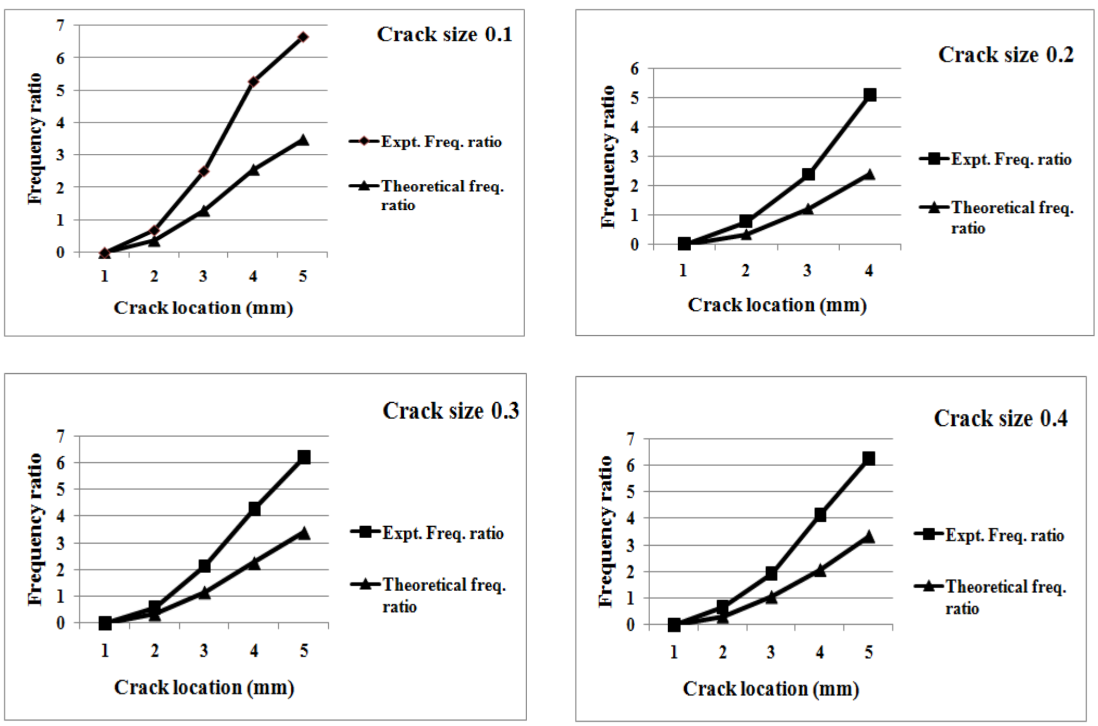

Figure 3. Comparison of Theoretical and Practical Results

Table 3. Comparison of Crack Location and Size Determined for Single Crack (Experimental method and FEA)

Actual (mm)		FEA (mm)		Experimental (mm)		% Error	
a/h	e	a/h	e	a/h	e	a/h	e
0.1	0.2	0.1101	0.1968	0.1206	0.1826	8.7065	-7.7766
0.1	0.4	0.1057	0.3856	0.1093	0.372	3.2937	-3.6559
0.1	0.6	0.1079	0.5897	0.1282	0.6175	15.835	4.50202
0.1	0.8	0.1096	0.7654	0.1205	0.6998	8.9876	-9.3741
0.2	0.2	0.2055	0.1939	0.2093	0.2057	1.8156	5.73651
0.2	0.4	0.1984	0.3749	0.2033	0.3778	2.4102	0.7676
0.2	0.6	0.1945	0.5659	0.2118	0.6107	8.1681	7.33584
0.2	0.8	0.1774	0.7568	0.1629	0.7487	-8.901	-1.0819
0.3	0.2	0.3049	0.1903	0.3124	0.1587	2.4008	-19.912
0.3	0.4	0.3004	0.3626	0.307	0.3285	2.1498	-10.381
0.3	0.6	0.2837	0.5417	0.3226	0.5013	12.058	-8.059
0.3	0.8	0.259	0.7463	0.2361	0.638	-9.699	-16.975
0.4	0.2	0.4064	0.1868	0.4191	0.1934	3.0303	3.41262
0.4	0.4	0.397	0.3447	0.3424	0.3701	-15.95	6.86301
0.4	0.6	0.3732	0.5125	0.3413	0.511	-9.347	-0.2935
0.4	0.8	0.3475	0.7347	0.3465	0.6632	-0.289	-10.781

Table 4. Comparison of Crack Properties Predicted by ANN with FEM for Single Crack in Simply Supported Beam

Actual (mm)		FEM Results (mm)		ANN Results (mm)		% Error in prediction	
a/h	e	a/h	e	a/h	e	a/h	e
0.1	0.2	0.1101	0.1968	0.1206	0.1826	8.7065	-7.7766
0.1	0.4	0.1057	0.3856	0.1093	0.372	3.2937	-3.6559
0.1	0.6	0.1079	0.5897	0.1282	0.6175	15.835	4.50202
0.1	0.8	0.1096	0.7654	0.1205	0.6998	8.9876	-9.3741
0.2	0.2	0.2055	0.1939	0.2093	0.2057	1.8156	5.73651
0.2	0.4	0.1984	0.3749	0.2033	0.3778	2.4102	0.7676
0.2	0.6	0.1945	0.5659	0.2118	0.6107	8.1681	7.33584
0.2	0.8	0.1774	0.7568	0.1629	0.7487	-8.901	-1.0819
0.3	0.4	0.3004	0.3626	0.307	0.3285	2.1498	-10.381
0.3	0.6	0.2837	0.5417	0.3226	0.5013	12.058	-8.059
0.3	0.8	0.259	0.7463	0.2361	0.638	-9.699	-16.975
0.4	0.2	0.4064	0.1868	0.4191	0.1934	3.0303	3.41262
0.4	0.4	0.397	0.3447	0.3424	0.3701	-15.95	6.86301
0.4	0.6	0.3732	0.5125	0.3413	0.511	-9.347	-0.2935
0.4	0.8	0.3475	0.7347	0.3465	0.6632	-0.289	-10.781

9. Conclusion

This work attempts to establish a systematic method of prediction of crack characteristics from measurement of natural frequencies using ANN. From the numerical and experimental study, following conclusions can be drawn

i. Variation in natural frequencies of first two to five modes is observed as they are predominant in crack properties.
ii. The results of Finite Element Analysis and experimental analysis are compared and they are in good agreement.
iii. For the same severity of crack, the frequency reduction is more for location of crack away from the support because of the stiffness of the structure; vibrations get suppressed near the supports.
iv. The error in prediction of crack location by theoretical analysis is in the range of 3% to 15% where as in case of experimental analysis; it is in the range of 5% to 20%. The variation in the experimental results is due to structure prone vibrations and vibrations getting transmitted through foundation.
v. The database obtained is used to as input to train the Neural Network. Appropriately trained Network can predict crack characteristics like depth and location by giving the natural frequency as input.
vi. The predictions of crack location and depth by ANN are verified with the results of FEM. The results are in good agreement with error of 1% to 5% for single crack whereas up to 15% for multiple cracks.
vii. In the present study, the beams under consideration have uniform cross section but this method can be extended to components with varying cross section, different geometry and any boundary condition.
viii. The proposed method can be extended for fault diagnosis in beams, shafts or rotating machine element.

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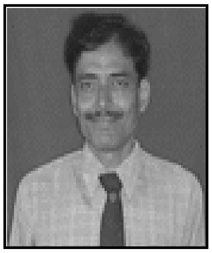

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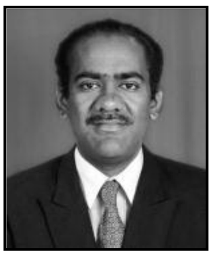

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