Parallel Soft Computing Control Optimization Algorithm for Uncertainty Dynamic Systems

Mansour Bazregar, Farzin Piltan, AliReza Nabaee and Mohammad Mahdi Ebrahimi

Senior Researcher at Research and Development Department, SSP Research and Development Company, Shiraz/ Iran; WWW.IRANSSP.COM
Piltan_f@iranssp.com

Abstract

This research contributes to the on-going research effort by exploring alternate methods for soft computing optimization the highly nonlinear and uncertain systems. This research addresses two basic issues related to the control of an uncertain system; (1) design of a robust feedback controller, and (2) the design of a parallel artificial intelligence based optimization to increase the result qualification. The robust backstepping controller proposed in this research is used to further demonstrate the appealing features exhibited by the continuum robot. Robust feedback controller is used to position control of continuum robot in presence of uncertainties. Using Lyapunov type stability arguments, a robust backstepping controller is designed to achieve this objective. The controller developed in this research is designed in two steps. Firstly, a robust stabilizing torque is designed for the nominal continuum robot dynamics derived using the constrained Lagrangian formulation. Next, the fuzzy logic methodology applied to it to solution uncertainty problem by parallel optimization. The fuzzy model free optimization is formulated to minimize the problem of nonlinear formulation of uncertain systems.

Keywords*: optimization, backstepping methodology, fuzzy inference engine, continuum robot manipulator*

1. INTRODUCTION AND BACKGROUND

The designed controller not only demonstrates the appealing features exhibited by the robot, but also demonstrates some of the nice features of backstepping-type controllers as well. The work done in this research contributes to bringing the continuum robot a step closer to practical industrial application. Controller is a device which can sense information from linear or nonlinear system (*e.g.*, continuum robot) to improve the systems performance [7-9]. The main targets in designing control systems are stability, good disturbance rejection, and small tracking error [7-12]. Several continuum robot are controlled by linear methodologies (*e.g.*, Proportional-Derivative (PD) controller, Proportional- Integral (PI) controller or Proportional- Integral-Derivative (PID) controller), but when robot works with various payloads and have uncertainty in dynamic models this technique has limitations. In some applications continuum robot are used in an unknown and unstructured environment, therefore strong mathematical tools used in new control methodologies to design nonlinear robust controller with an acceptable performance (*e.g.*, minimum error, good trajectory, disturbance rejection) [8-10].

Advanced control techniques such as integrator backstepping, feedback linearization, adaptive and robust have been applied to the control of numerous single -axis machines and robotic manipulators. Since available control techniques for continuum robot and robotic manipulators are so broad, the review in this research is restricted to some of the nonlinear control techniques for continuum robot. Most of the authors referenced on the nonlinear control techniques for continuum robot also do a significant amount of work on the control of robotic manipulators. Kokotovic [13] published one of the pioneering works on the backstepping control technique and Qu, *et al.*, [14] extended this technique and developed a robust backstepping-type controller for a one-link robot with the motor dynamics taken into consideration. Carroll, *et al.*, [15] also extended the work of Kokotovic [16] to design an embedded computed torque and output feedback controller for pennanent magnet brush de (BDC) motors. Hemati, *et al.*, [16] developed a robust feedback linearizing controller for a single-link robot actuated by a brushless de motor (BLOC). In [17], Carroll et al. also developed a robust tracking controller for a BLOC, which achieved globally bounded results for rotor position tracking error despite parametric uncertainties and additive bounded disturbances. In addition to DC machines, SR and PM stepper motors are also candidates for advanced nonlinear controllers. In [18], Die'-Spong, *et al.*, introduced a detailed nonlinear model and an electronic commutation strategy for the SR motor and applied a state feedback control algorithm which compensated for all the nonlinearities of the system. The work in [18] was then generalized to a direct-drive manipulator with SR actuation by Taylor, *et al.*, [19]. Carroll, *et al.*, [20] also used a backstepping technique to develop an adaptive tracking controller for the SR motor. Bodson [21] developed a model-based control law for the PM stepper motor using an exact linearization methodology while considering practical issues such as voltage saturation. Even though some of the above control techniques are not of the backstepping type controller, the backstepping-type controller developed in this thesis was somewhat inspired by them.

Continuum robots represent a class of robots that have a biologically inspired form characterized by flexible backbones and high degrees-of-freedom structures [1]. The idea of creating “trunk and tentacle” robots, (in recent years termed continuum robots [1]), is not new [2]. Inspired by the bodies of animals such as snakes [3], the arms of octopi [4], and the trunks of elephants [5, 6], researchers have been building prototypes for many years. A key motivation in this research has been to reproduce in robots some of the special qualities of the biological counterparts. This includes the ability to “slither” into tight and congested spaces, and (of particular interest in this work) the ability to grasp and manipulate a wide range of objects, via the use of “whole arm manipulation” *i.e.*, wrapping their bodies around objects, conforming to their shape profiles. Hence, these robots have potential applications in whole arm grasping and manipulation in unstructured environments such as rescue operations. Theoretically, the compliant nature of a continuum robot provides infinite degrees of freedom to these devices. However, there is a limitation set by the practical inability to incorporate infinite actuators in the device. Most of these robots are consequently underactuated (in terms of numbers of independent actuators) with respect to their anticipated tasks. In other words they must achieve a wide range of configurations with relatively few control inputs. This is partly due to the desire to keep the body structures (which, unlike in conventional rigid-link manipulators or fingers, are required to directly contact the environment) "clean and soft", but also to exploit the extra control authority available due to the continuum contact conditions with a minimum number of actuators. For example, the Octarm VI continuum manipulator, discussed frequently in this paper, has nine independent actuated degrees-of-freedom with only three sections. Continuum manipulators differ fundamentally from rigid-link and hyper-redundant robots by having an unconventional structure that lacks links and joints. Hence, standard techniques like the Denavit-Hartenberg (D-H) algorithm cannot be directly applied for developing continuum arm kinematics. Moreover, the design of each continuum arm varies with respect to the flexible backbone present in the system, the positioning, type and number of actuators. The constraints imposed by these factors make the set of reachable configurations and nature of movements unique to every continuum robot. This makes it difficult to formulate generalized kinematic or dynamic models for continuum robot hardware. Chirikjian and Burdick were the first to introduce a method for modeling the kinematics of a continuum structure by representing the curve-shaping function using modal functions [6]. Mochiyama used the Serret- Frenet formulae to develop kinematics of hyper-degrees of freedom continuum manipulators [5]. For details on the previously developed and more manipulator-specific kinematics of the Rice/Clemson "Elephant trunk" manipulator, see [1, 2, 5]. For the Air Octor and Octarm continuum robots, more general forward and inverse kinematics have been developed by incorporating the transformations of each section of the manipulator (using D-H parameters of an equivalent virtual rigid link robot) and expressing those in terms of the continuum manipulator section parameters [4]. The net result of the work in [6, 3-5] is the establishment of a general set of kinematic algorithms for continuum robots. Thus, the kinematics (*i.e.*, geometry based modeling) of a quite general set of prototypes of continuum manipulators has been developed and basic control strategies now exist based on these. The development of analytical models to analyze continuum arm dynamics (*i.e.*, physicsbased models involving forces in addition to geometry) is an active, ongoing research topic in this field. From a practical perspective, the modeling approaches currently available in the literature prove to be very complicated and a dynamic model which could be conveniently implemented in an actual device's real-time controller has not been developed yet. The absence of a computationally tractable dynamic model for these robots also prevents the study of interaction of external forces and the impact of collisions on these continuum structures. This impedes the study and ultimate usage of continuum robots in various practical applications like grasping and manipulation, where impulsive dynamics [1], [4] are important factors. Although continuum robotics is an interesting subclass of robotics with promising applications for the future, from the current state of the literature, this field is still in its stages of inception.

Although the fuzzy-logic control is not a new technique, its application in this current research is considered to be novel since it aimed for an automated dynamic-less response rather than for the traditional objective of uncertainties compensation [14]. The intelligent tracking control using the fuzzy-logic technique provides a cost-and-time efficient control implementation due to the automated dynamic-less input. This in turn would further inspire multi-uncertainties testing for continuum robot [9-14].

This paper is organized as follows; Section 2, is served as an introduction to the feedback backstepping controller formulation algorithm and its application to control of continuum robot and dynamic of continuum robot. Part 3, introduces and describes the methodology. Section 4 presents the simulation results and discussion of this algorithm applied to a continuum robot and the final section is describing the conclusion.

2. Theory

The Continuum section analytical model developed here consists of three modules stacked together in series. In general, the model will be a more precise replication of the behavior of a continuum arm with a greater of modules included in series. However, we will show that three modules effectively represent the dynamic behavior of the hardware, so more complex models are not motivated. Thus, the constant curvature bend exhibited by the section is incorporated inherently within the model. The model resulting from the application of Lagrange's equations of motion obtained for this system can be represented in the form

$$\boldsymbol{F_{coeff}}\ \underline{\boldsymbol{\tau}} = \boldsymbol{D}\left(\underline{\boldsymbol{q}}\right)\underline{\ddot{\boldsymbol{q}}} + \boldsymbol{C}\left(\underline{\boldsymbol{q}}\right)\underline{\dot{\boldsymbol{q}}} + \boldsymbol{G}\left(\underline{\boldsymbol{q}}\right) \quad \textbf{(1)}$$

where τ is a vector of input forces and q is a vector of generalized co-ordinates. The force coefficient matrix F_{coeff} transforms the input forces to the generalized forces and torques in the system. The inertia matrix, D is composed of four block matrices. The block matrices that correspond to pure linear accelerations and pure angular accelerations in the system (on the top left and on the bottom right) are symmetric. The matrix C contains coefficients of the first order derivatives of the generalized co-ordinates. Since the system is nonlinear, many elements of C contain first order derivatives of the generalized co-ordinates. The remaining terms in the dynamic equations resulting from gravitational potential energies and spring energies are collected in the matrix G. The coefficient matrices of the dynamic equations are given below,

$$\boldsymbol{Fcoeff} = \quad \textbf{(2)}$$

$$\begin{bmatrix} 1 & 1 & cos(\theta_1) & cos(\theta_1) & cos(\theta_1+\theta_2) & cos(\theta_1+\theta_2) \\ 0 & 0 & 1 & 1 & cos(\theta_2) & cos(\theta_2) \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1/2 & -1/2 & 1/2 & -1/2 & 1/2 + s_2 sin(\theta_2) & -1/2 + s_2 sin(\theta_2) \\ 0 & 0 & 1/2 & -1/2 & 1/2 & -1/2 \\ 0 & 0 & 0 & 0 & 1/2 & -1/2 \end{bmatrix}$$

$$\boldsymbol{D}\left(\underline{\boldsymbol{q}}\right) = \quad \textbf{(3)}$$

$$\begin{bmatrix} \begin{matrix} m_1+m_2 \\ +m_3 \end{matrix} & \begin{matrix} m_2 cos(\theta_1) \\ +m_3 cos(\theta_1) \end{matrix} & m_3 cos(\theta_1+\theta_2) & \begin{matrix} -m_2 s_2 sin(\theta_1) \\ -m_3 s_2 sin(\theta_1) \\ -m_3 s_3 sin(\theta_1+\theta_2) \end{matrix} & -m_3 s_3 sin(\theta_1+\theta_2) & 0 \\ \begin{matrix} m_2 cos(\theta_1) \\ +m_3 cos(\theta_1) \end{matrix} & m_2+m_3 & m_3 cos(\theta_2) & -m_3 s_3 sin(\theta_2) & -m_3 s_3 sin(\theta_2) & 0 \\ m_3 cos(\theta_1+\theta_2) & m_3 cos(\theta_2) & m_3 & m_3 s_3 sin(\theta_2) & 0 & 0 \\ \begin{matrix} -m_2 s_2 sin(\theta_1) \\ -m_3 s_2 sin(\theta_1) \\ -m_3 s_3 sin(\theta_1+\theta_2) \end{matrix} & -m_3 s_3 sin(\theta_2) & m_3 s_2 sin(\theta_2) & \begin{matrix} m_2 s_2^2 + I_1 + I_2 \\ +I_3 + m_3 s_2^2 + m_3 s_3^2 \\ +2 m_3 s_3 cos(\theta_2) s_2 \end{matrix} & \begin{matrix} I_2 + m_3 s_3^2 + I_3 \\ +m_3 s_3 cos(\theta_2) s_2 \end{matrix} & I_3 \\ -m_3 s_3 sin(\theta_1+\theta_2) & -m_3 s_3 sin(\theta_2) & 0 & \begin{matrix} I_2 + m_3 s_3^2 + I_3 \\ +m_3 s_3 cos(\theta_2) s_2 I \end{matrix} & I_2 + m_3 s_3^2 + I_3 & I_3 \\ 0 & 0 & 0 & I_3 & I_3 & I_3 \end{bmatrix}$$

$$\boldsymbol{C}\left(\underline{\boldsymbol{q}}\right) = \tag{4}$$

$$\begin{bmatrix} c_{11}+c_{21} & \begin{matrix} -2m_2 sin(\theta_1)\dot{\theta}_1 \\ -2m_3 sin(\theta_1)\dot{\theta}_1 \end{matrix} & \begin{matrix} -2m_3 sin(\theta_1+\theta_2) \\ (\dot{\theta}_1+\dot{\theta}_2) \end{matrix} & \begin{matrix} -m_2 s_2 \\ cos(\theta_1)(\dot{\theta}_1) \\ +(1/2)(c_{11}+c_{21}) \\ -m_3 s_2 \\ cos(\theta_1)(\dot{\theta}_1) \\ -m_3 s_3 \\ cos(\theta_1+\theta_2)(\dot{\theta}_1) \end{matrix} & -m_3 s_3 sin(\theta_1+\theta_2) & 0 \\ 0 & c_{12}+c_{22} & \begin{matrix} -2m_3 sin(\theta_2) \\ (\dot{\theta}_1+\dot{\theta}_2) \end{matrix} & \begin{matrix} -m_3 s_3(\dot{\theta}_1) \\ +(1/2) \\ (c_{12}+c_{22}) \\ -m_3 s_2(\dot{\theta}_1) \\ -m_3 s_3 \\ cos(\theta_2)(\dot{\theta}_1) \end{matrix} & \begin{matrix} -2m_3 s_3 \\ cos(\theta_2)(\dot{\theta}_1) \\ -m_3 s_3 \\ cos(\theta_2)(\dot{\theta}_2) \end{matrix} & 0 \\ 0 & 2m_3 sin(\theta_2)(\dot{\theta}_1) & c_{13}+c_{23} & \begin{matrix} -m_3 s_3 s_2 \\ cos(\theta_2)(\dot{\theta}_1) \\ -m_3 s_3(\dot{\theta}_1) \end{matrix} & \begin{matrix} -2m_3 s_3(\dot{\theta}_1) \\ -m_3 s_3(\dot{\theta}_2) \end{matrix} & \begin{matrix} (1/2) \\ (c_{13}+c_{23}) \end{matrix} \\ \begin{matrix} (1/2) \\ (c_{11}+c_{21}) \end{matrix} & \begin{matrix} 2m_3 s_3 cos(\theta_2)(\dot{\theta}_1) \\ -2m_3 s_2(\dot{\theta}_1) \\ +2m_2 s_2(\dot{\theta}_1) \end{matrix} & \begin{matrix} 2m_3 s_3(\dot{\theta}_1+\dot{\theta}_2) \\ -2m_3 s_2 cos(\theta_2) \\ (\dot{\theta}_1+\dot{\theta}_2) \end{matrix} & \begin{matrix} 2m_3 s_3 s_2 \\ sin(\theta_2)(\dot{\theta}_2) \\ +(1^2/4) \\ (c_{11}+c_{21}) \end{matrix} & \begin{matrix} m_3 s_3 s_2 \\ sin(\theta_2)(\dot{\theta}_2) \end{matrix} & 0 \\ 0 & \begin{matrix} (1/2)(c_{12}+c_{22})+ \\ 2m_3 s_3 cos(\theta_2)(\dot{\theta}_1) \end{matrix} & \begin{matrix} 2m_3 s_3 \\ (\dot{\theta}_1+\dot{\theta}_2) \end{matrix} & \begin{matrix} m_3 s_3 s_2 \\ sin(\theta_2)(\dot{\theta}_1) \end{matrix} & \begin{matrix} (1^2/4) \\ (c_{12}+c_{22}) \end{matrix} & 0 \\ 0 & 0 & (1/2)(c_{13}-c_{23}) & 0 & 0 & \begin{matrix} (1^2/4) \\ (c_{13}+c_{23}) \end{matrix} \end{bmatrix}$$

$$\boldsymbol{G}\left(\underline{\boldsymbol{q}}\right) = \tag{5}$$

$$\begin{bmatrix} -m_1 g - m_2 g + k_{11}(s_1 + (1/2)\theta_1 - s_{01}) + k_{21}(s_1 - (1/2)\theta_1 - s_{01}) - m_3 g \\ -m_2 g cos(\theta_1) + k_{12}(s_2 + (1/2)\theta_2 - s_{02}) + k_{22}(s_2 - (1/2)\theta_2 - s_{02}) - m_3 g cos \\ -m_3 g cos(\theta_1+\theta_2) + k_{13}(s_3 + (1/2)\theta_3 - s_{03}) + k_{23}(s_3 - (1/2)\theta_3 - s_{03}) \\ \begin{matrix} m_2 s_2 g sin(\theta_1) + m_3 s_3 g sin(\theta_1+\theta_2) + m_3 s_2 g sin(\theta_1) + k_{11}(s_1 + (1/2)\theta_1 - s_{01})(\\ +k_{21}(s_1 - (1/2)\theta_1 - s_{01})(-1/2) \end{matrix} \\ m_3 s_3 g sin(\theta_1+\theta_2) + k_{12}(s_2 + (1/2)\theta_2 - s_{02})(1/2) + k_{22}(s_2 - (1/2)\theta_2 - s_{02})(- \\ k_{13}(s_3 + (1/2)\theta_3 - s_{03})(1/2) + k_{23}(s_3 - (1/2)\theta_3 - s_{03})(-1/2) \end{bmatrix}$$

Backstepping Methodology: The continuum robot dynamics in Equations have the appropriate structure for the so-called backstepping controller design method. With the position error defined as $Z_1 = X_d - X_a$, all joints will track the desired specified state X_d if the error dynamics are given as follows:

$$\left(\dot{\boldsymbol{Z}}_{\boldsymbol{1}} + [\boldsymbol{K}_{\boldsymbol{p}}]\boldsymbol{Z}_{\boldsymbol{1}}\right) = \boldsymbol{0} \tag{6}$$

where $[K_p]$ is a positive definite gain matrix. The error dynamics in Equation (6) can be rewritten as:

$$\boldsymbol{X}_{\boldsymbol{2}} = \dot{\boldsymbol{X}}_{\boldsymbol{d}} + [\boldsymbol{K}_{\boldsymbol{p}}]\boldsymbol{Z}_{\boldsymbol{1}} \tag{7}$$

Substitution of Equation (7) into Equation (1) makes the position error dynamics go to zero. Since the state vector x_2 is not a control variable, Equation (7) cannot be directly substituted into Equation (1). The expression in Equation (7) is therefore defined as a fictitious control input and expressed below as X_{2_d}.

$$\boldsymbol{X}_{\boldsymbol{2}_{\boldsymbol{d}}} = \dot{\boldsymbol{X}}_{\boldsymbol{1}_{\boldsymbol{d}}} + [\boldsymbol{K}_{\boldsymbol{p}}](\boldsymbol{X}_{\boldsymbol{d}} - \boldsymbol{X}_{\boldsymbol{a}}) \tag{8}$$

The fictitious control input in Equation (8) is selected as the specified velocity trajectory and hence the velocity error can be defined as $Z_2 = X_{2_d} - X_{2_a}$. With the following dynamics

$$\left(\dot{\boldsymbol{Z}}_{\boldsymbol{2}} + [\boldsymbol{K}_{\boldsymbol{p}}]\boldsymbol{Z}_{\boldsymbol{2}}\right) = \boldsymbol{0} \tag{9}$$

the joint position error will approach zero asymptotically, which will lead to the eventual asymptotic convergence of the joint position error. The error dynamics in equation (9) can be rewritten as:

$$\boldsymbol{X}_{\boldsymbol{2}} = \dot{\boldsymbol{X}}_{\boldsymbol{d}} + [\boldsymbol{K}_{\boldsymbol{p}}]\boldsymbol{Z}_{\boldsymbol{2}} \tag{10}$$

Substitution of Equation (9) into Equation (1) leads to the following expression as the desired stabilizing torque:

$$\boldsymbol{\tau} = [\boldsymbol{H}](\dot{\boldsymbol{X}}_{\boldsymbol{2}_{\boldsymbol{d}}} + [\boldsymbol{K}_{\boldsymbol{p}}]\boldsymbol{Z}_{\boldsymbol{2}}) + \boldsymbol{C}(\boldsymbol{X}_{\boldsymbol{1}}, \boldsymbol{X}_{\boldsymbol{2}}) \tag{11}$$

The desired torque control input is a nonlinear compensator since it depends on the dynamics of the spherical motor. The time derivative of desired velocity vector is calculated using Equation (9). In terms of the desired state trajectory, and its time derivatives and the position and velocity state variables, the desired torque can be rewritten in following form:

$$\boldsymbol{\tau} = [\boldsymbol{H}]\boldsymbol{y} + \boldsymbol{C}(\boldsymbol{X}_{\boldsymbol{1}}, \boldsymbol{X}_{\boldsymbol{2}}) \tag{12}$$

Where

$$\boldsymbol{y} = \ddot{\boldsymbol{X}}_{\boldsymbol{1}_{\boldsymbol{d}}} + \left([\boldsymbol{K}_{\boldsymbol{p}}] + [\boldsymbol{K}_{\boldsymbol{d}}]\right)\left(\dot{\boldsymbol{X}}_{\boldsymbol{1}_{\boldsymbol{d}}} - \dot{\boldsymbol{X}}_{\boldsymbol{1}}\right) + ([\boldsymbol{K}_{\boldsymbol{p}}][\boldsymbol{K}_{\boldsymbol{d}}]\boldsymbol{X}_{\boldsymbol{d}} - \boldsymbol{X}_{\boldsymbol{a}} \tag{13}$$

The backstepping controller developed above is very similar to inverse dynamics control algorithm developed for robotic manipulators. The backstepping controller is ideal from a control point of view as the nonlinear dynamics of the continuum robot are cancelled and replaced by linear subsystems. The drawback of the backstepping controller is that it requires perfect cancellation of the nonlinear continuum robot dynamics. Accurate real time representations of the robot dynamics are difficulty due to uncertainties in the system dynamics resulting from imperfect knowledge of the robot mechanical parameters; existence of unmodeled dynamics and dynamic uncertainties due to payloads. The requirement for perfect dynamic cancellation raises sensitivity and robustness issues that are addressed in the design of a robust backstepping controller. Another drawback of the backstepping controller is felt during real-time implementation of the control algorithm. Implementation of the backstepping controller requires the computation of the exact robot dynamics at each sampling time. This computational burden has an effect on the performance of the control algorithm and imposes constraints on the hardware/software architecture of the control system. By only computing the dominant parts of the robot dynamics, this computational burden can be reduced. These drawbacks of the backstepping controller makes it necessary to consider control algorithms that compensate for both model uncertainties and for approximations made during the on-line computation of robot dynamics. Figure 1 shows the backstepping optimization controller.

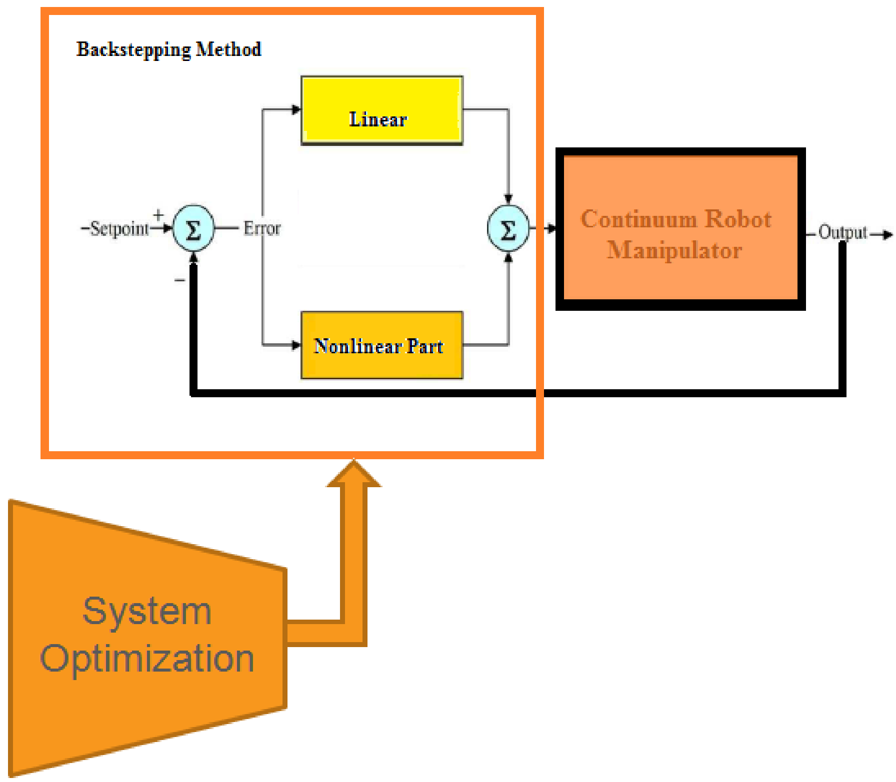

Figure 1: Backstepping Optimization Control Methodology with Application to Continuum Robot

Fuzzy Logic Theory: This section provides a review about foundation of fuzzy logic based on [11-16]. Supposed that U is the universe of discourse and x is the element of U, therefore, a crisp set can be defined as a set which consists of different elements (x) will all or no membership in a set. A fuzzy set is a set that each element has a membership grade, therefore it can be written by the following definition;

$$\boldsymbol{A = \{x, \mu_A(x) | x \in X\}; A \in U} \quad \textbf{(14)}$$

Where an element of universe of discourse is x, μ_A is the membership function (MF) of fuzzy set. The membership function ($\mu_A(x)$) of fuzzy set A must have a value between zero and one. If the membership function $\mu_A(x)$ value equal to zero or one, this set change to a crisp set but if it has a value between zero and one, it is a fuzzy set. Defining membership function for fuzzy sets has divided into two main groups; namely; numerical and functional method, which in numerical method each number has different degrees of membership function and functional method used standard functions in fuzzy sets. The membership function which is often used in practical applications includes triangular form, trapezoidal form, bell-shaped form, and Gaussian form.

Linguistic variable can open a wide area to use of fuzzy logic theory in many applications (e.g., control and system identification). In a natural artificial language all numbers replaced by words or sentences.

$If - then$ Rule statements are used to formulate the condition statements in fuzzy logic. A single fuzzy $If - then$ rule can be written by

$$\boldsymbol{If\ x\ is\ A\ Then\ y\ is\ B} \quad \textbf{(15)}$$

where A and B are the Linguistic values that can be defined by fuzzy set, the $If - part$ of the part of *"$x\ is\ A$"* is called the antecedent part and the $then - part$ of the part of *"$y\ is\ B$"* is called the Consequent or Conclusion part. The antecedent of a fuzzy if-then rule can have multiple parts, which the following rules shows the multiple antecedent rules:

$$\boldsymbol{if\ e\ is\ NB\ and\ \dot{e}\ is\ ML\ then\ T\ is\ LL} \quad \textbf{(16)}$$

where e is error, $\dot{e}$ is change of error, NB is Negative Big, ML is Medium Left, T is torque and LL is Large Left. $If - then$ rules have three parts, namely, fuzzify inputs, apply fuzzy operator and apply implication method which in fuzzify inputs the fuzzy statements in the antecedent replaced by the degree of membership, apply fuzzy operator used when the antecedent has multiple parts and replaced by single number between 0 to 1, this part is a degree of support for the fuzzy rule, and apply implication method used in consequent of fuzzy rule to replaced by the degree of membership. The fuzzy inference engine offers a mechanism for transferring the rule base in fuzzy set which it is divided into two most important methods, namely, Mamdani method and Sugeno method. Mamdani method is one of the common fuzzy inference systems and he designed one of the first fuzzy controllers to control of system engine. Mamdani's fuzzy inference system is divided into four major steps: fuzzification, rule evaluation, aggregation of the rule outputs and defuzzification. Michio Sugeno uses a singleton as a membership function of the rule consequent part. The following definition shows the Mamdani and Sugeno fuzzy rule base

$$\begin{array}{llllll} \boldsymbol{Mamdani} & \boldsymbol{F.R^1: if} & \boldsymbol{x\ is\ A\ and} & \boldsymbol{y\ is\ B} & \boldsymbol{then} & \boldsymbol{z\ is\ C} \\ \boldsymbol{Sugeno} & \boldsymbol{F.R^1: if} & \boldsymbol{x\ is\ A\ and} & \boldsymbol{y\ is\ B} & \boldsymbol{then} & \boldsymbol{f(x,y) is\ C} \end{array} \quad \textbf{(17)}$$

When x and y have crisp values fuzzification calculates the membership degrees for antecedent part. Rule evaluation focuses on fuzzy operation (AND/OR) in the antecedent of the fuzzy rules. The aggregation is used to calculate the output fuzzy set and several methodologies can be used in fuzzy logic controller aggregation, namely, Max-Min aggregation, Sum-Min aggregation, Max-bounded product, Max-drastic product, Max-bounded sum, Max-algebraic sum and Min-max. Two most common methods that used in fuzzy logic controllers are Max-min aggregation and Sum-min aggregation. Max-min aggregation defined as below

$$\boldsymbol{\mu_U(x_k, y_k, U) = \mu_{\bigcup_{i=1}^{r} FR^i}(x_k, y_k, U) = \max\left\{\min_{i=1}^{r}\left[\mu_{R_{pq}}(x_k, y_k), \mu_{p_m}(U)\right]\right\}} \tag{18}$$

The Sum-min aggregation defined as below

$$\boldsymbol{\mu_U(x_k, y_k, U) = \mu_{\bigcup_{i=1}^{r} FR^i}(x_k, y_k, U) = \sum \min_{i=1}^{r}\left[\mu_{R_{pq}}(x_k, y_k), \mu_{p_m}(U)\right]} \tag{19}$$

where r is the number of fuzzy rules activated by x_k and y_k and also $\mu_{\bigcup_{i=1}^{r} FR^i}(x_k, y_k, U)$ is a fuzzy interpretation of $i - th$ rule. Defuzzification is the last step in the fuzzy inference system which it is used to transform fuzzy set to crisp set. Consequently defuzzification's input is the aggregate output and the defuzzification's output is a crisp number. Centre of gravity method (COG) and Centre of area method (COA) are two most common defuzzification methods, which COG method used the following equation to calculate the defuzzification

$$\boldsymbol{COG(x_k, y_k) = \frac{\sum_i U_i \sum_{j=1}^{r} . \mu_u(x_k, y_k, U_i)}{\sum_i \sum_{j=1}^{r} . \mu_u(x_k, y_k, U_i)}} \tag{20}$$

and COA method used the following equation to calculate the defuzzification

$$\boldsymbol{COA(x_k, y_k) = \frac{\sum_i U_i . \mu_u(x_k, y_k, U_i)}{\sum_i \mu_U . (x_k, y_k, U_i)}} \tag{21}$$

Where $COG(x_k, y_k)$ and $COA(x_k, y_k)$ illustrates the crisp value of defuzzification output, $U_i \in U$ is discrete element of an output of the fuzzy set, $\mu_U.(x_k, y_k, U_i)$ is the fuzzy set membership function, and r is the number of fuzzy rules.

Based on foundation of fuzzy logic methodology; fuzzy logic controller has played important rule to design nonlinear controller for nonlinear and uncertain systems [20-32]. However the application area for fuzzy control is really wide, the basic form for all command types of controllers consists of:

- Input fuzzification (binary-to-fuzzy[B/F]conversion)
- Fuzzy rule base (knowledge base)
- Inference engine
- Output defuzzification (fuzzy-to-binary [F/B] conversion).

Figure 2 shows the blockdigram of fuzzy logic control methodology based on two inputs and one output.

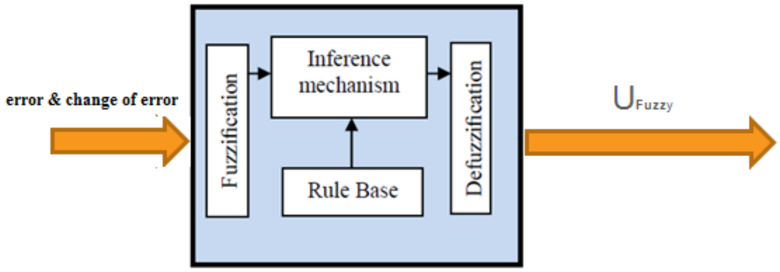

Figure 2. Block Diagram of Fuzzy Logic Control Methodology

3. Methodology

This step is focused on the design Mamdani's fuzzy optimization backstepping controller with application to continuum robot manipulator. As mentioned above pure backstepping controller has nonlinear dynamic parameters limitation in presence of uncertainty and external disturbances. In order to solve this challenge this work is used Mamdani's fuzzy inference engine optimization and adds to backstepping controller. The backstepping method is based on mathematical formulation which this method is introduced new variables into it in form depending on the dynamic equation of continuum robot arm. This method is used as feedback linearization in order to solve nonlinearities in the system. In this research fuzzy methodology is used to estimate the part of nonlinearity term in backstepping controller. The backstepping controller for continuum robot is calculated by;

$$\boldsymbol{U_{B.S} = U_{eq_{B.S}} + D.I} \tag{22}$$

Where $\boldsymbol{U_{B.S}}$ is backstepping output function, $\boldsymbol{U_{eq_{B.S}}}$ is backstepping nonlinear equivalent function which can be written as (22) and $\boldsymbol{I}$ is backstepping control law which calculated by (23)

$$\boldsymbol{U_{eq_{B.S}}} = [\boldsymbol{C} + \boldsymbol{G}] \tag{23}$$

We have

$$\boldsymbol{I = \left[\ddot{\theta} + K_1(K_1 - 1)\cdot e + (K_1 + K_2)\cdot \dot{e}\right]} \tag{24}$$

Based on (11) and (24) the fuzzy optimization is considered as

$$\boldsymbol{U_{fuzzy} = \sum_{l=1}^{M} \theta^T \zeta(x)} \tag{25}$$

Based on (23) the formulation of fuzzy backstepping filter can be written as;

$$\boldsymbol{U = U_{eqB.S} + DI + U_{fuzzy}} \tag{26}$$

Where $\boldsymbol{U}_{\boldsymbol{eq}_{B.S}} + \boldsymbol{U}_{\boldsymbol{fuzzy}} = [(\boldsymbol{C} + \boldsymbol{G})] + \sum_{l=1}^{M} \boldsymbol{\theta}^{\boldsymbol{T}}\, \boldsymbol{\zeta}(\boldsymbol{x})$

Most robust control designs are based on the assumption that even though the uncertainty vector η is unknown, some information is available on its bound.

Figure 3 shows a block diagram detailing the steps in the robust Mamdani parallel fuzzy optimization backstepping control of continuum robot.

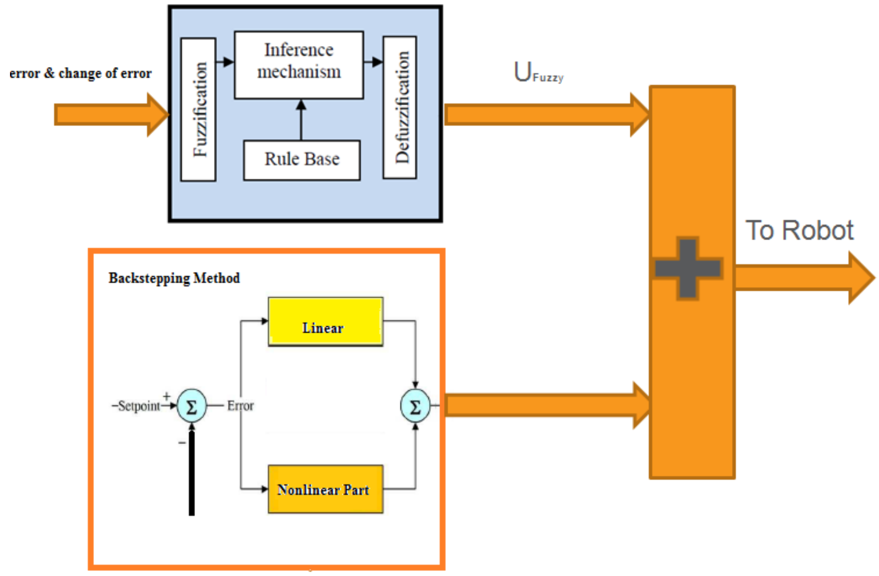

Figure 3. Block Diagram of Mamdani Parallel Fuzzy Optimization Backstepping for Continuum Robot

4. Result and Discussion

Robust parallel Mamdani fuzzy optimization backstepping controller (proposed method) was tested to Step response trajectory. The simulation was implemented in MATLAB/SIMULINK environment.

Tracking Performances: Figure 4 shows tracking performance for proposed method and backstepping controller (BSC) without disturbance.

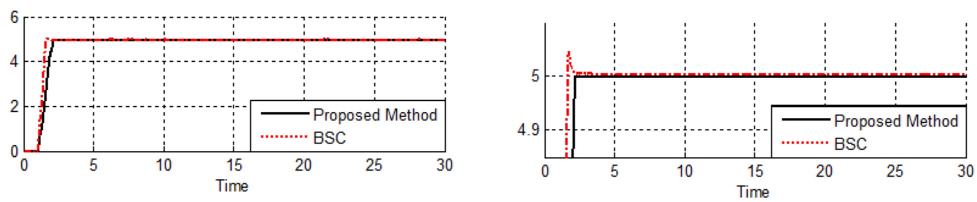

Figure 4. Proposed Method and BSC for Three Joints Trajectory

Disturbance Rejection: Figure 5 shows the power disturbance elimination in BSC and proposed method. The main target in this controller is disturbance rejection as well as the other responses. A band limited white noise with predefined of 40% the power of input signal is applied to BSC and proposed method. It found fairly fluctuations inBSC responses. As mentioned earlier, BSC works very well when all parameters are known, or we have a limitation uncertainty in parameters.

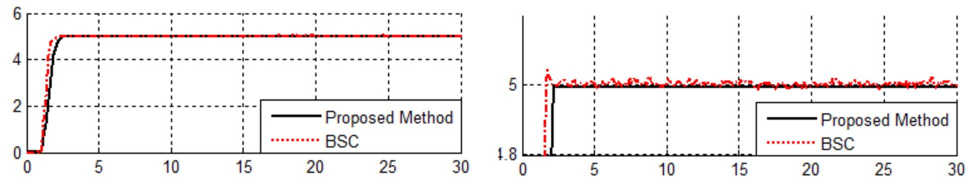

Figure 5. Proposed Method and BSC for Three Joints Trajectory with Disturbance

5. Conclusion

One of the most active research areas in field of continuum robot is artificial nonlinear control of this system because it is a nonlinear, time variant and uncertain system. To control of this system robust backstepping methodology is introduced. Backstepping controller (BSC) is an influential robust nonlinear controller to certain and partly uncertain systems. When all dynamic and physical parameters are known BSC works superbly; practically a large amount of systems have uncertainties and Mamdani parallel fuzzy inference optimization add to backstepping controller. Based on above discussion this method has acceptable performance against to pure backstepping controller in presence of uncertainty.

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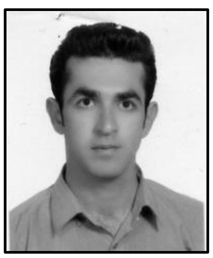
Authors

Mansour Bazregar is an industrial management researcher of research and development company SSP. Co. He is now pursuing his Master in indus trial management. He is an expert Industrial and Quality Management in this company. His research activities deal with the IC engine control, robot control and supply chain management.

Farzin Piltan was born on 1975, Shiraz, Iran. In 2004 he is jointed the research and development company, SSP Co, Shiraz, Iran. In addition to 7 textbooks, Farzin Piltan is the main author of more than 70 scientific papers in refereed journals. He is editorial board of international journal of control and automation (IJCA), editorial board of International Journal of Intelligent System and Applications (IJISA), editorial board of IAES international journal of robotics and automation, editorial board of International Journal of Reconfigurable and Embedded Systems and reviewer of (CSC) intern ational journal of robotics and automation. His main areas of research interests are nonlinear control, artificial control system and applied to FPGA, robotics and artificial nonlinear control and IC engine modelling and control.

AliReza Nabaee is an electrical electronic researcher of research and development company SSP. Co. His main areas of research interests are no nlinear control, artificial control system and robotics.



Mohamma d Mahdi Ebrahimi is an electrical communication researcher of research and development company SSP. Co. His main areas of research interests are nonlinear control, artificial control system and robotics.