Fatigue Life Prediction of a Parabolic Spring under Non-constant Amplitude Proportional Loading using Finite Element Method

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Abstract

This study presents a fatigue life prediction based on finite element analysis under non constant amplitude proportional loading. Parabolic spring is the vital component in a vehicle suspension system, commonly used in trucks. It needs to have excellent fatigue life and recently, manufacturers rely on constant loading fatigue data. The objective of this study is to simulate the non constant amplitude proportional loading for the fatigue life analysis. The finite element method (FEM) was performed on the spring model to observe the distribution of stress and damage. The fatigue life simulation was performed and analyzed for materials SAE1045-450-QT, SAE1045-595-QT, and SAE5160-825-QT. when using the loading sequences is predominantly tensile in the nature; the life of mounting in Goodman approach is more conservative. When the loading is predominantly tensile in nature, the life of the component in Morrow approach is more sensitive and is therefore recommended. It can be concluded that material SAE 1045-595-QT gives constantly higher life than material SAE5160-825-QT SAE1045-450-QT for all loading conditions under both methods.

Keywords: *Fatigue life; Non constant amplitude proportional loading; Parabolic spring; FEM*

1. Introduction

A spring is an elastic object that is used to store mechanical energy, usually made out of hardened steel. Originally called laminated or carriage spring, a leaf spring is a simple form of spring, commonly used for the suspension in wheeled vehicles. Advantage of a leaf spring over a helical spring is that end of the leaf spring may be guided along a definite path. It takes the form of a slender arc shaped length of spring steel of rectangular cross section. The large vehicles need a good suspension system that can deliver a good ride and handling. At the same time, that component needs to be lightweight and have an excellent of fatigue life. Fatigue is one of the major issues in automotive component. It must withstand numerous numbers of cycles before it can fail, or never fail at all during the service period. From the viewpoint of engineering applications, the purpose of fatigue research consists of the prediction of fatigue life on structures, increasing fatigue life and simplifying fatigue tests [1]. In reality, the most engineering components and structures are subjected to non constant amplitude proportional loading conditions at which stress–strain cycles fluctuate with time [2]. At this condition, component tends to fail under various sources of loading. Fatigue failure of mechanical components is a process of cyclic stress/strain evolutions and redistributions in the critical stressed volume [3].Using the FEM approach the fatigue life of parabolic spring is estimated for stress/strain life. Parabolic spring is widely used in automotive and one of the components of suspension system. It consists of one or more leaves

[4]. As a general rule, the leaf spring must be regarded as a safety component [5] as failure could lead to severe accidents [6]. Parabolic springs are subjected to cyclic compression and tension load when the heavy vehicle runs on the road. In industry, only manufacturer manages to test the fatigue life of these springs using constant amplitude loading. This is because non constant amplitude proportional loading fatigue test is time consuming and adds more cost. The aim of this paper is finding both stress life approach and strain life approach for different materials and mentioning which material having maximum life and which approach is gives optimum value for stress approach and strain approach. The model of leaf spring which is shown in Figure 1

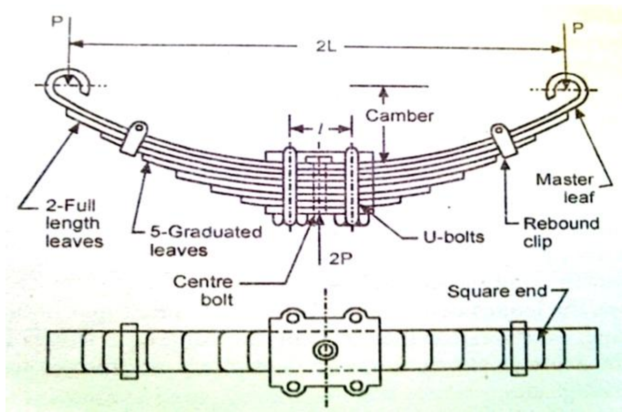

Figure1. Model of Leaf Spring

2. Finite Element Based Fatigue Analysis

The fatigue analysis is used to compute the fatigue life at one location in a structure. For multiple locations the process is repeated using geometry information applicable for each location. Necessary inputs for the fatigue analysis are shown in Figure 2. The three input information boxes are descriptions of the material properties, loading history and local geometry. All of these inputs are being discussed in the following sections.

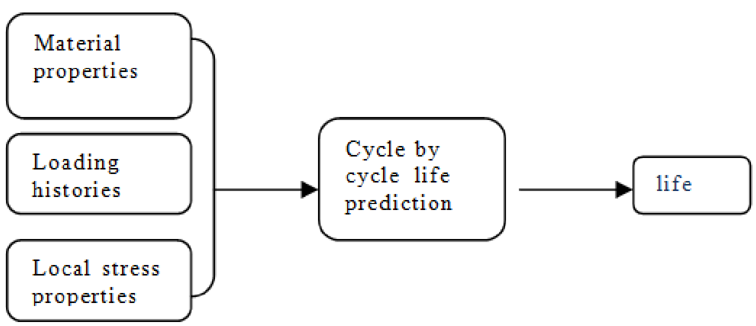

Figure 2. Fatigue Analysis Prediction Strategies

- Material information-cycle or repeated data is used on constant amplitude testing.
- Load histories information-measured or simulated load histories applied to a component. The term loads used to represent forces, displacements, accelerations, etc.

- Geometry information-relates the applied load histories to the local stresses and strains at the location of interest. The geometry information is usually derived from finite element (FE) results.

An integrated FE based durability analysis is considered a complete analysis of an entire component. Fatigue life can be estimated for every element in the finite element model and contour plots of life. Geometry information provided by FE results define how an applied load is provided by FE for each load case applied independently. Data provided for the desired fatigue analysis method. The schematic diagram of the integrated finite element based fatigue life prediction analysis is shown in Figure 3. The mechanical properties for the materials are mentioned in Table 1.

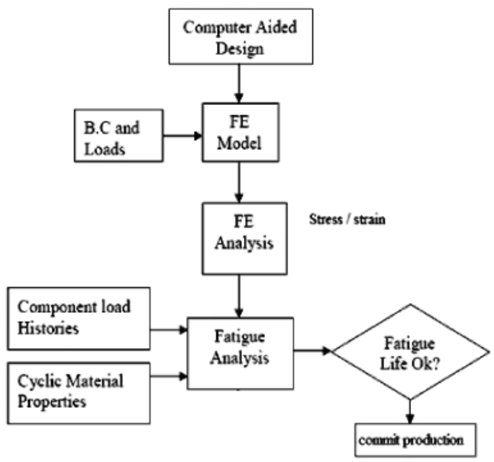

Figure 3. The Finite Element Based Fatigue Analysis Cycle

Table 1. Mechanical and Cyclic Properties of SAE1045-450- QT, SAE5160-825-QT &SAE1045-595-QT

Properties	Materials		
	SAE1045-450-QT	SAE5160-825-QT	SAE1045-595-QT
Yield strength(Mpa)	1515	1070	1860.
Ultimate tensile strength(Mpa)	1584	1550	2239
Elastic modulus(Mpa)	207000	207000	207000
Fatigue strength coefficient(S_f)	1686	2063	3047
Fatigue strength exponent(b)	-0.06	-0.08	-0.10
Fatigue ductility exponent(c)	-0.83	-1.05	-0.79
Fatigue ductility coefficient(ε_f')	0.79	9.56	0.13
Cyclic-strain hardening exponent(n')	0.09	0.10	0.10
Cyclic strength coefficient(k')	1874	2000	3498

2.1. Fatigue Analysis Methods

Analysis of fatigue can be carried out by one of the three basic approaches i.e., the total life (stress-life) approach and crack propagation approach, the crack initiation approach and crack propagation approach. The total-life (stress-life) approach was first applied over a hundred years ago (Wohler, 1867) and consider nominal elastic stresses and how they are related to life. The crack-initiation (stress-life) approach considers elastic-plastic local stresses and strains. It represents more fundamental approach and is used to determine the number of cycles required to initiate a small engineering cracks. Crack-propagation or linear elastic fracture mechanics (LEFM) approach is used to predict how quickly pre-existing cracks grow and to estimate how many loading cycles are required to grow these to a critical size when catastrophic failure would occur. First two methods are used in this study are briefly discussed these two methods in the following sections. The fatigue total-life(S-N) approach is usually used for the life prediction of components subjected to high cycle fatigue, where stresses are mainly elastic. This approach emphasizes nominal stresses rather than local stresses. It uses the material stress-life curve and employs fatigue notch factors to account for stress concentrations, empirical modification factors for surface finish effects and analytical equations such as modified Goodman and Gerber equations are given below.

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u} = 1 \qquad \text{-(1)}$$

$$\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_u}\right)^2 = 1 \qquad \text{-(2)}$$

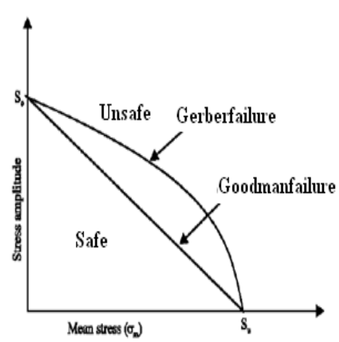

Figure 4. Representation of these Mean Stress Correction Methods

Where σ_a, Se, σ_m and S_a are the alternating stress and mean stress, alternating stress for equivalent completely reversed loading, and the mean stress and the ultimate tensile strength, respectively. The typical representation of these mean stress correction method shown in fig4. The Basquin (1910) showed that alternating stress verses number of cycles to failure (S-N) in finite life region could be represented as a log-log linear relationship. Basquin equation was then used to obtain the fatigue life using the material properties listed in Table 4. S-N approach uses to estimate the fatigue life for combined loading by determining an equivalent axial stress (Zoroufi and Fatemi, 2004) using one of the common failure criteria such as Tresca, von-mises, or maximum principal stress. The S-N equation is mathematically given by:

$$S_e = \sigma_f'(2N_f)^b \qquad -(3)$$

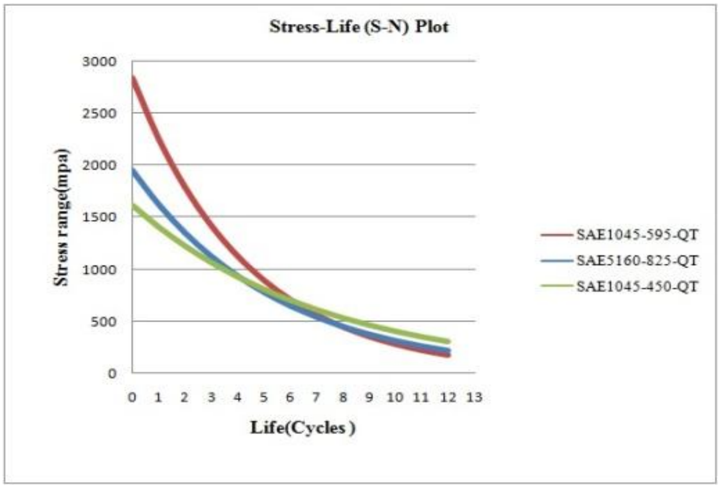

Figure 5. Stress-life (S-N) Plot

Where $S_{e,}$ σ_f',$2N_f$ and b are the stress amplitude, the fatigue strength coefficient, the reversals to failure and the fatigue strength exponent, respectively Figure 5 Shows comparison between the two materials with respect to S-N behaviour. It can be seen that these curves exhibit different life behaviour depending on the stress range experienced. From the figure, it is observed that in the long life area (high cycle fatigue), the different is lower while in the short life area (low cycle fatigue), the difference is higher. An important aspect of the fatigue process is plastic deformation. Fatigue cracks are initiated from the plastic straining in localized regions. Significant localized plastic deformation is often present, total-life approach doesn't account for plastic strain. Main advantage of this method is that it accounts for changes in local mean and residual stresses. Therefore, cyclic strain-controlled fatigue method could better characterize the fatigue behaviour of materials than cyclic strain controlled fatigue particularly in notched members where the significant localized plastic deformation is often present. In the crack initiation approach the plastic strain is directly measured and quantified. The total- life approach does not account for plastic strain. One of the main advantages of this method is that it accounts for changes in local mean and residual stresses. In strain-life when the load history contains large over loads, significant plastic deformation can exist, particularly at stress concentrations and the load sequence effects can be significant. In these cases, the crack initiation approach is generally superior to the total life approach for fatigue life prediction analysis. However, when the load levels are relatively low such that the resulting strains are mainly elastic, the crack initiation and total life approaches usually result in similar predictions. The crack initiation approach to the fatigue problem is widely used at present especially when the parabolic leaf spring are started or stopped then it is subjected to a very high stress range. The fatigue crack initiation approach involves the techniques for converting load history, geometry and material properties (monotonic and cyclic) input in to the fatigue life prediction. The operations involved in the prediction must be performed sequentially. First, the stress and strain at the critical site are estimated and rain flow cycle counting method is then used to reduce the load time history based on the peak –valley sequential. The next step is to use the finite element method to convert a reduced load time history in to a strain time history and calculate the stress and strain in the highly stressed area then the crack initiation methods are employed for predicting fatigue life. Following this, a simple linear damage hypothesis is used to accumulate the fatigue damage finally. The damage values for all cycles are summed until a critical damage sum (failure criteria) is reached. In order to perform the fatigue analysis and to implement the stress strain approach in complex structures strain life results are simulated using the 3D models to assess fatigue damage. After the complex load history was reduced to an elastic stress history for each critical element, a Neuber plasticity correction method was used to correct plastic behaviour. Elastic unit load analysis, using strength of material and an elastic finite element analysis model combined with a super position procedure of each load points service history verifies the local strain approach for fatigue evolution. In this study, it was observed that the local strain approach using the Smith-Watson-Topper (SWT) strain-life model is able to represent and to estimate many factors explicitly. These include mean stress effects, load sequence effects above and below the endurance limit and manufacturing process effects such as surface roughness and residual stresses.The fatigue resistance of metals can be characterized by a strain life curve. These curves are derived from the polished laboratory specimens tested under completely reversed strain control. The relationship between the total strain amplitude ($\Delta\varepsilon/2$) and reversals to failure ($2N_f$) can be expressed in following form (Coffin, 1954; Manson, 1953 [12, 13]) that represents the typical total strain-life curves.

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f^{'}}{E}\left(2N_f{}^{b}\right) + \varepsilon_f^{'}\left(2N_f{}^{c}\right) \qquad \text{- (4)}$$

Where, N_f is the fatigue life; σ'_f is the fatigue strength coefficient; E is the modules of elasticity; ε'_f is the fatigue ductility coefficient and c is fatigue ductility exponent. Morrow (1968 [14]) suggested that mean stress effects are considered by modifying the elastic term in the strain-life equation by mean stress (σ_m).

$$\varepsilon_{a} = \frac{\sigma_f^{'} - \sigma_m}{E}\,(2N_f)^{b} + \varepsilon_f^{'}(2N_f)^{c} \qquad \text{-- (5)}$$

Smith (1970) introduced another mean stress model which is called SWT mean stress correction model. It is mathematically defined as

$$\sigma_{max}\varepsilon_a E = \left(\sigma_f^{'}\right)^2 (2N_f)^{2b} + \sigma_f^{'}\varepsilon_f^{'} E (2N_f)^{b+c} \qquad \text{- (6)}$$

Where, σ_{max} is the maximum stress and ε_a is the strain amplitude.

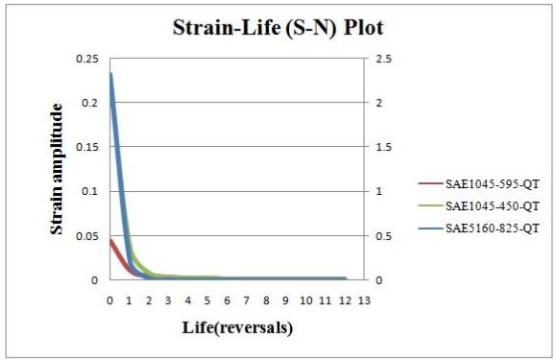

Figure 6. Strain-life (S-N) Plot

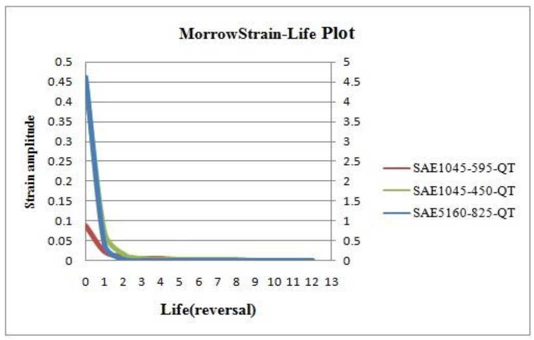

Figure 7. Morrow Strain-life (S-N) Plot

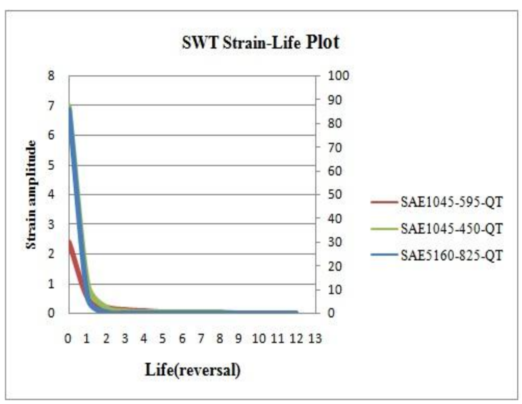

Figure 8. SWT Strain-life (S-N) Plot

Figure 6 represent the stain life curve indicates the different fatigue life behaviour for both materials. It is plotted based on the Coffin-Manson relationship. From the figure, it can be seen that in long life area (high cycle fatigue) the difference is lower while in the short life area (low cycle fatigue) the difference is higher. Figure 7 and Figure 8 Show the other strain life curves this are based on Morrow and SWT models, respectively.

3. Loading Information

Loading is another major input for the finite element based fatigue analysis. Unlike static stress, which is analyzed with calculations for a single stress state, fatigue damage occurs when stress at a point changes over time. There are essentially four classes of fatigue loading, with the ANSYS Fatigue Module currently supporting the first three:

- Constant amplitude, proportional loading
- Constant amplitude, non-proportional loading
- Non-constant amplitude, proportional loading
- Non-constant amplitude, non-proportional loading

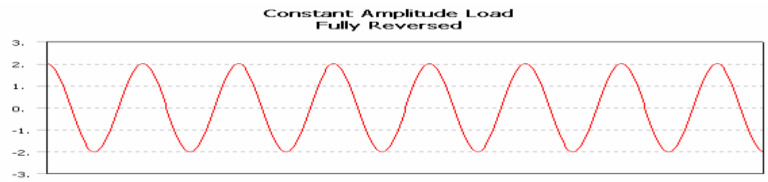

Figure 9. Constant Amplitude Load Fully Reversed

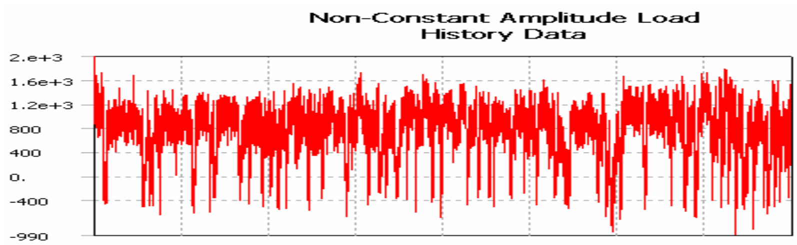

Figure 10. Non Constant Amplitude Loading

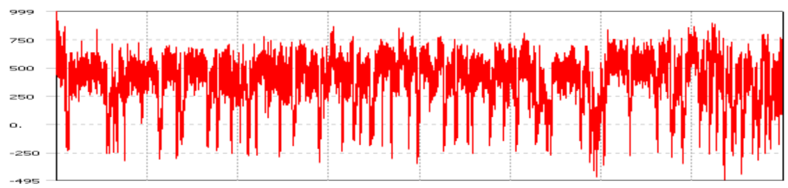

Figure 11. Transmission Loading Histories

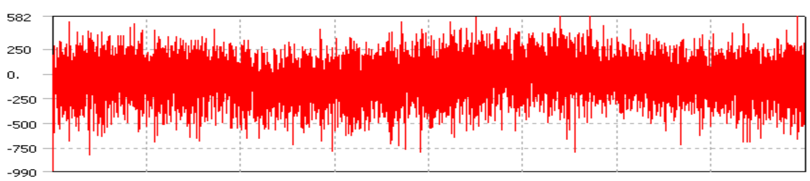

Figure 12. Bracket Loading Histories

Non-constant Amplitude, proportional loading within the ANSYS Fatigue Module uses a "quick counting" technique to substantially reduce runtime and memory. In quick counting, alternating and mean stresses are sorted into bins before partial damage is calculated. Without quick counting, data is not sorted into bins until after partial damages are found. The accuracy of quick counting is usually very good if a proper number of bins are used when counting. The bin size defines in to how many divisions the cycle counting history should be organized into for the history data loading type. Strictly speaking, bin size specifies the number of divisions of the rain flow matrix. A larger bin size has greater precision but will take longer to solve and use more memory. Bin size defaults to 32, meaning that the Rain flow Matrix is 32 x 32 in dimension. For Stress Life, another available option when conducting a variable amplitude fatigue analysis is the ability to set the value used for infinite life. In constant amplitude loading, if the alternating stress is lower than the lowest alternating stress on the fatigue curve, the fatigue tool will use the life at the last point. This provides for an added level of safety because many materials do not exhibit an endurance limit. However, in non-constant amplitude loading, cycles with very small alternating stresses may be present and may incorrectly predict too much damage if the number of the small stress cycles is high enough. To help control this, the user can set the infinite life value that will be used if the alternating stress is beyond the limit of the S-N curve. Setting a higher value will make small stress cycles less damaging if they occur many times. The Rain flow and damage matrix results can be helpful in determining the effects of small stress cycles in loading history. The component was loaded with two random time histories corresponding to typical histories for transmission and bracket components at different load levels. The detailed information about these loading histories of length is discussed in literature (Tucker and Bussa, 1977). These loading histories scaled to two peak strain levels are used as full length histories. Raw loading histories of the component are shown in figure8 and figure 9. The terms of SAETRN, SAEBRAKT represent the loading-time history for the transmission and bracket respectively. The considered load histories are based on the SAEs profile. The abscissa uses the time in seconds.

4. Finite Element Analysis

Numerical techniques are necessary to stimulate the physical behaviour and to evaluate the structural integrity of the different designs. The objective of the current study is to calculate the fatigue life for a leaf spring of a heavy vehicle using total life and crack initiation methods, to investigate the effect of mean stress on fatigue life and the probabilistic nature of fatigue on the S-N curve via design criteria.

5. Results and Discussion.

The linear static finite element analysis was performed using ANSYS workbench finite element code. The equivalent von-mises stress contours and critical locations shown in Figure 13. The eyes of parabolic spring were found to be areas of high stresses.

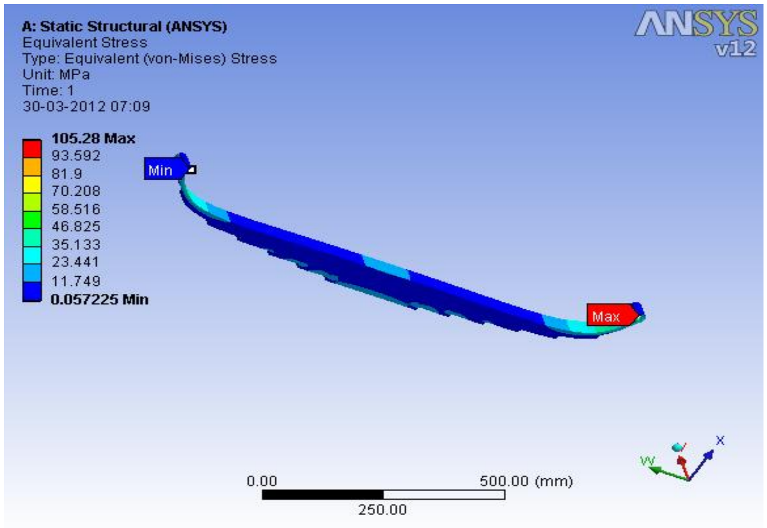

Figure 13. Stress Concentration Location

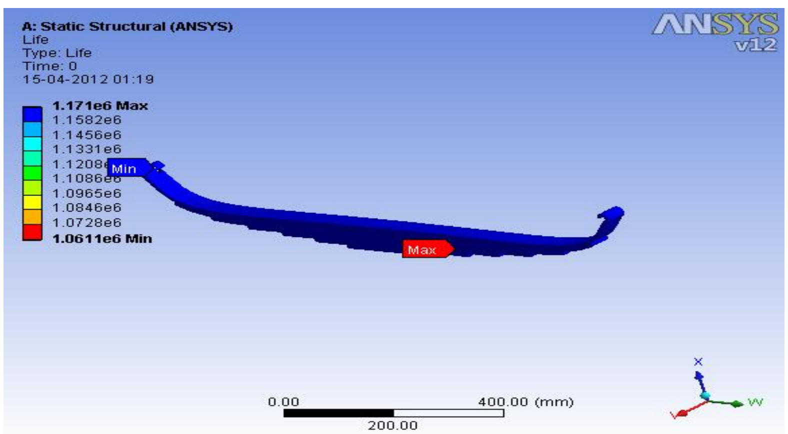

Figure 14. Life of the Parabolic Spring

The von-mises equivalent stresses are used for subsequent fatigue life analysis and comparisons. However, in table 2, it can be seen that when using the loading sequences are predominantly tensile in the nature; the Goodman approach is more conservative. Gerber mean stress correction has been found to give conservative when the time histories predominantly zero mean.

From the table 3, it is also seen that two mean stress methods, SWT and Marrow give lives less than that achieved using no mean stress correction with the Marrow method being the most conservative for loading sequences which are predominantly tensile in nature. When using the time history has a roughly zero mean (SAEBRAKT) then two methods have given approximately the same results. It can be also seen that SAE1045-595-QT is consistently higher life than SAE5160-825-QT, SAE1045-450-QT for all loading conditions.

Table 2. Predicted Fatigue Life using Total-life Approach

Loading Conditions / Materials	SAETRN x 10^5			SAEBRAKT x10^5		
	No Mean	Goodman	Gerber	No Mean	Goodman	Gerber
SAE1045-450-QT	10.611	2.9201	9.8807	2.4635	2.463	2.4087
SAE5160-825-QT	10.852	4.5605	9.9148	2.692	2.6588	2.6545
SAE1045-595-QT	11.595	11.024	11.57	3.2998	3.2996	3.2923

Table 3. Predicted Fatigue Life using Crack-initiation Approach

Loading Conditions / Materials	SAETRN			SAEBRAKT		
	No Mean	Marrow	SWT	No Mean	Marrow	SWT
SAE1045-450-QT	106.98	37.452	80.713	118.19	21.864	31.694
SAE5160-825-QT	139.18	44.404	103.69	129.079	24.81	86.365
SAE1045-595-QT	418.5	176.41	231.77	420.81	68.072	93.29

The three-dimensional cycle histogram and corresponding damage histogram for materials using SAETRN loading histories is shown in the Figures 15 and 16 given below. Figure 16 shows the results of the rain flow cycle count for the component. It can be seen that a lot of cycles with a low stress range and fewer with a high range. The height of each tower represents the number of cycles at that particular stress range and mean. Each tower is used to obtain damage on the S-N curve and damage is summed over all towers. Figure 16 shows that lower stress ranges produced zero damage. It is also showed that the high stress ranges were found to give the most of the damage and a fairly wide damage distribution at the higher ranges which mean that it cannot point to a single event causing damage. Most realistic service situations involve nonzero mean stresses, it is, therefore, very important to know the influence that mean stress has on the fatigue process so that the fully reversed (zero mean stress) laboratory data are usefully employed in the assignment of real situations.

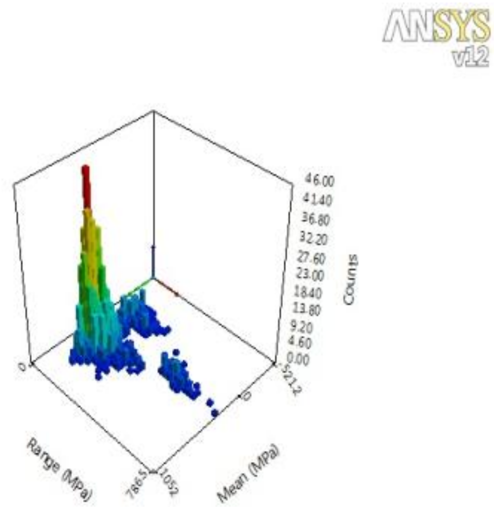

Figure15. Fatigue Sensitivity

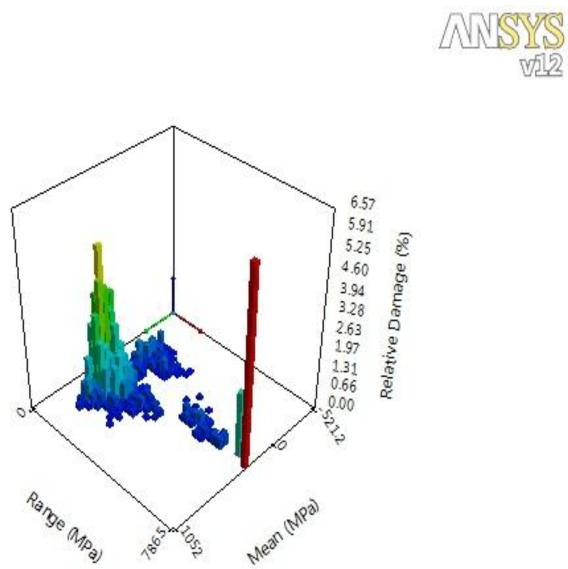

Figure 16. Damage Matrix

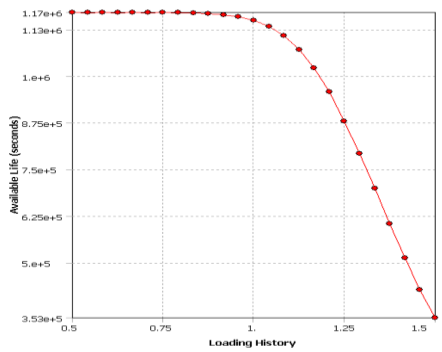

Figure 17. Rain Flow Matrix

6. Conclusions

A computational numerical model for the fatigue life assessment for leaf spring of the parabolic leaf spring is presented in this study. Through the study, several conclusions can be drawn with regard to the fatigue life of a component when subjected to complex variable amplitude loading conditions.

- The fatigue life was estimated based on Palmgren-Miner rule which is non-conservative SWT correction and Morrows methods, and damage rule can be applied to improve the estimation. It can be seen that when using the loading sequences are predominantly tensile in the nature; the life of leaf spring in Goodman approach is $2.9201x10^5$ sec which is more conservative.
- It can be seen that when using the loading sequences are predominantly zero mean (SAEBRAKT), the value of life of the leaf spring is 2.4087x105 sec in Gerber mean stress correction which has found to be more sensitive.
- It can be concluded that the influence of mean stress correction is more sensitive to tensile mean stress for total life approach. It is also seen that the two mean stress methods give lives less than that achieved using no mean stress correction.
- It is concluded for crack initiation approach that when the loading is predominantly tensile in nature, the life of the component in Morrow approach is $176.41x10^5$sec which is more sensitive and is therefore recommended. When using the time histories has zero mean (SAEBRAKT) then all three methods have been given approximately the same results.

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