

Effect of Rule Base on the Fuzzy-Based Tuning Fuzzy Sliding Mode Controller: Applied to 2nd Order Nonlinear System

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Abstract

Design a nonlinear controller for second order nonlinear uncertain dynamical systems is one of the most important challenging works. This paper focuses on the design of a robust rule base fuzzy-based tuning fuzzy sliding mode controller for second order nonlinear system in presence of uncertainties. Rule base is one of the important key factors in design of fuzzy based tuning controller. In order to provide high performance nonlinear methodology, fuzzy sliding mode controller is selected. Fuzzy sliding mode controller can be used to control of partly unknown nonlinear dynamic parameters of nonlinear system. Error-based fuzzy sliding mode controller has difficulty in handling unstructured model uncertainties. To solve this problem applied improve rule base fuzzy-based tuning method to error-based fuzzy sliding mode controller for adjusting the rule base in adaptive fuzzy-based tuning. This controller has acceptable performance in presence of uncertainty (e.g., overshoot=0%, rise time=0.4 second, steady state error = $1.3e-11$ and RMS error= $1.2e-11$).

Keywords: *fuzzy sliding mode controller, fuzzy-based tuning, rule base, sliding mode controller, uncertainties*

1. Introduction and Background

Controller is a device which can sense information from linear or nonlinear system to improve the systems performance [1-3]. The main targets in designing control systems are stability, good disturbance rejection, and small tracking error [5]. Several industrial systems are controlled by linear methodologies (e.g., Proportional-Derivative (PD) controller, Proportional- Integral (PI) controller or Proportional- Integral-Derivative (PID) controller), but when system works with various payloads and have uncertainty in dynamic models this technique has limitations. From the control point of view, uncertainty is divided into two main groups: uncertainty in unstructured inputs (e.g., noise, disturbance) and uncertainty in structure dynamics. In some applications systems are used in an unknown and unstructured environment, therefore strong mathematical tools used in new control methodologies to design nonlinear robust controller with an acceptable performance (e.g., minimum error, good trajectory, disturbance rejection). Sliding mode controller is an influential nonlinear controller to certain and uncertain systems which it is based on system's dynamic model. Sliding mode controller is a powerful nonlinear robust controller under condition of partly uncertain dynamic parameters of system [6, 7]. This controller is used to control of highly nonlinear systems. Chattering phenomenon and nonlinear equivalent dynamic formulation in uncertain

dynamic parameter are two main drawbacks in pure sliding mode controller [8-20]. The chattering phenomenon problem in pure sliding mode controller and fuzzy sliding mode controller is reduced by using linear saturation boundary layer function but prove the stability is very difficult. The nonlinear equivalent dynamic formulation problem in uncertain system is solved by using fuzzy logic theorem [8]. Fuzzy logic theory is used to remove the system's dynamics. To eliminate the system's dynamic, 49 rules Mamdani inference system is design and applied to sliding mode methodology with saturation function. This methodology is worked based on applied fuzzy logic in equivalent nonlinear dynamic part to eliminate unknown dynamic parameters. To estimate the system dynamics, error-based fuzzy sliding mode controller is designed. Proposed MIMO error-based fuzzy sliding mode controller with saturation function is a chattering free dynamic model-free controller. This methodology is based on applied fuzzy logic in equivalent nonlinear dynamic part to estimate unknown parameters. Pure sliding mode controller and error-based fuzzy sliding mode controller have difficulty in handling unstructured model uncertainties. It is possible to solve this problem by combining fuzzy sliding mode controller and fuzzy rule base tuning . This method is based on resolve the on line sliding surface gain (λ) as well as improve the output performance by change the rule base to tuning the sliding surface slope updating factor (α). Fuzzy rule base tuning error-based fuzzy sliding mode controller is stable model-free controller which does not need to limits the dynamic model of systems and eliminate the chattering phenomenon without to use the boundary layer saturation function. Lyapunov stability is proved in fuzzy-rule base tuning fuzzy sliding mode controller based on switching (sign) function [21-44].

Sliding mode controller (SMC) is a powerful nonlinear controller which has been analyzed by many researchers especially in recent years. This theory was first proposed in the early 1950 by Emelyanov and several co-workers and has been extensively developed since then with the invention of high speed control devices [2]. The main reason to opt for this controller is its acceptable control performance in wide range and solves two most important challenging topics in control which names, stability and robustness [7, 17-20]. Sliding mode control theory for control of robot manipulator was first proposed in 1978 by Young to solve the set point problem ($\dot{q}_d = \mathbf{0}$) by discontinuous method in the following form [45-65];

$$\tau_{(q,t)} = \begin{cases} \tau_i^+(q,t) & \text{if } S_i > 0 \\ \tau_i^-(q,t) & \text{if } S_i < 0 \end{cases} \quad (1)$$

where S_i is sliding surface (switching surface), $i = 1, 2, \dots, n$ for nonlinear system, $\tau_i(q, t)$ is the i^{th} torque of joint. Sliding mode controller is divided into two main sub controllers: discontinues controller(τ_{dis}) and equivalent controller(τ_{eq}). Discontinues controller causes an acceptable tracking performance at the expense of very fast switching. Conversely in this theory good trajectory following is based on fast switching, fast switching is caused to have system instability and chattering phenomenon. Fine tuning the sliding surface slope is based on nonlinear equivalent part [1, 6]. However, this controller is used in many applications but, pure sliding mode controller has two most important challenges: chattering phenomenon and nonlinear equivalent dynamic formulation in uncertain parameters[20]. Chattering phenomenon (Figure 1) can causes some problems such as saturation and heat the mechanical parts of systems or drivers. To reduce or eliminate the chattering, various papers have been reported by many researchers which classified into two most important methods: boundary layer saturation method and estimated uncertainties method [1, 66-79].

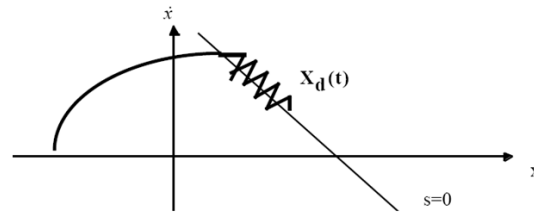


Figure 1. Chattering as a Result of Imperfect Control Switching [1]

In boundary layer saturation method, the basic idea is the discontinuous method replacement by saturation (linear) method with small neighborhood of the switching surface. This replacement caused to increase the error performance against with the considerable chattering reduction. Slotine and Sastry have introduced boundary layer method instead of discontinuous method to reduce the chattering [21]. Slotine has presented sliding mode with boundary layer to improve the industry application [22]. Palm has presented a fuzzy method to nonlinear approximation instead of linear approximation inside the boundary layer to improve the chattering and control the result performance [23]. Moreover, Weng and Yu improved the previous method by using a new method in fuzzy nonlinear approximation inside the boundary layer and adaptive method [24]. As mentioned [24] sliding mode fuzzy controller (SMFC) is fuzzy controller based on sliding mode technique to most exceptional stability and robustness. Sliding mode fuzzy controller has the two most important advantages: reduce the number of fuzzy rule base and increase robustness and stability. Conversely sliding mode fuzzy controller has the above advantages, define the sliding surface slope coefficient very carefully is the main disadvantage of this controller. Estimated uncertainty method used in term of uncertainty estimator to compensation of the system uncertainties. It has been used to solve the chattering phenomenon and also nonlinear equivalent dynamic. If estimator has an acceptable performance to compensate the uncertainties, the chattering is reduced. Research on estimated uncertainty to reduce the chattering is significantly growing as their applications such as industrial automation and robot manipulator. For instance, the applications of artificial intelligence, neural networks and fuzzy logic on estimated uncertainty method have been reported in [25-28]. Wu et. al., [30] have proposed a simple fuzzy estimator controller beside the discontinuous and equivalent control terms to reduce the chattering. Their design had three main parts i.e. equivalent, discontinuous and fuzzy estimator tuning part which has reduced the chattering very well. Elmali et. al., [27] and Li and Xu [29] have addressed sliding mode control with perturbation estimation method (SMCPE) to reduce the classical sliding mode chattering. This method was tested for the tracking control of the first two links of a SCARA type HITACHI robot. In this technique, digital controller is used to increase the system's response quality. However this controller's response is very fast and robust but it has chattering phenomenon. In recent years, artificial intelligence theory has been used in sliding mode control systems. Neural network, fuzzy logic and neuro-fuzzy are synergically combined with nonlinear classical controller and used in nonlinear, time variant and uncertain plant. Fuzzy logic controller (FLC) is one of the most important applications of fuzzy logic theory [80-97]. This controller can be used to control nonlinear, uncertain, and noisy systems. This method is free of some model techniques as in model-based controllers. As mentioned that fuzzy logic application is not only limited to the modelling of nonlinear systems [31-36] but also this method can help engineers to design a model-free controller. Control nonlinear systems using model-based controllers are based on system's dynamic model. These controllers often have many problems for modelling. Conventional controllers require accurate information of dynamic

model, but most of time these models are MIMO, nonlinear and partly uncertain therefore calculate accurate dynamic model is complicated [32]. The main reasons to use fuzzy logic methodology are able to give approximate recommended solution for uncertain and also certain complicated systems to easy understanding and flexible. Fuzzy logic provides a method to design a model-free controller for nonlinear plant with a set of IF-THEN rules [32]. The applications of artificial intelligence such as neural networks and fuzzy logic in modelling and control are significantly growing especially in recent years. For instance, the applications of artificial intelligence, neural networks and fuzzy logic, on control of robot manipulator have reported in [37-39]. Wai et. al., [37-38] have proposed a fuzzy neural network (FNN) optimal control system to learn a nonlinear function in the optimal control law. This controller is divided into three main groups: artificial intelligence controller (neuro-fuzzy) which it is used to compensate the system's nonlinearity and improves by adaptive method, robust controller to reduce the error and optimal controller which is the main part in this controller. Mohan and Bhanot [40] have addressed comparative study between some adaptive fuzzy and a new hybrid fuzzy control algorithm for manipulator control. They found that self-organizing fuzzy logic controller and proposed hybrid integrator fuzzy give the best performance as well as simple structure. Research on combinations of fuzzy logic systems with sliding mode controller in presence of nonlinear dynamic part is significantly growing as nonlinear control applications. For instance, the applications of fuzzy logic on sliding mode controller have reported in [24, 41-45]. Fuzzy sliding mode controller (FSMC) is a nonlinear controller based on sliding mode method when fuzzy logic methodology applied to sliding mode controller to reduce the high frequency oscillation (chattering) and compensate the dynamic model of uncertainty based on nonlinear dynamic model [42-43]. Temeltas [46] has proposed fuzzy adaption techniques and applied to SMC to have robust controller and solves the chattering problem. In this method however system's performance is better than sliding mode controller but it is depended on nonlinear dynamic equations. Hwang and Chao [47] have proposed a Tagaki-Sugeno (TS) fuzzy model based sliding mode controller based on neuro-fuzzy based linear state-space to estimate the uncertainties. A MIMO fuzzy sliding mode controller to reduces the chattering phenomenon and estimate the nonlinear equivalent part has been presented for a robot manipulator [42]. Sliding mode fuzzy controller (SMFC) is an artificial intelligence controller based on fuzzy logic methodology when, sliding mode controller is applied to fuzzy logic controller to reduce the fuzzy rules and refine the stability of close loop system in fuzzy logic controller [23, 48-50]. Lhee et al. [48] have presented a fuzzy logic controller based on sliding mode controller to more formalize and boundary layer thickness. Emami et. al., [51] have proposed a fuzzy logic approximate inside the boundary layer. H.K.Lee et. al., [52] have presented self tuning SMFC to reduce the fuzzy rules, increase the stability and to adjust control parameters control automatically. In comparison, to reduce the number of fuzzy rule base, increase the robustness and stability sliding mode fuzzy controller is more suitable than fuzzy logic controller [52]. However the SMFC has a good condition but the main drawback compared to FSMC is calculation the value of sliding surface slope coefficient pre-defined very carefully and FSMC is more suitable for implementation.

This paper is organized as follows: In Section 2, main subject of case study (robot manipulator) dynamic formulation, sliding mode controller and error-based fuzzy sliding mode controller are presented. Detail of fuzzy rule base tuning error-based fuzzy sliding mode controller is presented in Section 3. In Section 4, the simulation result is presented and finally in Section 5, the conclusion is presented.

2. Theorem: Case Study (Dynamic Formulation of Robot Manipulator), Modified Sliding Mode Controller

Case study (Dynamic of robot manipulator): Robot manipulator is a collection of links that connect to each other by joints, these joints can be revolute and prismatic that revolute joint has rotary motion around an axis and prismatic joint has linear motion around an axis. Each joint provides one or more degrees of freedom (DOF). Dynamic modeling of robot manipulators is used to describe the behavior of robot manipulator such as linear or nonlinear dynamic behavior, design of model based controller such as pure sliding mode which design this controller is based on nonlinear dynamic equations, and for simulation. The dynamic modeling describes the relationship between joint motion, velocity, and accelerations to force/torque or current/voltage and also it can be used to describe the particular dynamic effects (e.g., inertia, coriolis, centrifugal, and the other parameters) to behavior of system [1]. The Unimation PUMA 560 serially links robot manipulator was used as a basis, because this robot manipulator is widely used in industry and academic. It has a nonlinear and uncertain dynamic parameters serial link 6 degrees of freedom (DOF) robot manipulator. A nonlinear robust controller design is major subject in this work. The equation of a multi degrees of freedom (DOF) robot manipulator is calculated by the following equation [6, 70-97]:

$$M(q)\ddot{q} + N(q, \dot{q}) = \tau \quad (2)$$

Where τ is $n \times 1$ vector of actuation torque, $M(q)$ is $n \times n$ symmetric and positive definite inertia matrix, $N(q, \dot{q})$ is the vector of nonlinearity term, and q is $n \times 1$ position vector. In equation 2.8 if vector of nonlinearity term derive as Centrifugal, Coriolis and Gravity terms, as a result robot manipulator dynamic equation can also be written as [80]:

$$N(q, \dot{q}) = V(q, \dot{q}) + G(q) \quad (3)$$

$$V(q, \dot{q}) = B(q)[\dot{q} \dot{q}] + C(q)[\dot{q}]^2 \quad (4)$$

$$\tau = M(q)\ddot{q} + B(q)[\dot{q} \dot{q}] + C(q)[\dot{q}]^2 + G(q) \quad (5)$$

Where, $B(q)$ is matrix of coriolis torques, $C(q)$ is matrix of centrifugal torque, $[\dot{q} \dot{q}]$ is vector of joint velocity that it can give by: $[\dot{q}_1 \cdot \dot{q}_2, \dot{q}_1 \cdot \dot{q}_3, \dots, \dot{q}_1 \cdot \dot{q}_n, \dot{q}_2 \cdot \dot{q}_3, \dots]^T$, and $[\dot{q}]^2$ is vector, that it can given by: $[\dot{q}_1^2, \dot{q}_2^2, \dot{q}_3^2, \dots]^T$.

In robot manipulator dynamic part the inputs are torques and the outputs are actual displacements, as a result in (5) it can be written as [1, 6, 80-81];

$$\ddot{q} = M^{-1}(q) \cdot \{\tau - N(q, \dot{q})\} \quad (6)$$

To implementation (6) the first step is implement the kinetic energy matrix (M) parameters by used of Lagrange's formulation. The second step is implementing the Coriolis and Centrifugal matrix which they can calculate by partial derivatives of kinetic energy. The last step to implement the dynamic equation of robot manipulator is to find the gravity vector by performing the summation of Lagrange's formulation.

The kinetic energy equation (M) is a $n \times n$ symmetric matrix that can be calculated by the following equation [90-97];

$$M(\theta) = m_1 J_{v1}^T J_{v1} + J_{\omega 1}^{TC1} I_1 J_{\omega 1} + m_2 J_{v2}^T J_{v2} + J_{\omega 2}^{TC2} I_2 J_{\omega 2} + m_3 J_{v3}^T J_{v3} + J_{\omega 3}^{TC3} I_3 J_{\omega 3} + m_4 J_{v4}^T J_{v4} + J_{\omega 4}^{TC4} I_4 J_{\omega 4} + m_5 J_{v5}^T J_{v5} + J_{\omega 5}^{TC5} I_5 J_{\omega 5} + m_6 J_{v6}^T J_{v6} + J_{\omega 6}^{TC6} I_6 J_{\omega 6} \quad (7)$$

As mentioned above the kinetic energy matrix in n DOF is a $n \times n$ matrix that can be calculated by the following matrix [1, 6]

$$M(q) = \begin{bmatrix} M_{11} & M_{12} & \dots & \dots & \dots & M_{1n} \\ M_{21} & \dots & \dots & \dots & \dots & M_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ M_{n.1} & \dots & \dots & \dots & \dots & M_{n.n} \end{bmatrix} \quad (8)$$

The Coriolis matrix (B) is a $n \times \frac{n(n-1)}{2}$ matrix which calculated as follows;

$$B(q) = \begin{bmatrix} b_{112} & b_{113} & \dots & b_{11n} & b_{123} & \dots & b_{12n} & \dots & \dots & b_{1.n-1.n} \\ b_{212} & \dots & \dots & b_{21n} & b_{223} & \dots & \dots & \dots & \dots & b_{2.n-1.n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ b_{n.1.2} & \dots & \dots & b_{n.1.n} & \dots & \dots & \dots & \dots & \dots & b_{n.n-1.n} \end{bmatrix} \quad (9)$$

and the Centrifugal matrix (C) is a $n \times n$ matrix;

$$C(q) = \begin{bmatrix} C_{11} & \dots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{n1} & \dots & C_{nn} \end{bmatrix} \quad (10)$$

And last the Gravity vector (G) is a $n \times 1$ vector;

$$G(q) = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} \quad (11)$$

Sliding mode controller and application for robot manipulator: Design a robust controller for robot manipulator is essential because robot manipulator has highly nonlinear dynamic parameters. In this section formulations of sliding mode controller for robot manipulator is presented based on [1, 6]. Consider a nonlinear single input dynamic system is defined by [6]:

$$\dot{x}^{(n)} = f(\vec{x}) + b(\vec{x})u \quad (12)$$

Where u is the vector of control input, $x^{(n)}$ is the n^{th} derivation of x , $x = [x, \dot{x}, \ddot{x}, \dots, x^{(n-1)}]^T$ is the state vector, $f(x)$ is unknown or uncertainty, and $b(x)$ is of known *sign* function. The main goal to design this controller is train to the desired state; $x_d = [x_d, \dot{x}_d, \ddot{x}_d, \dots, x_d^{(n-1)}]^T$, and trucking error vector is defined by [6]:

$$\tilde{x} = x - x_d = [\tilde{x}, \dots, \tilde{x}^{(n-1)}]^T \quad (13)$$

A time-varying sliding surface $s(x, t)$ in the state space R^n is given by [6]:

$$s(x, t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x} = 0 \quad (14)$$

where λ is the positive constant. To further penalize tracking error, integral part can be used in sliding surface part as follows [6]:

$$s(x, t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \left(\int_0^t \tilde{x} dt\right) = 0 \quad (15)$$

The main target in this methodology is kept the sliding surface slope $s(x, t)$ near to the zero. Therefore, one of the common strategies is to find input U outside of $s(x, t)$ [6].

$$\frac{1}{2} \frac{d}{dt} s^2(x, t) \leq -\zeta |s(x, t)| \quad (16)$$

where ζ is positive constant.

$$\text{If } S(0) > 0 \rightarrow \frac{d}{dt} S(t) \leq -\zeta \quad (17)$$

To eliminate the derivative term, it is used an integral term from $t=0$ to $t=t_{reach}$

$$\int_{t=0}^{t=t_{reach}} \frac{d}{dt} S(t) \leq - \int_{t=0}^{t=t_{reach}} \eta \rightarrow S(t_{reach}) - S(0) \leq -\zeta(t_{reach} - 0) \quad (18)$$

Where t_{reach} is the time that trajectories reach to the sliding surface so, suppose $S(t_{reach} = 0)$ defined as

$$0 - S(0) \leq -\eta(t_{reach}) \rightarrow t_{reach} \leq \frac{S(0)}{\zeta} \quad (19)$$

and

$$\text{if } S(0) < 0 \rightarrow 0 - S(0) \leq -\eta(t_{reach}) \rightarrow S(0) \leq -\zeta(t_{reach}) \rightarrow t_{reach} \leq \frac{|S(0)|}{\eta} \quad (20)$$

Equation (20) guarantees time to reach the sliding surface is smaller than $\frac{|S(0)|}{\zeta}$ since the trajectories are outside of $S(t)$.

$$\text{if } S_{t_{reach}} = S(0) \rightarrow \text{error}(x - x_d) = 0 \quad (21)$$

suppose S is defined as

$$s(x, t) = \left(\frac{d}{dt} + \lambda\right) \tilde{x} = (\dot{x} - \dot{x}_d) + \lambda(x - x_d) \quad (22)$$

The derivation of S , namely, \dot{S} can be calculated as the following;

$$\dot{S} = (\ddot{x} - \ddot{x}_d) + \lambda(\dot{x} - \dot{x}_d) \quad (23)$$

suppose the second order system is defined as;

$$\ddot{x} = f + u \rightarrow \dot{S} = f + U - \ddot{x}_d + \lambda(\dot{x} - \dot{x}_d) \quad (24)$$

Where f is the dynamic uncertain, and also since $S = 0$ and $\dot{S} = 0$, to have the best approximation, \hat{U} is defined as

$$\hat{U} = -\hat{f} + \ddot{x}_d - \lambda(\dot{x} - \dot{x}_d) \quad (25)$$

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

$$\mathbf{U}_{dis} = \hat{\mathbf{U}} - \mathbf{K}(\bar{\mathbf{x}}, \mathbf{t}) \cdot \mathbf{sgn}(\mathbf{s}) \quad (26)$$

where the switching function $\mathbf{sgn}(\mathbf{S})$ is defined as [1, 6]

$$\mathbf{sgn}(s) = \begin{cases} \mathbf{1} & s > 0 \\ -\mathbf{1} & s < 0 \\ \mathbf{0} & s = 0 \end{cases} \quad (27)$$

and the $\mathbf{K}(\bar{\mathbf{x}}, \mathbf{t})$ is the positive constant. Suppose by (16) the following equation can be written as,

$$\frac{1}{2} \frac{d}{dt} \mathbf{s}^2(\mathbf{x}, \mathbf{t}) = \dot{\mathbf{S}} \cdot \mathbf{S} = [\mathbf{f} - \hat{\mathbf{f}} - \mathbf{Ksgn}(\mathbf{s})] \cdot \mathbf{S} = (\mathbf{f} - \hat{\mathbf{f}}) \cdot \mathbf{S} - \mathbf{K}|\mathbf{S}| \quad (28)$$

and if the equation (20) instead of (19) the sliding surface can be calculated as

$$s(\mathbf{x}, \mathbf{t}) = \left(\frac{d}{dt} + \lambda \right)^2 \left(\int_0^t \tilde{\mathbf{x}} dt \right) = (\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) + 2\lambda(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) - \lambda^2(\mathbf{x} - \mathbf{x}_d) \quad (29)$$

in this method the approximation of \mathbf{U} is computed as [6]

$$\hat{\mathbf{U}} = -\hat{\mathbf{f}} + \ddot{\mathbf{x}}_d - 2\lambda(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) + \lambda^2(\mathbf{x} - \mathbf{x}_d) \quad (30)$$

Based on above discussion, the sliding mode control law for a multi degrees of freedom robot manipulator is written as [1, 6]:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{eq} + \boldsymbol{\tau}_{dis} \quad (31)$$

Where, the model-based component $\boldsymbol{\tau}_{eq}$ is the nominal dynamics of systems and $\boldsymbol{\tau}_{eq}$ for first 3 DOF PUMA robot manipulator can be calculate as follows [1]:

$$\boldsymbol{\tau}_{eq} = [\mathbf{M}^{-1}(\mathbf{B} + \mathbf{C} + \mathbf{G}) + \dot{\mathbf{S}}] \mathbf{M} \quad (32)$$

and $\boldsymbol{\tau}_{dis}$ is computed as [1];

$$\boldsymbol{\tau}_{dis} = \mathbf{K} \cdot \mathbf{sgn}(\mathbf{S}) \quad (33)$$

by replace the formulation (33) in (31) the control output can be written as;

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{eq} + \mathbf{K} \cdot \mathbf{sgn}(\mathbf{S}) \quad (34)$$

Figure 2 shows the position classical sliding mode control for PUMA 560 robot manipulator. By (34) and (32) the sliding mode control of PUMA 560 robot manipulator is calculated as;

$$\boldsymbol{\tau} = [\mathbf{M}^{-1}(\mathbf{B} + \mathbf{C} + \mathbf{G}) + \dot{\mathbf{S}}] \mathbf{M} + \mathbf{K} \cdot \mathbf{sgn}(\mathbf{S}) \quad (35)$$

where $S = \lambda e + \dot{e}$ in PD-SMC and $S = \lambda e + \dot{e} + \left(\frac{\lambda}{2}\right)^2 \int e$ in PID-SMC.

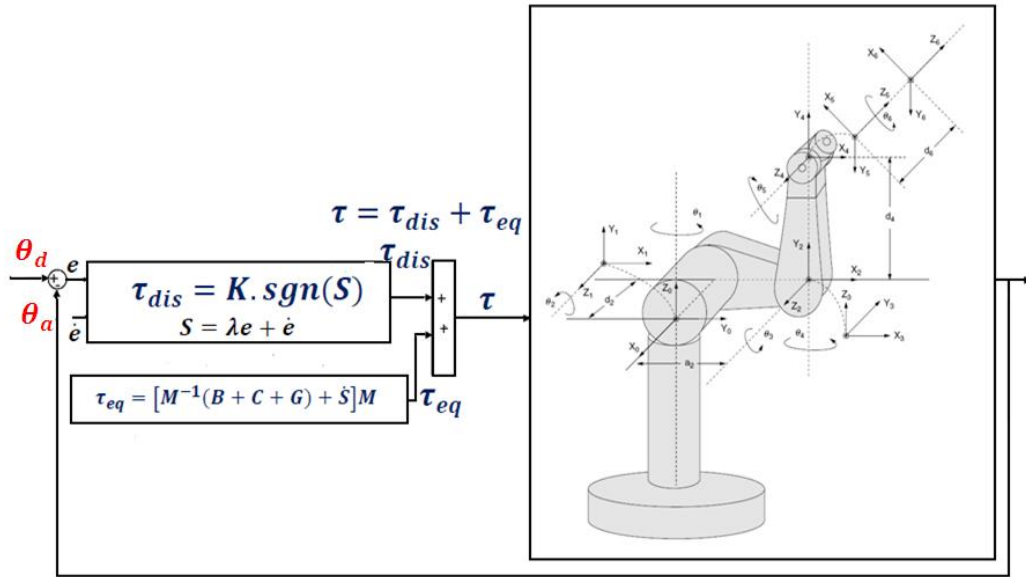


Figure 2. Block Diagram of Pure Sliding Mode Controller with Switching Function

Reduce the chattering in pure Sliding mode controller: Chattering phenomenon is one of the most important challenges in pure sliding mode controller with regards to the certain and partly uncertain systems. To reduce or eliminate the chattering in sliding mode controller this research is focused on boundary layer method. In boundary layer method the basic idea is replaced the discontinuous method by saturation (linear) method with small neighborhood of the switching surface [6, 60-77].

$$B(t) = \{x, |S(t)| \leq \phi\}; \phi > 0 \quad (36)$$

Where ϕ is the boundary layer thickness. Therefore the saturation function $\text{Sat}(S/\phi)$ is added to the control law as

$$U = K(\vec{x}, t) \cdot \text{Sat}(S/\phi) \quad (37)$$

Where $\text{Sat}(S/\phi)$ can be defined as [1]

$$\text{sat}(S/\phi) = \begin{cases} 1 & (S/\phi > 1) \\ -1 & (S/\phi < -1) \\ S/\phi & (-1 < S/\phi < 1) \end{cases} \quad (38)$$

Based on above discussion, the control law for multi degrees of freedom robot manipulator is written as [1, 6]:

$$\tau = \tau_{eq} + \tau_{sat} \quad (39)$$

Where, the model-based component τ_{eq} is the nominal dynamics of systems and to position control of PUMA robot manipulator (only first three degrees) τ_{eq} can be calculate as follows:

$$\tau_{eq} = [M^{-1}(B + C + G) + \dot{S}]M \quad (40)$$

and τ_{sat} is computed as [1, 6];

$$\tau_{sat} = K \cdot \text{sat}\left(\frac{S}{\phi}\right) \quad (41)$$

by replace the formulation (41) in (39) the control output can be written as [51-55];

$$\tau = \tau_{eq} + K \cdot \text{sat}\left(\frac{S}{\phi}\right) = \begin{cases} \tau_{eq} + K \cdot \text{sgn}(S) & , |S| \geq \phi \\ \tau_{eq} + K \cdot \frac{S}{\phi} & , |S| < \phi \end{cases} \quad (42)$$

Figure 3 shows the position classical sliding mode control for PUMA 560 robot manipulator. By (42) and (40) the sliding mode control of PUMA 560 robot manipulator is calculated as;

$$\tau = [M^{-1}(B + C + G) + \dot{S}]M + K \cdot \text{sat}\left(\frac{S}{\phi}\right) \quad (43)$$

where $S = \lambda e + \dot{e}$ in PD-SMC and $S = \lambda e + \dot{e} + \left(\frac{\lambda}{2}\right)^2 \int e$ in PID-SMC.

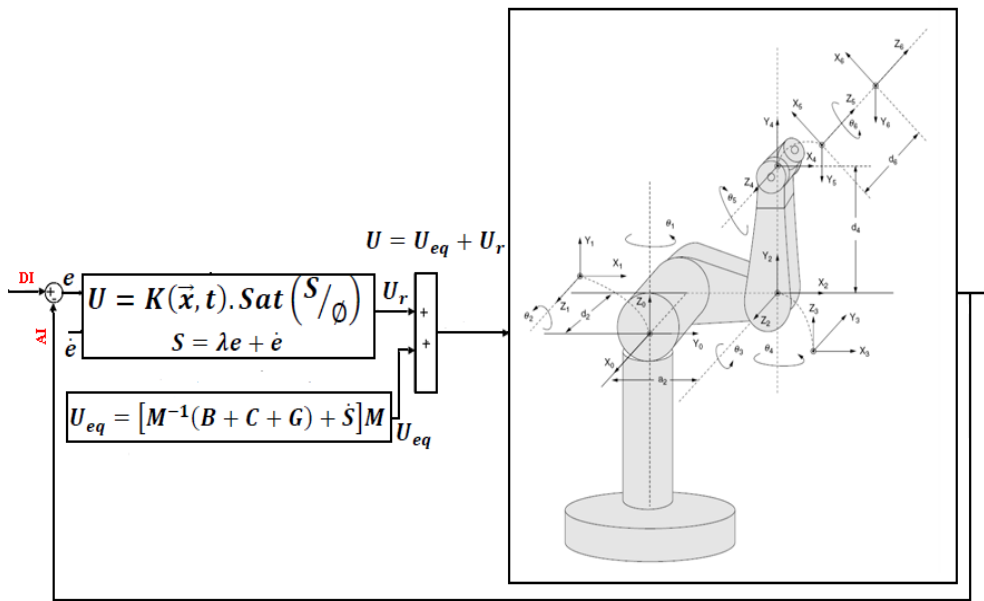


Figure 3. Block Diagram of Sliding Mode Controller with Saturation Function

3. Methodology: Design Fuzzy-Based Rule Base Evaluation Tuning Error-Based Fuzzy Sliding Mode Controller

Design fuzzy sliding mode controller: As shown in Figure 3, sliding mode controller is divided into two main parts: continuous part and equivalent part. Continuous part is based on linear saturation function which this method is used to reduce the chattering. Equivalent part is based on robot manipulator's dynamic formulation which these formulations are nonlinear; MIMO and some of them are unknown. Reduce or eliminate the chattering phenomenon and reduce the error are played important role, therefore boundary layer method is used beside the equivalent part to solve the chattering problem and reduce the error. Equivalent part of sliding mode controller is based on nonlinear dynamic formulations of robot manipulator. Robot manipulator's dynamic formulations are highly nonlinear and some of parameters are unknown therefore design a controller based on dynamic formulation is complicated. To solve this challenge fuzzy logic methodology is applied to sliding mode controller. Based on literature [43, 44, 58-61], most of researchers are designed fuzzy model-based sliding mode

controller and model-based sliding mode fuzzy controller. In this method fuzzy logic method is used to estimate some dynamic formulation that they are used in equivalent part. In researcher's method fuzzy logic method is applied to equivalent part to estimate it. Figures 4 and 5 show the fuzzy model-based sliding mode controller and model-based sliding mode fuzzy controller.

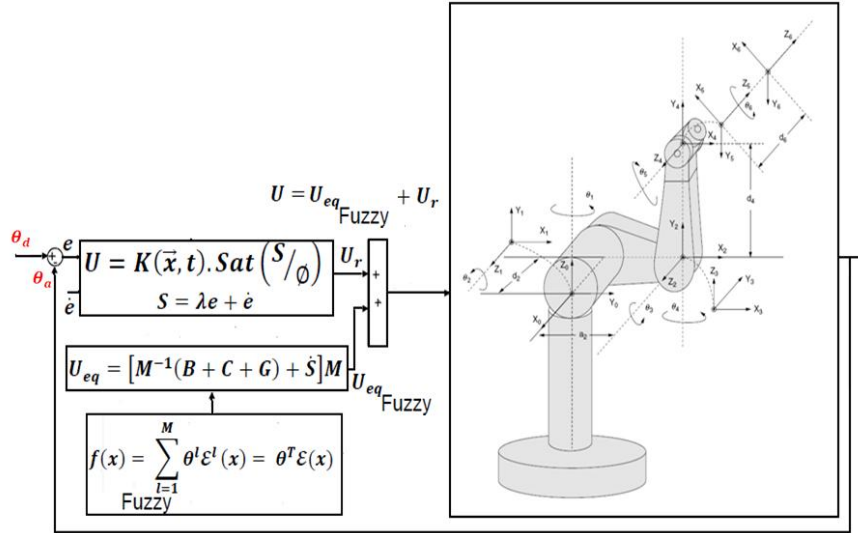


Figure 4. Fuzzy Model-based Sliding Mode Controller

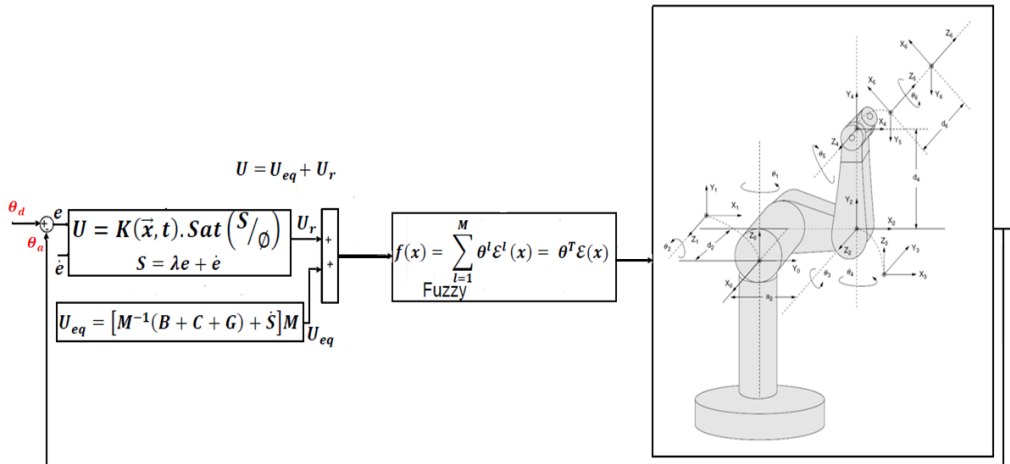


Figure 5. Model-based Sliding Mode Fuzzy Controller

To solve the challenge of sliding mode controller based on nonlinear dynamic formulation this research is focused on eliminate the nonlinear equivalent formulation. In this method; dynamic nonlinear equivalent part is replaced by performance/error-based fuzzy logic controller. In fuzzy error-based sliding mode controller; error based Mamdani's fuzzy inference system has considered with two inputs, one output and totally 49 rules instead of the dynamic equivalent part. Figure 6 is shown error-based fuzzy sliding mode controller. In this method a model free Mamdani's fuzzy inference system has considered based on error-based fuzzy logic controller instead of equivalent control.

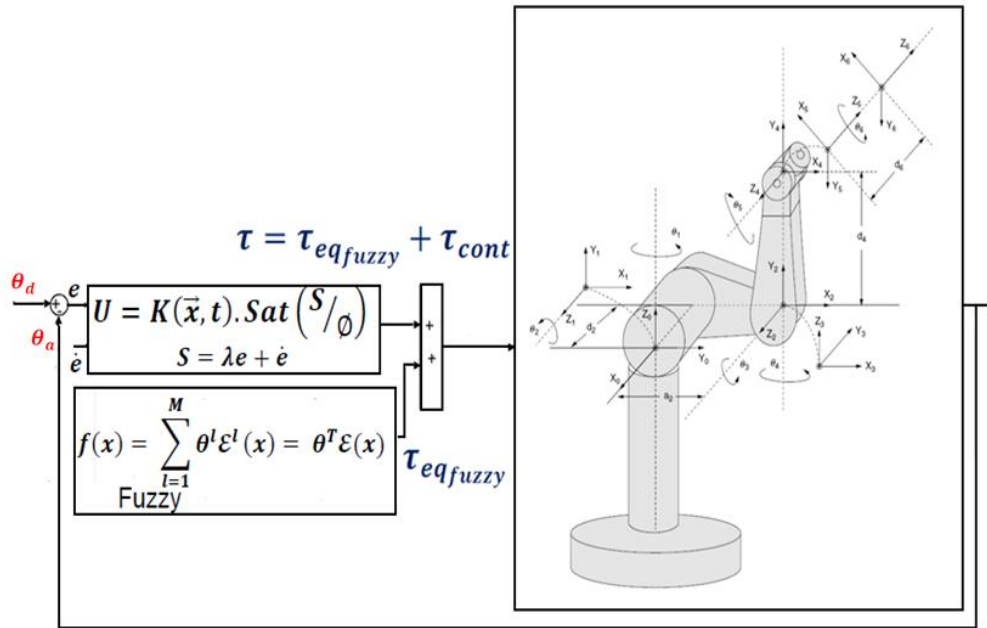


Figure 6. Error-based Fuzzy Sliding Mode Controller

For both sliding mode controller and fuzzy sliding mode controller applications the system performance is sensitive to the sliding surface slope coefficient(λ). For instance, if large value of λ is chosen the response is very fast the system is unstable and conversely, if small value of λ is considered the response of system is very slow but system is stable. Therefore to have a good response, compute the best value sliding surface slope coefficient is very important. Eksin et. al [83] have designed mathematical model-free sliding surface slope in fuzzy sliding mode controller. In above method researchers are used saturation function instead of switching function therefore the proof of stability is very difficult. Figure 6 has two main parts: sliding function and error-based fuzzy part. In fuzzy sliding mode controller based on (38) the PD-sliding surface is defined as follows [80-97]:

$$S = \dot{e} + \lambda_1 e \quad (44)$$

where $\lambda_1 = \text{diag}[\lambda_{11}, \lambda_{12}, \lambda_{13}]$. The time derivative of S is computed;

$$\dot{S} = \ddot{q}_d + \lambda_1 \dot{e} \quad (45)$$

Based on Figure 6, the fuzzy error-based sliding mode controller's output is written;

$$\hat{\tau} = \tau_{eq_fuzzy} + \tau_{cont} \quad (46)$$

Based on fuzzy logic methodology

$$f(x) = U_{fuzzy} = \sum_{l=1}^M \theta^l \zeta(x) \quad (47)$$

where θ^T is adjustable parameter (gain updating factor) and $\zeta(x)$ is defined by;

$$\zeta(x) = \frac{\sum_i \mu(x_i) x_i}{\sum_i \mu(x_i)} \quad (48)$$

Where $\mu(x_i)$ is membership function. τ_{fuzzy} is defined as follows;

$$\tau_{fuzzy} = \sum_{l=1}^M \theta^T \zeta(x) = [M^{-1}(B + C + G) + \dot{S}]M \quad (49)$$

Based on [80-81] to compute dynamic parameters of PUMA560;

$$\tau_{fuzzy} = \begin{bmatrix} \tau_{1fuzzy} \\ \tau_{2fuzzy} \\ \tau_{3fuzzy} \end{bmatrix}, M^{-1} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & 0 & 0 & 0 \\ M_{21} & M_{22} & M_{23} & 0 & 0 & 0 \\ M_{31} & M_{32} & M_{33} & 0 & M_{35} & 0 \\ 0 & 0 & 0 & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix}^{-1}$$

$$B + C + G = \begin{bmatrix} b_{112}\dot{q}_1\dot{q}_2 + b_{113}\dot{q}_1\dot{q}_3 + 0 + b_{123}\dot{q}_2\dot{q}_3 \\ 0 + b_{223}\dot{q}_2\dot{q}_3 + 0 + 0 \\ 0 \\ b_{412}\dot{q}_1\dot{q}_2 + b_{413}\dot{q}_1\dot{q}_3 + 0 + 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} C_{12}\dot{q}_2^2 + C_{13}\dot{q}_3^2 \\ C_{21}\dot{q}_1^2 + C_{23}\dot{q}_3^2 \\ C_{31}\dot{q}_1^2 + C_{32}\dot{q}_2^2 \\ 0 \\ C_{51}\dot{q}_1^2 + C_{52}\dot{q}_2^2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ g_2 \\ g_3 \\ 0 \\ g_5 \\ 0 \end{bmatrix}$$

$$\dot{S} = \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \\ \dot{S}_4 \\ \dot{S}_5 \\ \dot{S}_6 \end{bmatrix} \text{ and } M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & 0 & 0 & 0 \\ M_{21} & M_{22} & M_{23} & 0 & 0 & 0 \\ M_{31} & M_{32} & M_{33} & 0 & M_{35} & 0 \\ 0 & 0 & 0 & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix}$$

Therefore, the error-based fuzzy sliding mode controller for PUMA robot manipulator is calculated by the following equation;

$$\begin{bmatrix} \widehat{\tau}_1 \\ \widehat{\tau}_2 \\ \widehat{\tau}_3 \end{bmatrix} = \begin{bmatrix} \tau_{1fuzzy} \\ \tau_{2fuzzy} \\ \tau_{3fuzzy} \end{bmatrix} + \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} \text{ Sat } \begin{bmatrix} s_1 \\ \phi_1 \\ s_2 \\ \phi_2 \\ s_3 \\ \phi_3 \end{bmatrix} \quad (50)$$

As mentioned in Figure 6, the design of error-based fuzzy instead of equivalent part based on Mamdani's fuzzy inference method has four steps, namely, fuzzification, fuzzy rule base and rule evaluation, aggregation of the rule output (fuzzy inference system) and defuzzification.

Fuzzification: the first step in fuzzification is determine inputs and outputs which, it has two inputs (e, \dot{e}) and one output (τ_{fuzzy}). The inputs are error (e) which measures the difference between desired and actual output position, and the change of error (\dot{e}) which measures the difference between desired and actual velocity and output is fuzzy equivalent torque. The second step is chosen an appropriate membership function for inputs and output which, to simplicity in implementation because it is a linear function with regard to acceptable performance triangular membership function is selected in this research as shown in Figure 3.6. The third step is chosen the correct labels for each fuzzy set which, in this research namely as linguistic variable. Based on experience knowledge the linguistic variables for error (e) are; Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (Z), Positive Small (PS), Positive Medium (PM), Positive Big (PB), and based on literature [40] and experience knowledge it is quantized into thirteen levels represented by: -1,

-0.83, -0.66, -0.5, -0.33, -0.16, 0, 0.16, 0.33, 0.5, 0.66, 0.83, 1 the linguistic variables for change of error (\dot{e}) are; Fast Left (FL), Medium Left (ML), Slow Left (SL), Zero (Z), Slow Right (SR), Medium Right (MR), Fast Right (FR), and it is quantized in to thirteen levels represented by: -6, -5, -0.4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, and the linguistic variables to find the output are; Large Left (LL), Medium Left (ML), Small Left (SL), Zero (Z), Small Right (SR), Medium Right (MR), Large Right (LR) and it is quantized in to thirteen levels represented by: -85, -70.8, -56.7, -42.5, -28.3, -14.2, 0, 14.2, 28.3, 42.5, 56.7, 70.8, 85.

Fuzzy rule base and rule evaluation: the first step in rule base and evaluation is to provide a least structured method to derive the fuzzy rule base which, expert experience and control engineering knowledge is used because this method is the least structure of the other one and the researcher derivation the fuzzy rule base from the knowledge of system operate and/or the classical controller. Design the rule base of fuzzy inference system can play important role to design the best performance of fuzzy sliding mode controller, that to calculate the fuzzy rule base the researcher is used to heuristic method which, it is based on the behavior of the control of robot manipulator suppose that two fuzzy rules in this controller are;

F.R¹: IF e is NB and \dot{e} is FL, THEN τ is LL. (51)

F.R²: IF e is PS and \dot{e} is FL THEN τ is ML

The complete rule base for this controller is shown in Table 1. Rule evaluation focuses on operation in the antecedent of the fuzzy rules in fuzzy sliding mode controller. This part is used AND/OR fuzzy operation in antecedent part which AND operation is used.

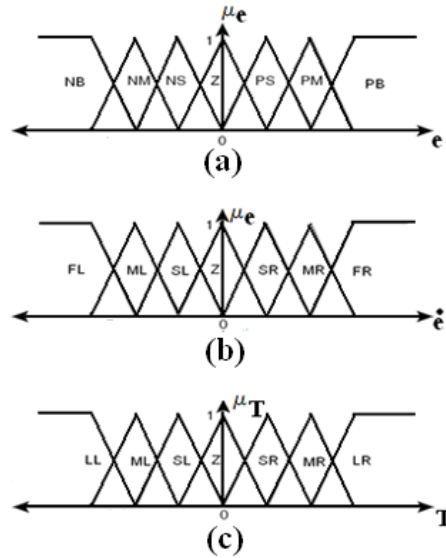


Figure 7. Membership Function: a) Error b) Change of Error c) Torque

Aggregation of the rule output (Fuzzy inference): Max-Min aggregation is used in this work.

Table 1. Modified Fuzzy Rule Base Table

	Decrease the overshoot →							
	← \dot{e}							
e	FL	ML	SL	Z	SR	MR	FR	
NB	LL	LL	LL	ML	SL	SL	Z	↑ Decrease the rise time
NM	LL	ML	ML	ML	SL	Z	SR	
NS	LL	ML	SL	SL	Z	SR	MR	
Z	LL	ML	SL	Z	SR	MR	LR	
PS	ML	SL	Z	SR	SR	MR	LR	
PM	SL	Z	SR	MR	MR	MR	LR	
PB	Z	SR	SR	MR	LR	LR	LR	
	↓							

Defuzzification: The last step to design fuzzy inference in our fuzzy sliding mode controller is defuzzification. This part is used to transform fuzzy set to crisp set, therefore the input for defuzzification is the aggregate output and the output of it is a crisp number. Center of gravity method (*COG*) is used in this research. Table 2 shows the lookup table in fuzzy sliding mode controller which is computed by *COG* defuzzification method. Table 2 has 169 cells to shows the error-based fuzzy instead of equivalent part behavior. For instance if $e = -1$ and $\dot{e} = -3.92$ then the output=-85. Based on Table 3.1 if two fuzzy rules are defined by

$F.R^1$: if e is *NB* and \dot{e} is *ML* then T is *LL*

$F.R^2$: if e is *NB* and \dot{e} is *FL* then T is *LL*

If all input fuzzy activated by crisp input values $e = -1$ and $\dot{e} = -3.92$ and fuzzy set to compute *NB*, *ML* and *FL* are defined as

$$e_{(NB)} = \{(0, -1.5), (0.25, -1.375), (0.5, -1.25), (0.75, -1.125), (1, -1), (0.75, -0.875), (0.5, -0.75), (0.25, -0.625), (0, -0.5)\}$$

$$\dot{e}_{(ML)} = \{(0, -5.8), (0.25, -5.17), (0.5, -4.55), (0.75, -3.92), (1, -3.3), (0.75, -2.67), (0.5, -2.05), (0.25, -1.42), (0, -0.83)\}$$

$$\dot{e}_{(FL)} = \{(0, -7.5), (0.25, -6.88), (0.5, -6.25), (0.75, -5.57), (1, -5), (0.75, -4.30), (0.5, -3.92), (0.25, -3.12), (0, -2.5)\}$$

while

$$T_{(LL)} = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.75, -94.5), (1, -85), (0.75, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\}$$

In this controller *AND* fuzzy operation is used therefore the output fuzzy set is calculated by using individual rule-base inference. Based on 2.63 the activation degrees is computed as

$$\mu_{FR_1} = \min[\mu_{e(N.B)}(-1), \mu_{\dot{e}(M.L)}(-3.92)] = \min[1, 0.75] = 0.75$$

$$\mu_{FR_2} = \min[\mu_{e(N.B)}(-1), \mu_{\dot{e}(F.L)}(-3.92)] = \min[1, 0.5] = 0.5$$

The activation degrees of the consequent parts for $F.R^1$ and $F.R^2$ are computed as:

$$\mu_{FR_1}(-1, -3.92, T) = \min[\mu_{FR_1}(-1, -3.92), \mu_{T(LL)}] = \min[0.75, \mu_{T(LL)}]$$

$$\mu_{FR_2}(-1, -3.92, T) = \min[\mu_{FR_2}(-1, -3.92), \mu_{T(LL)}] = \min[0.5, \mu_{T(LL)}]$$

Fuzzy set $T_{LL(1)}$ and $T_{LL(2)}$ have nine elements:

$$F.F^1(-1, -3.92, T) = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.75, -94.5), (0.75, -85), (0.75, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\}$$

$$F.F^2(-1, -3.92, T) = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.5, -94.5), (0.5, -85), (0.5, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\}$$

Max-min aggregation is used to find the output of fuzzy set:

$$\begin{aligned} \mu_{U_{12}}(-1, -3.92, T) &= \mu_{U_{i=1}^2 FR^1}(-1, -3.92, T) \\ &= \max\{\mu_{FR}^1(-1, -3.92, T)_{L.L}, \mu_{FR}^2(-1, -3.92, T)_{L.L}\} \end{aligned}$$

$$U_{12} = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.75, -94.5), (0.75, -85), (0.75, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\}$$

Based on (2.71) the COG defuzzification is selected as;

$$COG = [(0.25 \times -113.5) + (0.5 \times -104) + (0.75 \times -94.5) + (0.75 \times -85) + 0.75 \times -75.5 + 0.5 \times -66 + 0.25 \times -56.5] / [0.25 + 0.5 + 0.75 + 0.75 + 0.5 + 0.25] = -318.75375 / 3 = -106.25125$$

Table 2. τ_{eq} fuzzy Performance: Lookup Table in Fuzzy Sliding Mode Controller by COG

$\begin{matrix} \rightarrow \\ \dot{e} \\ \leftarrow \\ e \\ \downarrow \end{matrix}$	Membership Function (τ_{eq} fuzzy)												
	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
-1	-85	-	-	-84	-	-81	-79	-71	-68	-65	-62	-60	-54
-	-	-84	-82	-80	-78	-77	-74	-70	-64	-60	-56	-54	-47
0.83	84.8	84.8	82.1	82.1	78	77	74	70	64	60	56	54	47
-	-78	-73	-70	-68	-64	-61	-60	-57	-55	-50	-47	-40	-38
0.66	78	73	70	68	64	61	60	57	55	50	47	40	38
-0.5	-70	-60	-58	-51	-42	-38	-34	-33	-31	-29	-	-	-28
-	-50	-48	-45	-40	-38	-34	-32	-30	-28	-26	-25	-21	-20
0.33	50	48	45	40	38	34	32	30	28	26	25	21	20
-	-30	-25	-21	-18	-16	-14	-10	-9	-8	-7	-6.8	-6	-5
0.16	30	25	21	18	16	14	10	9	8	7	6.8	6	5
0	-10	-8	-6	-1	2	3	6	7	8	10	12	15	17
0.16	10	8	6	1	2	3	6	7	8	10	12	15	17
0.33	15	18	21	22	23	25	27	28	29	30	30.5	30.8	31
0.5	29	29.8	31	33	34	34.6	35	35.2	36	37	38	39	42
0.66	40	41	42	43	45	45	46	46.3	46.8	47	48	51	52
0.83	48	49	50	52	53	55	56	57	58	59	60	61	63
1	60	61	62	63	64	66	67	68	68.5	69	70	70.8	71
1	66	68.7	68.9	70	72	74	75	77	78	79	81	83	84

Design fuzzy based rule base evaluation tuning error-based fuzzy sliding mode controller:

In most of industrial robot manipulators, controllers are still usually classical linear, but the manipulator dynamics is highly nonlinear and have uncertain or variation in parameters (e.g., structure and unstructured), as a result design a classical linear controllers for this system is very difficult and sometimes impossible. The first solution is to make the robust algorithm in order to reduce the uncertainty problems in a limit variation (e.g., sliding mode controller and computed torque like controller). Conversely the first solution is used in many applications it

has some limitations such as nonlinear dynamic part in controller. The second solution is applied artificial intelligence method (e.g., fuzzy logic) in conventional nonlinear method to reduce or eliminate the challenges. However the second solution is a superior to reduce or eliminate the dynamic nonlinear part with respect to have stability and fairly good robustness but it has a robust in a limit variation. The third solution is used the on-line fuzzy sliding mode controller (e.g., fuzzy-based tuning sliding surface slope in error-based fuzzy sliding mode controller). Adaptive (on-line) control is used in systems whose dynamic parameters are varying and need to be training on line. Error-based fuzzy sliding mode controller has difficulty in handling unstructured model uncertainties and this controller's performance is sensitive to sliding surface slope coefficient. It is possible to solve above challenge by combining fuzzy-based tuning method and error-based fuzzy sliding mode controller which this methodology can help to improve system's tracking performance by on-line tuning (fuzzy-based tuning) method.

Based on above discussion, compute the best value of sliding surface slope coefficient has played important role to improve system's tracking performance especially the system parameters are unknown or uncertain. This problem is solved by tuning the surface slope coefficient (λ) of the error-based fuzzy sliding mode controller continuously in real-time. In this methodology, the system's performance is improved with respect to the classical sliding mode controller and error-based fuzzy sliding mode controller. Figure 8 shows the fuzzy-based tuning fuzzy sliding mode controller.

Based on (47) and (48) to adjust the sliding surface slope coefficient we define $\hat{f}(x|\lambda)$ as the fuzzy based tuning.

$$\hat{f}(x|\lambda) = \lambda^T \zeta(x) \tag{52}$$

If minimum error (λ^*) is defined by;

$$\lambda^* = \mathit{arg\ min} [(\mathit{Sup})\{\hat{f}(x|\lambda) - f(x)\}] \tag{53}$$

Where λ^T is adjusted by an adaption law and this law is designed to minimize the error's parameters of $\lambda - \lambda^*$. adaption law in fuzzy-based tuning error-based fuzzy sliding mode controller is used to adjust the sliding surface slope coefficient. Fuzzy-based tuning part is a supervisory controller based on Mamdani's fuzzy logic methodology. This controller has two inputs namely; error (e) and change of error (\dot{e}) and an output namely; gain updating factor(α). As a summary design a fuzzy-based tuning based on fuzzy logic method in fuzzy based tuning error-based fuzzy sliding mode controller has five steps:

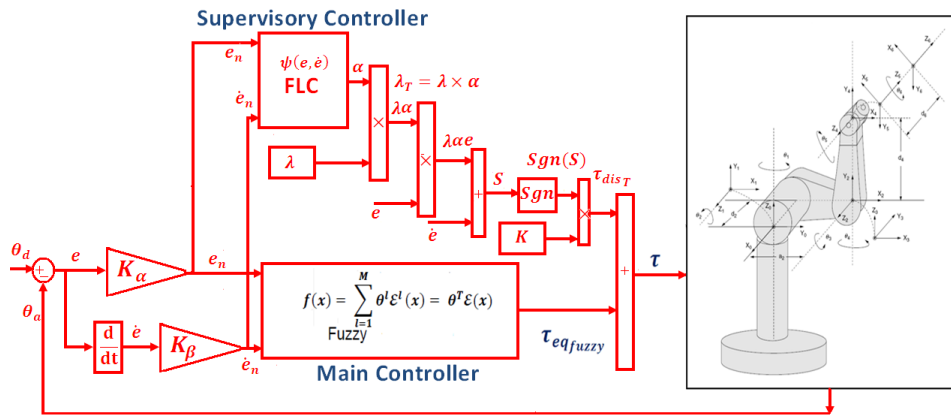


Figure 8. Fuzzy-based Tuning Error-based Fuzzy Sliding Mode Fuzzy Controller

1. **Determine inputs and outputs:** it has two inputs error and change of error (e, \dot{e}) and the output name's is sliding surface slope updating factor (α).

2. **Find linguistic variable:** The linguistic variables for error(e) are; Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (Z), Positive Small (PS), Positive Medium (PM), Positive Big (PB), and it is quantized into thirteen levels represented by: -1, -0.83, -0.66, -0.5, -0.33, -0.16, 0, 0.16, 0.33, 0.5, 0.66, 0.83, 1, the linguistic variables for change of error(\dot{e}) are ;Fast Left (FL), Medium Left (ML), Slow Left (SL),Zero (Z), Slow Right (SR), Medium Right (MR), Fast Right (FR), and it is quantized in to thirteen levels represented by: -6, -5, -0.4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, and the linguistic variables for sliding surface slope updating factor (α) are; Zero (ZE), Very Small (VS), Small (S), Small Big (SB), Medium Big (MB), Big (B), and Very Big (VB) and they are defined on [0.5,1] and quantized into thirteen levels respected by: 0.5, 0.5417, 0.583, 0.625, 0.667, 0.7087, 0.7503, 0.792, 0.834, 0.876, 0.917, 0.959, 1.

3. **Type of membership function:** In this research triangular membership function is selected because it has linear equation with regard to has a high-quality response.

4. **Design fuzzy rule table:** the rule base for sliding surface slope updating factor of fuzzy-based tuning error-based fuzzy sliding mode controller is based on
F.R¹: *IF e is NB and \dot{e} is NB, THEN α is VB.* (53)

The complete rule base for supervisory controller is shown in Table 3.

5. **Defuzzification:** COG method is used to defuzzification in this research.

Table 3. Fuzzy Rule Base for Sliding Surface Slope Updating Factor (α)

$e \backslash \dot{e}$	FL	ML	SL	Z	SR	MR	FR
NB	VB	VB	VB	B	SB	S	ZE
NM	VB	VB	B	B	MB	S	VS
NS	VB	MB	B	VB	VS	S	VS
Z	S	SB	MB	ZE	MB	SB	S
PS	VS	S	VS	VB	B	MB	VB
PM	VS	S	MB	B	B	VB	VB
PB	ZE	S	SB	B	VB	VB	VB

Based on Figure 8, supervisory controller is a controller to solve the unstructured uncertainties and tuning the sliding surface slope coefficient. This controller consists of two parts: fuzzy logic controller and scaling factor. Fuzzy logic controller is a Mamdani's error base inference system which has error (e) and change of error (\dot{e}) as inputs and sliding surface slope updating factor (α) as output. Each inputs has seven linguistic variables thus the controller's output has 49 rules, the output is defined between [0.5 1] based on Table 3 and it is quantized into thirteen levels. Scaling factor (k_α and k_β) are used to limit error between [-1 1] and change of error between [-6 6]. To normalize the error and change of error scaling factors are applied (k_α and k_β) to e and \dot{e} as bellows:

$$e_n = k_\alpha \times e \tag{54}$$

$$\dot{e}_n = k_\beta \times \dot{e} \tag{55}$$

Sliding surface slope updating factor (α) is used to tuning the main controller to give the best possible results. It is required because the robot manipulator's dynamic equations are highly nonlinear, the rules formulated in fuzzy sliding mode controller through user experience are not always correct under defined and also to unstructured uncertainties. It is independent of robot manipulator dynamic parameters and depends only on current system's performance; it is based on error and change of error and rule base. In this method the actual sliding surface slope coefficient (λ) is obtained by multiplying the sliding surface with sliding surface slope updating factor (α). The sliding surface slope updating factor (α) is calculated on-line by 49 rules Mamdani's error-based fuzzy logic methodology in each rule base part. To limitation the error between [-1 1] and change of error between [-6 6], the best values for scaling factors are; $K_\alpha = 1.5$ and $K_\beta = 3$ based on Table 3.4.

Table 4. Best Value of k_α and k_β to Tuning the α

k_α	k_β	e_{band}	\dot{e}_{band}	α_{band}
0.25	0.5	[-0.166 0.18]	[-0.98 1.1]	[-0.1 0.1]
0.5	1	[-0.332 0.37]	[-1.96 2.2]	[-0.05 0.2]
0.75	1.5	[-0.498 0.54]	[-2.94 3.3]	[0 0.3]
1	2	[-0.664 0.733]	[-3.92 4.4]	[0.12 0.38]
1.25	2.5	[-0.83 0.9]	[-4.9 5.5]	[0.3 0.85]
1.5	3	[-0.995 1.1]	[-5.88 6.6]	[0.49 1.01]
1.75	3.5	[-1.162 1.26]	[-6.86 7.7]	[0.58 1.38]
2	4	[-1.328 1.44]	[-7.84 8.8]	[0.63 1.6]

Table 5 shows the sliding surface slope updating factor (α) lookup table in fuzzy-based tuning part by COG defuzzification method. It has 169 cells to shows the fuzzy-based tuning to on-line tuning the sliding surface slope coefficient. For instance if $e = -1$ and $\dot{e} = -3.92$ then the output=0.5. Based on Table 3 if two fuzzy rules are defined by

$F.R^1$: if e is NB and \dot{e} is ML then α is VB

$F.R^2$: if e is NB and \dot{e} is FL then α is VB

If all input fuzzy activated by crisp input values $e = -1$ and $\dot{e} = -3.92$ and fuzzy set to compute NB , ML and FL are defined as

$e_{(NB)} = \{(0, -1.5), (0.25, -1.375), (0.5, -1.25), (0.75, -1.125), (1, -1), (0.75, -0.875), (0.5, -0.75), (0.25, -0.625), (0, -0.5)\}$

$\dot{e}_{(ML)} = \{(0, -5.8), (0.25, -5.17), (0.5, -4.55), (0.75, -3.92), (1, -3.3), (0.75, -2.67), (0.5, -2.05), (0.25, -1.42), (0, -0.83)\}$

$\dot{e}_{(FL)} = \{(0, -7.5), (0.25, -6.88), (0.5, -6.25), (0.75, -5.57), (1, -5), (0.75, -4.30), (0.5, -3.92), (0.25, -3.12), (0, -2.5)\}$

Table 5. Sliding Surface Slope Updating Factor (α): Fuzzy-based Tuning Fuzzy Sliding Mode Controller Lookup Table by COG Method

$\begin{matrix} \rightarrow \\ \dot{e} \\ \leftarrow \\ e \\ \downarrow \end{matrix}$	Membership Function (α)												
	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
-1	0.5	0.5	0.5	0.5	0.55	0.55	0.55	0.6	0.65	0.65	0.7	0.7	0.75
- 0.83	0.5	0.5	0.5	0.5	0.55	0.55	0.6	0.6	0.65	0.7	0.7	0.75	0.75
- 0.66	0.5	0.5	0.5	0.55	0.55	0.6	0.6	0.65	0.7	0.7	0.75	0.75	0.8
-0.5	0.5	0.5	0.55	0.55	0.6	0.6	0.65	0.7	0.7	0.75	0.75	0.8	0.8
- 0.33	0.5	0.55	0.55	0.6	0.6	0.65	0.7	0.7	0.75	0.75	0.8	0.8	0.85
- 0.16	0.55	0.55	0.6	0.6	0.65	0.7	0.75	0.75	0.75	0.8	0.8	0.85	0.9
0	0.55	0.6	0.6	0.65	0.7	0.75	0.75	0.75	0.8	0.85	0.85	0.9	0.9
0.16	0.6	0.65	0.7	0.75	0.75	0.75	0.75	0.8	0.8	0.85	0.85	0.9	0.9
0.33	0.65	0.7	0.75	0.75	0.75	0.8	0.8	0.8	0.85	0.85	0.9	0.9	0.95
0.5	0.7	0.75	0.75	0.75	0.8	0.8	0.8	0.85	0.85	0.9	0.9	0.95	0.95
0.66	0.75	0.75	0.75	0.8	0.8	0.85	0.85	0.85	0.9	0.9	0.95	1	1
0.83	0.75	0.75	0.8	0.85	0.85	0.9	0.9	0.95	0.95	1	1	1	1
1	0.75	0.8	0.85	0.9	0.95	0.95	1	1	1	1	1	1	1

while $\alpha_{(VB)} = \{(0,0.4165), (0.25,0.4403), (0.5,0.4641), (0.75,0.4879), (1,0.5), (0.75,0.5238), (0.5,0.5476), (0.25,0.5714), (0,0.5834)\}$

In this controller *AND* fuzzy operation is used therefore the output fuzzy set is calculated by using individual rule-base inference. Based on 2.63 the activation degrees is computed as

$$\mu_{FR_1} = \min[\mu_{e(N.B)}(-1), \mu_{\dot{e}(M.L)}(-3.92)] = \min[1, 0.75] = 0.75$$

$$\mu_{FR_2} = \min[\mu_{e(N.B)}(-1), \mu_{\dot{e}(F.L)}(-3.92)] = \min[1, 0.5] = 0.5$$

The activation degrees of the consequent parts for $F. R^1$ and $F. R^2$ are computed as:

$$\mu_{FR_1}(-1, -3.92, \alpha) = \min[\mu_{FR_1}(-1, -3.92), \mu_{\alpha(VB)}] = \min[0.75, \mu_{\alpha(VB)}]$$

$$\mu_{FR_2}(-1, -3.92, \alpha) = \min[\mu_{FR_2}(-1, -3.92), \mu_{\alpha(VB)}] = \min[0.5, \mu_{\alpha(VB)}]$$

Fuzzy set $\alpha_{L.L(1)}$ and $\alpha_{L.L(2)}$ have nine elements:

$$F. F^1(-1, -3.92, \alpha) = \{(0,0.4165), (0.25,0.4403), (0.5,0.4641), (0.75,0.4879), (1,0.5), (0.75,0.5238), (0.5,0.5476), (0.25,0.5714), (0,0.5834)\}$$

$$F. F^2(-1, -3.92, T) = \{(0,0.4165), (0.25,0.4403), (0.5,0.4641), (0.75,0.4879), (1,0.5), (0.75,0.5238), (0.5,0.5476), (0.25,0.5714), (0,0.5834)\}$$

Based on 2.69, Max-min aggregation is used to find the output of fuzzy set:

$$\begin{aligned} \mu_{U_{12}}(-1, -3.92, \alpha) &= \mu_{U_{i=1}^2 FR^1}(-1, -3.92, \alpha) \\ &= \max\{\mu_{FR}^1(-1, -3.92, \alpha)_{VB}, \mu_{FR}^2(-1, -3.92, \alpha)_{VB}\} \end{aligned}$$

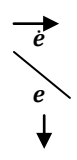
$$U_{12} = \{(0,0.4165), (0.25,0.4403), (0.5,0.4641), (0.75,0.4879), (0.75,0.5), (0.75,0.5238), (0.5,0.5476), (0.25,0.5714), (0,0.5834)\}$$

Based on (2.71) the COG defuzzification is selected as;

$$COG = (0.25 \times 0.4403) + (0.5 \times 0.4641) + (0.75 \times 0.4879) + (0.75 \times 0.5) + (0.75 \times 0.5238) + (0.5 \times 0.5476) + (0.25 \times 0.5714) \div [0.25 + 0.5 + 0.75 + 0.75 + 0.75 + 0.5 + 0.25] = 1.875375 \div 1 = 1.875375 = 0.5$$

Table 6 shows the fuzzy equivalent torque performance ($\tau_{eq\ fuzzy}$) lookup table in fuzzy-based tuning error-based fuzzy sliding mode controller by COG defuzzification method.

Table 6. Fuzzy Equivalent Torque Performance ($\tau_{eq\ fuzzy}$): Fuzzy-based Tuning Fuzzy Sliding Mode Controller Lookup Table by COG Method

	Membership Function												
	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
-1	-85	-85	-85	-85	-84	-84	-83	-71	-57	-43	-28	-14	0
-0.83	-85	-85	-85	-85	-71	-57	-56	-57	-43	-28	-14	0	0
-0.66	-85	-85	-85	-71	-57	-43	-43	-43	-28	-14	0	0	14
-0.5	-85	-85	-71	-57	-43	-28	-28	-28	-14	0	0	14	28
-0.33	-85	-71	-57	-43	-28	-14	-14	-14	0	0	14	28	43
-0.16	-71	-57	-43	-28	-14	-14	0	0	0	14	28	43	57
0	-71	-57	-43	-28	-14	0	0	0	14	28	43	43	71
0.16	-43	-28	-14	0	0	0	0	14	14	28	43	57	70
0.33	-28	-14	0	0	0	14	14	14	28	43	57	71	84
0.5	-14	0	0	0	14	28	28	28	43	57	70	84	85
0.66	0	0	0	14	28	43	43	43	57	70	84	85	85
0.83	0	0	14	28	43	57	57	57	70	85	85	85	85
1	0	14	28	43	57	71	85	85	85	85	85	85	85

Based on Tables 1, 5 and 6 where $e = -1$ and $\dot{e} = -3.92$ and fuzzy set to compute NB , ML and FL are defined as

$$e_{(NB)} = \{(0, -1.5), (0.25, -1.375), (0.5, -1.25), (0.75, -1.125), (1, -1), (0.75, -0.875), (0.5, -0.75), (0.25, -0.625), (0, -0.5)\}$$

$$\dot{e}_{(ML)} = \{(0, -5.8), (0.25, -5.17), (0.5, -4.55), (0.75, -3.92), (1, -3.3), (0.75, -2.67), (0.5, -2.05), (0.25, -1.42), (0, -0.83)\}$$

$$\dot{e}_{(FL)} = \{(0, -7.5), (0.25, -6.88), (0.5, -6.25), (0.75, -5.57), (1, -5), (0.75, -4.30), (0.5, -3.92), (0.25, -3.12), (0, -2.5)\}$$

while

$$T_{(LL)} = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.75, -94.5), (1, -85), (0.75, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\}$$

Based on 2.63 the activation degrees is computed as

$$\mu_{FR_1} = \min[\mu_{e(NB)}(-1), \mu_{\dot{e}(ML)}(-3.92)] = \min[1, 0.75] = 0.75$$

$$\mu_{FR_2} = \min[\mu_{e(NB)}(-1), \mu_{\dot{e}(FL)}(-3.92)] = \min[1, 0.5] = 0.5$$

The activation degrees of the consequent parts for $F.R^1$ and $F.R^2$ are computed as:

$$\mu_{FR_1}(-1, -3.92, T) = \min[\mu_{FR_1}(-1, -3.92), \mu_{T(LL)}] = \min[0.75, \mu_{T(LL)}]$$

$$\mu_{FR_2}(-1, -3.92, T) = \min[\mu_{FR_2}(-1, -3.92), \mu_{T(LL)}] = \min[0.5, \mu_{T(LL)}]$$

Fuzzy set $T_{LL(1)}$ and $T_{LL(2)}$ have nine elements:

$$F.F^1(-1, -3.92, T) = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.75, -94.5), (0.75, -85), (0.75, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\}$$

$$F.F^2(-1, -3.92, T) = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.5, -94.5), (0.5, -85), (0.5, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\}$$

Based on 2.69, Max-min aggregation is used to find the output of fuzzy set:

$$\begin{aligned} \mu_{U_{12}}(-1, -3.92, T) &= \mu_{U_{i=1}^{FR^1}}(-1, -3.92, T) \\ &= \max\{\mu_{FR}^1(-1, -3.92, T)_{L,L}, \mu_{FR}^2(-1, -3.92, T)_{L,L}\} \end{aligned}$$

$$U_{12} = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.75, -94.5), (0.75, -85), (0.75, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\}$$

Based on (2.71) the COG defuzzification is selected as;

$$COG = [(0.25 \times -113.5) + (0.5 \times -104) + (0.75 \times -94.5) + (0.75 \times -85) + 0.75 \times -75.5 + 0.5 \times -66 + 0.25 \times -56.5] / [0.25 + 0.5 + 0.75 + 0.75 + 0.75 + 0.5 + 0.25] = -318.75375 = -85$$

Based on Figures 3.8, torque performance is calculated by;

$$\tau = \tau_{equ \text{ fuzzy}} + (\tau_{dis})_{tuning} \tag{56}$$

where $(\tau_{dis})_{tuning}$ is a discontinuous part which tuning by fuzzy-based tuning method. Table 7 shows the torque performance in fuzzy-based tuning error-based fuzzy sliding mode controller look up table.

Table 7. Torque (τ) Performance: Fuzzy-based Tuning Fuzzy Sliding Mode Controller Lookup Table by COG Method

$\begin{matrix} \rightarrow \\ e \\ \swarrow \\ e \\ \downarrow \end{matrix}$	Membership Function (τ)												
	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
-1	-92	-92	-92	-92	-90	-90	-80	-77	-70	-55	-50	-20	0
-0.83	-92	-92	-92	-92	-80	-77	70	-55	-50	-30	-25	0	0
-0.66	-92	-92	-92	-80	-77	-77	-70	-55	-50	-25	0	0	25
-0.5	-92	-92	-71	-57	-43	-28	-28	-28	-25	0	0	25	43
-0.33	-92	-71	-57	-43	-28	-25	-25	-25	0	0	25	43	57
-0.16	-71	-57	-43	-28	-25	-25	0	0	0	14	28	43	57
0	-71	-57	-43	-28	-25	0	0	0	14	28	43	43	71
0.16	-43	-28	-25	0	0	0	0	14	14	28	43	57	70
0.33	-28	-25	0	0	0	14	14	14	28	43	57	71	84
0.5	-25	0	0	0	14	28	28	28	43	57	70	84	92
0.66	0	0	0	14	28	43	43	43	57	70	84	92	92
0.83	0	0	14	28	43	57	57	57	70	92	92	92	92
1	0	14	28	43	57	71	92	92	92	92	92	92	92

Based on Figure 8, fuzzy-based tuning error-based fuzzy sliding mode controller for PUMA560 robot manipulator is calculated by the following equation;

$$\begin{bmatrix} \widehat{\tau}_1 \\ \widehat{\tau}_2 \\ \widehat{\tau}_3 \end{bmatrix} = \begin{bmatrix} \tau_{1eqfuzzy} \\ \tau_{2eqfuzzy} \\ \tau_{3eqfuzzy} \end{bmatrix} + \begin{bmatrix} \lambda_1 \times \alpha_1 \\ \lambda_2 \times \alpha_2 \\ \lambda_3 \times \alpha_3 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} \text{sgn} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} \quad (57)$$

Where $\alpha = [\alpha_1, \alpha_2, \alpha_3]$ is sliding surface slope updating factor and it is calculated based on error-based fuzzy logic methodology. Based on Tables 2 and 6; fuzzy-based tuning is used to modified the torque performance in error-based fuzzy sliding mode controller with regard to the uncertainties by on-line tuning the sliding surface slope coefficient which caused to increase the stability and robust in FTFSMC.

Rule base Modification: let n is total number of shifting in rows obtained from K and y. For (7×7) rule base the shifting is rows 1, 2, 3 is done as mentioned in Table 1 and shown in Tables 8, 9 and 10.

Table 8. Fuzzy Rule Base for Sliding Surface Slope Updating Factor (α)

$e \backslash \dot{e}$	FL	ML	SL	Z	SR	MR	FR
NB	VB	VB	VB	VB	B	SB	S
NM	VB	VB	VB	B	B	MB	S
NS	VB	VB	MB	B	VB	VS	S
Z	S	S	SB	MB	ZE	MB	SB
PS	VS	VS	S	VS	VB	B	MB
PM	VS	VS	S	MB	B	B	VB
PB	ZE	ZE	S	SB	B	VB	VB

Table 9. Fuzzy Rule Base for Sliding Surface Slope Updating Factor (α)

$e \backslash \dot{e}$	FL	ML	SL	Z	SR	MR	FR
NB	VB	VB	VB	VB	VB	B	SB
NM	VB	VB	VB	VB	B	B	MB
NS	VB	VB	VB	MB	B	VB	VS
Z	S	S	S	SB	MB	ZE	MB
PS	VS	VS	VS	S	VS	VB	B
PM	VS	VS	VS	S	MB	B	B
PB	ZE	ZE	ZE	S	SB	B	VB

Table 9. Fuzzy Rule Base for Sliding Surface Slope Updating Factor (α)

$\begin{matrix} \dot{e} \\ e \end{matrix}$	FL	ML	SL	Z	SR	MR	FR
NB	VB	VB	VB	VB	VB	VB	B
NM	VB	VB	VB	VB	VB	B	B
NS	VB	VB	VB	VB	MB	B	VB
Z	S	S	S	S	SB	MB	ZE
PS	VS	VS	VS	VS	S	VS	VB
PM	VS	VS	VS	VS	S	MB	B
PB	ZE	ZE	ZE	ZE	S	SB	B

4. Results

Error based fuzzy tuning fuzzy sliding mode controller, 1 row shifting error based fuzzy tuning fuzzy sliding mode controller and 3 rows shifting error based fuzzy tuning fuzzy sliding mode controller are compared to each other. In this simulation, to control position of PUMA robot manipulator the first, second, and third joints are moved from home to final position without and with external disturbance. The simulation was implemented in MATLAB/SIMULINK environment. Trajectory performance, torque performance, disturbance rejection, steady state error and RMS error are compared in these controllers. These systems are tested by band limited white noise with a predefined 40% of relative to the input signal amplitude. This type of noise is used to external disturbance in continuous and hybrid systems and applied to nonlinear dynamic of these controllers.

Tracking performances: In fuzzy-based tuning error-based fuzzy sliding mode controller the sliding surface gain is adjusted online depending on the last values of error (e) and change of error (\dot{e}) and rule base by sliding surface slope updating factor (α). Figure 9 shows tracking performance in error based fuzzy tuning fuzzy sliding mode controller (zero), 1 row shifting error based fuzzy tuning fuzzy sliding mode controller (one) and 3 rows shifting error based fuzzy tuning fuzzy sliding mode controller (three) without disturbance for step trajectory.

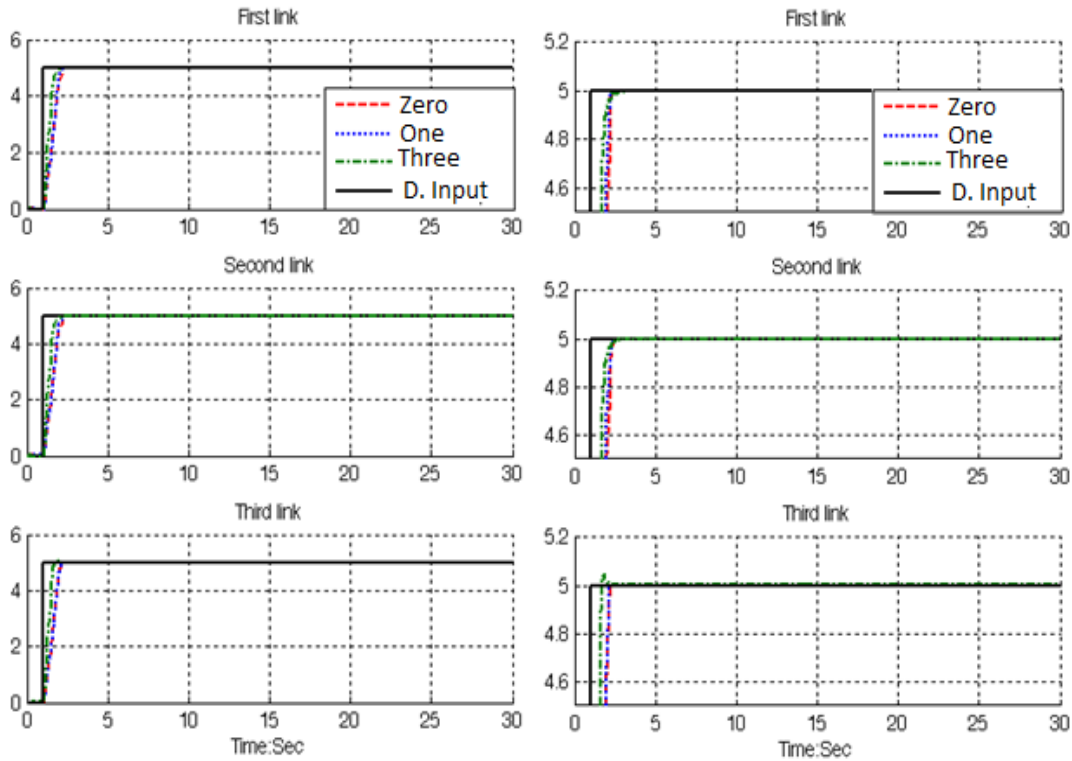


Figure 9. Zero shifting FTFSMC, one row shifting FTFSMC, three rows shifting FTFSMC and desired input for first, second and third link step trajectory performance without disturbance

Based on Figure 9 it is observed that, the overshoot in zero shifting FTFSMC is 0%, one row shifting FTFSMC's is 1% and in three rows shifting FTFSMC's is 0. From the trajectory MATLAB simulation for all three controllers in certain system, it was seen that all of them have acceptable performance.

Disturbance rejection: Figure 10 show the power disturbance elimination in zero shifting FTFSMC, one row shifting FTFSMC and three rows shifting FTFSMC with disturbance for step trajectory. The disturbance rejection is used to test the robustness comparisons of these three controllers for step trajectory. A band limited white noise with predefined of 40% the power of input signal value is applied to the step trajectory. It found fairly fluctuations in trajectory responses.

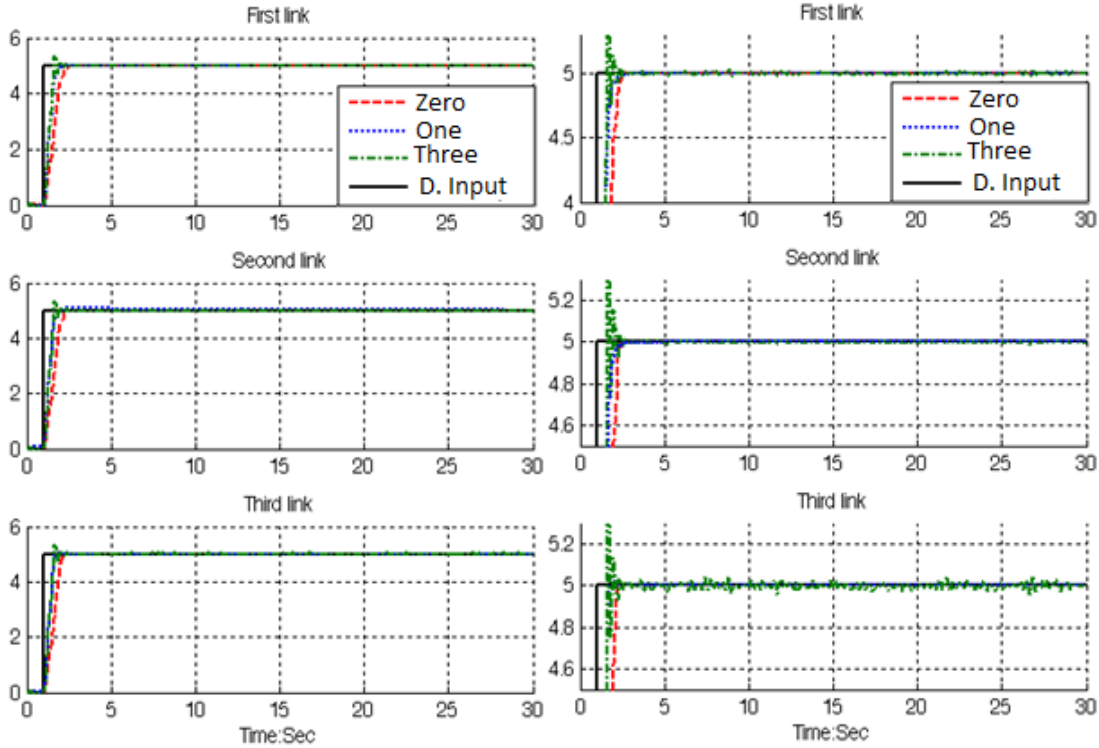


Figure 10. Desired inputs, zero shifting FTFSMC, one row shifting FTFSMC and three rows shifting FTFSMC for first, and second and third link trajectory with 40% external disturbance: step trajectory

Based on Figure 10; by comparing step response trajectory with 40% disturbance of relative to the input signal amplitude in zero shifting FTFSMC, one row shifting FTFSMC and three rows shifting FTFSMC, zero shifting FTFSMC's overshoot about (0%) is lower than one row shifting FTFSMC's (0.5%) and three rows shifting FTFSMC's (1%). Besides the Steady State and RMS error in zero shifting FTFSMC, one row shifting FTFSMC and three rows shifting FTFSMC it is observed that, error performances in zero shifting FTFSMC (Steady State error = $1.08e-12$ and RMS error= $1.5e-12$) are about lower than one row shifting FTFSMC (Steady State error = $1.08e-6$ and RMS error= $1.5e-6$) and three rows shifting FTFSMC (Steady State error= $1.6e-6$ and

Torque performance: Figure 11 has indicated the power of chattering rejection in zero shifting FTFSMC, one row shifting FTFSMC and three rows shifting FTFSMC with 40% disturbance.

Figure 11 have indicated the robustness in torque performance for first three links PUMA robot manipulator in zero shifting FTFSMC, one row shifting FTFSMC and three rows shifting FTFSMC in presence of 40% disturbance.

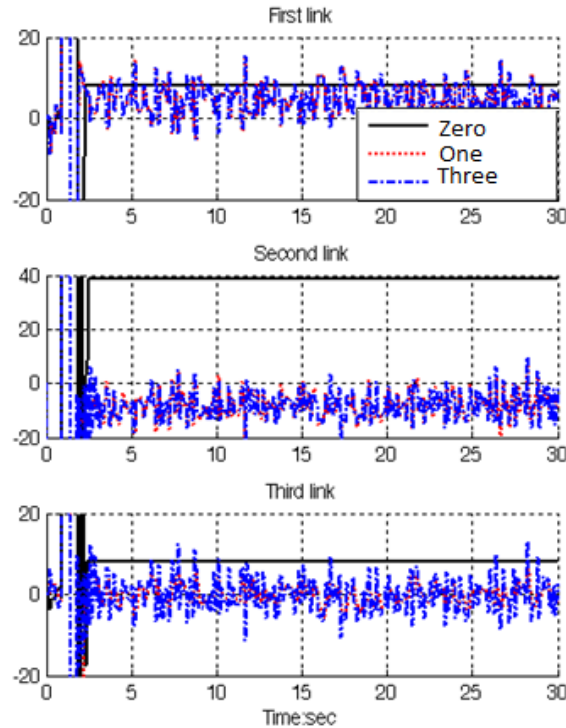


Figure 11. Zero Shifting FTFSMC, one row shifting FTFSMC and three rows Shifting FTFSMC for first, second and third link torque performance with 40% Disturbance

5. Conclusion

Refer to the research, effect of rule base on the fuzzy-based tuning fuzzy sliding mode controller is design and applied to robot manipulator in presence of structure and unstructured uncertainties. The rule bases obtained after the shifting of elements are also given in table form and are tested on robot manipulator. The simulation results obtained are using the modified rule bases given improvement than the typical fuzzy based tuning fuzzy sliding mode controller. Also the results obtained for large changes in tuning factor using modified rule base controller is better than fuzzy sliding mode controller. The results criteria is selected for analytical comparison of the controllers are given in tabular form showing the influence of modified rule base on the performance of the process. Obviously robot manipulator is nonlinear and MIMO system so in proposed controller in first step design free model controller based on fuzzy sliding mode controller and after that disturbance rejection is improved by rule base fuzzy tunable gain. This implementation considerably reduces the output oscillation response in the presence of uncertainties. As a result, this controller will be able to control a wide range of robot manipulator with a high sampling rates because its easy to implement versus high speed markets.

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