Uncertainty Handling using Fuzzy Logic in Rule Based Systems

Poonam[1], Surya Prakash Tripathi[2] and Praveen Kumar Shukla[3]

[1]*Department of Information Technology, Northern India Engineering College, Lucknow (rupali_poo@rediffmail.com)*

[2]*Department of Computer Science & Engineering, Institute of Engineering & Technology (A Constituent College of GBTU), Lucknow (tripathee_sp@yahoo.co.in)*

[3]*Department of Computer Science & Engineering, Babu Banarasi Das University, Lucknow (praveenshuklak@rediffmail.com)*

Abstract

In real world computing environment, the information is not complete, precise and certain, making very difficult to derive an actual decision. To deal with processing and modeling of such information, fuzzy techniques are applied to exercise the proper conclusion. This paper focuses on the basics of Fuzzy Logic and its application in Rule Based Systems to make them capable to handle the real world problems. Also, different research issues associated with FRBSs have been discussed. Also, a traffic control system is proposed and evaluated using MATLAB.

Keywords: *Fuzzy Logic (FL), Fuzzy Sets, Linguistic Variables, Control Systems, Fuzzy Rule Based Systems (FRBS)*

1. Introduction

Humans are capable to use linguistic information precisely in their decision making. Due to imprecise and uncertain nature of the linguistic information, machines are not capable to use them in decision making processes using traditional methods. To make the machines intelligent, like humans in this regard, Fuzzy Techniques are used.

The idea of the Fuzzy Logic was first introduced by Professor Lotfi Ahmad Zadeh, at University of Berkeley, California in his seminal paper "Fuzzy Sets" [1]. Fuzzy Logic [2, 3] is a form of multi-valued logic derived from fuzzy set theory to deal with approximate reasoning. It provides the means to represent and process the linguistic information and subjective attributes of the real world. Fuzzy Logic is the extension of Boolean Crisp Logic to deal with the concept of partial truth. Fuzzy Logic is applied in the number of areas, i.e. engineering applications, medical applications, economics and management, industrial applications and many more. It is also integrated with other soft computing techniques, like Neural Network (an approach that mimics the functionality of human brain), Genetic Algorithms (a nature inspired search and optimization technique), PSO (Particle Swarm Optimization) etc.

In the early stage of the Fuzzy Logic, a number of misconceptions have been created. Here we are going to introduce few points about fuzzy logic to make the concept very clear.

1. Fuzzy logic is not fuzzy.

2. Fuzzy logic is precise.

3. Fuzzy Logic is a precise system of reasoning, deduction and computation in which the objects of discourse and analysis are associated with information, which is or is allowed to be imperfect.

4. Any formal system can be fuzzified.

Rule Base Systems [4] are highly applicable in decision making, control systems and forecasting. To deal with imprecise, uncertain and inexact real world knowledge, in rule based systems, fuzzy techniques are used. Fuzzy logic is the way to represent the complex situations in terms of simple natural languages.

This paper introduces the Fuzzy Rule Based Systems (FRBS) and different research issues associated with them. In section 2, the basic mathematical concept of the Fuzzy Logic has been introduced. Section 3 introduces the two basic types of FRBS, Mamdani FRBS and TSK FRBS. Also, the authors have discussed five research issues with the fuzzy rule based systems. In section 4, a traffic control system is modeled and evaluated using MATLAB. In section 5, conclusion and future scope of the research issues has been discussed.

2. Basic Concepts of Fuzzy Logic

The theory of Fuzzy Logic [5] can be developed using the concepts of Fuzzy Sets similar to how theory of crisp bivalent logic can be developed using the concept of crisp sets. Specifically, there exists an isomorphism between sets and logic. In view of this, a good foundation of the fuzzy sets is necessary to develop the theory of Fuzzy Logic.

A fuzzy set is a set without clear or sharp (crisp) boundaries. Partial membership degree is possible in fuzzy sets. In other words, softness is associated with the membership of elements associated.

Examples may include, like TEMPERATURE. This fuzzy variable may take fuzzy values, like COLD, COOL, WARM, HOT. A fuzzy value such as 'WARM' is a fuzzy descriptor.

2.1 Universe of Discourse

If universe of discourse is represented by X, is a set that contains every set of interest in the context of a given class of a problem. The vein diagram of the fuzzy set is given in Figure 1.

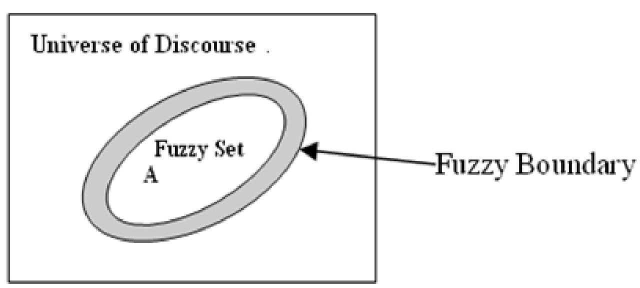


Figure 1. Venn Diagram of Fuzzy Set

2.2 Representation of Fuzzy Sets

There are two representation techniques of fuzzy sets, membership function method and symbolic representation.

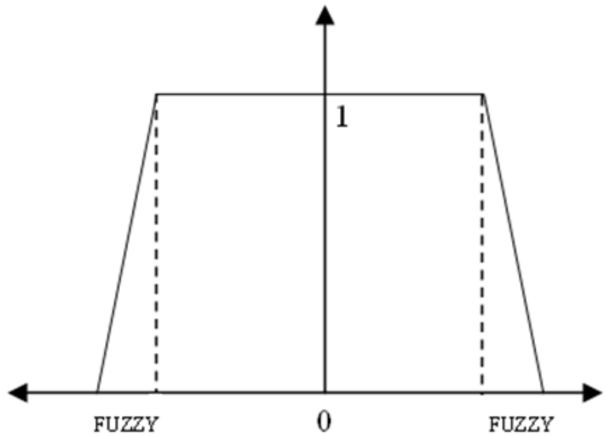


Figure 2. Membership Function

2.2.1 Membership Function Method

This function gives the grade (degree) of membership within the set of any element of Universe of Discourse. The membership function maps the elements of the universe on to the numerical values in the interval [0, 1].

$\mu_A(x): X \rightarrow [0,1]$

Here, $\mu_A(x)$ is the membership function of the fuzzy set A in the universe X. It is defined as follows:

$A = \{(x, \mu_A(x)); x \in X, \mu_A(x) \in [0,1]\}$

The membership function represents the grade of possibility that an element x belongs to the set A. It is a possibility function, not a probability function.

2.2.2 Symbolic Representation

A fuzzy set may be symbolically represented as follows: $A = \{x \mid \mu_A(x)\}$. They can also be represented as a formal series, when the universe is discrete in the nature.

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots\ldots\ldots\ldots$$
$$\mu_A(x_{i1})/x_i + \ldots\ldots\ldots\ldots\ldots\ldots$$

or

$$A = \sum_{x_{i \in X}} \frac{\mu_A(x_i)}{x_i}$$

2.3 Algebraic Operations on Fuzzy Sets

Let X is a set of objects with elements denoted by x, i.e. X={x}

A fuzzy set A in X is characterized by a membership function $\mu_A(x)$, which maps each point in X on to the real interval [0.0, 1.0]. As $\mu_A(x)$ approaches 1.0, the grade of membership of X in A increases.

1. A is empty iff for all x, $\mu_A(x)=0.0$
2. If A and B are the two fuzzy sets, then A=B iff for all x: $\mu_A(x)=\mu_B(x)$.
3. $\mu_A(x)'=1-\mu_A(x)$
4. A is contained in B iff $\mu_A(x) <= \mu_B(x)$
5. $C = A \cup B \Rightarrow AUNIONB$, where $\mu_C(x)=\max(\mu_A(x), \mu_B(x))$
6. $C = A \cup B \Rightarrow AINTERSECTIONB$, where $\mu_C(x)=\min(\mu_A(x), \mu_B(x))$

2.4 Support Set

This is a crisp set of a fuzzy set containing all the elements (in the universe) whose membership grade is greater than 0. The support set S of a fuzzy set A with membership function $\mu_A(x)$ is given by

$$S=\{x \in X \mid \mu_A(x)>0\}$$

2.5 α-cut of the Fuzzy Set

The α-cut of the fuzzy set A is the crisp set denoted by A_α formed by those elements of A whose membership function grade is greater than or equal to a specified threshold value α.

$$A_\alpha=\{x \in X \mid \mu_A(x) \geq \alpha\}, \alpha \in [0,1]$$

The strong α-cut is defined by

$$A_\alpha=\{x \in X \mid \mu_A(x) > \alpha\}, \alpha \in [0,1]$$

When α =0 then α-cut will become the support set of a fuzzy set.

Also, the fuzzy AND, OR and NOT operations can be generalized. The generalized FUZZY AND operation is called Triangular Norm (T-Norm) and generalized FUZZY UNION is called T-Conorm (S- Norm). The basic concepts of Fuzzy Set Theory may be studied in [3, 5].

3. Fuzzy Rule Based Systems (FRBS)

Fuzzy Rule Based Systems (FRBS) constitute an extension to classical rule based systems, because they deal with IF – THEN rules whose antecedents and consequents are composed of fuzzy logic statements, in place classical logical ones.

The most common applications of FRBS includes, Fuzzy Modeling [6], Fuzzy Control [7] and Fuzzy Classification [8]. In a FRBS, fuzzy logic used to perform the operations like, representation of different form of knowledge, modeling the interactions and relationships that exist among its variables. The main features of the knowledge captured by fuzzy sets involve handling of uncertainty. Due to this, inference methods have become more robust and flexible with the approximate reasoning methods using Fuzzy Logic.

Linguistic variables and values are used for the enhancement of the Knowledge Representation. These variables and their values are defined by the context dependent fuzzy sets whose meanings are specified by gradual membership function.

Two major types of FRBSs proposed are, Mamdani Fuzzy Rule Based Systems [9] and Takagi-Sugeno-Kang FRBSs [10].

3.1 Mamdani Fuzzy Rule – Based Systems

These are the FRBS with fuzzifier and defuzzifier, more commonly these are known as Fuzzy Logic Controllers (FLC). The major constituents of the Mamdani FRBS are shown in Figure 3.

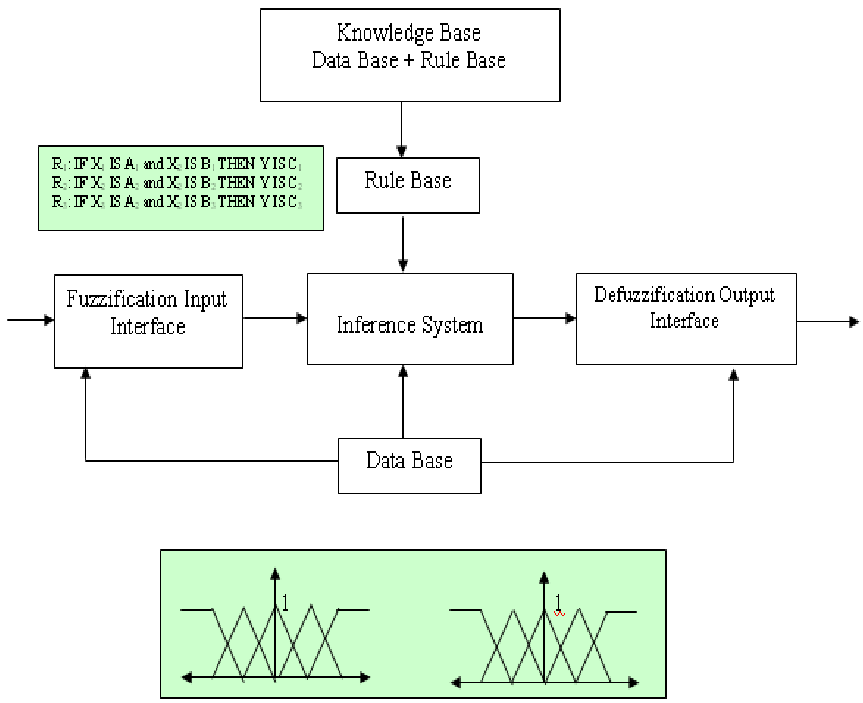


Figure 3. Mamdani Fuzzy Rule Based Systems [36]

1. Knowledge Base: The Knowledge Base (KB) stores the available knowledge about the problem in the form of fuzzy “IF THEN” rules. It composed of two main components, Data Base (DB) and Rule Base (RB). Data Base (DB) stores the membership functions of fuzzy sets and scaling functions for context adaptation purpose. Rule Base (RB) stores the FUZZY IF THEN rules for the purpose inference and decision making. Multiple rules can be fired simultaneously for the same input.

2. Fuzzification Interface: It transforms the crisp input data into fuzzy values that acts as input to fuzzy reasoning process.

3. Inference System: It infers from the fuzzy input to several resulting output fuzzy sets according to the information stored in the Knowledge Base (KB).

4. Defuzzification Interface: It converts the fuzzy sets obtained from the inference process into a crisp action that constitutes the global output of the FRBS.

3.2 TSK Fuzzy Rule Based Systems

A new FRBS model (Figure 4) is proposed, based on rules whose antecedent is composed of the linguistic variables and the consequent is represented by a function of the input variables.

IF X_1 is A_1 and…………and X_n is A_n THEN $Y=p_1.X_1+\ldots\ldots+p_n.X_n+p_0$

Here, X_i is the system input variable, Y as the output variable p= $(p_0, p_1, \ldots\ldots, p_n)$ is a vector of real parameters. A_i is the direct specification of a fuzzy set or linguistic label that points to a one particular member of a fuzzy partition of a linguistic variable.

The output of a TSK FRBS using a KB composed of m rules is obtained as a weighted sum of the individual outputs provided by each rule, Y_i is given by

$$\frac{\sum_{i=1}^{m} h_i .Y_i}{\sum_{i=1}^{m} h_i}$$

Here, i=1, 2,………,m. $h_i = T(A_{i1}(x_1),\ldots\ldots, A_{in}(x_n))$ is the matching degree between the antecedent part of the ith rule and the current inputs to the systems, $x_0 = (x_1,\ldots\ldots x_n)$. T stands for the conjunctive operator modeled by a t-norm.

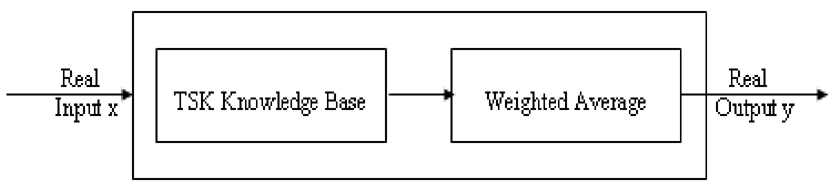


Figure 4. TSK Type Fuzzy Rule Based Systems

3.3 Research Issues and Challenges in Fuzzy Rule Based Systems

Application of fuzzy computing in rule based systems provides a mathematical platform to deal with information imprecision in decision making processes. Several issues have been raised by the researchers in this field. We are presenting here the burning issues in the FRBS research.

3.1 Context Adaptation

Human decisions and perceptions are extremely dependent on the context. Therefore, when Fuzzy Logic models the human decision making capabilities in machines, it is mandatory to apply context in Fuzzy Systems.

Context adaptation in Fuzzy Systems has been approached as scaling of fuzzy sets from one universe of discourse to another by means of non-linear scaling functions, whose parameters are identified by the data.

The idea behind this research issue is to develop new context aware approaches to automatic development of the fuzzy systems from data. Other objectives includes, interpretability oriented adaptation, identification of context variable, high level of rule identification.

The abstract idea of this research issue [11] is given in Figure 5.

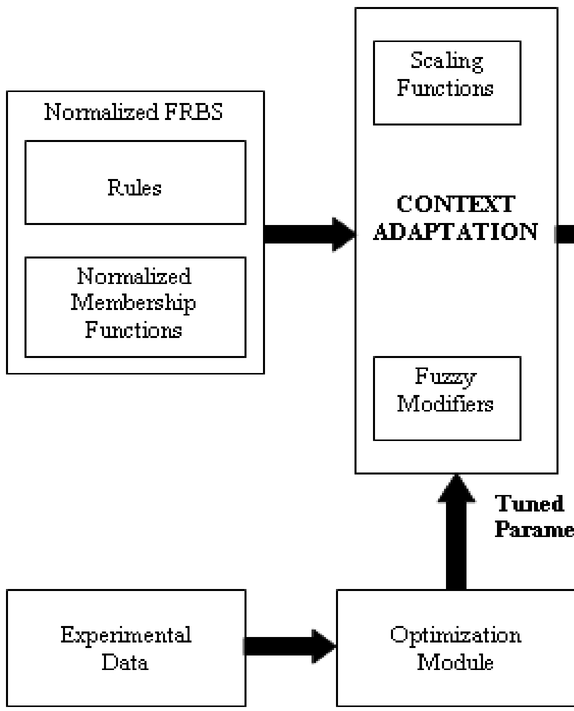


Figure 5. Context Adaptation Process [11]

In [12], a method of achieving context adaptation by adjusting an initial normalized fuzzy rule based systems through the use of operators that appropriately change the representation of linguistic variables.

A multi-objective evolutionary algorithm based Pareto-optimum context adapted Mamdani Type FRBSs with different trade – offs between accuracy and interpretability is investigated in [13]. In this proposal, a novel index is proposed based on fuzzy ordering relations, in-order to provide a measure of interpretability.

Other approaches related to context adaptation can be studied in [14, 15, 16, 17, 18].

3.2 Interpretability Accuracy Trade-Off

The trade – off between interpretability and accuracy is an important research issue. The definition of the accuracy is straight forward in different applications, but the definition of the interpretability is rather problematic [19].

The main purpose is to build fuzzy systems with a user controllable trade off between accuracy and interpretability. Interpretability is maintained by the structural choices regarding the type of membership functions, rules and inference mechanism. Interpretability can be maintained or enhanced during the fuzzy systems generation or obtained by post processing of the resulting data driven fuzzy systems [20, 21].

A new linguistic rule representation model [22, 23] was proposed to perform a genetic lateral tuning of membership functions, based on linguistic 2- tuple representation model that allow the lateral displacement of a label considering a unique parameter. It provides reduction of search space that eases the derivation of optimal path.

A user-controllable interpretability-accuracy trade off for fuzzy systems has been discussed in [24].

The interpretability- accuracy trade off issue is discussed with multi-objective fuzzy genetics-based machine learning in [25].

3.3 Fuzzy Rule Selection

In high dimensional data problems [26, 27], the number of rules in the Rule Base grows exponentially as more inputs are added. Hence, it is required to have a fuzzy rule generation method. It is likely to derive fuzzy rule sets, including following types of rules;

Redundant rules**:** The actions of these rules are covered by the other rules.

Wrong rules: These are ill defined rules and perturb the systems performance.

Conflicting rules: These rules worsen the system performance, when co-existing with other rules in the RB.

Other solutions for the problem of data dimensionality and rule overflow, are rule reduction methods. Two approaches are proposed for this rule reduction:

Approach 1: To combine the membership functions of two or more rules, reducing to a single one's (Scatter partition FRBS).

Approach 2**:** To select the fuzzy rules, we get the rule subsets with a good cooperation from the initial RB (descriptive and scatter FRBS).

Several methods can be obtained from selection, with different search algorithms that are providing most successful combination of fuzzy rules [28,29,30].

A genetic rule selection process in order to obtain a compact and accurate fuzzy rule based classification systems is discussed in [31].

3.4 Optimization of Membership Function and Scaling Function

Optimization of the membership function results in the improvement of interpretability and accuracy of the fuzzy systems. Also, the scaling functions are optimized to maintain and precise the context related issues in the fuzzy rule based systems.

Scaling functions apply the universe of discourse of the input and output variables to the domain where fuzzy sets are defined. Their tuning and adaptation allows the scaled universe of discourse to match the variable range in a better way. Several parameters are considered for the purpose of optimization. These may include, 1. Scaling Functions, 2. Upper and lower bounds (Linear Scaling Functions), 3. Contraction/dilation operators (Nonlinear scaling functions).

Also the optimization of the membership functions is an important research issue. The tuning process [32] slightly adjusts the shapes of the membership functions of the preliminary data base definition. Different types of membership functions are considered for this purpose, i.e. 1. Triangular, 2. Trapezoidal, etc.

For the purpose of semantic interpretability of linguistic fuzzy models, an index is proposed [33]. Tuning of the membership function and rule selection is performed using a multi-objective evolutionary algorithms.

3.5 Fuzzy Partition Granularity

Obtaining good uniform fuzzy partition granularity [32] that improves the FRBS behavior is an important research issue. The granularity selection plays an important role in many characteristics of the FRBS, such as the accuracy in fuzzy modeling or the smoothening in fuzzy control. Also, the granularity of the input variables specifies the maximum number of fuzzy rules that may compose the Rule Base (RB). So, it has a strong influence on aspects, like complexity of rule learning, interpretability of the FRBS obtained or its accuracy.

The issue of the fuzzy partition granularity is a fuzzy rule base classification systems are discussed in [35].

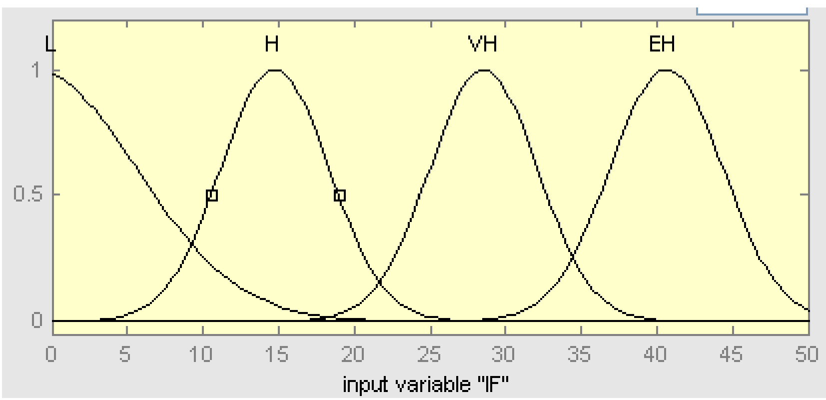
4. Proposed Urban Traffic Control System

Traffic control system has been proposed and implemented using the concept of fuzzy techniques. The overall system is defined as follows.

4.1 Definition of Membership Functions

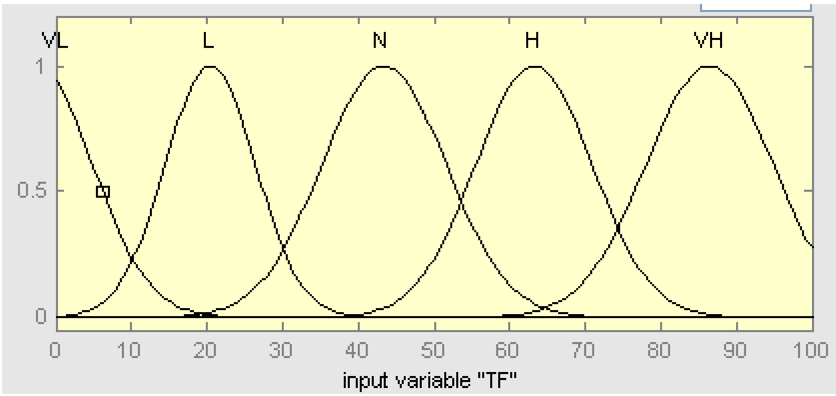
For MATLAB SIMULATION of the proposed traffic control systems, three input fuzzy membership functions and one output fuzzy membership functions are developed. At input, we have Traffic Flow (TF), Traffic Queue (TQ) and Incoming Flow (IF) and at output the membership function is Green Time (GT).

1. Input Membership Functions



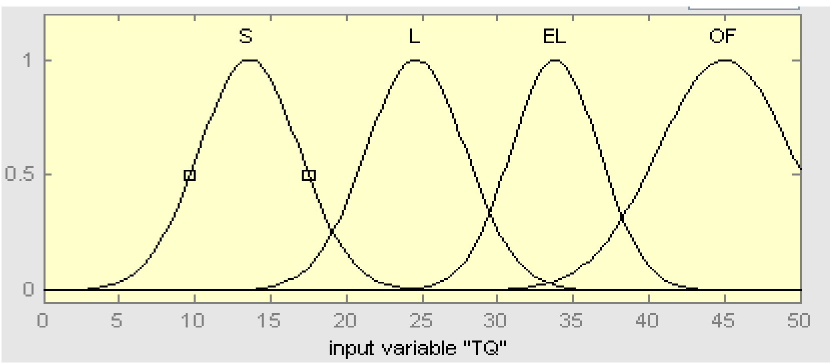
L-Low, H-High, VH- Very High, EH- Extremely High

Figure 6. MF of Incoming Flow (IF)



VL-Very Low, L-Low, N-Normal, H-High, VH-Very High

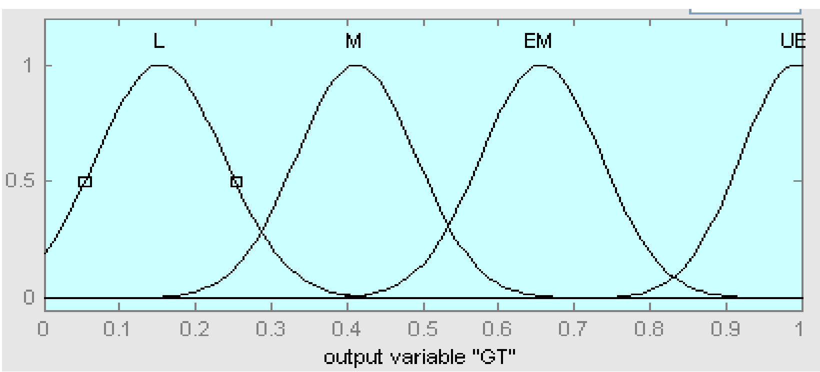
Figure 7. MF of Traffic Flow (TF) Variable



S-Small, L-Large, EL- Extra Large, OF-Over Flow

Figure 8. MF of Traffic Queue (TQ)

2. Output Membership Functions



L-Less, M- More, EM-Extremely More, UE- Unacceptable

Figure 9. MF of the Green Time (GT)

4.2 Knowledge Base Definition

The knowledge base of the system is defined by the following rule matrices.

Table 1. Rule Matrix 1

Traffic Flow (TF)	Incoming Flow (IF)	Green Time (GT)
VL	L	L
L	VL	L
H	L	M
N	L	L
H	N	M
N	H	L
L	VH	EM
L	H	M
VL	VH	EM
VH	H	M
VH	H	U

The phase sequence change is realized by the Traffic Queue data on the remaining phases, as discussed in TABLE II. P1, P2 and P3 are the phases (T1, T3), (T2, T4), (R1, R3), where as the phase P4 (R2, R4) is assumed to be fixed.

Table 2. Rule Matrix Phase Sequence Change

TQ1	TQ2	TQ3	NEXT PHASE
S	S	S	P1
S	L	S	P2
S	S	L	P3
L	S	S	P1
L	EL	L	P2
EL	L	S	P1
L	S	EL	P3
S	EL	L	P2
EL	L	L	P1
EL	EL	L	P1
L	EL	S	P2

4.3 System Evaluation

The proposed system has been implemented using MATLAB 7.0. The system is simulated with two input membership functions and one output membership function. The system is developed using Mamdani Type Fuzzy System, having multiple rules in the knowledge base (KB).

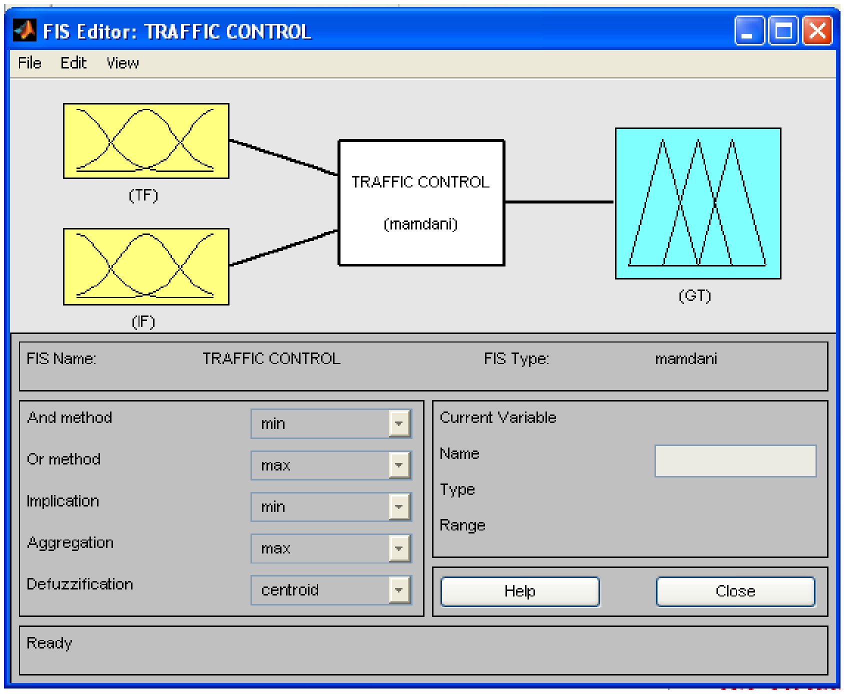


Figure 10. MATLAB Fuzzy Inference System (FIS)

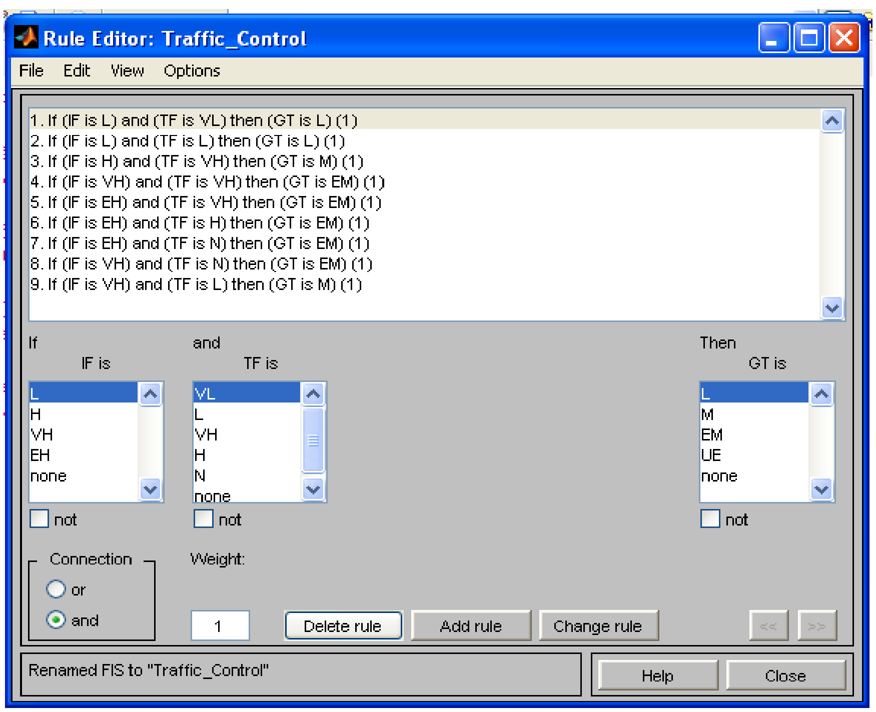


Figure 11. Rule Editor in MATLAB for Proposed System

4.4 Result Analysis

The following results have been obtained by the simulation and the results are found satisfactory. The Traffic Flow, Incoming Flow and Green Time are discussed and realized in TABLE III.

TABLE 3. Simulation Result I

S. No.	Input1 [Traffic Flow]	Input2 [Incoming Flow]	Output [Green Time]
1	10	5	20
2	10	10	27
3	20	17	50
4	30	25	100
5	50	55	120
6	60	65	155
7	70	74	170
8	80	70	240
9	90	95	270
10	100	110	320

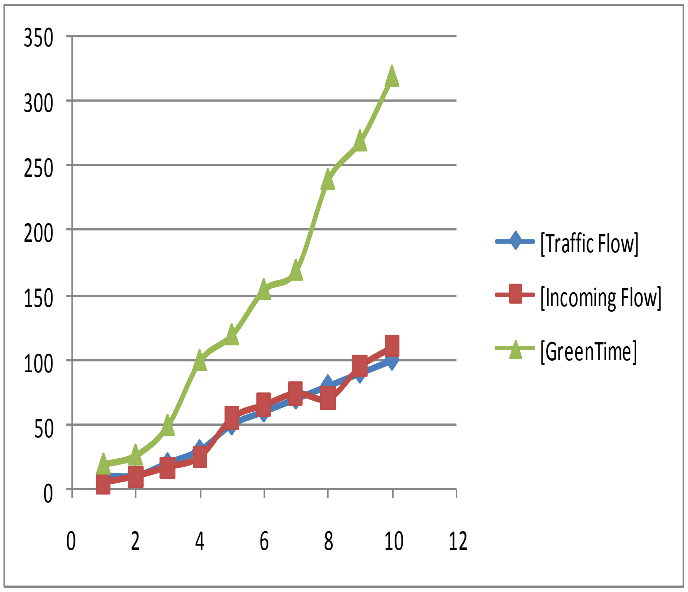


Figure 12. Relations among Traffic Flow, Incoming Flow and Green Time

The phase sequence change is realized in TABLE IV which is based on the rules of Knowledge Base.

Table 4. Simulation Result

TQ1	TQ2	TQ3	NEXT PHASE
6	6	6	P1
8	15	8	P2
9	9	18	P3
25	21	21	P1
21	59	21	P2
56	10	23	P1
15	45	75	P3
30	75	40	P2

5. Conclusion & Future Scope

Modern engineering, medical and business applications are requiring to enhance their capability to deal with imprecise and uncertain information, enabling them to have a strong reasoning and decision power. It makes them to handle more complex and linguistic computations easily and efficiently. All these requirements lead to rapid development and integration of Fuzzy Logic in control systems.

In future, authors would like to implement the solutions for the problems addressed in the section IV, by Evolutionary Computation techniques. The use of multi-objective Evolutionary Algorithms and Memetic Genetic Algorithms will be preferred.

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