

On the Carrying capacity of Enemy Species, Inhibition coefficient of Ammensal Species and Dominance reversal time in An Ecological Ammensalism - A Special case study with Numerical approach

K. V. L. N. Acharyulu¹ and N. Ch. Pattabhi Ramacharyulu²

¹Faculty of Science, Department of Mathematics
Bapatla Engineering College, Bapatla-522101, India

²Professor (Retd.) of Mathematics, Department of Mathematics & Humanities
National Institute of Technology, Warangal – 506004, India
kvlna@yahoo.com, patabhi1933@yahoo.com

Abstract

The paper aims to study numerically on some special cases of mathematical model of An Ecological Ammensalism with multiple constraints. The mathematical model comprises of Ammensal-enemy species pair with cover for Ammensal, alternative resources for enemy and migration for both the species. The model is formed by a couple of first order non linear ordinary differential equations. Some relations among the carrying capacity of enemy species, inhibition coefficient of Ammensal species and the dominance reversal time are investigated.

Key words: Non-linear system, Enemy species, Ammensal species, Carrying capacity, Dominance reversal time

AMS Classificates: 92D25, 92D40

1. Introduction

In general, we analyze any mathematical model with mathematical techniques to get a thorough knowledge of the problems of real life in the field of Bio science. Recently majority of the mathematicians drew their attention to comprehend the situations in mathematical ecology, mathematical bio-economics, mathematical demography, mathematical medical sciences, mathematical agriculture and so on. Advanced computing methods are the backbone of modern scientific development. Several authors like Kapur [15, 16], Lakiminarayan and Pattabhi Ramacharyulu [17], N.C Srinivas [18] etc. applied various computational techniques to examine Local stability and Global stability. Acharyulu. K.V.L.N and Pattabhi Ramacharyulu N.Ch. [1-15] studied manifold dimensions of stability phenomena in ecological Ammensalism.

The present paper is a numerical study on some special cases of mathematical model of An Ecological Ammensalism with multiple constraints. The mathematical model consists of Ammensal-enemy species pair with cover for Ammensal, alternative resources for enemy and migration for both the species. The model is obtained by a couple of first order non linear ordinary differential equations. In continuation of Authors earlier work [3] cited in “Ecological Ammensalism with multifarious restraints-A numerical Study”. In this paper some relations among Ammensal coefficient of Ammensal species, the carrying capacity of enemy species and the dominance reversal time are investigated. The authors computed the numerical solutions of the model with the help of the classical RK method of fourth order. The dominance reversal time is derived for observing the interactions between the Ammensal

and enemy species in few considered cases. The figures are drawn with the help of Matlab wherever needed.

Notation Adopted:

- $N_1(t)$: The population of the Ammensal species (S_1) at time t
- $N_2(t)$: The population of the Enemy species (S_2) at time t
- a_i : The natural growth rates of S_i , $i = 1, 2$.
- a_{ii} : The rate of decrease of S_i ; due to its own insufficient resources , $i=1,2$.
- a_{12} : The inhibition coefficient of S_1 due to S_2 i.e The Ammensal coefficient.
- a_{21} : The inhibition coefficient of S_2 due to S_1
- $H_1(t)$: The replenishment or renewal of S_1 per unit time
- $H_2(t)$: The replenishment or renewal of S_2 per unit time
- K_i : a_i/a_{ii} are the carrying capacity of N_i , $i = 1, 2$.
- α : a_{12}/a_{11} is the coefficient of Ammensalism.
- h_1 : $a_{11} H_1$ is the rate of harvest of the Ammensal
- h_2 : $a_{22} H_2$ is the rate of harvest of the enemy.
- m_1 : Rate of decrease of the Ammensal due to harvesting.
- m_2 : Rate of decrease of the enemy due to harvesting.
- m : a constant characterized by the cover provided for the Ammensal species.

The state variables N_1 and N_2 as well as the model parameters $a_1, a_2, a_{11}, a_{22}, K_1, K_2, \alpha, h_1, h_2, m_1, m_2$ and m are assumed to be non-negative constants.

2. The Basic Model Equations

The model equations are obtained as

$$\frac{dN_1(t)}{dt} = (1 - m_1)a_1N_1(t) - a_{11}N_1^2(t) - (1 - m)a_{12}N_1(t)N_2(t) - h_1(t) \tag{1}$$

$$\frac{dN_2(t)}{dt} = (1 - m_2)a_2N_2(t) - a_{22}N_2^2(t) - h_2(t) \tag{2}$$

with the conditions $N_i(0) = N_{i0} \geq 0, i = 1, 2$;

3. Relation between Ammensal Coefficient (α) and Dominance Reversal Time

The numerical solutions of the mathematical model are calculated by utilizing the fourth order Runge-Kutta method. The fixed parameters are considered as **$a_1=1.592739, a_{11}=3.30158, a_2=3.110636, a_{22}= 1.959451, N_{10}=3.762642, N_{20}=0.429724, m=0.5, m_1 = 0.6, m_2 = 0.7, h_1 = 0.8; h_2 = 0.8$** and the varying variable is the inhibition coefficient (a_{12}) of Ammensal species. (3)

Table 1.

S.No	Carrying capacity of S_1 i.e $K_1=a_1/a_{11}$ ($a_1=1.592739$, $a_{11}=3.30158$)	Carrying capacity of S_2 i.e $K_2=a_2/a_{22}$ ($a_2=3.110636$, $a_{22}=1.959451$)	Inhibition Coefficient(a_{12}) of Ammensal species	Ammensal coefficient $\alpha : a_{12}/a_{11}$	Dominance Reversal time(t^*)
1	0.4824	1.5875	0.773829	0.2343	0.243
2	0.4824	1.5875	1.773829	0.5372	0.237
3	0.4824	1.5875	2.773829	0.8401	0.225
4	0.4824	1.5875	3.773829	1.143	0.219
5	0.4824	1.5875	4.773829	1.4459	0.192
6	0.4824	1.5875	5.773829	1.7488	0.171
7	0.4824	1.5875	6.773829	2.0516	0.156
8	0.4824	1.5875	7.773829	2.3545	0.145
9	0.4824	1.5875	8.773829	2.6574	0.131
10	0.4824	1.5875	9.773829	2.9603	0.125
11	0.4824	1.5875	10.773829	3.2632	0.117

The obtained dominance reversal time (t^*) with corresponding numerical solutions can be observed in the following figures from Figure 1 to Figure 11.

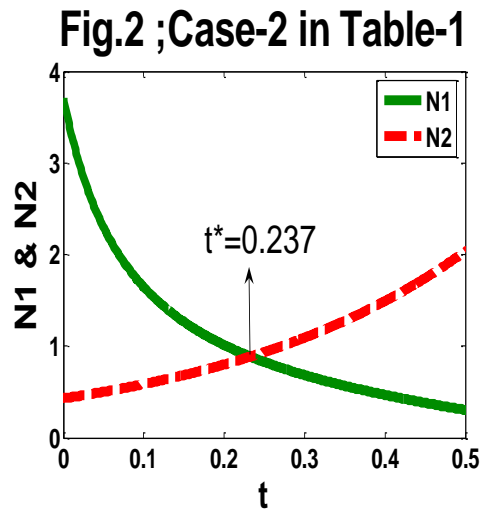
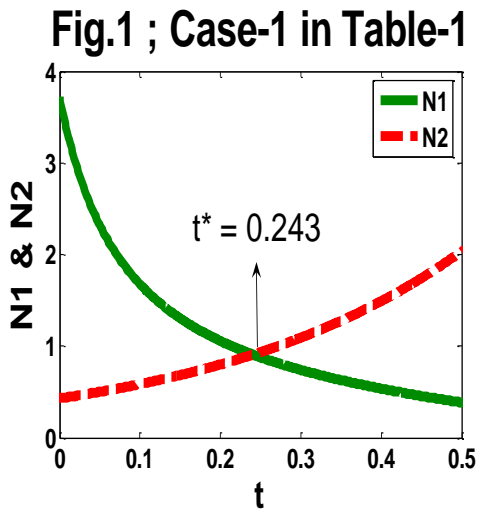


Fig.3 ; Case-3 in Table-1

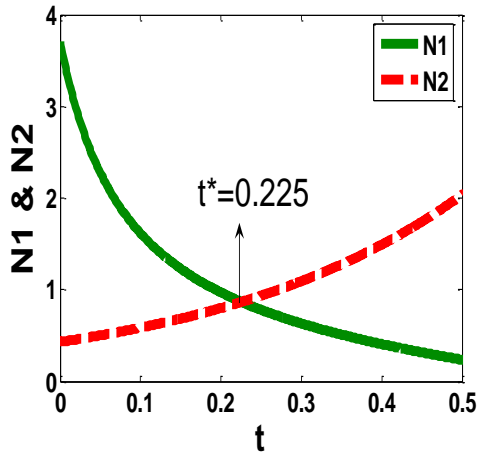


Fig.4 ; Case-4 in Table-1

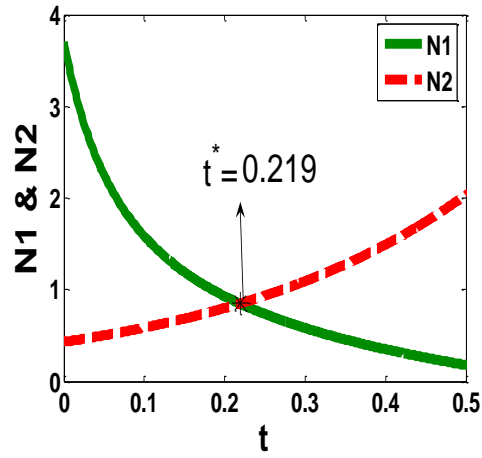


Fig.5 ; Case-5 in Table-1

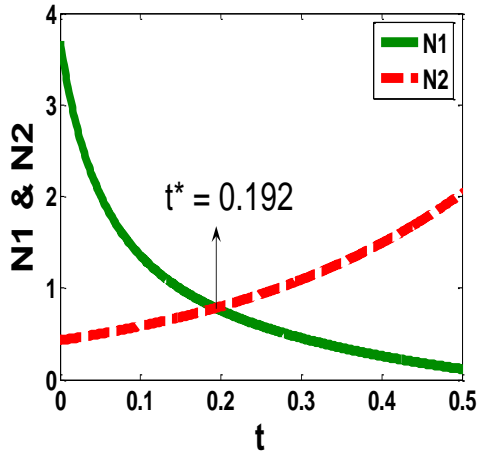


Fig.6 ; Case-6 in Table-1

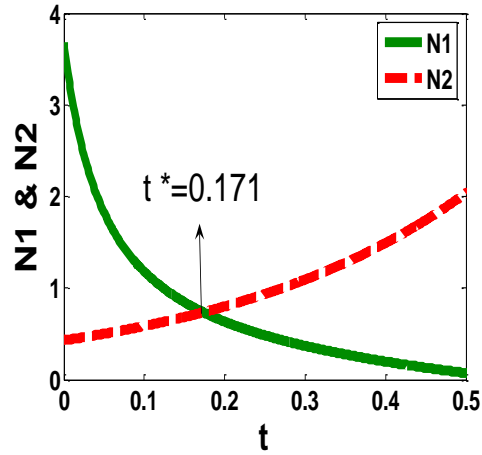


Fig.7 ; Case-7 in Table-1

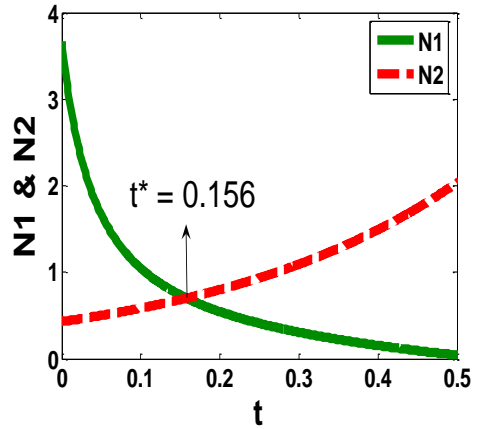


Fig.8 ; Case-8 in Table-1

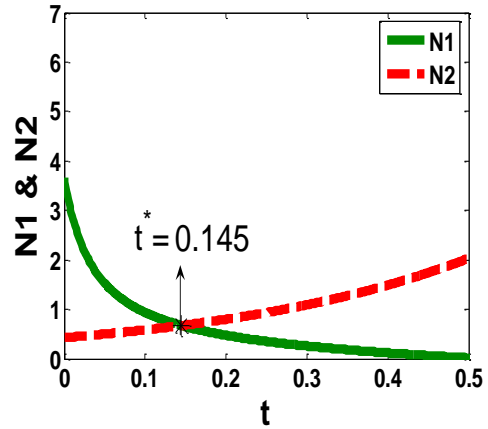


Fig.9 ; Case-9 in Table-1

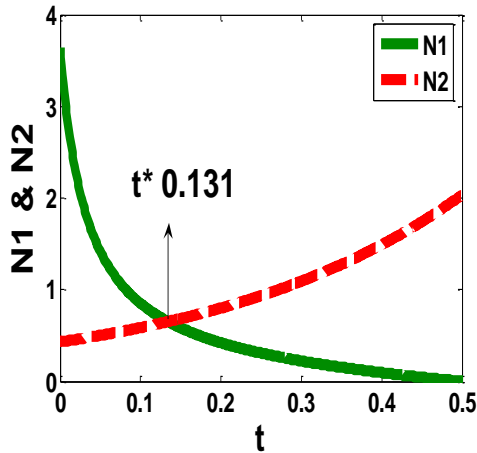


Fig.10 ; Case-10 in Table-1

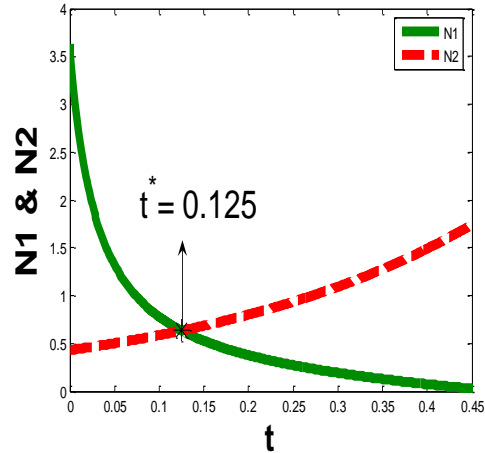
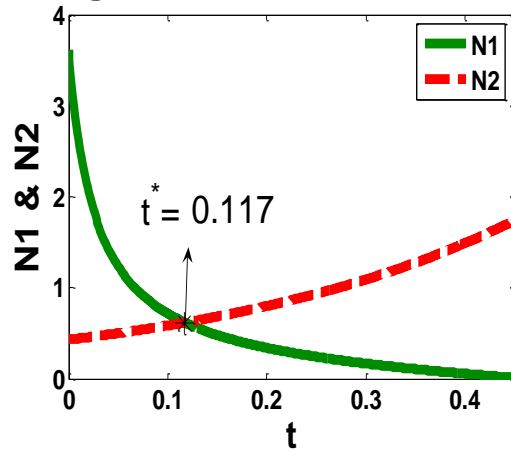


Fig.11 ; Cae-11 in Table-1



4. Conclusions

In all the above Cases, Ammensal species outnumbers the enemy species till the dominance reversal time. The enemy outnumbers the Ammensal after dominance reversal time (t^*). Further the Ammensal species declines and moves asymptotically to the equilibrium point. The enemy species flourishes with exponential growth rate throughout the interval. In addition to this, it is observed that the dominance reversal time gradually decreases in both the cases, having increase in inhibition coefficient as well as in Ammensal coefficient of Ammensal Species.

5. Relation between Carrying capacity of Enemy Species (K_2) and Dominance reversal time (t^*)

The numerical computations of the mathematical model are calculated by using the classical fourth order of Runge-Kutta method. The fixed parameters are considered as

$a_1=1.592739, a_{11}=3.30158, a_{12}=1.773829, a_2=3.110636, a_{22}=1.959451, N_{10}=3.762642, N_{20}=0.429724, m=0.5, m_1 = 0.6, m_2 = 0.7, h_1 = 0.8; h_2 = 0.8$ and the varying variable is the natural growth rate (a_2) of the Enemy species. (4)

Table 2.

S.No	Carrying capacity of S_1 i.e $K_1=a_1/a_{11}$ ($a_1=1.592739,$ $a_{11}=3.30158$)	The natural growth rate of S_2 i.e a_2	Carrying capacity of S_2 i.e $K_2=a_2/a_{22}$ ($a_{22}=1.959451$)	Dominance reversal time (t^*)
1	0.4824	0.110636	0.0564	0.454
2	0.4824	1.110636	0.5668	0.340
3	0.4824	2.110636	1.0771	0.275
4	0.4824	3.110636	1.5875	0.228
5	0.4824	4.110636	2.0978	0.203
6	0.4824	5.110636	2.6082	0.183
7	0.4824	6.110636	3.1185	0.165
8	0.4824	7.110636	3.6288	0.145
9	0.4824	8.110636	4.1392	0.138
10	0.4824	9.110636	4.6495	0.129
11	0.4824	10.110636	5.1599	0.119

The derived solutions with dominance reversal time (t^*) are shown from Figure 12 to Figure 22.

Fig.12 ; Case-1 in Table-2

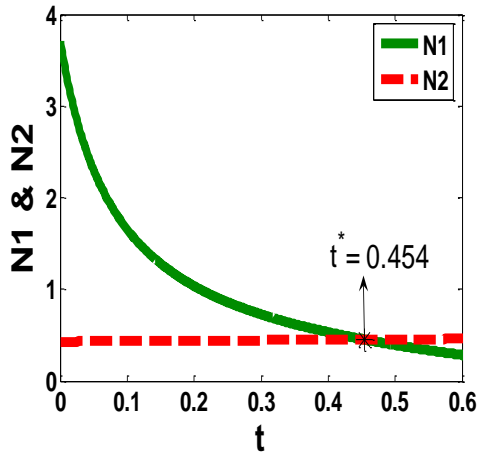


Fig.13 ; Case-2 in Table-2

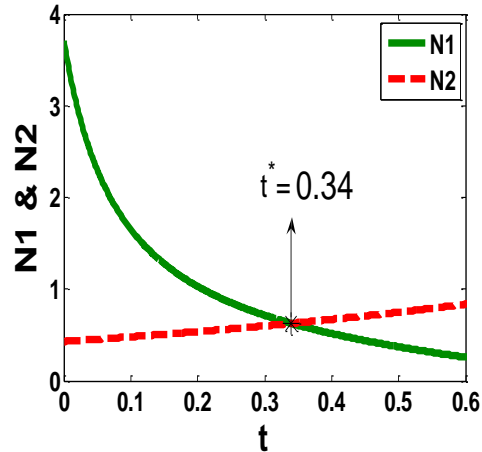


Fig.14 ; Case3 in Table-2

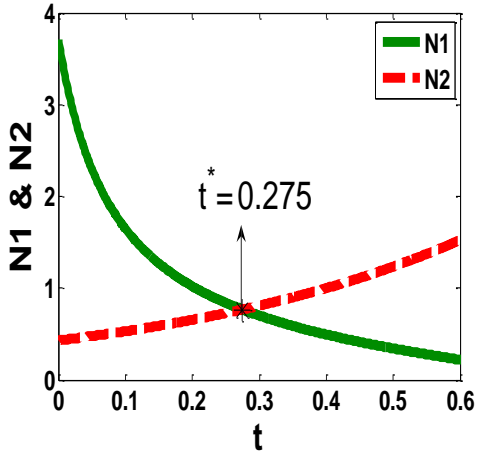


Fig.15 ; Case-4 in Table-2

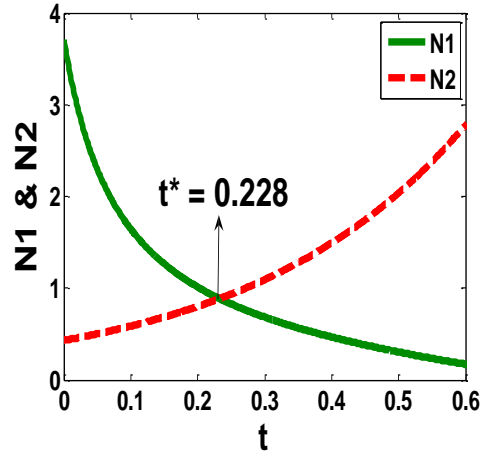


Fig.16 ; Case-5 in Table-2

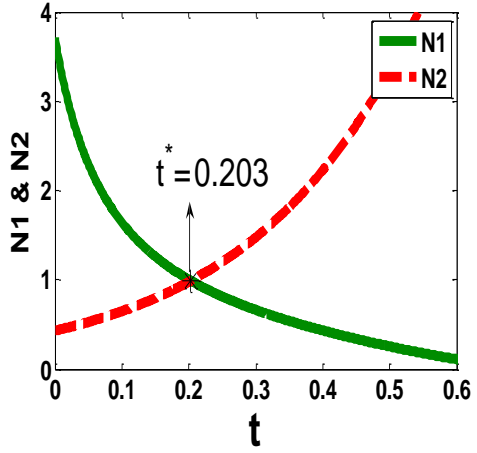


Fig.17 ; Case-6 in Table-2

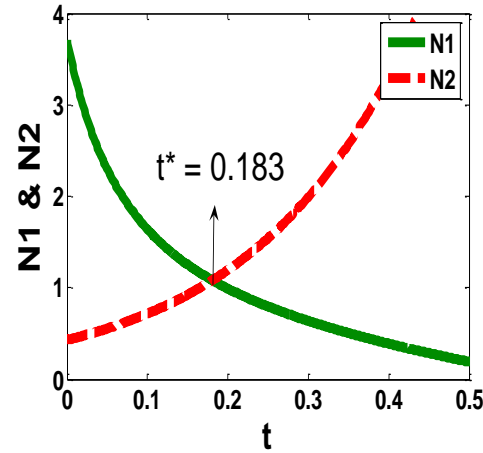


Fig.18 ; Case-7 in Table-2

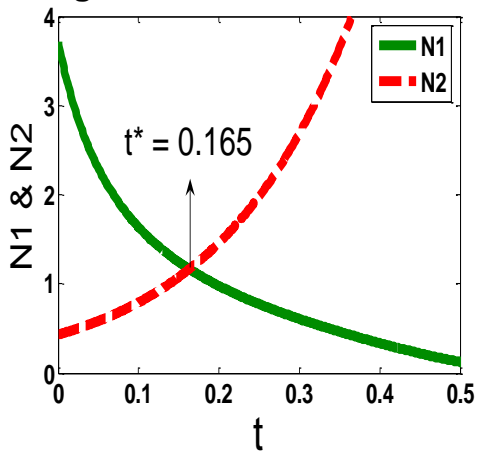


Fig.19 ; Case-8 in Table-2

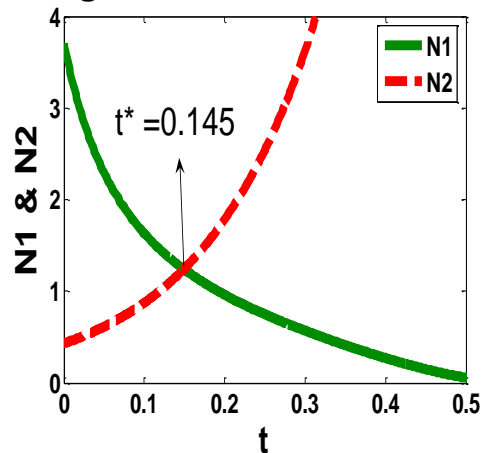


Fig.20 ; Case-9 in Table-2

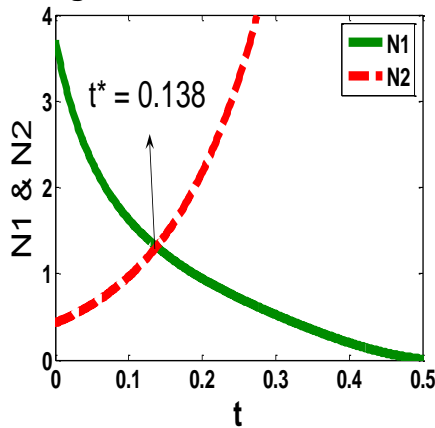


Fig.21; Case-10 in TABLE-2

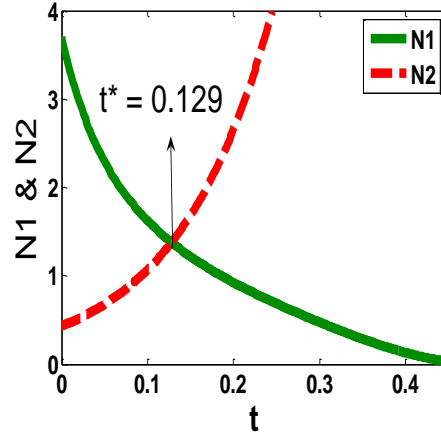
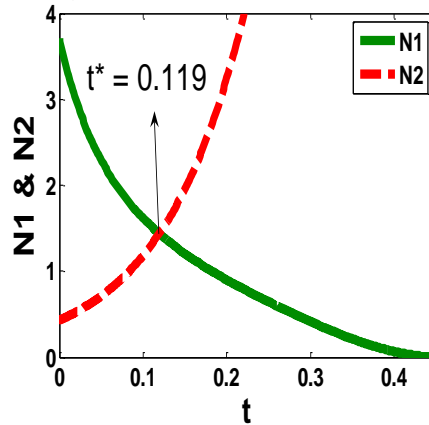


Fig.22 ; Case-11 in Table-2



6. Conclusions

The Ammensal species eclipses the enemy species up to the dominance reversal time. The enemy commands the Ammensal after dominance reversal time (t^*). Further the Ammensal species gradually declines towards the equilibrium point. The enemy species prospers with prominent growth rate throughout the interval. It is also noticed that the dominance reversal time decreases stepwise in both the cases having increase in natural growth rate as well as in Carrying capacity of Enemy Species. The overall conclusions are stated in Table-4

Table 4.

critierion	Conclusion
The carrying capacity of (K_2)Enemy species increases	The dominance reversal time (t^*) gradually decreases
The growth rate(a_2) of Enemy species increases	
The Ammensal Coefficient(α) increases	
The inhibition coefficient(a_{12}) of Ammensal species increases	

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Authors



K. V. L. N. Acharyulu: He is working as a Lecturer in the Department of Mathematics, Bapatla Engineering College, Bapatla which is a prestigious institution of Andhra Pradesh. He took his M. Phil. Degree in Mathematics from the University of Madras and stood first in R.K.M. Vivekananda College, Chennai. Nearly for the last ten years he is rendering his services to the students and he is applauded by one and all for his best way of teaching. He has participated in some seminars and presented his papers on various topics. More than 35 papers were published in International Journals. He has submitted his thesis for the award of Ph.D under the able guidance of Prof. N.Ch.Pattabhi Ramacharyulu.



N. Ch. Pattabhi Ramacharyulu: He is a retired professor in the Department of Mathematics & Humanities, National Institute of Technology, Warangal. He is a stalwart of Mathematics. His yeoman services as a Lecturer, Professor, Professor Emeritus and Deputy Director enriched the knowledge of thousands of students. He has nearly 38 PhDs and plenty number of M.Phils to his credit. His International and National research papers more than 200 were published in various esteemed national and international journals. He is a Member of Various Professional Bodies. He published four books on Mathematics. He received so many prestigious awards and rewards. He is a walking Encyclopedia on various subjects of modern Mathematics.