

## Design Robust Backstepping on-line Tuning Feedback Linearization Control Applied to IC Engine

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### Abstract

*Design a nonlinear controller for second order nonlinear uncertain dynamical systems (e.g., Internal Combustion Engine) is one of the most important challenging works. This paper focuses on the design of a robust backstepping adaptive feedback linearization controller (FLC) for internal combustion (IC) engine in presence of uncertainties. In order to provide high performance nonlinear methodology, feedback linearization controller is selected. Pure feedback linearization controller can be used to control of partly unknown nonlinear dynamic parameters of IC engine. In order to solve the uncertain nonlinear dynamic parameters, implement easily and avoid mathematical model base controller, Mamdani's performance/error-based fuzzy logic methodology with two inputs and one output and 49 rules is applied to pure feedback linearization controller. The results demonstrate that the error-based fuzzy feedback linearization controller is a model-free controllers which works well in certain and partly uncertain system. Pure feedback linearization controller and error-based feedback linearization like controller with have difficulty in handling unstructured model uncertainties. To solve this problem applied backstepping-based tuning method to error-based fuzzy feedback linearization controller for adjusting the feedback linearization controller gain ( $K_p, K_v$ ). This controller has acceptable performance in presence of uncertainty (e.g., overshoot=1%, rise time=0.48 second, steady state error =  $1.3e-9$  and RMS error= $1.8e-11$ ).*

**Keywords:** IC engine, backstepping controller, robust controller, adaptive methodology, feedback linearization controller, fuzzy logic methodology, feedback linearization like controller

### 1. Motivation, Introduction and Background

**Motivation:** Internal combustion (IC) engines are optimized to meet exhaust emission requirements with the best fuel economy. Closed loop combustion control is a key technology that is used to optimize the engine combustion process to achieve this goal. In order to conduct research in the area of closed loop combustion control, a control oriented cycle-to-cycle engine model, containing engine combustion information for each individual engine cycle as a function of engine crank angle, is a necessity. Air-to-fuel (A/F) ratio is the mass ratio of air to fuel trapped inside a cylinder before combustion begins, and it affects engine emissions, fuel economy, and other performances. In this research, a fuzzy backstepping adaptive MIMO fuzzy estimator feedback linearization control scheme is used to simultaneously control the mass flow rate of port fuel injection (PFI) systems to regulate the A/F ratio of PFI to desired levels. One of the most important challenges in the field of IC engine is IC engine control; because this system is MIMO, nonlinear, time variant parameter

and uncertainty [63-71]. Presently, IC engines are used in different (unknown and/or unstructured) situation consequently caused to provide complicated systems, as a result strong mathematical theory are used in new control methodologies to design nonlinear robust controller. Conventional and soft computing methods are two main categories of nonlinear plant control, where the conventional control theory uses the model-based method and the soft computing control theory (e.g., fuzzy logic, neural network, and neuro fuzzy) uses the artificial intelligence methods. However both of conventional and artificial intelligence theories have applied effectively in many areas, but these methods also have some limitations [1-2].

**Introduction and Background:** Modeling of an entire IC engine is a very important and complicated process because engines are nonlinear, multi inputs-multi outputs and time variant. One purpose of accurate modeling is to save development costs of real engines and minimizing the risks of damaging an engine when validating controller designs. Nevertheless, developing a small model, for specific controller design purposes, can be done and then validated on a larger, more complicated model. [63-71]. Dynamic modeling of IC engines is used to describe the behavior of this system, design of model based controller, and for simulation. The dynamic modeling describes the relationship between nonlinear output formulation to electrical or mechanical source and also it can be used to describe the particular dynamic effects to behavior of system[1]. In an internal combustion engine, a piston moves up and down in a cylinder and power is transferred through a connecting rod to a crank shaft. The continual motion of the piston and rotation of the crank shaft as air and fuel enter and exit the cylinder through the intake and exhaust valves is known as an engine cycle. The first and most significant engine among all internal combustion engines is the Otto engine, which was developed by Nicolaus A. Otto in 1876 [63-71].

To control of this system nonlinear control methodology (feedback linearization controller) is introduced. Feedback linearization controller (FLC) is an influential nonlinear controller to certain and partly uncertain systems which it is based on feedback linearization and computes the required arm torques using the nonlinear feedback control law. When all dynamic and physical parameters are known feedback linearization controller works superbly. Conversely, pure feedback linearization controller is used in many applications; it has an important drawback namely; nonlinear equivalent dynamic formulation in uncertain dynamic parameter. The nonlinear equivalent dynamic formulation problem in uncertain system can be solved by using artificial intelligence theorem. Fuzzy logic theory is used to estimate the system dynamic. However fuzzy logic controller is used to control complicated nonlinear dynamic systems, but it cannot guarantee stability and robustness. Fuzzy logic controller is used in adaptive methodology and this method is also can applied to nonlinear conventional control methodology to improve the stability, increase the robustness, reduce the fuzzy rule base and estimate the system's dynamic parameters. To reduce the fuzzy rule base with regards to improve the stability and robustness feedback linearization fuzzy controller (feedback linearization like controller) is introduced. In feedback linearization fuzzy controller feedback linearization controller is applied to fuzzy logic controller to reduce the fuzzy rules and increase the stability and robustness. The main disadvantage in feedback linearization fuzzy controller is calculation the value of controllers gain coefficient pri-defined very carefully. To estimate the system dynamics, fuzzy feedback linearization controller is introduced. Most of researchers are applied fuzzy logic theorem in feedback linearization controller to design a fuzzy model-based controller. This methodology is based on applied fuzzy logic in equivalent nonlinear dynamic part to estimate unknown parameters. Pure feedback linearization controller, feedback linearization fuzzy controller and fuzzy feedback linearization controller

have difficulty in handling unstructured model uncertainties. It is possible to solve this problem by combining fuzzy feedback linearization controller or feedback linearization fuzzy controller and adaption law which this method can help improve the system's tracking performance by online tuning method [1-40].

Research on feedback linearization controller is significantly growing on nonlinear systems application which has been reported in [1, 6, 15, 16]. Vivas and Mosquera [15] have proposed a predictive functional controller and compare to feedback linearization controller for tracking response in uncertain environment. However both controllers have been used in feedback linearization, but predictive strategy gives better result as a performance. A feedback linearization control with non parametric regression models have been presented for a nonlinear second order system [16]. This controller also has been problem in uncertain dynamic models. Based on [1, 6] and [15, 16] feedback linearization controller is a significant nonlinear controller to certain systems which it is based on feedback linearization and computes the required arm torques using the nonlinear feedback control law [41-60].

This paper is organized as follows: In section 2, main subject of modelling IC engine formulation and feedback linear formulation are presented. Detail of fuzzy methodology is presented in section 3. In section 4, the simulation result is presented and finally in section 5, the conclusion is presented.

## 2. Theorem: Dynamic Formulation of IC Engine, Feedback Linearization Formulation Applied to IC Engine

**Dynamic of IC engine:** In developing a valid engine model, the concept of the combustion process, abnormal combustion, and cylinder pressure must be understood. The combustion process is relatively simple and it begins with fuel and air being mixed together in the intake manifold and cylinder. This air-fuel mixture is trapped inside cylinder after the intake valve(s) is closed and then gets compressed [13]. When the air-fuel mixture is compressed it causes the pressure and temperature to increase inside the cylinder. Unlike normal combustion, the cylinder pressure and temperature can rise so rapidly that it can spontaneously ignite the air-fuel mixture causing high frequency cylinder pressure oscillations. These oscillations cause the metal cylinders to produce sharp noises called knock, which it caused to abnormal combustion. The pressure in the cylinder is a very important physical parameter that can be analyzed from the combustion process. Since cylinder pressure is very important to the combustion event and the engine cycle in spark ignition engines, the development of a model that produces the cylinder pressure for each crank angle degree is necessary. A cylinder pressure model that calculates the total cylinder pressure over 720 crank angle degrees was created based upon the following formulation [63-71]:

$$P_{cyl}(\theta) = P_m(\theta) + P_{net}(\theta) \quad (1)$$

where  $P_{cyl}(\theta)$  is pressure in cylinder,  $P_m(\theta)$  is Wiebe function, and  $P_{net}(\theta)$  is motoring pressure of a cylinder. Air fuel ratio is the mass ratio of air and fuel trapped inside the cylinder before combustion starts. Mathematically it is the mass of the air divided by the mass of the fuel as shown in the equation below:

$$Air\ to\ Fuel = \frac{\dot{m}_{air}}{\dot{m}_{fuel}} \quad (2)$$

If the ratio is too high or too low, it can be adjusted by adding or reducing the amount of fuel per engine cycle that is injected into the cylinder. The fuel ratio can be used to determine which fuel system should have a larger impact on how much fuel is injected into the cylinder.

Since a direct fuel injector has immediate injection of its fuel with significant charge cooling effect, it can have a quicker response to the desired amount of fuel that is needed by an engine [66].

**Feedback linearization methodology:** It is assumed that the desired motion trajectory for the manipulator  $q_d(t)$ , as determined, by a path planner. Define the tracking error as:

$$e(t) = q_d(t) - q_a(t) \quad (3)$$

Where  $e(t)$  is error of the plant,  $q_d(t)$  is desired input variable, that in our system is desired displacement,  $q_a(t)$  is actual displacement. If an alternative linear state-space equation in the form  $\dot{x} = Ax + BU$  can be defined as

$$\dot{x} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I \end{bmatrix} U \quad (4)$$

With  $U = -M^{-1}(q) \cdot (P_m(\theta) + P_{net}(\theta)) + M^{-1}(q) \cdot \tau$  and this is known as the Brunousky canonical form. By equation (3) and (4) the Brunousky canonical form can be written in terms of the state  $x = [e^T \ \dot{e}^T]^T$  as:

$$\frac{d}{dt} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} U \quad (5)$$

$$\text{With } U = \ddot{q}_d + M^{-1}(q) \cdot \{(P_m(\theta) + P_{net}(\theta)) - \tau\} \quad (6)$$

Then compute the required IC engine total energy using inverse of equation (6), namely, [2]

$$U_T = M(q)(\ddot{q}_d - U) + (P_m(\theta) + P_{net}(\theta)) \quad (7)$$

Where

$$M^{-1} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1} \quad M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

This is a nonlinear feedback control law that guarantees tracking of desired trajectory. Selecting proportional-plus-derivative (PD) feedback for  $U(t)$  results in the feedback linearization controller; [1-2]; [6]; [7-8]

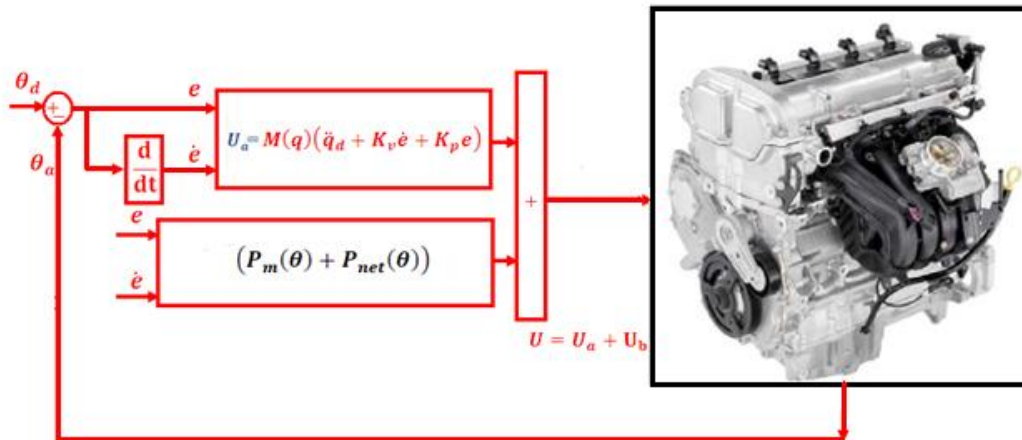
$$U_T = M(q)(\ddot{q}_d + K_v \dot{e} + K_p e) + (P_m(\theta) + P_{net}(\theta)) \quad (8)$$

and the resulting linear error dynamics are

$$(\ddot{q}_d + K_v \dot{e} + K_p e) = 0 \quad (9)$$

According to linear system theory, convergence of the tracking error to zero is guaranteed [2, 9].

Where  $K_p$  and  $K_v$  are the controller gains. The resulting schemes is shown in Figure 1, in which two feedback loops, namely, inner loop and outer loop, which an inner loop is a compensate loop and an outer loop is a tracking error loop. However, mostly parameter  $(P_m(\theta) + P_{net}(\theta))$  is all unknown. So the control cannot be implementation because non linear parameters cannot be determined. In the following section error-based fuzzy feedback linearization controller will be introduced to overcome the problems.



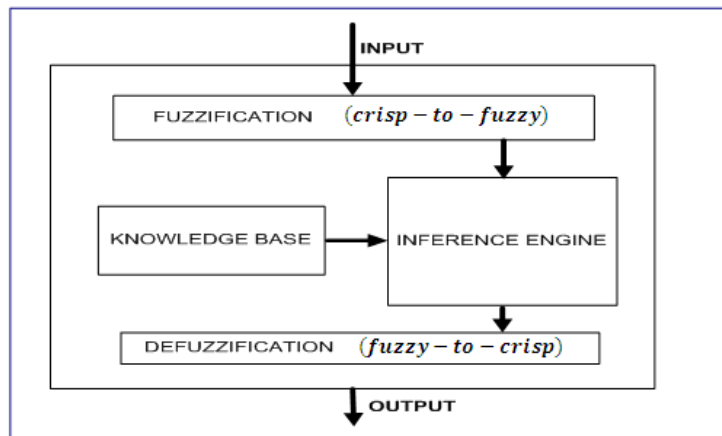
**Figure 1. Block Diagram of Feedback Linearization Controller: Applied to IC Engine**

### 3. Methodology: Design Fuzzy-Based Backstepping Error-Based Fuzzy Feedback Linearization Controller

**Design fuzzy feedback linearization controller:** Based on foundation of fuzzy logic methodology; fuzzy logic controller has played important rule to design nonlinear controller for nonlinear and uncertain systems [53]. However the application area for fuzzy control is really wide, the basic form for all command types of controllers consists of;

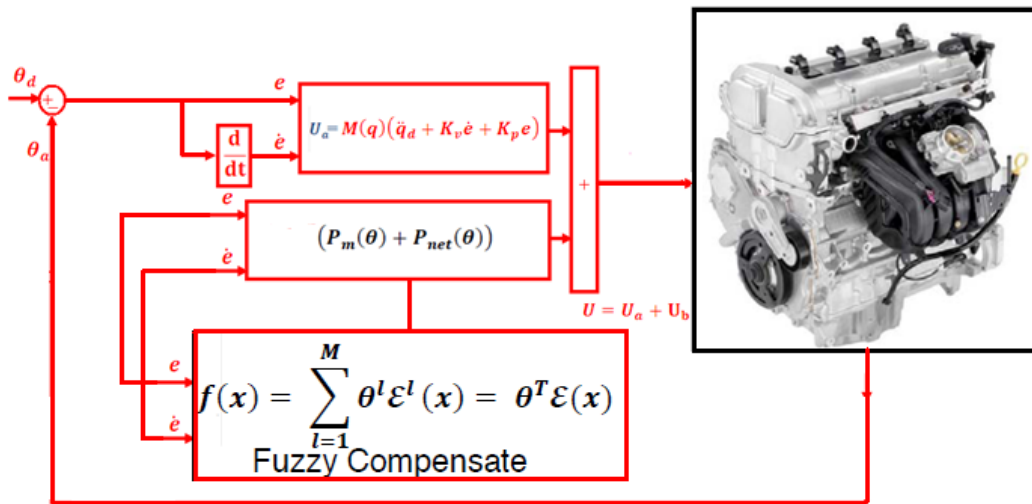
- Input fuzzification (binary-to-fuzzy[B/F]conversion)
- Fuzzy rule base (knowledge base)
- Inference engine
- Output defuzzification (fuzzy-to-binary[F/B]conversion).

The basic structure of a fuzzy controller is shown in Figure 2.



**Figure 2. Structure of Fuzzy Logic Controller (FLC)**

Based on Figure 2; fuzzification is used to change the crisp set into fuzzy set. Knowledge base is used to rule evaluation and determine the membership degree and if all fuzzy inputs activated by the known input values. Fuzzy inference engine is used to transferring the rule base into fuzzy set by Mamdani's or Sugeno method based on aggregation of the rules output. Defuzzification is the last part to calculate the fuzzy inference system. As shown in Figure 1, feedback linearization controller is divided into two main parts: linear part and nonlinear equivalent part. Nonlinear equivalent part is based on IC engine's dynamic formulation which these formulations are nonlinear; MIMO and some of them are unknown. IC engine's dynamic formulations are highly nonlinear and some of parameters are unknown therefore design a controller based on dynamic formulation is complicated. To solve this challenge fuzzy logic methodology is applied to feedback linearization controller. In this method fuzzy logic method is applied to nonlinear equivalent part to estimate it. To solve the challenge of feedback linearization controller based on nonlinear dynamic formulation this research is focused on eliminate the nonlinear equivalent formulation. In this method; dynamic nonlinear equivalent part is replaced by performance/error-based fuzzy logic methodology. In fuzzy error-based feedback linearization controller; error based Mamdani's fuzzy inference system has considered with two inputs, one output and totally 49 rules instead of the nonlinear dynamic equivalent part. Figure 3 is shown error-based fuzzy feedback linearization controller. In this method a model free Mamdani's fuzzy inference system has considered based on error-based fuzzy logic controller instead of nonlinear equivalent control.



**Figure 3. Block Diagram of Error-based Fuzzy Feedback Linearization Controller: Applied to IC Engine**

For both feedback linearization controller and fuzzy feedback linearization controller the system performance is sensitive to the controller gain coefficient ( $K_p, K_v$ ). Therefore to have a good response, compute the best value of controller gain coefficient is very important. Figure 3 has two main parts: linear based controller and error-based fuzzy part. Based on Figure 3, the fuzzy error-based feedback linearization controller's output is written;

$$\hat{U} = U_{eq\_fuzzy} + U_{Linear} \quad (10)$$

Based on fuzzy logic methodology

$$f(x) = U_{fuzzy} = \sum_{l=1}^M \theta^T \zeta(x) \quad (11)$$

where  $\theta^T$  is adjustable parameter (gain updating factor) and  $\zeta(x)$  is defined by;

$$\zeta(x) = \frac{\sum_i \mu(x_i) x_i}{\sum_i \mu(x_i)} \quad (12)$$

Where  $\mu(x_i)$  is membership function.  $\tau_{fuzzy}$  is defined as follows;

$$\tau_{fuzzy} = \sum_{l=1}^M \theta^T \zeta(x) = (P_m(\theta) + P_{net}(\theta)) \quad (13)$$

As mentioned in Figure 2, design of error-based fuzzy instead of equivalent part based on Mamdani's fuzzy inference method has four steps, namely, fuzzification, fuzzy rule base and rule evaluation, aggregation of the rule output (fuzzy inference system) and defuzzification.

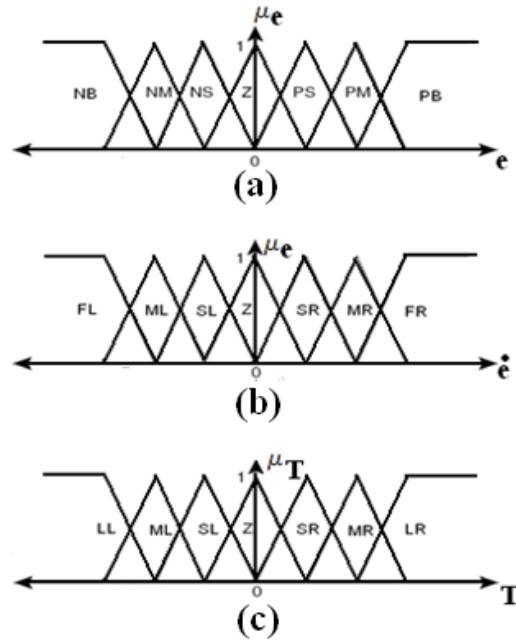
**Fuzzification:** the first step in fuzzification is determine inputs and outputs which, it has two inputs ( $e, \dot{e}$ ) and one output ( $u_{fuzzy}$ ). The inputs are error ( $e$ ) which measures the difference between desired and actual output, and the change of error ( $\dot{e}$ ) which measures the difference between desired change of input and actual change of output and output is fuzzy equivalent part. The second step is chosen an appropriate membership function for inputs and output which, to simplicity in implementation because it is a linear function with regard to acceptable performance triangular membership function is selected in this research as shown in Figure 4. The third step is chosen the correct labels for each fuzzy set which, in this research namely as linguistic variable. Based on experience knowledge the linguistic variables for error ( $e$ ) are; Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (Z), Positive Small (PS), Positive Medium (PM), Positive Big (PB), and based on literature [40] and experience knowledge it is quantized into thirteen levels represented by: -1, -0.83, -0.66, -0.5, -0.33, -0.16, 0, 0.16, 0.33, 0.5, 0.66, 0.83, 1 the linguistic variables for change of error ( $\dot{e}$ ) are; Fast Left (FL), Medium Left (ML), Slow Left (SL), Zero (Z), Slow Right (SR), Medium Right (MR), Fast Right (FR), and it is quantized in to thirteen levels represented by: -6, -5, -0.4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, and the linguistic variables to find the output are; Large Left (LL), Medium Left (ML), Small Left (SL), Zero (Z), Small Right (SR), Medium Right (MR), Large Right (LR) and it is quantized in to thirteen levels represented by: -85, -70.8, -56.7, -42.5, -28.3, -14.2, 0, 14.2, 28.3, 42.5, 56.7, 70.8, 85.

**Fuzzy rule base and rule evaluation:** the first step in rule base and evaluation is to provide a least structured method to derive the fuzzy rule base which, expert experience and control engineering knowledge is used because this method is the least structure of the other one and the researcher derivation the fuzzy rule base from the knowledge of system operate and/or the conventional controller. Design the rule base of fuzzy inference system can play important role to design the best performance of fuzzy feedback linearization controller, that to calculate the fuzzy rule base the researcher is used to heuristic method which, it is based on the behavior of the control of IC engine suppose that two fuzzy rules in this controller are;

$$\mathbf{F.R}^1: \text{IF } e \text{ is NB and } \dot{e} \text{ is FL, THEN } U \text{ is LL.} \quad (14)$$

$$\mathbf{F.R}^2: \text{IF } e \text{ is PS and } \dot{e} \text{ is FL THEN } U \text{ is ML}$$

The complete rule base for this controller is shown in Table 1. Rule evaluation focuses on operation in the antecedent of the fuzzy rules in fuzzy feedback linearization controller. This part is used *AND/OR* fuzzy operation in antecedent part which *AND* operation is used.



**Figure 4. Membership Function: a) error b) change of error c) output**  
**Aggregation of the rule output (Fuzzy inference): based on (15), Max-Min**  
**aggregation is used in this work.**

$$\mu_U(x_k, y_k, U) = \mu_{U_{i=1}^r FR^i}(x_k, y_k, U) = \max \left\{ \min_{i=1}^r \left[ \mu_{R_{pq}}(x_k, y_k), \mu_{P_m}(U) \right] \right\} \quad (15)$$

**Table 1. Modified Fuzzy Rule Base Table**

Decrease the overshoot		$\dot{e}$						
		←	←	←	←	←	←	←
$\dot{e}$	$e$	FL	ML	SL	Z	SR	MR	FR
	NB	LL	LL	LL	ML	SL	SL	Z
	NM	LL	ML	ML	ML	SL	Z	SR
	NS	LL	ML	SL	SL	Z	SR	MR
	Z	LL	ML	SL	Z	SR	MR	LR
	PS	ML	SL	Z	SR	SR	MR	LR
	PM	SL	Z	SR	MR	MR	MR	LR
	PB	Z	SR	SR	MR	LR	LR	LR
		Decrease the rise time						

**Defuzzification:** The last step to design fuzzy inference in our fuzzy feedback linearization controller is defuzzification. This part is used to transform fuzzy set to crisp set, therefore the input for defuzzification is the aggregate output and the output of it is a crisp number. Based on (16) centre of gravity method (COG) is used in this research.

$$COG(x_k, y_k) = \frac{\sum_i U_i \sum_{j=1}^r \mu_u(x_k, y_k, U_i)}{\sum_i \sum_{j=1}^r \mu_u(x_k, y_k, U_i)} \quad (16)$$



Table 2 shows the lookup table in fuzzy feedback linearization controller which is computed by COG defuzzification method. Table 2 has 169 cells to shows the error-based fuzzy instead of equivalent part behavior. For instance if  $e = -1$  and  $\dot{e} = -3.92$  then the output=-85. Based on Table 1 if two fuzzy rules are defined by

$$F.R^1: \text{if } e \text{ is } NB \text{ and } \dot{e} \text{ is } ML \text{ then } U \text{ is } LL$$

$$F.R^2: \text{if } e \text{ is } NB \text{ and } \dot{e} \text{ is } FL \text{ then } U \text{ is } LL$$

If all input fuzzy activated by crisp input values  $e = -1$  and  $\dot{e} = -3.92$  and fuzzy set to compute  $NB$ ,  $ML$  and  $FL$  are defined as

$$e_{(NB)} = \{(0, -1.5), (0.25, -1.375), (0.5, -1.25), (0.75, -1.125), (1, -1), (0.75, -0.875), (0.5, -0.75), (0.25, -0.625), (0, -0.5)\}$$

$$\dot{e}_{(ML)} = \{(0, -5.8), (0.25, -5.17), (0.5, -4.55), (0.75, -3.92), (1, -3.3), (0.75, -2.67), (0.5, -2.05), (0.25, -1.42), (0, -0.83)\}$$

$$\dot{e}_{(FL)} = \{(0, -7.5), (0.25, -6.88), (0.5, -6.25), (0.75, -5.57), (1, -5), (0.75, -4.30), (0.5, -3.92), (0.25, -3.12), (0, -2.5)\}$$

$$\text{while } U_{(LL)} = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.75, -94.5), (1, -85), (0.75, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\}$$

In this controller  $AND$  fuzzy operation is used therefore the output fuzzy set is calculated by using individual rule-base inference. Based on (14) the activation degrees is computed as

$$\mu_{FR_1} = \min[\mu_{e(N.B)}(-1), \mu_{\dot{e}(M.L)}(-3.92)] = \min[1, 0.75] = 0.75$$

$$\mu_{FR_2} = \min[\mu_{e(N.B)}(-1), \mu_{\dot{e}(F.L)}(-3.92)] = \min[1, 0.5] = 0.5$$

The activation degrees of the consequent parts for  $F.R^1$  and  $F.R^2$  are computed as:

$$\mu_{FR_1}(-1, -3.92, U) = \min[\mu_{FR_1}(-1, -3.92), \mu_{U(LL)}] = \min[0.75, \mu_{U(LL)}]$$

$$\mu_{FR_2}(-1, -3.92, U) = \min[\mu_{FR_2}(-1, -3.92), \mu_{U(LL)}] = \min[0.5, \mu_{U(LL)}]$$

Fuzzy set  $U_{L.L(1)}$  and  $U_{L.L(2)}$  have nine elements:

$$\begin{aligned} F.F^1(-1, -3.92, U) \\ = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.75, -94.5), (0.75, -85), \\ (0.75, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\} \end{aligned}$$

$$\begin{aligned} F.F^2(-1, -3.92, U) = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.5, -94.5), (0.5, -85), \\ (0.5, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\} \end{aligned}$$

Based on 14, Max-min aggregation is used to find the output of fuzzy set:

$$\begin{aligned} \mu_{U_{12}}(-1, -3.92, U) &= \mu_{\bigcup_{i=1}^2 FR^i}(-1, -3.92, U) \\ &= \max\{\mu_{FR^1}^1(-1, -3.92, U)_{L.L}, \mu_{FR^2}^2(-1, -3.92, U)_{L.L}\} \end{aligned}$$

$$U_{12} = \{(0, -123), (0.25, -113.5), (0.5, -104), (0.75, -94.5), (0.75, -85), (0.75, -75.5), (0.5, -66), (0.25, -56.5), (0, -47)\}$$

Based on (15) the COG defuzzification is selected as;

$$COG = [(0.25 \times -113.5) + (0.5 \times -104) + (0.75 \times -94.5) + (0.75 \times -85) + (0.75 \times -75.5) + 0.5 \times -66 + 0.25 \times -56.5] / [0.25 + 0.5 + 0.75 + 0.75 + 0.5 + 0.25] - 1 = -318.753.75 = -85$$

**Table 2.  $U_{eq\ fuzzy}$  Performance: Lookup Table in Fuzzy Feedback Linearization Controller by COG**

$\begin{matrix} \rightarrow \\ e \\ \downarrow \end{matrix}$	Membership Function ( $U_{eq\ fuzzy}$ )												
	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
<b>-1</b>	-85	-	-	-84	-	-81	-79	-71	-68	-65	-62	-60	-54
<b>-</b>	-	-84	-82	-80	-78	-77	-74	-70	-64	-60	-56	-54	-47
<b>0.83</b>	84.8	84.8	84.8	84.8	84.8	84.8	84.8	84.8	84.8	84.8	84.8	84.8	84.8
<b>-</b>	-78	-73	-70	-68	-64	-61	-60	-57	-55	-50	-47	-40	-38
<b>0.66</b>	-70	-60	-58	-51	-42	-38	-34	-33	-31	-29	-	-	-28
<b>-0.5</b>	-70	-60	-58	-51	-42	-38	-34	-33	-31	-29	28.4	28.1	-28
<b>-</b>	-50	-48	-45	-40	-38	-34	-32	-30	-28	-26	-25	-21	-20
<b>0.33</b>	-30	-25	-21	-18	-16	-14	-10	-9	-8	-7	-6.8	-6	-5
<b>-</b>	-30	-25	-21	-18	-16	-14	-10	-9	-8	-7	-6.8	-6	-5
<b>0.16</b>	-10	-8	-6	-1	2	3	6	7	8	10	12	15	17
<b>0</b>	-10	-8	-6	-1	2	3	6	7	8	10	12	15	17
<b>0.16</b>	15	18	21	22	23	25	27	28	29	30	30.5	30.8	31
<b>0.33</b>	29	29.8	31	33	34	34.6	35	35.2	36	37	38	39	42
<b>0.5</b>	40	41	42	43	45	45	46	46.3	46.8	47	48	51	52
<b>0.66</b>	48	49	50	52	53	55	56	57	58	59	60	61	63
<b>0.83</b>	60	61	62	63	64	66	67	68	68.5	69	70	70.8	71
<b>1</b>	66	68.7	68.9	70	72	74	75	77	78	79	81	83	84

**Design error-based fuzzy backstepping adaptive fuzzy feedback linearization controller:** The other step is focused on design error-based Mamdani's fuzzy [30-40] backstepping adaptive fuzzy estimator feedback linearization control. As mentioned above pure feedback linearization controller has nonlinear dynamic equivalent limitations in presence of uncertainty and external disturbances in order to solve these challenges this work applied Mamdani's fuzzy inference engine estimator in feedback linearization controller. However proposed MIMO fuzzy estimator feedback linearization control has satisfactory performance but calculate the controller gain by try and error or experience knowledge is very difficult, particularly when system has structure or unstructured uncertainties; MIMO Mamdani's fuzzy backstepping fuzzy estimator feedback linearization controller is recommended. The backstepping method is based on mathematical formulation which this method is introduced new variables into it in form depending on the dynamic equation of IC engine. This method is used as feedback linearization in order to solve nonlinearities in the system. To use of nonlinear fuzzy filter this method in this research makes it possible to create dynamic nonlinear backstepping estimator into the adaptive fuzzy estimator feedback

linearization process to eliminate or reduce the challenge of uncertainty in this part. The backstepping controller is calculated by;

$$U_{B.S} = U_{eq_{B.S}} + M \cdot I \tag{17}$$

Where  $U_{B.S}$  is backstepping output function,  $U_{eq_{B.S}}$  is backstepping nonlinear equivalent function which can be written as (18) and  $I$  is backstepping control law which calculated by (19)

$$U_{eq_{B.S}} = [(P_m(\theta) + P_{net}(\theta))] \tag{18}$$

$$I = [\ddot{\theta} + K_1(K_1 - 1) \cdot e + (K_1 + K_2) \cdot \dot{e}] \tag{19}$$

Based on (3) and (18) the fuzzy backstepping filter is considered as

$$(P_m(\theta) + P_{net}(\theta)) = \sum_{l=1}^M \theta^l \zeta(x) - \lambda S - K \tag{20}$$

Based on (17) the formulation of fuzzy backstepping filter can be written as;

$$U = U_{eq_{B.S}fuzzy} + MI \tag{21}$$

Where  $U_{eq_{B.S}fuzzy} = [(P_m(\theta) + P_{net}(\theta))] + \sum_{l=1}^M \theta^l \zeta(x) + K$

As a result MIMO fuzzy backstepping adaptive fuzzy estimation feedback linearization control is very stable which it is one of the most important challenges to design a controller with suitable response. Figure 5 is shown the block diagram of proposed MIMO fuzzy backstepping adaptive fuzzy estimation feedback linearization control.

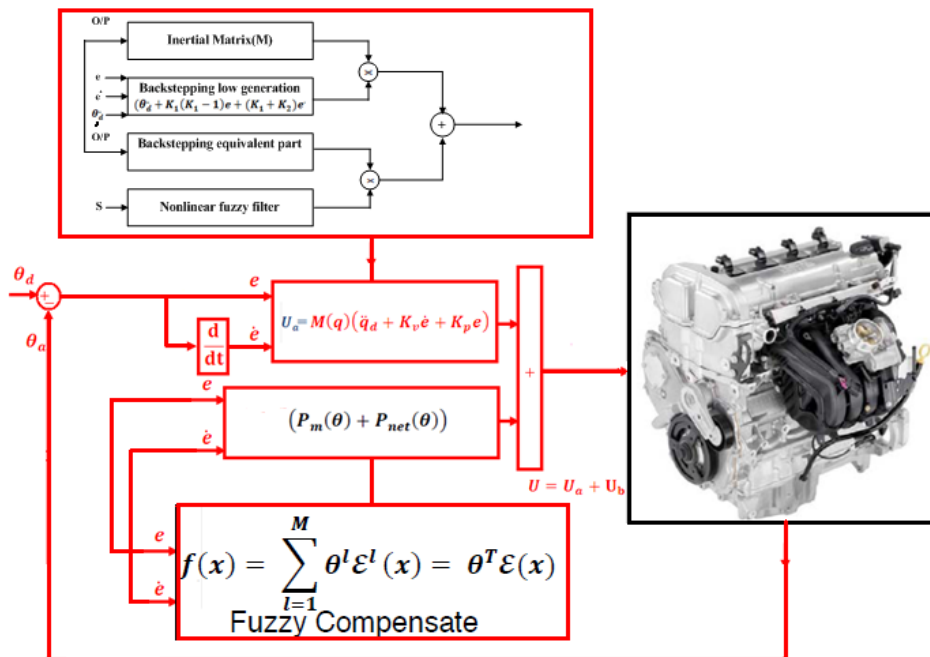
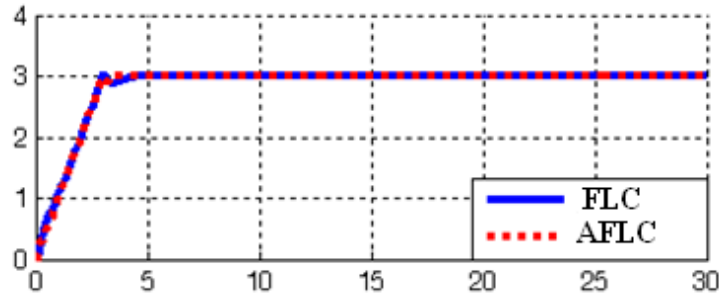


Figure 5. Block Diagram of a MIMO Fuzzy Backstepping Adaptive Fuzzy Estimator Feedback Linearization Controller

#### 4. Results

To validation of this work it is used IC engine and implements proposed fuzzy backstepping adaptive fuzzy estimator feedback linearization controller (AFLC) and feedback linearization controller (FLC) in this IC engine. The simulation was implemented in Matlab/Simulink environment. Fuel ratio and disturbance rejection are compared in these controllers. It is noted that, these systems are tested by band limited white noise with a predefined 40% of relative to the input signal amplitude. This type of noise is used to external disturbance in continuous and hybrid systems.

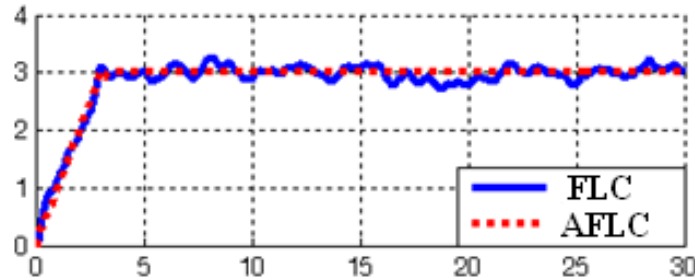
**Fuel ratio trajectory:** Figure 6 is shown the fuel ratio in proposed AFLC and FLC in uncertain environment but without disturbance for desired.



**Figure 6. FLC Vs. AFLC: Fuel Ratio**

By comparing this response, Figure 6, in FLC and AFLC, conversely the AFLC's overshoot is lower than FLC's, the rise time in both of methodologies have identical response. The Settling time in AFLC is fairly lower than FLC.

**Disturbance Rejection:** Figure 7 is indicated the power disturbance removal in FLC and AFLC. As mentioned by, FLC is a robust nonlinear controller which it is used as a base controller in this work. Besides a band limited white noise with predefined of 40% the power of input signal is applied to the trajectory response FLC and AFLC; it found slight oscillations in classical FLC trajectory responses.



**Figure 7. FLC Vs. AFLC: Fuel Ratio with External Disturbance**

Among above graph, relating to desired trajectory following with structure and unstructured disturbance, FLC has slightly fluctuations. AFLC's overshoot is lower than

FLC's, FLC and AFLC's rise time are the same and finally the Settling time in AFLC is quite lower than FLC.

## 5. Conclusion

Refer to the research, an error-based fuzzy backstepping on-line tune fuzzy feedback linearization controller is design and applied to IC engine in presence of structure and unstructured uncertainties. Regarding to the positive points in pure feedback linearization controller and fuzzy feedback linearization methodology and on-line tuning algorithm, the response is improved. Fuzzy logic method by adding to the feedback linearization controller has covered negative points. Obviously IC engine is nonlinear and MIMO system so in proposed controller in first step design free model controller based on fuzzy feedback linearization controller and after that disturbance rejection is improved by on-line fuzzy backstepping tunable gain. This implementation considerably reduces the output oscillation response in the presence of uncertainties. As a result, this controller will be able to control a wide range of IC engine with a high sampling rates because its easy to implement versus high speed markets.

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