Numerical Simulation of the Process of Bone Remodeling in the Context of Damaged Elastic

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Abstract

In this paper we propose a model of bone remodeling which takes in consideration the elasticity with damage properties of the material. Also the non linear equation of the bone apparent density is solved by a finite difference method, particularly a model with n unit elements. We will study the influence of damage damping on the adaptation of the structure under the effect of a controlled mechanical loading.

Keywords: *finite difference method, bone remodeling, apparent density, damage, osteocytes*

1. Introduction

The bone is continually renewed by local apposition and resorption of bone matrix in order to adapt to its environment [10]. This is due to the different cells involved in bone remodelling : osteocytes, osteoclasts and osteoblasts [6, 15, 5].

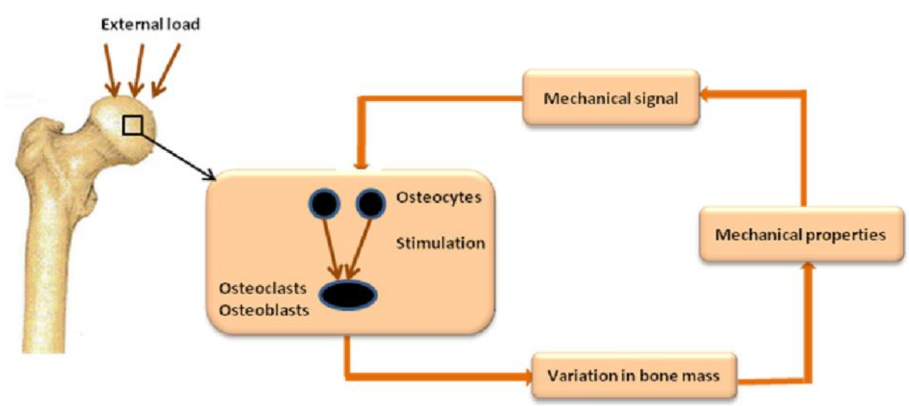


Figure 1: Schematic Proposed by Cowin and Mullender

In response to external mechanical loading, the osteocytes network is sensitive to the state of local deformation of bone tissue and is able to alert other specialized cells in the remodeling.

These cells can easily change the density of the bone and has the ability to cause a variation in mechanical properties (Figure 1) [6,11, 12, 17].

Also, the behavior of bone is a combination of several mechanisms, accompanied by micro and / or macro cracks. Those effects are modeled by the damage with strain [4].

The scientific community: mathematicians, engineers, biomechanicians among others are interested in modeling of such processes [9, 16, 20, 21, 23, 24, 26, 27]. In this work, we retain the model proposed by Mullender [18, 22]. The evolution equation of the model is solved by finite difference. The related results show the influence of the damage and distribution of osteocytes.

2. Model Geometry with n unit Elements

According to Zidi [28], we consider a solid discretized into n elements uniformly loaded by a compressive stress (one-dimensional case) [7]. The study of the bone fragment is made, so that one is located in the one-dimensional case, we will discretize into n unit elements for which we will apply a compressive force evenly distributed over the various units as depicted in figure 2 [3].

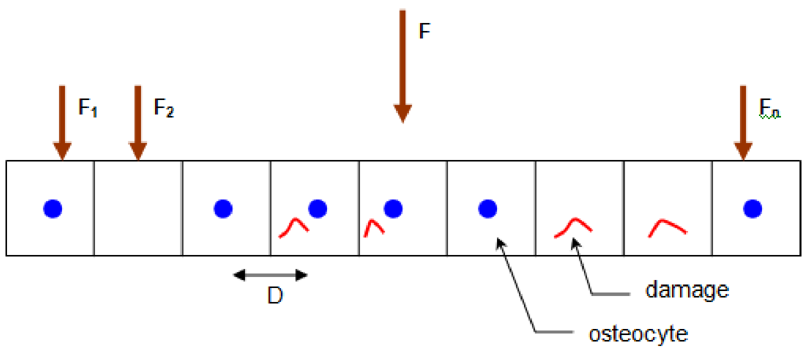


Figure 2 : Model Geometry with n unit Elements

3. Evolution Law of Bone Density

The evolution law of the apparent bone density is an extension of the one proposed by Mullender [18, 2] and is given by:

$$\frac{\partial \rho_i}{\partial t} = \tau . \sum_{k=0}^{k=m} e^{-\frac{d(i,I_k)}{D}} \left(\frac{S_k}{\rho_k^{\beta}} - \bar{S}_0 \right)$$

- with $\rho_{min} \leq \rho \leq \rho_{max}$ and $1 \leq i \leq n$. Where ρ_{min} is the density of completely resorbed bone, ρ_{max} is the maximum density defined for a compact bone and τ is a positive constant related to the reaction time of bone tissue.
- m ($m \leq n$) is the total number of osteocytes in the solid.

- I_k ($1 \leq m \leq k$) corresponds to the series of numbers of the elements containing an osteocyte.
- S_k represents the density of deformation energy in I_k and S_0 is the constant determined from the energy density of deformation, S_0 that doesn't generate no remodeling.
- Moreover, β is a parameter reflecting the intensity of the stimulus cell.
- D is a normalization factor limiting the area of influence of osteocyte.
- $d(i,I_k)$ is the distance between the centers of geometric element i and the element I_k.

4. The Young's Modulus

To simplify the study, we assume that the bone is an isotropic material and inhomogeneous [14, 25].

- Bone structure without damage has a Young's modulus given by:

  $$E = c\,\rho^{\alpha} \quad [8, 19]$$

  We note that Young's modulus E is related to apparent bone density, with c =100 and α = 3 are two constants characteristic of the bone.

- With damage, the Young's modulus becomes:

  $$\tilde{E} = (1-d)\,E$$

  Where d defines the degree of damage and $0 \leq d \leq 1$ and then we have:

  $$\tilde{E} = (1-d)\,c\,\rho^{\alpha}$$

  Hypothesis proposed by Abdali : [13]

  $$\tilde{E} = c\,\tilde{\rho}^{\alpha}$$

  $\tilde{\rho}$ is the bone density that takes into account the damage: $\tilde{\rho} = \rho(1-d)^{\frac{1}{\alpha}}$

5. Resolution

To discretize the previous equation, we use the method of finite difference with an implicit scheme and the fixed point method by Abdali. [1, 13]

We obtain:

$$\frac{\rho^{n+1}{}_i - \rho^n{}_i}{\Delta t} = \tau.\sum_{k=0}^{k=m} e^{-\frac{d(i,I_k)}{D}} \left(\frac{S_k}{\rho_k^{\beta,n+1}} - \bar{S}_0\right) \qquad i = 1,2,....n$$

$$\rho^0 = \rho_0 \qquad \rho_0 \text{ initial data}$$

Some stages of the resolution:

To understand the significance of certain parameters of the law of evolution, we have:

$$\frac{\partial \rho_i}{\partial t} = \tau.\sum_{k=0}^{k=m} e^{-\frac{d(i,I_k)}{D}} \left(\frac{S_k}{\rho_k^{\beta}} - \overline{S}_0\right)$$

n : is the number of cells (= 50)

m: corresponds to the number of the cells containing an osteocyte (●)

<u>The hypotheses</u> (H):

H1- n=m : all the cells contain an osteocyte.

H2- n ≠ m : with cases to check:

- Package of osteocytes in the central part.
- Two packs of osteocytes at the ends.
- Two or three packs of osteocytes alternated with empty boxes.

H3- we have the same deformation in each box:

$$F_1 = F_2 = F_3 = \ldots = F_n \quad \Rightarrow \quad F = nF_1 \quad \Rightarrow \quad F_1 = \frac{F}{n}$$

We have the stress tensor : $\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$

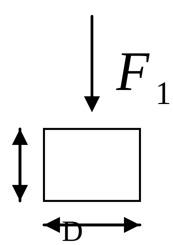
Since the stress is uniaxial along the vertical axis.

$\sigma_{11} = \sigma_{33} = 0$,

$\sigma_{12} = \sigma_{13} = \sigma_{21} = \sigma_{23} = \sigma_{31} = \sigma_{32} = 0$

and $\boldsymbol{\sigma_{22}} = \frac{F_1}{D\,x\,1} = \frac{F_1}{D}$ this comes from that:

TK = 1



Thickness (TK)

Therefore the stress tensor :

So $\sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{F_1}{D} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

For $\overline{S}_0$ =0.04 MPa

S_k is the density of deformation energy, so :

$$S_k = \frac{1}{2}\left(\sigma_{ij}\,\varepsilon_{ij}\right)_k$$

The calculation of S_k we have :

$$\sigma_{ij} = \frac{E}{1+\gamma}\left[\varepsilon_{ij} + \frac{\gamma}{1-2\gamma}\varepsilon_{kk}\,\delta_{ij}\right]$$

We have : $\varepsilon_{kk} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$

And :

$$\varepsilon = \begin{pmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix}$$

For $\delta_{ij} \rightarrow \begin{cases} =1 & si\,, i = j \\ =0 & si\,, i \neq j \end{cases}$

6. Discussion of Results

We simulated the case of an uniform distribution of the osteocytes cells with (n=m=50), and of another heterogeneous case (n ≠ m). The values of the other parameters used during the digital simulations are given in the table 1.

Table 1 : The values of the parameters used during the digital simulations.

Data	Symbol	Values	Units
Maximum density:	ρ_{max}	1,75	g/Cm^3
Minimal density:	ρ_{min}	0,01	g/Cm^3
Initial density:	ρ_0	0,6	g/Cm^3
The step of time:	Δt	5.10^{-3}	UT
The total force:	F	10	N
The distance between 2 centers:	D	25	mm
The signal of reference:	S_0	0,04	MPA
Constants:	$\alpha = 3$	β= 0,5	n = 50
The degree of damage: The damage doesn't follow a law of evolution, is given a randomly d = 0 - 0.2 - 0.5 - 0.8			

We studied in the range of elasticity of bone tissue by varying the number and position of osteocytes in the elements unity. Also Young's modulus will reflect the state of damage of bone tissue.

6.1. Case 1: n ≠ m, d = 0 (without damage)

(m=30: alternate packages 3 x 10 ostéocytes)

The evolution of the bone density compared to the position of the osteoytes is illustrated as shown in the figure 3.

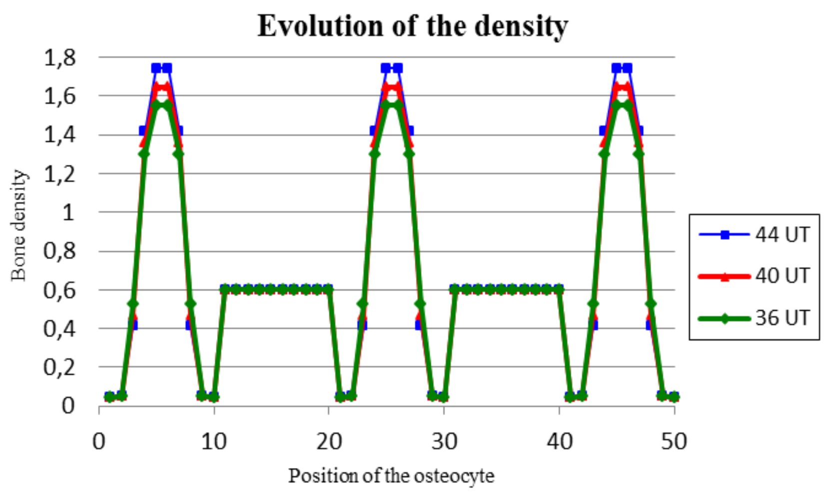


Figure 3: The Evolution of the Density in Case 1: n ≠ m, d = 0

The model converges at a density peak in three areas that correspond to packets of osteocytes, and retains the initial density in two areas empty of the osteocytes.

According to this study, we have made the following findings:

In the areas filled by osteocytes, the calculation converged at the end of 44 units of time (UT) with a density peak of 1.74 g/cm3.

The component elements of this zone reached the maximum density and thus support the imposed loading.

Whereas in the private areas of osteocytes, the calculation converged at the initial density of 0.6 g/cm3.

We observe that the results of the simulation reflects the localization of osteocytes in bone like the results of Zidi [28]; and the bone adapts to its mechanical environment (figure 1).

Compared to the approch of Zidi [28], our model converges only after 44 units of time (UT) instead of 125 units of time (UT).

6.2. Case 2: n ≠ m, d ≠ 0 (with damage)

(m=30: alternate packages 3 x 10 ostéocytes)

The evolution of the bone density with damage compared to the position of the osteoytes is illustrated as shown in the figure 4.

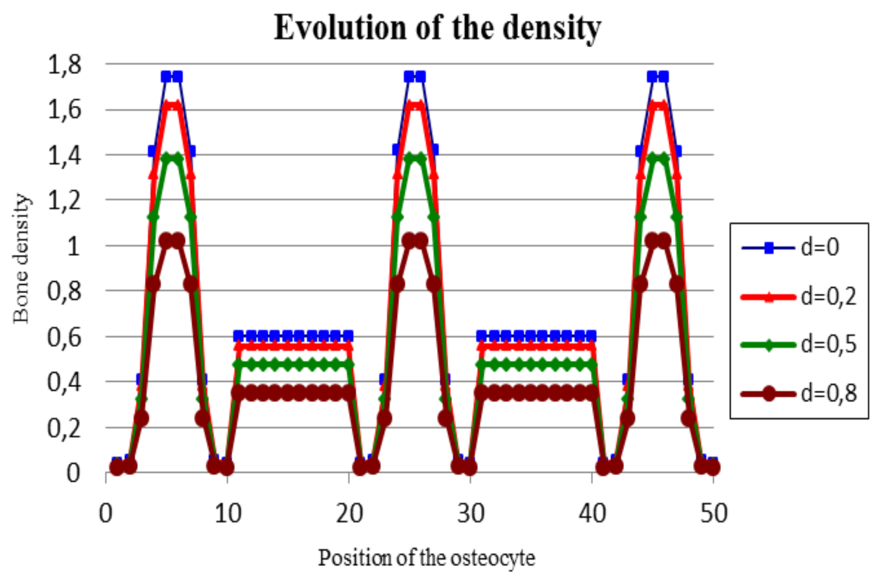


Figure 4: The Evolution of the Density in Case 2: n ≠ m, d ≠ 0

In this case, we introduce the notion of the degree of damage d in the equation of evolution of bone density; and we keep the same approach of the simulation.

Figure 4 presents the same look as the previous case (Figure 3) : we have three areas that correspond to packets of osteocytes with a density peak and two areas that retain the initial density.

When d=0 without damage, we have the same results.

When d increases (for exemple d=0,5) bone density decreases :

- In the areas filled by osteocytes, the value of bone density is equal to 1.4 g/cm3.
- In the private areas of osteocytes, the value of bone density is equal to 0.5 g/cm3.

The findings are:

The value of the bone density with damage depends on the position of the osteoytes as the previous case (case 1).

The values of the damage variable influence on the bone density: When d increases bone density decreases, as mentioned in the results of Hazelwood and Alexander. [29, 30]

6.3. Case 3: n ≠ m, d = 0 (without damage)

(m=30: alternate packages 3 x 10 ostéocytes)

The evolution of the Young's modulus without damage compared to the position of the osteoytes is illustrated as shown in the figure 5.

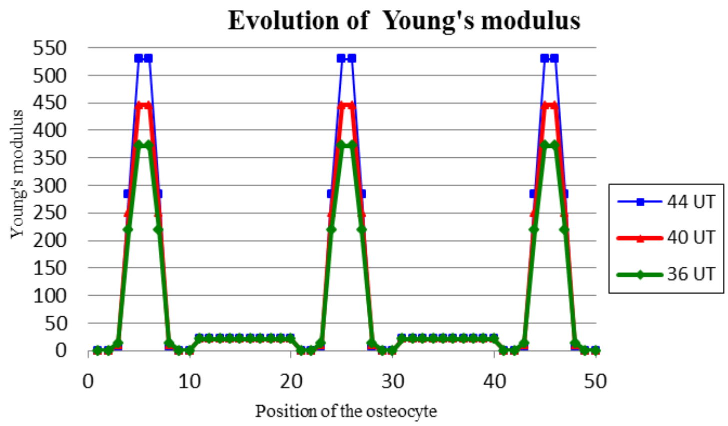


Figure 5: The Evolution of the Young's Modulus in Case 3: n ≠ m, d ≠ 0

The model converges at an Young's modulus peak in three areas that correspond to packets of osteocytes, and retains the initial Young's modulus in two areas empty of the osteocytes. A high Young's modulus means that the material is stiffer.

The findings are:

In the areas filled by osteocytes, the calculation converged at the end of 44 units of time (UT) with an Young's modulus peak.

Whereas in the private areas of osteocytes, the calculation converged at the initial Young's modulus where the bone is less stiff and less strong.

In this case, the results of the simulation reflects the localization of osteocytes in bone and that the bone adapts to its mechanical environment (figure 1).

6.4. Case 4: n ≠ m, d ≠ 0 (with damage)

(m=30: alternate packages 3 x 10 ostéocytes)

The evolution of the Young's modulus with damage compared to the position of the osteoytes is illustrated as shown in the figure 6.

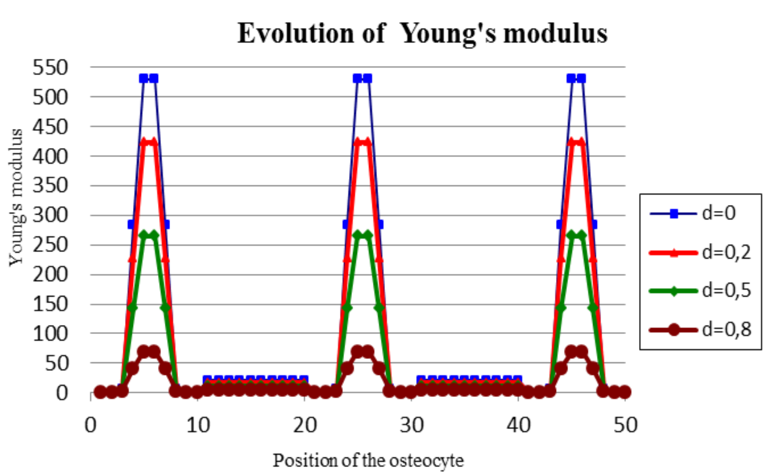


Figure 6: The Evolution of the Young's Modulus in Case 4: n ≠ m, d ≠ 0

We introduce the notion of the degree of damage in the equation of evolution of the Young's modulus; and we keep the same approach of the simulation.

Figure 6 presents the same look as the previous case (Figure 5).

The findings are:

The value of the Young's modulus with damage depends on the position of the osteoytes as the previous case (case 3).

When the damage increases, the Young's modulus decreases, as mentioned in the results of Martínez-Reina and Palchik [31, 32]

7. Conclusion

The evolution equation of the model is solved by finite differences and the results presented show :

- The influence of the distribution of osteocytes in the remodeling process: The private areas of osteocytes are insensitive to mechanical load and that only the parts containing osteocytes that meets this burden.
- The damage influence on the bone density and the stiffness of bone.

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