Efficient BER Analysis of OFDM System over Nakagami-m Fading Channel

[1]Mukesh Kumar Mishra, [2]Neetu Sood and [3]Ajay K Sharma

[1,2]Department of Electronics & Communication Engineering, [3]Department of Computer Science Engineering National Institute of Technology Jalandhar, India

Abstract

In this paper, we present an efficient technique for the BER of OFDM system over Nakagami–m fading channels, using well known characteristics function based analysis approach. The average BER expressed in terms of the higher transcendental function such as the confluent hyper geometric functions. Our numerical results show that depending on the number of channel taps, the BER performance may degrade with increasing values of Nakagami–m fading parameters.

Keywords: *OFDM, Nakagami-m fading channel, multipath propagation, characteristics function (CHF), BER performance*

1. Introduction

Orthogonal frequency division multiplexing is being used more and more in telecommunication, wired and wireless. DVB and DAB already use this modulation technique and ADSL is based on it. The advantage of this modulation is the reason for its increasing usage. OFDM can be implemented easily, it is spectrally efficient and can provide high data rates with sufficient robustness to channel imperfections [1, 2].In an OFDM [3] scheme a large number of sub channels or subcarriers are used to transmit digital data. Each sub channel is orthogonal to each other. They are closely spaced and narrow band. The separation of the sub channels is as minimal as possible to obtain high spectral efficiency. OFDM is being used because of its capability to handle with multipath interference at the receiver.

A great number of channel models have been proposed to describe the statistics of the amplitude and phase of multipath fading signals. Among them Nakagami-m distribution [4] is a generalized distribution which can model different fading environments. It has greater flexibility and accuracy in matching some experimental data than the Rayleigh, lognormal or Rice distributions [5, 6]. For example, Suzuki [7] showed that Nakagami distributions fit some urban radio multipath channel data better than Rayleigh, Rice, or lognormal distributions. It also has the advantage of including the Rayleigh and the one-sided Gaussian distribution as special cases.

Rayleigh and Rician fading channels have already been deployed and studied in depth for OFDM systems. In [8] used the pilot symbol along with the previously known channel coefficients for fast Rayleigh faded channels. In [9] BER performance in frequency selective Rician fading channel is studied. In [10] purpose of this paper is to clarify quantitatively system performances of the MPSK signal transmitted through a slow and flat m-distributed fading channel. In [11] Alouini and Goldsmith used moment generating function (MGF) technique to study the error performance of different modulation techniques over Nakagami-m fading channel. In [12] used the characteristics function approach to evaluate the BER performance and outage probability over Nakagami-m and Rician fading channel and summarized the advantages of this approach over MGF technique. In [13] claimed that the distribution of samples of the frequency-domain channel impulse response can be approximated by another Nakagami distribution with a new fading parameter different from the time-domain fading parameter. In [14] modeled the OFDM-BPSK system by presenting an approximate BER performance of OFDM system in frequency selective Nakagami-m channel using CHF approach and claimed that for Nakagami-m distribution, if we increase the value of m it is not necessary that the performance will always increases.

So our motivation behind this paper is to study and analyze the performance of OFDM system over Nakagami-m fading channel using CHF method to remove the complexity of the system, and within a single common framework, developed a general method for calculating the average error rates of single channel and multiple channel reception.

The main intention of paper is to derive the analytical expression for error rates of OFDM systems over Nakagami-m fading channel. To accomplish this, in first step we derive the PDF of Nakagami-m random phase vectors in an integral form by using CHF method. Further this PDF is used to evaluate the BER performance of an OFDM system over Nakagami-m fading channels. Here the average BER is expressed in terms of higher transcendental function such as gauss hyper geometric function. Finally it is observed through numerical results, that BER performance of an OFDM signal over multi propagation Nakagami-m channels does not improve with increasing Nakagami fading parameter m.

The rest of the paper organized as follows. The section 2 concerns with the system model of the OFDM communication systems. In section 3, we derive an integral expression for the PDF of Nakagami-m random phase vectors. In section 4 BER expressions for fading channels are derived. The section 5 discusses about the numerical results and finally section 6 concludes the work.

2. OFDM Model

Consider an OFDM system with N sub-carriers. Let X (k) is the k^{th} OFDM data block to be transmitted with N subcarriers. These data are used to modulate N orthogonal sub carriers. Then IDFT is used to modulate the input signal. After modulation signal can be represented as:

$$x(n) = 1/N \sum_{k=0}^{N-1} X(k) \exp\left(\frac{j2\pi kn}{N}\right) \quad n=0, 1....N-1 \tag{1}$$

According to the nature of the interaction between the wave and obstacle the signal can be reflected diffracted or diffused. This phenomenon referred to as multipath propagation. The channel impulse response of a multipath fading channel is modeled as a $\left|h(n)e^{-j\theta(n)}\right|$. In our analysis we assume that frequency synchronization are achieved at the receiver side r (n) can be represented in frequency domain as:

$$R(k) = X(k) * H(k) + N \tag{2}$$

Where N is an independent identically distributed (i.i.d) complex Gaussian noise component with zero mean and unit variance. The amplitude of H (k) is modeled as a Nakagami RV with PDF.

3. PDF of Nakagami-m RV

In this section, we derive a closed form expression for the first-order PDF and for that we use characteristics function approach (CHF). Let we have an auxiliary function Z=R cos(θ). Where R is the fading amplitude and (θ) is the random phase distributed uniformly over [0, 2π]. The Nakagami-m distribution is given by [5]:

$$f(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} e^{\frac{-mr^2}{\Omega}}, r \geq 0 \tag{3}$$

The mathematical definition for finding CHF of X is given as [20]:

$$\Psi_X(\eta) = \text{E [exp (j}\eta\text{X)]} \tag{4}$$

$$\Psi_X(\eta) = \int_{-\infty}^{\infty} f_X(x) e^{j\eta x} dx \tag{5}$$

The standard derivation for the characteristic function, as derived in [15, 19] is given by:

$$\Psi_{X\,n} = \int_0^{\infty} f(r) J_0(\eta r) dr \tag{6}$$

Where $J_0(.)$ is the zeroth order Bessel function of the first kind [18].

Using the eq. (3), eq. (6) can be written as:

$$\Psi_{X_n}(\eta) = \int_0^{\infty} \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} e^{\frac{-mr^2}{\Omega}} J_0(\eta r) dr \tag{7}$$

To simplify the eq. (7) we use integral identity [17, eq. 6.631.1] and eq. (7) can be expressed as:

$$\Psi_{X\,n}(\eta) = {}_1F_1\left(m_n; 1; \frac{-\Omega_n}{m_n} \eta^2 \right) \tag{8}$$

Where ${}_1F_1$ (. ;.;) is confluent hyper geometric function. Equation (8) is the CHF which is same as derived in [15] for Nakagami-m fading channel.

Because channel tap coefficients are independent so joint CHF can be given as [23]:

$$\Psi_{X\,n}(\eta) = \prod_0^{M-1} {}_1F_1\left(m_n; 1; \frac{-\Omega_n \eta^2}{m_n} \right) \approx \Psi(\eta) \tag{9}$$

To obtain the PDF for X the inverse Fourier Transform is applied to get the equation as given in [15]:

$$f_X(x) = \frac{1}{2\pi}\int_0^{\infty} e^{-j\eta x}\Psi(\eta)d(\eta)$$

(10)

$$f_X(x) = \frac{1}{\pi}\int_0^{\infty} \cos(\eta x)\Psi(\eta)d(\eta)$$

(11)

$$f_X(x) = \frac{1}{\pi}\int_0^{\infty}\prod_0^{M-1} {}_1F_1\left(m_n;1;\frac{-\Omega_n\eta^2}{m_n}\right)\cos(\eta x)d\eta$$

(12)

Equation (12) is the expression for the PDF in a integral form,that is further used in error rate calaculation.

4. BER Performance Analysis

The conditional BER of a particular modulation is given by Q (Sx), where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{\left(\frac{-t^2}{2}\right)}dt$ Using the alternative representation of above equation as given in [14].

$$Q(Sx) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{\left(\frac{-S^2x^2}{2\sin^2(\phi)}\right)}d\phi$$

(13)

Where S is a factor related to signal to noise ratio (SNR),as given in [22] $S = \sqrt{\frac{2gE_b}{N_0}}$,

where g=1 for coherent binary phase shift keying(BPSK),g=1/2 for coherent orthogonal binary frequency shift keying (BFSK), and g= $\frac{1}{2}+\frac{1}{3\pi}$ for coherent BFSK with minimum correlation. The parameter *x* is the fading amplitude.

The error rate denoted by P(S) can be expressed as:

$$\mathrm{P(S)} = \int_0^{\infty} Q(Sx)f_X(x)dx$$

(14)

$$\mathrm{P(S)} = \int_0^{\infty}\left(\frac{1}{\pi}\int_0^{\pi/2}\left(\exp\left(\frac{-s^2x^2}{2\sin^2(\phi)}\right)\right)d\phi\right)\left(\frac{1}{\pi}\int_0^{\infty}\cos(\eta x)\Psi(\eta)d\eta\right)dx$$

(15)

$$\mathrm{P(S)} = \frac{1}{\pi^2}\int_0^{\infty}\Psi(\eta)d\eta\left(\int_0^{\pi/2}\int_0^{\infty}\cos(\eta x)\exp\left(\frac{-S^2x^2}{2\sin^2(\phi)}\right)dxd\phi\right)$$

(16)

Using the identity [17,3.896.4] eq. (16) can be wrriten as:

$$P(S) = \frac{1}{\pi^2}\int_0^{\infty}\Psi(\eta)d\eta\left(\int_0^{\pi/2}\frac{1}{2}\sqrt{\frac{2\pi\sin^2(\theta)}{S^2}}\exp\left(\frac{-\eta^2\sin^2(\phi)}{2S^2}\right)d\phi\right) \quad (17)$$

After integration and some simplification eq. (20) can be expressed as:

$$\text{P(S)}=\left(\frac{1}{2\pi}\right)\int_0^{\infty}\Psi(\eta)e^{\frac{-\eta^2}{2S^2}}\frac{erfi\left(\frac{\eta}{\sqrt{2S}}\right)}{\eta}d\eta \quad (18)$$

$$\text{P(S)}=\left(\frac{1}{2\pi}\right)\int_0^{\infty}\prod_0^{M-1}{}_1F_1\left(m_n;1;\frac{-\Omega_n}{m_n}\eta^2\right)e^{\frac{-\eta^2}{2S^2}}\frac{erfi\left(\frac{\eta}{\sqrt{2S}}\right)}{\eta}d\eta \quad (19)$$

Where *erfi* is imaginary error function, eq. (19) can be used for numerical computation of BER for OFDM signals transmitted over Nakagami-m fading channels. Further eq. (19) is used to express the BER expression for the single tap and two tap multipath channels. For single tap BER expression can be given as:

$$\text{P(S)}=\left(\frac{1}{2\pi}\right)\int_0^{\infty}{}_1F_1\left(m_0;1;\frac{-\Omega_0}{m_0}\eta^2\right)e^{\frac{-\eta^2}{2S^2}}\frac{erfi\left(\frac{\eta}{\sqrt{2S}}\right)}{\eta}d\eta \quad (20)$$

In the similar manner for the two tap multipath channel we have expression as:

$$\text{P(S)}=\left(\frac{1}{2\pi}\right)\int_0^{\infty}{}_1F_1\left(m_0;1;\frac{-\Omega_0}{m_0}\eta^2\right){}_1F_1\left(m_1;1;\frac{-\Omega_1\eta^2}{m_1}\right)e^{\frac{-\eta^2}{2S^2}}\frac{erfi\left(\frac{\eta}{\sqrt{2S}}\right)}{\eta}d\eta \quad (21)$$

5. Results & Discussions

In this section the BER performance of an OFDM system over Nakagami-m fading channel is analytically evaluated. By varying the fading parameter m obtaining the BER vs SNR is plotted as shown in figure 1 and figure 2 for single tap channel. Figure 3 and figure 4 for two tap channel with m_0=m_1=m and Ω_0= Ω_1 = Ω=1.In figure 1 and figure 3 we consider BFSK modulation technique and figure 2 and figure 4 shows the BPSK-OFDM system.

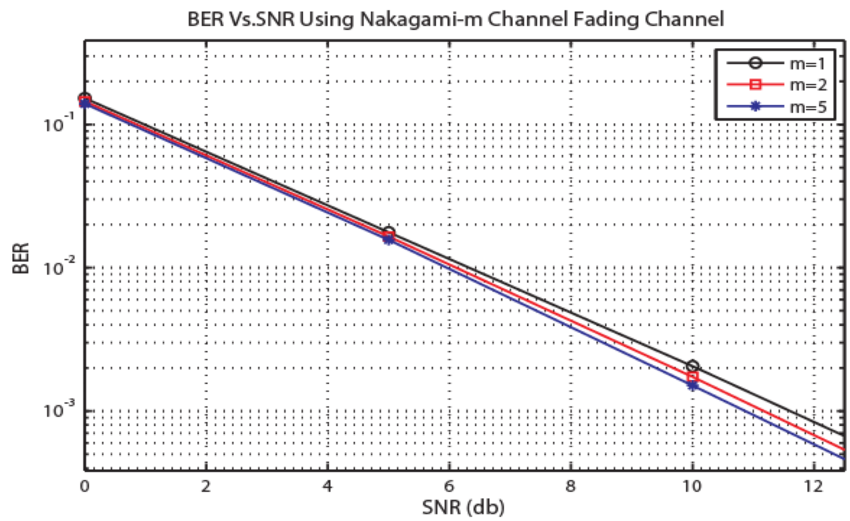

Figure.1 BER Vs SNR for OFDM-BFSK system with single tap

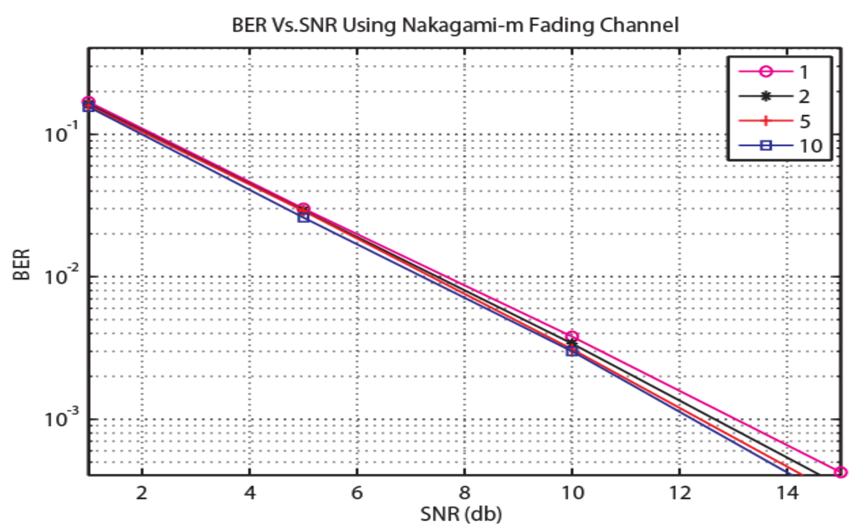

Figure.2 BER Vs SNR for OFDM-BPSK system with single tap

As increasing the fading parameter m, BER starts decreasing as expected for single tap. This fact is already reported in many research papers for Nakagami-m fading channels [10], [11] and [15]. When we consider two tap Nakagami-m channel, slopes of the error rate performance increases if fading parameter m increases. Further, if we increase m, no reduction in BER has been reported rather it starts increasing. So this put a limit to increase the value of m beyond the certain value. This interesting fact is already reported by [22, 23] for the sum of two RVs frequency selective Nakagami-m fading channel and for the frequency non-selective Nakagami-m fading channel respectively. Here we prove this fact for the one RV frequency selective Nakagami-m fading channel and the threshold value for Nakagami-m is achieved to be 1.4 through analytical results.

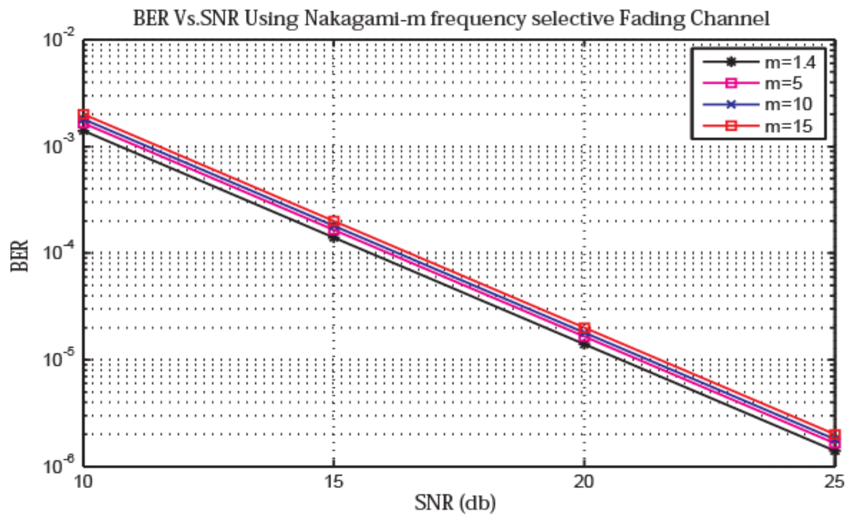

Figure.3 BER Vs SNR for OFDM-BFSK system with two tap

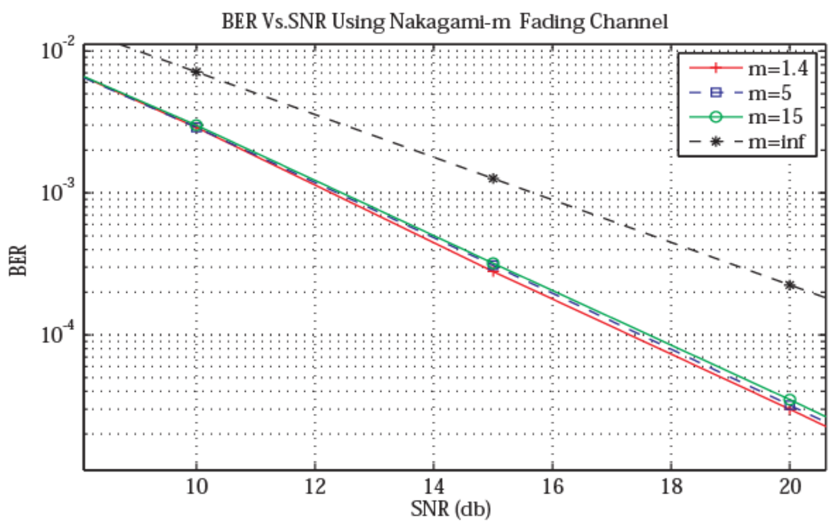

Figure.4 BER Vs SNR for OFDM-BPSK system with two tap

6. Conclusion

In this paper, a closed form integral PDF has been derived using CHF approach and further utilized for evaluating the closed form expression for average error rate performance of OFDM system over Nakagami-m fading channel with the fading severity index. Based on this unified approach, either numerically or analytically, performance of various modulations over multipath fading channels can be evaluated. Finally it has been found that, depending on the number of channel taps, larger Nakagami-m fading parameters do not necessarily give smaller error rates.

References

[1] Orlandos Grigoriadis, H. Srikanth Kamath, "BER Calculation Using Mat lab Simulation for OFDM Transmission", Proc. Of the International multi conference of engineering and computer Scientists, 19-21 March 2008, Hong Kong.

[2] J. Lee, H.L. Lou, D. Toumpakaris, and J. M. Cioffi, "SNR Analysis of OFDM Systems in the Presence of Carrier Frequency Offset for Fading Channels" IEEE Trans. on wireless commun.vol. 5, no. 12, pp.3360-3364, Dec. 2006

[3] Ramjee Prasad, "*OFDM for Wireless Communications Systems*", Artech House 2004.

[4] Popovic, H., Stefanovic, D., Mitic, A., Stefanovic, I., "Some Statistical Characteristics of Nakagami-*m* Distribution", 8th International Conf. on Telecom. In Modern Satellite, Cable and Broadcasting Services , 26-28 Sept. 2007 Serbia.

[5] Marvin K. Simon, Mohamed- Slim Alouini, "*Digital Communication over Fading Channels"*, John Willy & Sons, 2000.

[6] John G. Proakis, "*Digital Communications,*" McGraw-Hill, l995.

[7] H. Suzuki, "A statistical model for urban multipath propagation", IEEE Trans. Commun., vol. COM-25, pp. 673-680, July 1977.

[8] Jin Goog Kim,Tae Joon and Jong Tae Lim, "Channel estimation for OFDM over Fast Rayleigh Fading Channels", Proceedings of world Academy of science and technology, vol. 21, pp. 455-458, Jan. 2007.

[9] Jun Lu, Thiang Tjhung,Fumiyuki Adachi and Cheng Li Huang, "BER performance of OFDM-MDPSK system in Frequency –Selective Rician Fading with Diversity Reception", IEEE Trans. On Vehicular Tech. vol. 49, no. 4, pp. 1216-1225, July 2000.

[10] Yoshiya Miyagaki, N Moinaga, "Error probability Characteristics for CPSK signal in Nakagami fading channel", IEEE Proc. vol. 26,no. 1, pp.88-100 Jan 1978.

[11] M.S. Alouini and A. J. Goldsmith, "A unified approach for calculating error rates of linearly modulated signals over generalized fading channels", IEEE Trans. Commun., vol. 47, no. 9, pp. 1324–1334, Sep. 1998

[12] A. Annamalai, C. Tellambura, and V. Bhargava, "Unified analysis of equal-gain diversity on Rician and Nakagami fading channels," in Proc. Wireless Commun. Networking Conf., 1999, pp.10–14.

[13] Z. Kang, K. Yao, and F. Lorenzelli, "Nakagami-m fading modeling in the frequency domain for OFDM system analysis", *IEEE Commun. Lett.* , vol. 7, no. 10, pp. 484–486, Oct. 2003.

[14] Zheng du,Julian Cheng and Norman C. Beaulieu, "Asymptotic BER performance of OFDM in Frequency Selective Nakagami-m Channels", IEEE Conference on Vehicular Technology, vol. 1, pp. 612-615,Sept.2004.

[15] N.C. Beaulieu and J. Cheng, "Precise error rate analysis of bandwidth efficient BPSK in Nakagami fading and co-channel interference", IEEE Trans. Commun. Vol.52, pp. 149-158, Jan. 2004.

[16] A. A. Abu-Dayya, N.C. Beaulieu, "Micro diversity on Rician Fading Channels", IEEE Trans on Commun., vol. 42, no. 6, June 1994.

[17] I. S. Gradshteyn and I. M. Ryzhik, "*Table of Integrals, Series, and Products*", 6th ed. San Diego, CA: Academic, 2000.

[18] M. Abramowitz and I. A. Stegun, "*Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*", New York: Dover, 1972.

[19] A. Abdi, H. Hashemi, and S. Nader-Esfahani, "On the PDF of the sum of random vectors", *IEEE Trans. Commun.*, vol. 48, no. 1, pp. 7–12, Jan. 2000.

[20] A. Papoulis, "*Probability, Random Variables, and Stochastic Processes"*, 3rd ed. New York: McGraw-Hill, 1991.

[21] Marvin K. Simon, Mohamed- Slim Alouini, "A Unified Approach to the Performance Analysis of Digital Communication over Generalized Fading Channels", Proceedings of IEEE ,vol.86 no.9, pp. 1860-1877, Sep. 1998.

[22] Zheng du,Julian Cheng and Norman C. Beaulieu, "Accurate Error Rate Performance Analysis of OFDM on Frequency Selective Nakagami-m Fading Channels," IEEE Trans. on commun..vol. 54, no. 2, pp.319-328, Feb. 2006.

[23] Neetu Sood, Ajay K Sharma, Moin Uddin, "BER Performance of OFDM-BPSK and -QPSK over Generalized Gamma Fading Channel", *International Journal of Computer Applications,* vol. 3, no.6, pp.13-16, June 2010.

Authors

Mukesh Kumar Mishra received the B.E. degree in Electronics and Tele-communication from Pt.Ravishankar Shukla University, India 2008.Currently, he is pursuing his M.Tech in National Institute of Technology, Jalandhar, India. His current research includes the analytical performance analysis of OFDM based wireless communication system, the simulation of wireless systems based on OFDM and simulation of fading channels.

Neetu Sood received the B.Tech. degree (with Honour) in Electronics and Communication from Sant Longowal Institute of Engineering and Technology, India 2000. In year Jan. 2002, she completed her M.E.Degree (With Honour) in Electronics and Communication from Thapar University, Patiala, India. From year 2002 to 2007, she worked with DAV institute of Engineering and Technology, Jalandhar and Guru Nanak Dev University (Regional Campus), Jalandhar.Currently, she is working as Assistant Professor at Dr. B R Ambedkar National Institute of Technology, Jalanadhar and working towards her PhD degree. Her current research includes the simulation of wireless systems based on OFDM and simulation of fading channels

Ajay K Sharma received his BE in Electronics and Electrical Communication Engineering from Punjab University Chandigarh, India in 1986, MS in Electronics and Control from Birla Institute of Technology (BITS), Pilani in the year 1994 and PhD in Electronics Communication and Computer Engineering in the year 1999. From 1986 to 1995 he worked with TTTI, DTE Chandigarh, Indian Railways New Delhi, SLIET Longowal and National Institute of technology (Erstwhile Regional Engineering College), Hamirpur HP at various academic and administrative positions. He has joined National Institute of Technology (Erstwhile Regional Engineering College) Jalandhar as Assistant Professor in the Department of Electronics and Communication Engineering in the year 1996. From November 2001, he has worked as Professor in the same department and presently him working as Professor in Computer Science & Engineering in the same institute. His major areas of interest are broadband optical communication systems, dispersion compensation, fiber nonlinearities, optical soliton transmission, WDM systems and networks, Radio-over-Fiber (RoF) and wireless communication and networks. He has published 202 research papers in the International/National Journals/Conferences and 12 books. He has completed two R&D projects funded by Government of India and one project is ongoing. Presently he is associated to implement the World Bank project of 209 Million For Technical Education Quality Improvement programme of the institute. He is technical reviewer of reputed international journals like: Optical Engineering, Optics letters, Optics Communication, Digital Signal Processing. He has been appointed as member of technical Committee on Telecom under International Association of Science and Technology Development (IASTD) Canada for the term 2004-2007 and he is Life member of Indian Society for Technical Education (I.S.T.E.), New Delhi.