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Analysis and Modelling of Static Synchronous Compensator (STATCOM): A comparison of Power Injection and Current Injection Models in Power Flow Study

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Abstract

This paper presents the mathematical steady-state modelling of Static Synchronous Compensator (STATCOM), which is the most widely used member of Flexible Alternating Current Transmission Systems (FACTS). STATCOM Power Injection Model (PIM), derived from one voltage source representation, is presented and analyzed in detailed. A program, Flexible Alternating Current Transmission Systems Power Flow (FACTSPF) in MATLAB has been developed which extends conventional Newton-Raphson (NR) algorithm based on the Power Injection Model (PIM).The STATCOM PIM and Current Injection Model (CIM) implemented in Power System Analysis Toolbox (PSAT) are incorporated in a 5-bus system. The results obtained from simulation of the 5-bus using FACTSPF are matched with those of PSAT in acceptable tolerance and thus confirms the robustness of the PIM. The PIM model represents a robust and feasible alternative when compared with Current Injection Model (CIM) implemented in Power System Analysis Toolbox (PSAT).

Keywords: *FACTS; STATCOM; NR; PIM; CIM; FACTSPF; PSAT*

1. Introduction

Flexible AC transmission system (FACTS) controllers are power electronics based controllers. With the applications of FACTS technology, bus voltage magnitude and power flow along the transmission lines can be more flexibly controlled [1],[2],[3]. Among the FACTS controllers, the most advanced type is the controller that employs Voltage Sourced Converter (VSC) as synchronous sources [1]. Representative of the VSC type FACTS controllers are the Static Synchronous Compensator (STATCOM), which is a shunt type controller, the Static Series Compensator (SSSC), which is a series type controller and the Unified Power Flow Controller (UPFC), a combined series-shunt type controller [4]. Of all the VSC the most widely used is the STATCOM. It can provide bus voltage magnitude control. Computation and control of power flow for power systems embedded with STATCOM appear to be fundamental for power system analysis and planning purposes. Power flow studies incorporating STATCOM requires accurate model in solution algorithms.

There are mainly two models of STATCOM which have well tested in power systems. There are the Current Injection Model (CIM) and the Power Injection Model (PIM) [5],[6],[7],[8],[9]. The CIM STATCOM has a current source connected in shunt the bus for voltage magnitude control. The PIM models the STATCOM as shunt voltage source behind an equivalent reactance or impedance, which is also referred to as voltage source model (VSM) [10]. This steady state power injection model of STATCOM has proved reliable when incorporated in power systems and is well documented [11],[12]. The use of this STATCOM in power system simulators has therefore increased over the last one decade and is therefore adopted implementation in this work with the voltage expressed in rectangular coordinate.

2. Static Synchronous Compensator (STATCOM) Power Flow Model

The STATCOM is a FACTS controller based on voltage sourced converter (VSC). A VSC generate a synchronous voltage of fundamental frequency, controllable magnitude and phase angle. If a VSC is shunt-connected to a system via a coupling transformer as shown in Fig. 1, the resulting STATCOM can inject or absorb reactive power to or from the bus to which it is connected and thus regulate the bus voltage magnitude [4]. This STATCOM model is known as Power Injection Model (PIM) or Voltage Source Model (VSM). Steady state modelling of STATCOM within the Newton-Raphson method in rectangular co-ordinates is carried out as follows:

The Thevenin equivalent circuit representing the fundamental frequency operation of the switched-mode voltage sourced converter and its transformer is shown in Figure 1.

$$V_{STC} = V_k + Z_{SC} I_{STC} \tag{1}$$

is expressed in Norton equivalent form

$$I_{STC} = I_N - Y_{SC} V_k \tag{2}$$

where

$$I_N = Y_{SC} V_{STC}$$

In these expressions, V_k represents bus k voltage and V_{STC} represents the voltage source inverter. I_N is the Norton's current while I_{STC} is the inverter's current. Also, Z_{SC} and Y_{SC} are the transformer's impedance and short-circuit admittance respectively.

The STATCOM voltage injection V_{STC} bound constraints is as follows:

$$V_{STC\ \min} \le V_{STC} \le V_{STC\ \max} \tag{3}$$

Where $V_{STC\ \min}$ and $V_{STC\ \max}$ are the STATCOM's minimum and maximum voltages.

The current expression in (2) is transformed into a power expression by the VSC and power injected into bus k as shown in equations (4) and (5) respectively.

$$S_{STC} = V_{STC} I_{STC}^* = V_{STC}^2 Y_{SC}^* - V_{STC} Y_{SC}^* V_k^* \tag{4}$$

$$S_k = V_k I_{STC}^* = V_{STC} Y_{SC}^* V_k^* - V_k^2 Y_{SC}^* \tag{5}$$

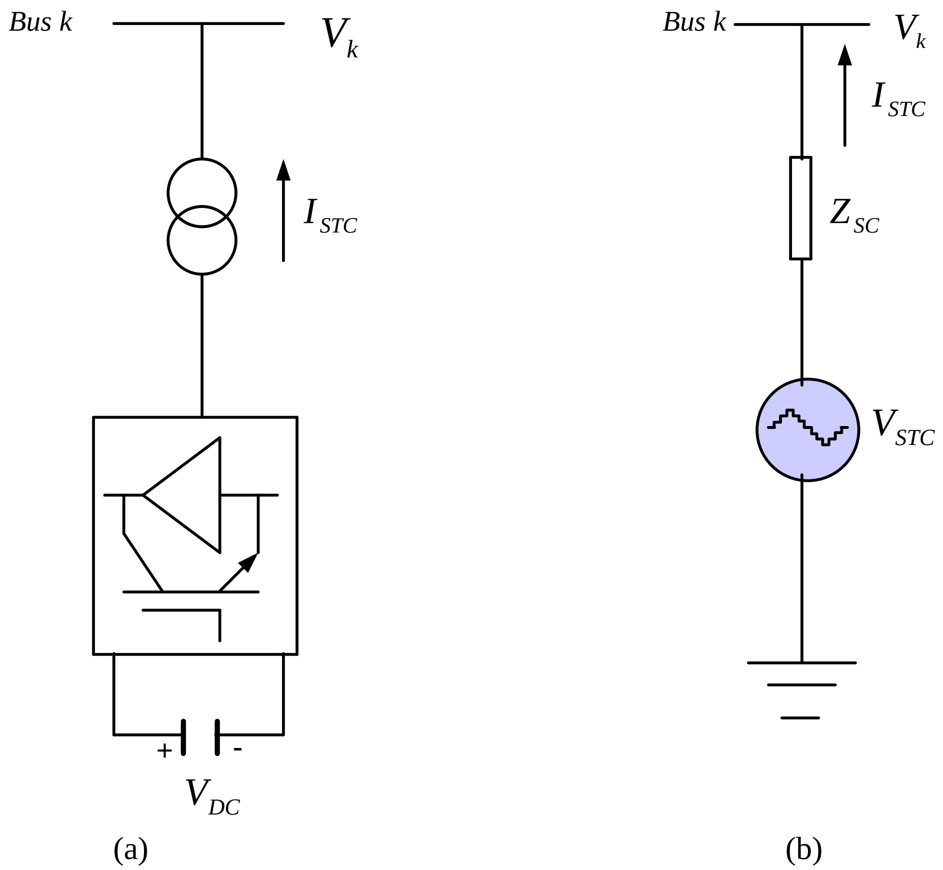

Figure 1: Thevenin Equivalent Circuit Diagram of STATCOM: (a) STATCOM Schematic Diagram; (b) STATCOM Equivalent Circuit

Using the rectangular coordinate representation,

$$V_k = e_k + jf_k$$

$$V_{STC} = e_{STC} + jf_{STC}$$

$$|V_{STC}| = \left(e_{STC}^2 + f_{STC}^2\right)^{\frac{1}{2}}$$

$$\delta_{STC} = \tan^{-1}\left(\frac{f_{STC}}{e_{STC}}\right)$$

Where $|V_{STC}|$ and δ_{STC} are the STATCOM voltage magnitude and angle respectively
e_k and f_k are the real and imaginary parts of the bus voltage respectively.
e_{STC} and f_{STC} are the real and imaginary parts of the STATCOM voltage respectively

The active and reactive powers for the STATCOM and node k respectively are:

$$P_{STC} = G_{SC}\left\{\left(e_{STC}^2 + f_{STC}^2\right) - \left(e_{STC}e_k + f_{STC}f_k\right)\right\} + B_{SC}\left(e_{STC}f_k - f_{STC}e_k\right) \tag{6}$$

$$Q_{STC} = G_{SC}\left(e_{STC}e_k - f_{STC}e_k\right) + B_{SC}\left(-e_{STC}^2 - f_{STC}^2 + e_{STC}e_k + f_{STC}f_k\right) \tag{7}$$

and

$$P_k = G_{SC}\left\{e_k^2 + f_k^2 - \left(e_k e_{STC} + f_k f_{STC}\right)\right\} + B_{SC}\left(e_k f_{STC} - f_k e_{STC}\right) \tag{8}$$

$$Q_k = G_{SC}\left(e_k f_{STC} - f_k e_{STC}\right) + B_{SC}\left\{\left(e_k f_{STC} + f_k e_{STC}\right) - \left(e_k^2 + f_k^2\right)\right\} \tag{9}$$

2.1. Linearised Power Equations

A single-phase power network with n-buses is described by $2\times(n-1)$ non-linear equations. The inclusion of one STATCOM model augments the number of equations by two. The solution of the combined system of non-linear equations is carried out by iteration using the full Newton-Raphson method.

The Jacobian used in conventional power flow is suitably extended to take account of the new elements contributed by the STATCOM. The set of linearised power flow equations for the complete system is

$$\begin{bmatrix} \Delta P_k \\ \Delta |V_k|^2 \\ \Delta P_{STC} \\ \Delta Q_{STC} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_k}{\partial e_k} & \frac{\partial P_k}{\partial f_k} & \frac{\partial P_k}{\partial e_{STC}} & \frac{\partial P_k}{\partial f_{STC}} \\ \frac{\partial |V_k|^2}{\partial e_k} & \frac{\partial |V_k|^2}{\partial f_k} & 0 & 0 \\ \frac{\partial P_{STC}}{\partial e_k} & \frac{\partial P_{STC}}{\partial f_k} & \frac{\partial P_{STC}}{\partial e_{STC}} & \frac{\partial P_{STC}}{\partial f_{STC}} \\ \frac{\partial Q_{STC}}{\partial e_k} & \frac{\partial Q_{STC}}{\partial f_k} & \frac{\partial Q_{STC}}{\partial e_{STC}} & \frac{\partial Q_{STC}}{\partial f_{STC}} \end{bmatrix} \begin{bmatrix} \Delta e_k \\ \Delta f_k \\ \Delta e_{STC} \\ \Delta f_{STC} \end{bmatrix} \tag{10}$$

The Jacobian elements in equation (10) are given in Appendix A

3. Implementation

A MATLAB based program was developed for the power flow analysis of electrical power systems without and with steady-state model of the FACTS controller, STATCOM. The program is referred to as “Flexible Alternating Current Transmission System Power Flow” (FACTSPF).

The procedure for power flow solution by the Newton-Raphson method without and with FACTS controllers is shown in flowchart of Figure 2. The input data includes the basic system data needed for conventional power flow calculation, i.e., the number and types of buses, transmission line data, generation and load data and the values of STATCOM control parameters. System admittance matrix and conventional Jacobian matrix is formed due to incoming of STATCOM. At the next step, Jacobian matrix and the mismatched power flow equations are modified. The bus voltages are updated at each iteration. Convergence is checked and if no, Jacobian matrix is modified and power equations are mismatched until convergence is achieved. If yes, power flow results are displayed.

Power Analysis Toolbox (PSAT) is a MATLAB toolbox for static, dynamic analysis and control of electric power systems. PSAT includes power flow; continuation power flow; optimal power flow, small signal stability analysis and time domain simulation [12],[13]. It has steady-state and dynamic models of three voltage sourced converter FACTS controllers, namely STATCOM, SSSC, HVDC and UPFC. The STATCOM model implemented in PSAT is a Current Injection Model (CIM) which is fully developed. In order to validate the results of FACTSPF, simulation of power system incorporating STATCOM were carried out using the two packages, PSAT and FACTSPF. The results when the two packages were applied to 5-bus system are subsequently presented.

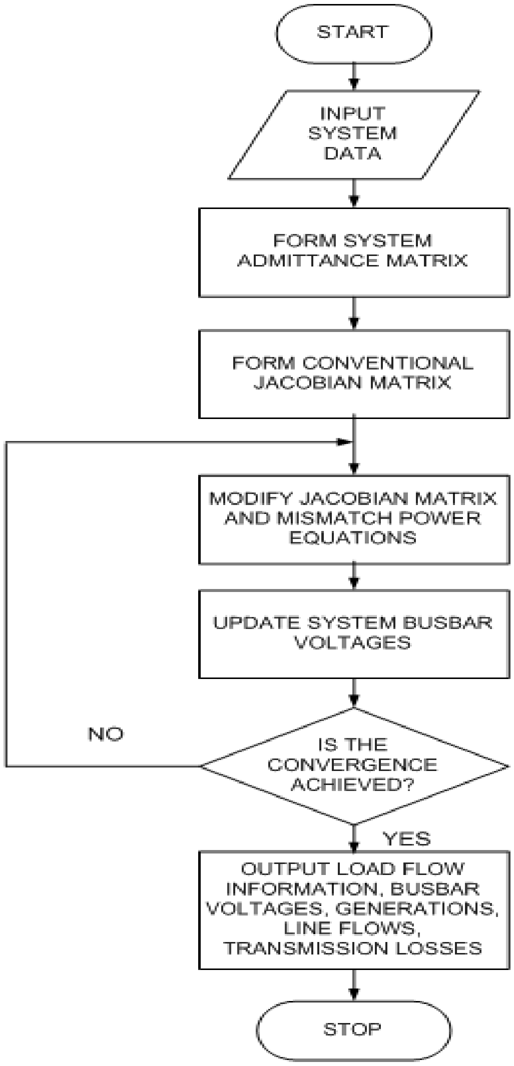

Figure 2: Flowchart for Power Flow Solution by Newton-Raphson with STATCOM Controller

3.1. Power Flow Analysis of Power Systems Incorporating of FACTS Controllers

In order to investigate the performance of the PIM of STATCOM, the CIM and PIM STATCOM were embedded in a standard 5-bus system. The test system is shown in Figure 3. The 5-bus power system data as well as the STATCOM data are given in the appendix.

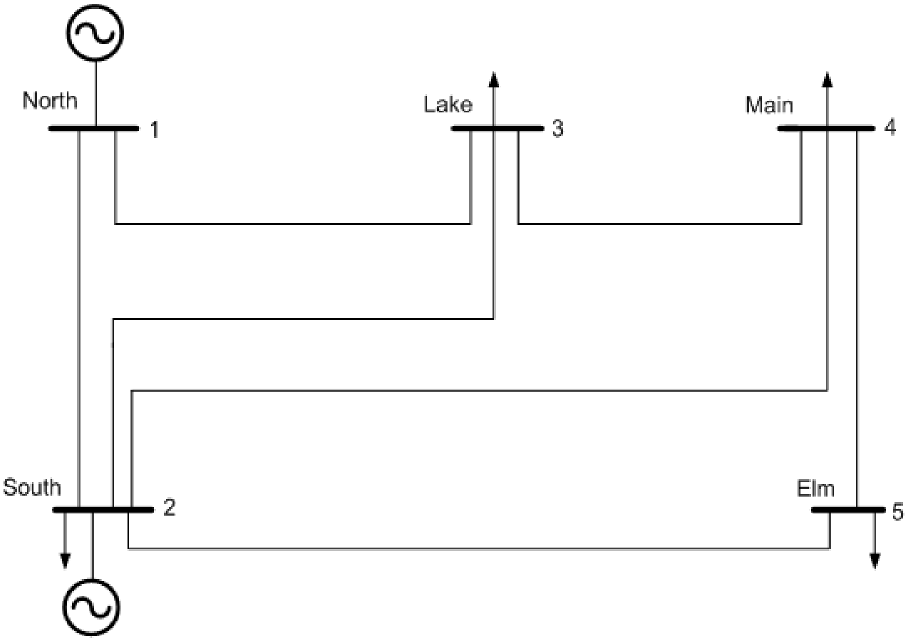

Figure 3: 5-bus Test System Source (Acha et al, 2004)

The STATCOM model (PIM) was installed in the 5-bus system for voltage magnitude control. The 5-bus was also simulated using PSAT and its STATCOM current injection model used for bus voltage control. The power flow analysis of FACTSPF and that of PSAT were then compared.

From the power flow results for the 5-bus system (Table 1), it can be observed that the voltage magnitudes at bus Lake, bus Main and bus Elm are lower than 1.0p.u. and are therefore potential buses for the application of STATCOM. The five-bus network was modified to include one STATCOM connected at Lake, to maintain the bus voltage magnitude at 1 p.u. (Figure 4a). The 5-bus system implementation in PSAT is shown in Figure 4b.The resulting power flow solution is shown in Table 2a, and it indicates an improvement in the voltage profile of the system with Lake Voltage regulated at 1.0p.u. Note that the STATCOM injected reactive power of 20.48Mvar at bus Lake while the STATCOM voltage magnitude and phase angle were maintained at $1.0205 p.u.$ and -4.83^0 respectively. The installation of the STATCOM resulted in improved network voltage profile.

The slack generator reduces its reactive power generation by 5.9% compared with the base case, and the reactive power flow from North to lake reduces by more than 32%. The reactive power absorbed by the south generator increased by 25% of the base case. In general, more reactive power is available in the network when compared with the base case due to the installation of STATCOM. As expected the active power flows were slightly affected. The system active power loss reduces to 6.06MW.

PSAT was also used to simulate the 5-bus system with STATCOM installed to control Lake bus voltage magnitude at 1.00p.u. The power flow results for the PIM model and CIM STATCOM are similar; the only difference can be seen in Lake Voltage angle with the VSM model being 4.83^0 while that of CIM is 4.84^0. The difference can be attributed to the computation errors which are different for each program.

The parameters of the STATCOM models are shown in Table 2b. In order to control the Lake bus voltage magnitude at 1.00 p.u., the VSM model injected a reactive power

20.47Mvar with voltage magnitude of 1.0205 p.u. and phase angle 4.83^0. For the CIM STATCOM, it injected a current of 0.2047p.u.

The power flow and the system loss for the PIM and CIM STATCOM are essentially the same to four significant figures. Two other scenarios were simulated using the two models to control voltage magnitude at bus 4 (Main) and bus 5 (Elm). The power flow analyses carried out produced similar output results. The two programs converged quadratically in five iterations to maximum absolute power mismatch of 1E-012 per unit as shown in Figure 5. Shown in Table 3 are the Power flow computation times for the two programs. It can be observed from Table 3 that FACTSPF completes the power flow computation in lesser time when compared to that of PSAT. It has been shown that the developed STATCOM model (PIM) is very effective in the control of bus voltage magnitude of a vulnerable bus.

Table 1: Power flow results of 5-bus system

Bus No	Bus Type	FACTSPF Bus Voltage		PSAT Bus Voltage	
		Magnitude (p.u.)	Phase angle (deg.)	Magnitude (p.u.)	Phase angle (deg.)
1	Swing	1.060	0.00	1.060	0.00
2	PV	1.000	-2.06	1.000	-2.06
3	PQ	0.987	-4.64	0.987	-4.64
4	PQ	0.984	-4.96	0.984	-4.96
5	PQ	0.972	-5.77	0.972	-5.77

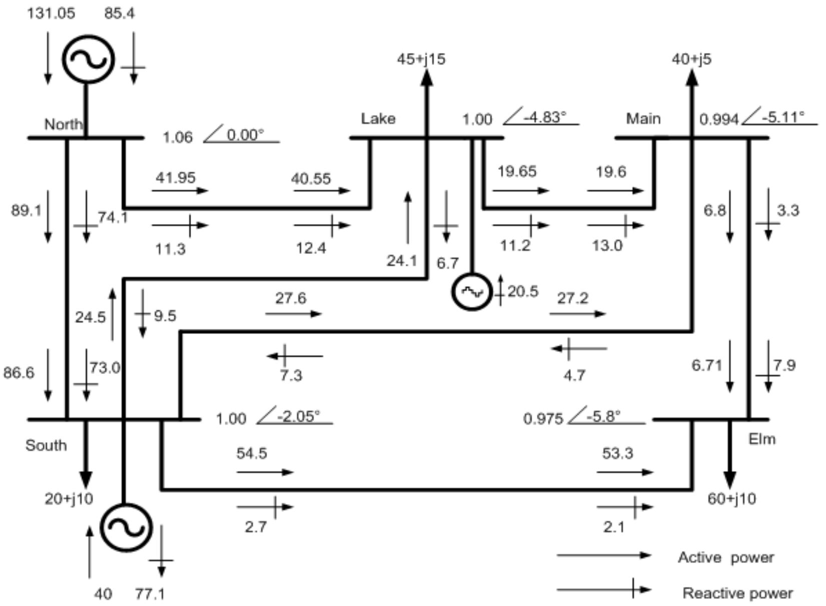

Figure 4a: Power Flow Results of 5-bus with STATCOM Installed at Bus 3

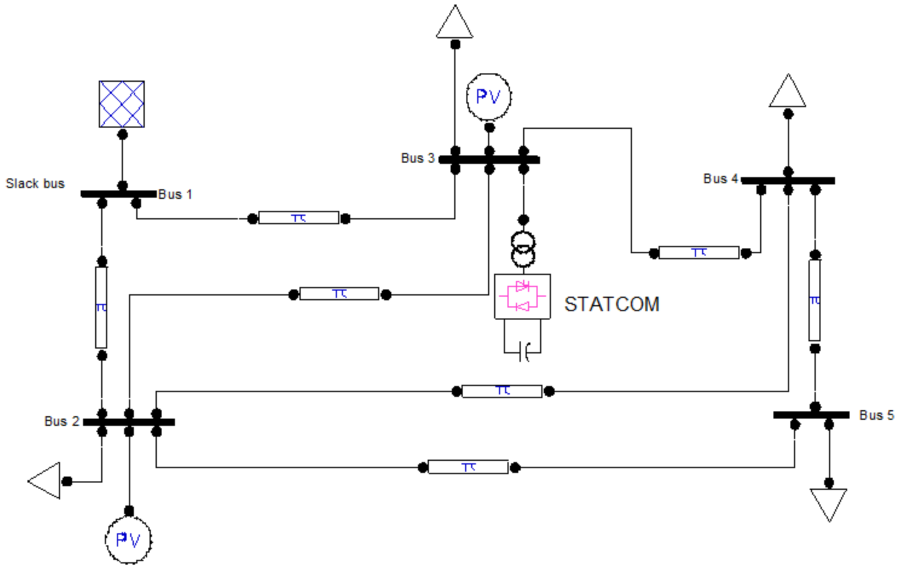

Figure 4(b): STATCOM-upgraded 5-bus system in PSAT

Table 2(a): Power Flow Results of 5-bus System without and with STATCOM

Bus No	Bus Type	Base case, FACTSPF (without STATCOM) Nodal Voltage		FACTSPF (with STATCOM) Nodal Voltage		PSAT(with STATCOM) Nodal Voltage	
		Magnitude (p.u.)	Phase angle (deg.)	Magnitude (p.u.)	Phase angle (deg.)	Magnitude (p.u.)	Phase angle (deg.)
1	Swing	1.060	0.00	1.060	0.00	1.060	0.00
2	PV	1.000	-2.06	1.000	-2.05	1.000	-2.05
3	**PQ**	**0.987**	**-4.64**	**1.000**	**-4.83**	**1.000**	**-4.84**
4	PQ	0.984	-4.96	0.994	-5.11	0.994	-5.11
5	PQ	0.972	-5.77	0.975	-5.80	0.975	-5.80

Table 2(b): Parameters of STATCOM

Bus No	FACTSPF, VSM STATCOM model				PSAT, CIM model		
	STATCOM Voltage Magnitude (p.u.)	STATCOM Voltage Phase angle (Deg.)	STATCOM Power Active (MW)	STATCOM Power Reactive (Mvar)	STATCOM injected current (p.u.)	STATCOM Power Active (MW)	STATCOM Power Reactive (Mvar)
3	1.0205	-4.64	0.00	-20.47	0.2047	0.00	-20.47

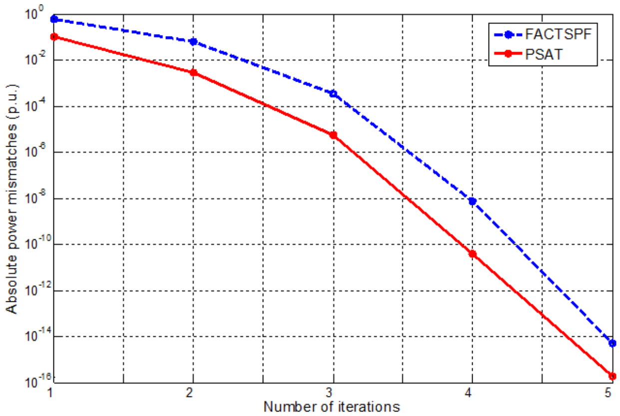

Figure 5: Absolute Power Mismatches as Function of Number of Iterations for PSAT and FACTSPF

Table 3: Power Flow computation time for FACTSPF and PSAT

Program	Computation time (sec.)
FACTSPF	0.359
PSAT	0.406

4. Conclusion

In this paper the PIM of STATCOM has been presented with the voltage expressed in rectangular form. A MATLAB based power flow program developed was extended to incorporate the STATCOM and named Flexible Alternating Current Transmission System Power Flow (FACTSPF). 5-bus power system with the incorporation of the PIM and CIM were simulated using the FACTSPF and PSAT respectively. The STATCOM was able to effectively regulate the bus voltage magnitude at which was connected. The results obtained by FACTSPF are matched with those of PSAT in acceptable tolerance and thus confirms the robustness of the PIM.

The PIM of STATCOM is effective and reliable in terms of computation speed and accuracy. It is a reliable substitute for CIM.

References

[1] Gyugyi L, Schauer C.D., Williams S.L., Rietman T.R., Torgerson D.R., Edris A., (1995), “The Unified Power Flow Controller: A New Approach to Power Transmission Control”, IEEE Transactions on Power Delivery, vol. 2, pp. 1085-1097.

[2] Edris A, Mehraban A.S., Rahman M, Gyugyi L, Arabi S, Reitman T., (1998),“Controlling The Flow of real and reactive power”, IEEE Computer Applications in power, 20-5.

[3] Haque M.H., Yam C.M., (2003), “A simple method of solving the controlled load flow problem of a power system in the presence of UPFC”, Electric Power Systems Research 65(1), pp. 55-62.

[4] Hingorani N.G. and Gyugyi, L. (2000), “Understanding FACTS”, The Institute of Electrical and Electronics Engineers, New York.

[5] Sen, K.K., (1999), “STATCOM-static synchronous compensator theory, modelling and applications”, IEEE PES Winter Meeting 2, pp.1177-1183.

[6] Chun, L., Qirong, J., Xiaorong, X. and Zhonghong, W. (1998), “Rule-based control for STATCOM to increase power system stability, Power System Technology”, Proceedings 1998 International Conference on POWERCON, pp. 372–376.

[7] Rahim, A. H. M. A., Al-Baiyat, S. A. and Al-Maghrabi, H. M.: 2002, Robust damping controller design for a static compensator, IEE Proceedings on Generation, Transmission and Distribution 149, 491–496.
[8] Haque, M. H.: (2004), “Improvement of first swing stability limit by utilizing full benefit of shunt FACTS devices”, IEEE Transactions on Power Systems 19(4), 1894–1902.
[9] Acha, E., Fuerte-Esquivel, C.R., Ambriz-Perez, H. and Angeles-Camacho, C. (2004), “*FACTS Modelling and Simulation in Power Networks*”, John Wiley & Sons, Chichester.
[10] Wei, X., Chow, J.H., Fardanesh, B. and Edris, A.A. (2004), “A Common modelling framework of voltage sourced converters for power flow, sensitivity, and dispatch analysis”, *IEEE Transactions Power on System*, vol.19, pp. 934-941.
[11] Zhang, X. –P., Rehtanz, C. and Pal, B. (2006) “*Flexible AC Transmission Systems: Modelling and Control*”, Springer-Verlag Berlin Heidelberg, Germany.
[12] Milano, F. (2005), “An Open Source Power System Analysis Toolbox, *IEEE Transactions on Power Systems*”, vol.20, no.3, pp.1199–1206.
[13] Milano F. (2008), “Power System Analysis Toolbox”, http://thunderbox.uwaterloo.ca/~fmilano.

APPENDIX A

Partial derivatives for the static compensator (STATCOM) model are:

$$\frac{\partial P_k}{\partial e_k} = G_{sc}\left(2e_k - e_{STC}\right) + B_{sc} f_{STC}$$

$$\frac{\partial P_k}{\partial f_k} = G_{sc}\left(2f_k - f_{STC}\right) - B_{sc} e_{STC}$$

$$\frac{\partial V_k}{\partial e_k} = \frac{e_l}{\sqrt{e_k^2 + f_k^2}}$$

$$\frac{\partial V_k}{\partial f_k} = \frac{f_k}{\sqrt{e_k^2 + f_k^2}}$$

$$\frac{\partial P_k}{\partial e_{STC}} = -G_{sc} e_k - B_{sc} f_k$$

$$\frac{\partial P_l}{\partial f_{STC}} = -G_{sc} f_l + B_{sc} e_l$$

$$\frac{\partial P_{STC}}{\partial e_l} = -G_{sc} e_{STC} - B_{sc} f_{STC}$$

$$\frac{\partial P_{STC}}{\partial F_l} = -G_{sc} F_{STC} + B_{sc} E_{STC}$$

$$\frac{\partial P_{STC}}{\partial e_{STC}} = G_{sc}\left(2e_{STC} - e_l\right) + B_{sc} f_l$$

$$\frac{\partial P_{STC}}{\partial F_{STC}} = G_{sc}\left(2F_{STC} - F_l\right) - B_{sc} E_l$$

$$\frac{\partial Q_{STC}}{\partial e_l} = -G_{sc} f_{STC} + B_{sc} e_{STC}$$

$$\frac{\partial Q_{STC}}{\partial f_l} = G_{sc} e_{STC} + B_{sc} f_{STC}$$

$$\frac{\partial Q_{STC}}{\partial e_{STC}} = G_{sc} f_l - B_{sc}\left(2e_{STC} - e_l\right)$$

$$\frac{\partial Q_{STC}}{\partial f_{STC}} = -G_{sc} e_l - B_{sc}\left(2f_{STC} - f_l\right)$$

APPENDIX B

Bus Data and Transmission line Data for power systems

B.1 Bus Data For 5-Bus System (Base: 100MVA, 330 kV, f = 50 Hz)

Bus No	Bus Type	Voltage Magnitude (P.U)	Voltage Phase Angle (Deg.)	P_g Generated (P.U)	Q_g Generated (P.U)	P_L Load (P.U)	Q_L Load (P.U)
1	Swing	1.60	0.00	0.00	0.00	0.00	0.00
2	PV	1.00	0.00	0.40	0.00	0.20	0.10
3	PQ	1.00	0.00	0.00	0.00	0.45	0.15
4	PQ	1.00	0.00	0.00	0.00	0.40	0.05
5	PQ	1.00	0.00	0.00	0.00	0.60	0.10

(Source:Acha et al, 2004)

B.2 Transmission Line Data For 5-Bus System (Base: 100MVA, 330 kV, f = 50 Hz)

S/N	BUS FROM	BUS TO	Resistance R (p.u.)	Reactance X (p.u.)	Susceptance B (p.u.)	Remark
01	1	2	0.0200	0.0600	0.0600	1 Line
02	1	3	0.0800	0.2400	0.0500	1 Line
03	2	3	0.0600	0.1800	0.0400	1 Line
04	2	4	0.0600	0.1800	0.0400	1 Line
05	2	5	0.0400	0.1200	0.0300	1 Line
06	3	4	0.0100	0.0300	0.0200	1 Line
07	4	5	0.0800	0.2400	0.0500	1 Line

(Source: Acha et al, 2004)

APPENDIX C

Table C STATCOM static data

R_{STC} (p.u.)	X_{STC} (p.u.)	$Q_{STC(\max)}$ (p.u.)	$Q_{STC(\min)}$ (p.u.)
0.0	0.10	-1.00	1.00

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