Economic Production Lot Size Model with Stochastic Demand and Shortage Partial Backlogging Rate under Imperfect Quality Items

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Abstract

In this paper we presents the EPLS (Economic Production Lot Size) model which accounts for a production system producing perfect and imperfect quality items. Also, a single period multi-item volume flexible production model for deteriorating items with stochastic demand and stochastic imperfect production. Deterioration is taken as constant. Linear holding cost is considered. Shortages are permitted in inventory with partial backlogging. Profit maximization techniques are also used. The problem parameter effects upon the optimal solutions are examined numerically.

Keyword:*Imperfect Quality Items, Volume Flexible, Partial Backlogging, Linear Holding Cost, Deterioration.*

1. Introduction

An Economic Production Quantity models deals with an inventory production system in which procurement of inventory occurs through production cycle within the cycle itself. Several researchers have been considered, production rate is taken as constant and demand dependent i.e. the rate of production is assumed to be predetermined and inflexible. But machine production rate can also easily be changed according to Schweitzer and Seidmann (1991); they also assumed that unit production cost is taken as a function of the production rate. In a real production environment, all the produced items can't of good quality. Some items will also defective in the produced quantities. Defective items as a result of imperfect production process. In an imperfect manufacturing process, a certain proportions of products become defective due to poor production quality and material defects and subsequently defective products are scrapped if they are not rework able or it is not cost effective to do so. In a multi stage production system, products move from one stage to next stage and every stage may yield a certain proportion of defectives items. This proportion of defectives may vary from stage to stage and from cycle to cycle. The non-reworked items become waste, creating additional costs for producers and the environmental in general. Goyal (1978) developed the effect of increased in-process inventory of economic batch production quantity model in multi-stage system, but they ignored defective items. Kumar and Vrat (1979) studied a model for optimal inventory of finished goods in multi-stage production system. Gupta and Chakraborty (1984) introduced the rework option of rejected items and recycling from last stage to first stage and obtained an economic batch quantity model. They did not consider the cost incurred due to shortage caused by rejection in various stages. Tay and Ballou (1988) considered an integrated production inventory model with reprocessing and inspection. Zhang and Gerchak (1990) determined a joint lot sizing and inspection policy under an EOQ model, where a random proportion of units are defective. They considered a model, where the defective units can't be used and thus must be replaced by non-defective ones. They found that a considerable deviation from the optimal quantity will generally result in only a small increase in objective function value. Agnihotri and Kenett (1995) worked on models considering defective items and studied the impact of defects an a process with rework. Lee et.al. (1997) considered a model for a multi-stage production system considering the various proportions of defective items produced in every stage. But they did not consider the rework option of defective items. Kim, C.H and Hong, Y.S. (1999) developed an optimal production run length in deteriorating production process. Duri, C. et al. (2000) studied the performance evaluation and design of a CONWIP system with inspections. Salameh and Jaber (2000) considered a model to determine the total profit per unit of time and the economic lot size for a product from a supplier. Each lot of the produced delivered by the supplier contains defective items with a known probability density function (p.d.f). The purchaser performs a 100% screening process immediately on receiving a lot. Items of poor quality detected in the screening process of a lot are sold at a discounted price at the end of the screening process of a lot. Abad,P.L. (2001) discussed a pricing and lot sizing problem for a product with a variable rate of deterioration. Goyal, S.K. et al. (2002) presented a practical approach for determining the optimal lot size. They assumed that poor items are withdrawn from stock and no shortage was allowed. Flapper et al. (2002) developed a logistic plan and control of the rework process and identified the rework characteristics in a process industry based on a frame work provided by Flapper and Jensen (2004). Ougang, Chen and Chang (2002) investigate the lot size, reorder point inventory model involving variable lead time with partial backorders, where the production process is imperfect. In this model, the options of investing in process quality improvement and setup cost reduction are included and lead time can be shortened at an extra crashing cost. The objective of that model is to simultaneously optimize the lot size, the reorder point, the process quality, the setup cost and the lead time. Chiu, Y.P. (2003) showed the effects of the reworking of defective items on the EPQ model with backlogging allowed. In this study, a random defective rate is considered, and when regular production ends, the reworking of defective items starts immediately. Not all of the defective items are reworked, a portion of them are scrap and are discarded. He derived optimal lot size that minimizes the overall costs for the imperfect quality EPQ model where backorders are permitted. Jaber,M.Y. et al. (2003) studied lot sizing with learning and forgetting in set-ups and in product quality. Goyal, Huang and Chen (2003) considered a simple approach for determining an optimal integrated vendor buyer inventory policy for an item with imperfect quality. Ouyang, Wu and Ho (2003) investigate the integrated vendor-buyer inventory problem and an arrival order lot may contain some defective items and the defective rate is a random variable. They derive an integrated mixture inventory model with back orders and lost sales in which the order quantity, reorder point, lead time and the number of shipment from vendor to buyer are decision variables. They first assume that the lead time demand follows a normal distribution and then relax the assumption about the form of the distribution function of the lead time demand and apply the minimax distribution free procedure to solve the problem. Stewart, D., Cheraghi, S.H. and Malzahn, D. (2004) showed the fuzzy defect avoidance system (FDAS) for product defect control. Chang (2004) proposed the inventory problem for items received with imperfect quality. Razaei, J. (2005) developed Economic Order Quantity (EOQ) model with imperfect quality items and good quality item have a selling price for per unit and defective items are sold as a single batch at a discounted price. Each lot received contains percentage defectives with a known probability density function. They also assumed that a 100% screening process of the lot is conducted. Shortages are allowed. Alfares, H.K. (2007) considered the inventory policy for an item with a stock-level dependent demand rate and a storage time dependent holding cost. The holding cost per unit of the item per unit time is assumed to be an increasing function of the time spent in storage. Wee, H.M. et al. (2007) studied an optimal inventory model for items with imperfect quality and shortage backordering. Each lot received or produced contains some percentage of defectives with a known probability density function. Sarker, B.R. et al. (2008) presents an inventory models for the optimum batch quantity in a multi-stage with rework process for different operational policies. Policy one deals with the rework within the same cycle with no shortage and policy two deals with the rework done after N cycles incurring shortages in each cycle.

In this paper, a single period multi-item volume flexible production model for deteriorating items has been derived with stochastic demand and stochastic imperfect production. Deterioration is taken as constant. Linear holding costs are considered. Shortages are permitted in inventory with partial backlogging. Profit maximization techniques are also used in this study. The model is illustrated through numerical examples.

2. Fundamental Assumptions And Notations

Assumptions

i). This is a single period inventory model.
ii). The inventory system is an imperfect production system and involves multiple items.
iii). The unit production cost is a function of the rate of production.
iv). The rate of production is considered to be a decision variable.
v). Percentage of imperfections is stochastic.
vi). Total demand over the period of cycle is stochastic and uniform over time.
vii). Shortages are permitted and partially backlogged.
viii). Deterioration rate is constant.
ix). Holding cost is a linear increasing function of time.
x). Screening costs for all items are same.

Notations: The inventory system involves n items and for i^{th} items (i =1, 2…n) are used:

1) A_i's, B_i's and R_i's are constants in the density function $f_i(x_i)$ is probability density function of the demand x_i $(0 < x_i < \infty)$.

$$f_i(x_i) = A_i + B_i x_i \ , \ 0 \le x_i \le R_i .$$
$$= 0 \qquad , elsewhere$$

2) Consider $g_i(e_i)$ is probability density function for the rate of defective units' e_i.

$$g_i(e_i) = d_i, \quad 0 \le e_i \le b_i$$
$$= 0, \quad \text{elsewhere}$$

Where b_i's and d_i's are constants. e_i is rate of imperfect units.

3) $(C_{1i} + \alpha t)$ is variable holding cost per unit item per unit time.

4) θ_i is constant deterioration rate.
5) C_{Pi} is production cost per unit.
6) C_{si} is shortage cost per unit item.
7) C_{Lsi} is lost sale cost per unit item.
8) EAP (Q_1, Q_2…Q_n) is total expected average profit.
9) EP_i is the expected profit for i^{th} item.
10) K_i is selling price per units item of imperfect quality.
11) L_i is salvage value per unit.
12) P_i is production rate per unit time.
13) Pii is production rate of good (perfect) units which satisfies the relation Pii = (1-ei) Pi.
14) qi (t) is on hand inventory at time t ≥ 0.
15) Q_i is total production.
16) Q_{ii} is total production of good units which satisfies the relation $Q_{ii} = (1-e_i) Q_i$.
17) Q_{si} is the shortage amount.
18) S_c is screening cost per unit item.
19) S_i is selling price per unit item of good quality.
20) S_M is maximum shortage cost allowed which is considered as stochastic.
21) t_{1i} is the production time.
22) t_{2i} is the time after which shortages occur.
23) T_i is fixed duration of the cycle.
24) x_i is total demand over time period (0, T_i) which is stochastic.
25) B are maximum budget (total production cost and screening cost) which are considered as stochastic.
26) Backlogging rate, $B_r = e^{-\delta t}$.
27) $C_{\theta i}$ is the deterioration cost.
28) The unit production cost is given by,

$$C_{Pi} = N + \frac{G}{Q_i} + HQ_i$$

Where N, G, H are all positive constants.

This cost function is based on the following factors:

a) The material cost N per unit item is fixed.
b) As the production rate increases, same costs like labor and energy costs are equally distributed over a large no. of units. Hence the unit production cost decreases, because (G/Q_i) decrease as the production rate (Q_i) increases.
c) The third term (H Q_i) associated with tool or die costs is proportional to the production rate.

3. Modeling And Analysis

Case I: When Shortages Do Not Occur:

The inventory level q_i (t) governed by the differential equations, i = 1, 2,…n.

$$\frac{dq_{1i}(t)}{dt} + \theta_i q_i(t) = (1-e_i)P_i - \frac{x_i}{T_i}, \qquad 0 \le t \le t_{1i} \quad \text{--------(1)}$$

$$\frac{dq_{2i}(t)}{dt} + \theta_i q_i(t) = -\frac{x_i}{T_i}, \qquad t_{1i} \le t \le T_i \quad \text{------(2)}$$

with the boundary conditions, $q_i(0) = 0$ and $q_i(T_i) = 0$
Solutions of equations (1) and (2) are given by,

$$q_{1i}(t) = \frac{1}{\theta_i}\left(P_{ii} - \frac{x_i}{T_i}\right)(1-e^{-\theta_i t}), \qquad 0 \le t \le t_{1i} \quad \text{---------(3)}$$

$$q_{2i}(t) = \left(\frac{P_{ii}\, t_{1i}\, e^{\theta_i (T_i - t)}}{\theta_i T_i} - \frac{x_i}{\theta_i T_i}\right), \qquad t_{1i} \le t \le T_i \quad \text{-----(4)}$$

Where, $P_{ii} = (1-e_i)P_i$ [(by notations (16)]

Since shortages do not occur, we must have, $q_i(T_i) \ge 0$,

$$\left(\frac{P_{ii}\, t_{1i}\, e^{\theta_i (T_i - t)}}{\theta_i T_i} - \frac{x_i}{\theta_i T_i}\right) \ge 0, \qquad x_i \le P_{ii}\, t_{1i} \quad \text{------(5)}$$

Now, $Q_i = P_i t_{1i}$ and $Q_{ii} = P_{ii} t_{1i}$

Expected holding cost for non-defective units of i^{th} item, using (3), (4) and (5) is given by,

$$= \int_0^1 \left[\int_0^{Q_{ii}} \left\{ \int_0^{t_{1i}} (C_{1i} + \alpha t) q_i(t)\, dt + \int_{t_{1i}}^{T_i} (C_{1i} + \alpha t) q_i(t)\, dt \right\} f_i(x_i)\, dx_i \right] g_i(e_i)\, de_i$$

$$= \int_0^1 \left[\int_0^{Q_{ii}} C_{1i} \left\{ \frac{Q_{ii}}{\theta_i} + \frac{P_{ii}\, e^{-\theta_i t_{1i}}}{\theta_i^2} - \frac{P_{ii}}{\theta_i^2} - \frac{x_i\, e^{-\theta_i t_{1i}}}{\theta_i^2 T_i} \right. \right.$$

$$\left. + \frac{x_i}{\theta_i^2 T_i} - \frac{Q_{ii}}{\theta_i^2 T_i} + \frac{Q_{ii}\, e^{\theta_i (T_i - t_{1i})}}{\theta_i^2 T_i} - \frac{x_i}{\theta_i} \right\}$$

$$+\alpha\left\{\frac{Q_{ii}\,t_{1i}}{2\theta_i}+\frac{Q_{ii}\,e^{-\theta_i t_{1i}}}{\theta_i^2}+\frac{P_{ii}\,e^{-\theta_i t_{1i}}}{\theta_i^3}-\frac{P_{ii}}{\theta_i^3}-\frac{x_i\,t_{1i}\,e^{-\theta_i t_{1i}}}{\theta_i^2\,T_i}\right.$$

$$-\frac{x_i\,e^{-\theta_i t_{1i}}}{\theta_i^3 T_i}+\frac{x_i}{\theta_i^3 T_i}-\frac{Q_{ii}}{\theta_i^2}+\frac{Q_{ii}\,t_{1i}\,e^{\theta_i(T_i-t_{1i})}}{\theta_i^2 T_i}$$

$$\left.\left.\left.-\frac{Q_{ii}}{\theta_i^3 T_i}+\frac{Q_{ii}\,e^{\theta_i(T_i-t_{1i})}}{\theta_i^3 T_i}-\frac{x_i T_i}{2\theta_i}\right\}\right]f_i(x_i)\,dx_i\right]g_i(e_i)\,de_i \qquad \text{-------(6)}$$

Expected holding cost for defective units of i^{th} item is given by,

$$=\int_0^1\left[\int_0^{t_{1i}}(C_{1i}+\alpha t)e_i P_i\,t\,dt\right]g_i(e_i)\,de_i \qquad (\text{using } Q_i=P_i\,t_{1i})$$

$$=\int_0^1\left[\frac{C_{1i}e_i Q_i^2}{2P_i}+\frac{\alpha e_i Q_i^3}{3P_i^2}\right]g_i(e_i)\,de_i \qquad \text{---------(7)}$$

The salvage value is given by

$$=L_i\int_0^1\left\{\int_0^{Q_{ii}}(Q_{ii}-x_i)f_i(x_i)\,dx_i\right\}g_i(e_i)\,de_i \qquad \text{------------------(8)}$$

Expected deterioration cost for non-defective units of i^{th} item is given by,

$$C_{\theta i}=\theta_i\int_0^1\left[\int_0^{Q_{ii}}\left\{\int_0^{t_{1i}}q_i(t)\,dt+\int_{t_{1i}}^{T_i}q_i(t)\,dt\right\}f_i(x_i)\,dx_i\right]g_i(e_i)\,de_i$$

$$=\int_0^1\left[\int_0^{Q_{ii}}\left\{Q_{ii}+\frac{P_{ii}\,e^{-\theta_i t_{1i}}}{\theta_i}-\frac{P_{ii}}{\theta_i}-\frac{x_i\,e^{-\theta_i t_{1i}}}{\theta_i T_i}\right.\right.$$

$$\left.\left.+\frac{x_i}{\theta_i T_i}-\frac{Q_{ii}}{\theta_i T_i}+\frac{Q_{ii}\,e^{\theta_i(T_i-t_{1i})}}{\theta_i T_i}-x_i\right\}f_i(x_i)\,dx_i\right]g_i(e_i)\,de_i \qquad \text{--------(9)}$$

Total production cost of i^{th} items is given by,

$$C_{Pi} = \left(N + \frac{G}{Q_i} + HQ_i \right) Q_i\, t_{1i} \qquad \left(t_{1i} = \frac{x_i}{P_{ii}} \right)$$

$$= (RQ_i + G + HQ_i^2)\, t_{1i} = (RQ_i + G + HQ_i^2) \frac{x_i}{P_{ii}} \qquad \text{--------(10)}$$

Case II: When Shortages Occur

The governing differential equations are,

$$\frac{dq_{1i}(t)}{dt} + \theta_i q_{1i}(t) = P_{ii} - \frac{x_i}{T_i}, \qquad 0 \le t \le t_{1i} \qquad \text{---------(11)}$$

$$\frac{dq_{2i}(t)}{dt} + \theta_i q_{2i}(t) = -\frac{x_i}{T_i}, \qquad t_{1i} \le t \le t_{2i} \qquad \text{--------(12)}$$

$$\frac{dq_{3i}(t)}{dt} = -e^{-\delta t} \frac{x_i}{T_i}, \qquad t_{2i} \le t \le t_{3i} \qquad \text{---------(13)}$$

$$\frac{dq_{4i}(t)}{dt} = P_{ii} - \frac{x_i}{T_i}, \qquad t_{3i} \le t \le T_i \qquad \text{---------(14)}$$

With the boundary conditions, $q_i(0) = 0,\ q_i(t_{2i}) = 0,\ q_i(T_i) = 0,$

Solutions of equations (11), (12), (13) and (14) are given by,

$$q_{1i}(t) = \frac{1}{\theta_i} \left(P_{ii} - \frac{x_i}{T_i} \right) \left(1 - e^{-\theta_i t} \right), \qquad 0 \le t \le t_{1i} \qquad \text{---------(15)}$$

$$q_{2i}(t) = \frac{1}{\theta_i} P_{ii} e^{-\theta_i t} (e^{\theta_i t_{1i}} - 1) + \frac{x_i}{\theta_i T_i} (e^{-\theta_i t} - 1), \quad t_{1i} \le t \le t_{2i} \qquad \text{--------(16)}$$

$$q_{3i}(t) = \frac{x_i}{\delta T_i} (e^{-\delta t} - e^{-\delta t_{2i}}), \qquad t_{2i} \le t \le t_{3i} \qquad \text{---------(17)}$$

$$q_{4i}(t) = (P_{ii} - \frac{x_i}{T_i})(t - T_i), \qquad t_{3i} \le t \le T_i \qquad \text{---------(18)}$$

At, $t = t_{3i}$, and with the help of equations (17) and (18), we get

$$\frac{x_i}{\delta T_i}(e^{-\delta t_{3i}} - e^{-\delta t_{2i}}) = (P_{ii} - \frac{x_i}{T_i})\,(t_{3i} - T_i)$$

$$t_{3i} = \frac{x_i t_{2i}}{P_{ii} T_i} + T_i - \frac{x_i}{P_{ii}} \qquad \text{-------(19)}$$

Expected holding cost for non-defective units of i[th] item [using (15), and (16)] is given by,

$$= \int_0^1 \left[\int_{Q_{ii}}^{\infty} \left\{ (C_{1i} + \alpha t) \int_0^{t_{1i}} q_{1i}(t)\,dt + \int_{t_{1i}}^{t_{2i}} (C_{1i} + \alpha t) q_{2i}(t)\,dt \right\} f_i(x_i)\,dx_i \right] g_i(e_i)\,de_i$$

$$= \int_0^1 \Bigg[\int_{Q_{ii}}^{\infty} \Bigg[C_{1i} \Bigg\{ \frac{Q_{ii}}{\theta_i} + \frac{x_i}{\theta_i^2 T_i} - \frac{P_{ii} e^{\theta_i (t_{1i} - t_{2i})}}{\theta_i^2}$$

$$+ \frac{P_{ii} e^{-\theta_i t_{2i}}}{\theta_i^2} - \frac{x_i e^{-\theta_i t_{2i}}}{\theta_i^2 T_i} - \frac{x_i t_{2i}}{\theta_i T_i} \Bigg\} + \alpha \Bigg\{ \frac{Q_{ii} t_{1i}}{2\theta_i} + \frac{x_i}{\theta_i^3 T_i}$$

$$- \frac{P_{ii} t_{2i} e^{-\theta_i (t_{1i} - t_{2i})}}{\theta_i^2} + \frac{Q_{ii}}{\theta_i^2} - \frac{P_{ii} e^{\theta_i (t_{1i} - t_{2i})}}{\theta_i^3} + \frac{P_{ii} t_{2i} e^{-\theta_i t_{2i}}}{\theta_i^2}$$

$$+ \frac{P_{ii} e^{-\theta_i t_{2i}}}{\theta_i^3} - \frac{x_i t_{2i} e^{-\theta_i t_{2i}}}{\theta_i^2 T_I} - \frac{x_i e^{-\theta_i t_{2i}}}{\theta_i^3 T_I}$$

$$- \frac{x_i t_{2i}^2}{2\theta_i^2 T_i} \Bigg\} \Bigg] f_i(x_i)\,dx_i \Bigg] g_i(e_i)\,de_i \qquad \text{--------(20)}$$

Expected deteriorating cost for non-defective of i[th] item is given by,

$$C_{\theta i} = \theta_i \int_0^1 \left[\int_{Q_{ii}}^{\infty} \left\{ \int_0^{t_{1i}} q_{1i}(t)\,dt + \int_{t_{1i}}^{t_{2i}} q_{2i}(t)\,dt \right\} f_i(x_i)\,dx_i \right] g_i(e_i)\,de_i$$

$$= \int_{0}^{1} \int_{Q_{ii}}^{\infty} \Big[\Big\{ Q_{ii} + \frac{x_i}{\theta_i T_i} - \frac{P_{ii} e^{\theta_i (t_{1i} - t_{2i})}}{\theta_i} + \frac{P_{ii} e^{-\theta_i t_{2i}}}{\theta_i} - \frac{x_i e^{-\theta_i t_{2i}}}{\theta_i T_i}$$

$$- \frac{x_i t_{2i}}{T_i} \Big\} f_i(x_i) dx_i \Big] f_i(e_i) de_i \qquad \text{----------(21)}$$

Expected shortages cost for i[th] item $= C_{si} Q_{si}$

$$= C_{si} \int_{0}^{1} \left[\int_{Q_{ii}}^{\infty} \left\{ \int_{t_{2i}}^{t_{3i}} \big(-q_{3i}(t)\big) dt + \int_{t_{3i}}^{T_i} \big(-q_{4i}(t)\big) dt \right\} f_i(x_i)\, dx_i \right] g_i(e_i)\, de_i$$

$$\Big[\text{ using, } t_{3i} = \frac{x_i t_{2i}}{P_{ii} T_i} + T_i - \frac{x_i}{P_{ii}} \Big]$$

$$= C_{si} \int_{0}^{1} \left[\int_{Q_{ii}}^{\infty} \left[\left\{ \frac{x_i t_{2i}}{\delta T_i} - \frac{x_i^2 t_{2i}}{\delta P_{ii} T_i^2} - \frac{x_i}{\delta} + \frac{x_i^2}{\delta P_{ii} T_i} \right. \right. \right.$$

$$\left. - \frac{x_i t_{2i} e^{-\delta t_{2i}}}{\delta T_i} + \frac{x_i^2 t_{2i} e^{-\delta t_{2i}}}{\delta P_{ii} T_i^2} + \frac{x_i e^{-\delta t_{2i}}}{\delta} - \frac{x_i^2 e^{-\delta t_{2i}}}{\delta P_{ii} T_i} \right\}$$

$$\left. \left. + \left\{ \frac{1}{2} (P_{ii} - \frac{x_i}{T_i}) (\frac{x_i^2}{P_{ii}^2} + \frac{x_i^2 t_{2i}^2}{P_{ii}^2 T_i^2} - \frac{2 x_i^2 t_{2i}}{P_{ii}^2 T_i}) \right\} \right] f_i(x_i) dx_i \right] g_i(e_i)\, de_i \qquad \text{--------(22)}$$

Where,

$$Q_{si} = \int_{0}^{1} \Big[\int_{Q_{ii}}^{\infty} \Big\{ \frac{x_i t_{2i}}{\delta T_i} - \frac{x_i^2 t_{2i}}{\delta P_{ii} T_i^2} - \frac{x_i}{\delta} + \frac{x_i^2}{\delta P_{ii} T_i}$$

$$- \frac{x_i t_{2i} e^{-\delta t_{2i}}}{\delta T_i} + \frac{x_i^2 t_{2i} e^{-\delta t_{2i}}}{\delta P_{ii} T_i^2} + \frac{x_i e^{-\delta t_{2i}}}{\delta} - \frac{x_i^2 e^{-\delta t_{2i}}}{\delta P_{ii} T_i}$$

$$+\frac{1}{2}(P_{ii}-\frac{x_i}{T_i})(\frac{x_i^2}{P_{ii}^2}+\frac{x_i^2 t_{2i}^2}{P_{ii}^2\ T_i^2}-\frac{2x_i^2 t_{2i}}{P_{ii}^2\ T_i^2})\ \}\ f_i(x_i)\,dx_i\ \ ]\,g_i(e_i)\,de_i$$

Expected lost sale cost for i^{th} item is given by,

$$=C_{Lsi}\int_0^1\ [\ \int_{Q_{ii}}^{\infty}\ \{\ \int_{t_{2i}}^{t_{3i}}\ (1-e^{-\delta t})\frac{x_i}{T_i}dt\ \}\ f_i(x_i)\,dx_i\ \ ]\,g_i(e_i)\,de_i$$

$$[\ \ \text{using, } t_{3i}=\frac{x_i t_{2i}}{P_{ii}T_i}+T_i-\frac{x_i}{P_{ii}}\ \ ]$$

$$=C_{Lsi}\int_0^1\ [\ \int_{Q_{ii}}^{\infty}\ \frac{x_i\delta}{2T_i}\{\ \ \frac{x_i^2 t_{2i}^2}{P_{ii}^2 T_i^2}+T_i^2+\frac{x_i^2}{P_{ii}^2}+\frac{2x_i\,t_{2i}}{P_{ii}}$$

$$-\frac{2x_i\,T_i}{P_{ii}}-\frac{2x_i^2 t_{2i}}{P_{ii}^2\ T_i}-t_{2i}^2\ \ \}f_i(x_i)\,dx_i\ \ ]\,g_i(e_i)\,de_i \qquad \text{---------(23)}$$

Total production cost of i^{th} item is given by,

$$C_{Pi}=(N+\frac{G}{Q_i}+HQ_i)Q_i(T_i+t_{1i}-t_{3i})$$

$$=(NQ_i+G+HQ_i^2)\ (T_i+\frac{x_i}{P_{ii}}-\frac{x_i t_{2i}}{P_{ii}T_i}-T_i+\frac{x_i}{P_{ii}})$$

$$=(NQ_i+G+HQ_i^2)\ (\frac{2x_i}{P_{ii}}-\frac{x_i t_{2i}}{P_{ii}T_i}) \qquad \text{-------------(24)}$$

Expected profit for i^{th} item = Epi

= revenue from sale of perfect units + salvage value + revenue from sale of imperfect units - holding cost for non defective units - holding cost for defective units – deterioration cost - shortages cost – lost sale cost – screening cost - production cost.

Hence for all the items total expected average profit is given by,

$$EAP(Q_1,Q_2...Q_n)=\sum_{1=1}^{n}\ \frac{1}{T_1}EP_i$$

$$= \sum_{i=1}^{n} \frac{1}{T_i} [\; S_i \int_0^1 \{ \int_0^{Q_{ii}} x_i f_i(x_i) dx_i + Q_{ii} \int_{Q_{ii}}^{\infty} f_i(x_i) dx_i \} g_i(e_i) de_i$$

$$+ L_i \int_0^1 \{ \int_0^{Q_{ii}} (Q_{ii} - x_i) f_i(x_i) dx_i \} g_i(e_i) de_i + K_i Q_i \int_0^1 e_i \; g_i(e_i) de_i$$

$$- \int_0^1 [\int_0^{Q_{ii}} [\; C_{1i} \{ \frac{Q_{ii}}{\theta_i} + \frac{P_{ii} e^{-\theta_i t_{1i}}}{\theta_i^2}$$

$$- \frac{P_{ii}}{\theta_i^2} - \frac{x_i e^{-\theta_i t_{1i}}}{\theta_i^2 T_i} + \frac{x_i}{\theta_i^2 T_i} - \frac{Q_{ii}}{\theta_i^2 T_i} + \frac{Q_{ii} e^{\theta_i (T_i - t_{1i})}}{\theta_i^2 T_i} - \frac{x_i}{\theta_i} \}$$

$$+ \alpha \{ \frac{Q_{ii} t_{1i}}{2\theta_i} + \frac{Q_{ii} e^{-\theta_i t_{1i}}}{\theta_i^2} + \frac{P_{ii} e^{-\theta_i t_{1i}}}{\theta_i^3} - \frac{P_{ii}}{\theta_i^3}$$

$$- \frac{x_i t_{1i} e^{-\theta_i t_{1i}}}{\theta_i^2 T_i} - \frac{x_i e^{-\theta_i t_{1i}}}{\theta_i^3 T_i} + \frac{x_i}{\theta_i^3 T_i} - \frac{Q_{ii}}{\theta_i^2} + \frac{Q_{ii} t_{ii} e^{\theta_i (T_i - t_{1i})}}{\theta_i^2 T_i}$$

$$- \frac{Q_{ii}}{\theta_i^3 T_i} + \frac{Q_{ii} e^{\theta_i (T_i - t_{1i})}}{\theta_i^3 T_i} - \frac{x_i T_i}{2\theta_i} \} \;] f_i(x_i) dx_i \;] g_i(e_i) de_i$$

$$- \int_0^1 [\; C_{1i} \frac{e_i Q_i^2}{2 P_i} + \frac{\alpha e_i Q_i^3}{3 P_i^2} \;] g_i(e_i) de_i$$

$$- \int_0^1 [\int_{Q_{ii}}^{\infty} C_{1i} \{ \frac{Q_{ii}}{\theta_i} + \frac{x_i}{\theta_i^2 T_i} - \frac{P_{ii} e^{\theta_i (t_{1i} - t_{2i})}}{\theta_i^2} + \frac{P_{ii} e^{-\theta_i t_{2i}}}{\theta_i^2}$$

$$- \frac{x_i e^{-\theta_i t_{2i}}}{\theta_i^2 T_i} - \frac{x_i t_{2i}}{\theta_i T_i} \} + \alpha \{ \frac{Q_{ii} t_{ii}}{2\theta_i} + \frac{x_i}{\theta_i^3 T_i} - \frac{P_{ii} t_{2i} e^{\theta_i (t_{1i} - t_{2i})}}{\theta_i^2}$$

$$+ \frac{Q_{ii}}{\theta_i^2} - \frac{P_{ii} e^{\theta_i (t_{1i} - t_{2i})}}{\theta_i^3} + \frac{P_{ii} t_{2i} e^{-\theta_i t_{2i}}}{\theta_i^2} + \frac{P_{ii} e^{-\theta_i t_{2i}}}{\theta_i^3}$$

$$-\frac{x_i t_{2i} e^{-\theta_i t_{2i}}}{\theta_i^2 T_i}-\frac{x_i e^{-\theta_i t_{2i}}}{\theta_i^3 T_i}-\frac{x_i t_{2i}^2}{2\theta_i T_i}\ \Big\}\ \Big] f_i(x_i)dx_i\ \Big] g_i(e_i)de_i$$

$$-\int_0^1 \Big[\int_0^{Q_{ii}} \Big\{ Q_{ii}+\frac{P_{ii}e^{-\theta_i t_{1i}}}{\theta_i}-\frac{P_{ii}}{\theta_i}-\frac{x_i e^{-\theta_i t_{1i}}}{\theta_i T_i}$$

$$+\frac{x_i}{\theta_i T_i}-\frac{Q_{ii}}{\theta_i T_i}+\frac{Q_{ii}e^{\theta_i(T_i-t_{1i})}}{\theta_i T_i}-x_i\ \Big\} f_i(x_i)dx_i\ \Big] g_i(e_i)de_i$$

$$-\int_0^1 \Big[\int_{Q_{ii}}^{\infty} \Big\{ Q_{ii}+\frac{x_i}{\theta_i T_i}-\frac{P_{ii}e^{\theta_i(t_{1i}-t_{2i})}}{\theta_i}$$

$$+\frac{P_{ii}e^{-\theta_i t_{2i}}}{\theta_i}-\frac{x_i e^{-\theta_i t_{2i}}}{\theta_i T_i}-\frac{x_i t_{2i}}{T_i}\ \Big\} f_i(x_i)dx_i\ \Big] g_i(e_i)de_i$$

$$-C_{si}\int_0^1 \Big[\int_{Q_{ii}}^{\infty} \Big\{ \frac{x_i t_{2i}}{\delta T_i}-\frac{x_i^2 t_{2i}}{\delta P_{ii}T_i^2}-\frac{x_i}{\delta}+\frac{x_i^2}{\delta P_{ii}T_i}-\frac{x_i t_{2i}e^{-\delta t_{2i}}}{\delta T_i}+\frac{x_i^2 t_{2i}e^{-\delta t_{2i}}}{\delta P_{ii}T_i^2}$$

$$+\frac{x_i e^{-\delta t_{2i}}}{\delta}-\frac{x_i^2 e^{-\delta t_{2i}}}{\delta P_{ii}T_i}+\frac{1}{2}(P_{ii}-\frac{x_i}{T_i})(\frac{x_i^2}{P_{ii}^2}+\frac{x_i^2 t_{2i}^2}{P_{ii}^2 T_i^2}-\frac{2x_i^2 t_{2i}}{P_{ii}^2 T_i})\ \Big\} f_i(x_i)dx_i\ \Big] g_i(e_i)de_i$$

$$-C_{Lsi}\int_0^1 \Big[\int_{Q_{ii}}^{\infty} \frac{x_i\delta}{2T_i}\Big\{ \frac{x_i^2 t_{2i}^2}{P_{ii}^2 T_i^2}+T_i^2+\frac{x_i^2}{P_{ii}^2}+\frac{2x_i t_{2i}}{P_{ii}}-\frac{2x_i T_i}{P_{ii}}-\frac{2x_i^2 t_{2i}}{P_{ii}^2 T_i}$$

$$-t_{2i}^2\ \Big\} f_i(x_i)dx_i\ \Big] g_i(e_i)de_i-S_c Q_i$$

$$-(NQ_i+G+HQ_i^2)\frac{x_i}{P_{ii}}-(NQ_i+G+HQ_i^2)(\frac{2x_i}{P_{ii}}-\frac{x_i t_{2i}}{P_{ii}T_i})\ \Big] \qquad \text{---------(26)}$$

We have to maximize the expected average profit subject to probabilistic imprecise limitations on total production cost along with screening cost under budget and shortage constraints and this is given by,

Max. EAP (Q_1, $Q_2 \ldots Q_n$)

$$\text{S.t. } \sum_{i=1}^{n} (C_{Pi} + S_c) Q_i \le B$$

$$\sum_{i=1}^{n} C_{si} Q_{si} \le S_M \qquad \text{------------(27)}$$

Here, we consider the particular density function for demand x_i and the percentage of defective units e_i in item Q_i.

We have taken the density functions for demand as linear i.e.

$$f_i(x_i) = \begin{cases} A_i + B_i x_i & , \ 0 \le x_i \le R_i \\ 0 & , \ elsewhere \end{cases} \qquad \text{--------(28)}$$

where, A_i's and B_i's are constants.
From the property of p.d.f.

i.e. $\int_{-\infty}^{\infty} f_i(x)\,dx = 1$, we have $\int_{0}^{R_i} f_i(x_i)\,dx_i = 1$

$$\int_{0}^{R_i} (A_i + B_i x_i)\,dx_i = 1$$

$$\left[A_i x_i + B_i \frac{x_i^2}{2} \right]_0^{R_i} d = 1$$

$$\Rightarrow A_i R_i + \frac{B_i R_i^2}{2} = 1 \quad , \quad i = 1, 2.....n \qquad \text{---------(29)}$$

We have taken the density functions for e_i's as

$$g_i(e_i) = \begin{cases} d_i & , \ 0 \le e_i \le b_i \\ 0 & , \ \text{elsewhere} \end{cases}$$

$$\int_{0}^{b_i} d_i\,de_i = 1 \qquad d_i [e_i]_0^{b_i} = 1 \quad b_i d_i = 1 \, , \ i = 1, 2.....n$$

Under these considerations, from (25), we have,

$$EAP(Q_1, Q_2....Q_n) = \sum_{i=1}^{n} \frac{1}{T_i}\Big[\ d_i \int_0^{b_i} \Big\{ S_i \frac{A_i Q_{ii}^2}{2} + \frac{B_i Q_{ii}^3}{3}$$

$$+A_i Q_{ii}(R_i - Q_{ii}) + \frac{B_i Q_{ii}}{2}(R_i^2 - Q_{ii}^2)\ \Big\} de_i$$

$$+d_i \int_0^{b_i} L_i Q_{ii}^2 \Big(\frac{A_i}{2} + \frac{B_i Q_{ii}}{6}\Big) de_i + d_i \int_0^{b_i} K_i Q_i e_i de_i$$

$$-\int_0^{b_i} \Big[\ A_i \Big[\ C_{1i} \Big\{ \frac{Q_{ii}^2}{\theta_i} - \frac{Q_{ii}^3}{6\theta_i P_{ii} T_i}\ \Big\}$$

$$+\alpha \Big\{ \frac{Q_{ii}^3}{4\theta_i P_{ii}} + \frac{Q_{ii}^2}{2\theta_i^2} - \frac{Q_{ii}^4}{12\theta_i P_{ii}^2 T_i} - \frac{Q_{ii}^2 T_i}{4\theta_i}\ \Big\}\ \Big] + B_i \Big[\ C_{ii} \Big\{ \frac{Q_{ii}^3}{3\theta_i} - \frac{Q_{ii}^4}{12\theta_i P_{ii} T}\ \Big\}$$

$$+\alpha \Big\{ \frac{Q_{ii}^4}{6\theta_i P_{ii}} + \frac{Q_{ii}^3}{6\theta_i^2} - \frac{Q_{ii}^5}{20\theta_i P_{ii}^2 T_i} - \frac{Q_{ii}^3 T_i}{6\theta_i} \Big\}\Big]\ \Big] g_i(e_i) de_i - \int_0^{b_i} \Big[\ \frac{C_{1i} e_i Q_i^2}{2P_i}$$

$$+\frac{\alpha e_i Q_i^3}{3P_i^2}\ \Big] g_i(e_i) de_i$$

$$-\int_0^{b_i} \Big[\ A_i \Big[\ C_{1i} \Big\{ \frac{Q_{ii} R_i}{\theta_i} - \frac{R_i^2}{2\theta_i} - \frac{Q_{ii}^2}{2\theta_i}\ \Big\} + \alpha \Big\{ \frac{Q_{ii} R_i^2}{4\theta_i P_{ii}} - \frac{Q_{ii}^3}{4\theta_i P_{ii}}$$

$$-\frac{t_{2i} R_i^2}{2\theta_i} + \frac{t_{2i} Q_{ii}^2}{3\theta_i} + \frac{Q_{ii} R_i}{\theta_i^2} - \frac{Q_{ii}^2}{2\theta_i^2} - \frac{R_i^2}{2\theta_i^2} + \frac{R_i^2 t_{2i}^2}{4\theta_i T_i} - \frac{Q_{ii}^2 t_{2i}^2}{4\theta_i T_i}\ \Big\}\ \Big]$$

$$+B_i \Big[\ C_{1i} \Big\{ \frac{Q_{ii} R_i^2}{2\theta_i} - \frac{R_i^3}{3\theta_i} - \frac{Q_{ii}^3}{6\theta_i}\ \Big\} + \alpha \Big\{ \frac{Q_{ii} R_i^3}{6\theta_i P_{ii}}$$

$$-\frac{Q_{ii}^4}{6\theta_i P_{ii}} - \frac{t_{2i} R_i^3}{3\theta_i} + \frac{t_{2i} Q_{ii}^3}{3\theta_i} + \frac{Q_{ii} R_i^2}{2\theta_i^2} - \frac{Q_{ii}^3}{6\theta_i^2} - \frac{R_i^3}{3\theta_i^2}$$

$$+\frac{R_i^3 t_{2i}^2}{6\theta_i T_i}-\frac{Q_{ii}^3 t_{2i}^2}{6\theta_i T_i}\ \}\ ]\ ]g_i(e_i)de_i-\int_0^{b_i}\ [\ A_i\{\ 2Q_{ii}x_i$$

$$-x_i^2+\frac{x_i^3}{3P_{ii}T_i}-\frac{Q_{ii}x_i^2}{2P_{ii}T_i}\ \}+B_i\{\ Q_{ii}x_i^2-\frac{2x_i^3}{3}+\frac{x_i^4}{4P_{ii}T_i}-\frac{Q_{ii}x_i^3}{3P_{ii}T_i}\ \}\ ]g_i(e_i)de_i$$

$$-\int_0^{b_i}\ [\ A_i(Q_{ii}R_i-\frac{Q_{ii}^2}{2}-\frac{R_i^2}{2})+B_i(\frac{Q_{ii}R_i^2}{2}-\frac{Q_{ii}^3}{6}-\frac{R_i^3}{3})\ ]g_i(e_i)de_i$$

$$-C_{Si}\int_0^{b_i}\ [\ A_i(\frac{R_i^2 t_{2i}^2}{2T_i}-\frac{Q_{ii}^2 t_{2i}^2}{2T_i}-\frac{R_i^3 t_{2i}^2}{6P_{ii}T_i^2}+\frac{Q_{ii}^3 t_{2i}^2}{6P_{ii}T_i^2}$$

$$-\frac{R_i^2 t_{2i}}{2}+\frac{Q_{ii}^2 t_{2i}}{2}+\frac{R_i^3}{6P_{ii}}-\frac{Q_{ii}^3}{6P_{ii}}+\frac{R_i^4 t_{2i}^2}{4P_{ii}^2 T_i^2}-\frac{Q_{ii}^4 t_{2i}^2}{4P_{ii}^2 T_i^2}-\frac{R_i^4}{8P_{ii}^2 T_i}$$

$$+\frac{Q_{ii}^4}{8P_{ii}^2 T_i}-\frac{R_i^4 t_{2i}^2}{8P_{ii}^2 T_i^3}+\frac{Q_{ii}^4 t_{2i}^2}{8P_{ii}^2 T_i^3})+B_i(\frac{R_i^3 t_{2i}^2}{3T_i}-\frac{Q_{ii}^3 t_{2i}^2}{3T_i}$$

$$-\frac{R_i^4 t_{2i}^2}{8P_{ii}T_i^2}+\frac{Q_{ii}^4 t_{2i}^2}{8P_{ii}T_i^2}-\frac{R_i^3 t_{2i}}{3}+\frac{Q_{ii}^3 t_{2i}}{3}+\frac{R_i^4}{8P_{ii}}-\frac{Q_{ii}^4}{8P_{ii}}+\frac{R_i^5 t_{2i}^2}{5P_{ii}T_i^2}-\frac{Q_{ii}^5 t_{2i}^2}{5P_{ii}^2 T_i^2}$$

$$-\frac{R_i^5}{10P_{ii}^2 T_i}+\frac{Q_{ii}^5}{10P_{ii}^2 T_i}-\frac{R_i^5 t_{2i}^2}{10P_{ii}^2 T_i^3}$$

$$+\frac{Q_{ii}^5 t_{2i}^2}{10P_{ii}^2 T_i^3})\ ]g_i(e_i)de_i-C_{LSi}\int_0^{b_i}\ [\ A_i\{\ \frac{R_i^4\delta t_{2i}^2}{8P_{ii}^2 T_i^3}$$

$$-\frac{Q_{ii}^4\delta t_{2i}^2}{8P_{ii}^2 T_i^3}+\frac{R_i^2\delta T_i}{4}-\frac{Q_{ii}^2\delta T_i}{4}+\frac{R_i^4\delta}{8P_{ii}^2 T_i}-\frac{Q_{ii}^4\delta}{8P_{ii}^2 T_i}$$

$$+\frac{R_i^3\delta t_{2i}}{3P_{ii}T_i}-\frac{Q_{ii}^3\delta t_{2i}}{3P_{ii}T_i}-\frac{R_i^3\delta}{3P_{ii}}+\frac{Q_{ii}^3\delta}{3P_{ii}}-\frac{R_i^4\delta t_{2i}}{4P_{ii}^2 T_i^2}+\frac{Q_{ii}^4\delta t_{2i}}{4P_{ii}^2 T_i^2}$$

$$-\frac{R_i^2\delta t_{2i}^2}{4T_i}+\frac{Q_i^2\delta t_{2i}^2}{4T_i}\ \}+B_i\{\ \frac{R_i^5\delta t_{2i}^2}{10P_{ii}^2 T_i^3}-\frac{Q_{ii}^5\delta t_{2i}^2}{10P_{ii}^2 T_i^3}+\frac{R_i^3\delta T_i}{6}$$

$$-\frac{Q_{ii}^3\delta T_i}{6}+\frac{R_i^5\delta}{10P_{ii}^2T_i}-\frac{Q_{ii}^5\delta}{10P_{ii}^2T_i}+\frac{R_i^4\delta t_{2i}}{4P_{ii}T_i}-\frac{Q_{ii}^4\delta t_{2i}}{4P_{ii}T_i}-\frac{R_i^4\delta}{4P_{ii}}+\frac{Q_{ii}^4\delta}{4P_{ii}}-\frac{R_i^5\delta t_{2i}}{5P_{ii}^2T_i^2}$$

$$+\frac{Q_{ii}^5\delta t_{2i}}{5P_{ii}^2T_i^2}-\frac{R_i^3\delta t_{2i}^2}{6T_i}+\frac{Q_{ii}^3\delta t_{2i}^2}{6T_i}\ \Big\}\ \Big]g_i(e_i)de_i$$

$$-S_cQ_i-(NQ_i+G+HQ_i^2)\left(\frac{3x_i}{P_{ii}}-\frac{x_it_{2i}}{P_{ii}T_i}\right)\ \Big] \qquad (30)$$

Now substitute the value of $Q_{ii}=(1-e_i)Q_i$ and $P_{ii}=(1-e_i)P_i$ in equation (29) and integrating with respect to e_i, we have,

$$EAP(Q_1,Q_2...Q_n)=\sum_{i=1}^{n}\frac{d_i}{T_i}\Big[\ S_i\left(b_i-\frac{b_i^2}{2}\right)\left(A_iR_i+\frac{B_iR_i^2}{2}\right)Q_i$$

$$+(L_i-S_i)\left\{\ \frac{A_iQ_i^2}{6}\left(\ 1-(1-b_i)^3\ \right)+\frac{B_iQ_i^3}{24}\left(\ 1-(1-b_i)^4\ \right)\ \right\}$$

$$+\frac{K_iQ_ib_i^2}{2}-A_i\Big[\ C_{1i}\left(\frac{Q_i^2}{\theta_i}-\frac{Q_i^3}{6\theta_iP_iT_i}\right)+\alpha\left(\frac{Q_i^3}{4\theta_iP_i}+\frac{Q_i^2}{2\theta_i^2}\right.$$

$$\left.-\frac{Q_i^4}{12\theta_iP_i^2T_i}-\frac{Q_i^2T_i}{4\theta_i}\right)\ \Big]\frac{\left(\ 1-(1-b_i)^3\ \right)}{3}-B_i\Big[\ C_{1i}\left(\frac{Q_i^3}{3\theta_i}\right.$$

$$\left.-\frac{Q_i^4}{12\theta_iP_iT_i}\right)+\alpha\left(\frac{Q_i^4}{6\theta_iP_i}+\frac{Q_i^3}{6\theta_i^2}-\frac{Q_i^5}{20\theta_iP_i^2T_i}-\frac{Q_i^3T_i}{6\theta_i}\right)\ \Big]$$

$$\frac{\left(\ 1-(1-b_i)^4\ \right)}{4}-\frac{C_{1i}Q_i^2b_i^2}{4P_i}-\frac{\alpha Q_i^3b_i^2}{6P_i^2}-C_{1i}\Big[\ \left(\frac{A_iQ_iR_i}{\theta_i}+\frac{B_iQ_iR_i^2}{2\theta_i}\right)$$

$$\left(b_i-\frac{b_i^2}{2}\right)-\left(\frac{A_iR_i^2}{2\theta_i}+\frac{B_iR_i^3}{3\theta_i}\right)b_i$$

$$-\frac{A_iQ_i^2}{6\theta_i}\left(\ 1-(1-b_i)^3\ \right)-\frac{B_iQ_i^3}{24\theta_i}\left(\ 1-(1-b_i)^4\ \right)\ \Big]$$

$$-\alpha\Big[\ \Big\{\ A_i\left(\frac{Q_iR_i^2}{4\theta_iP_i}-\frac{t_{2i}R_i^2}{2\theta_i}-\frac{R_i^2}{2\theta_i^2}+\frac{R_i^2t_{2i}^2}{4\theta_iT_i}\right)$$

$$+B_i\left(\frac{Q_i R_i^3}{6\theta_i P_i}-\frac{t_{2i} R_i^3}{3\theta_i}-\frac{R_i^3}{3\theta_i^2}+\frac{R_i^3 t_{2i}^2}{6\theta_i T_i}\right)\Big\} b_i+\Big\{ A_i \frac{Q_i R_i}{\theta_i^2}+B_i \frac{Q_i R_i^2}{2\theta_i^2}\Big\}$$

$$(b_i-\frac{b_i^2}{2})+A_i\left(\frac{t_{2i} Q_i^2}{2\theta_i}-\frac{Q_i^3}{4\theta_i P_i}-\frac{Q_i^2}{2\theta_i^2}-\frac{Q_i^2 t_{2i}^2}{4\theta_i T_i}\right)$$

$$\frac{\left(1-(1-b_i)^3\right)}{3}+B_i\left(\frac{t_{2i} Q_i^3}{3\theta_i}-\frac{Q_i^4}{6\theta_i P_i}-\frac{Q_i^3}{6\theta_i^2}-\frac{Q_i^3 t_{2i}^2}{6\theta_i T_i}\right)\frac{\left(1-(1-b_i)^4\right)}{4}\Big]$$

$$-A_i\left(Q_i^2-\frac{Q_i^3}{6P_i T_i}\right)\frac{\left(1-(1-b_i)^3\right)}{3}-B_i\left(\frac{Q_i^3}{3}-\frac{Q_i^4}{12P_i T_i}\right)\frac{\left(1-(1-b_i)^4\right)}{4}$$

$$-(A_i Q_i R_i+\frac{B_i Q_i R_i^2}{2})(b_i-\frac{b_i^2}{2})$$

$$+\left(\frac{A_i R_i^2}{2}+\frac{B_i R_i^3}{3}\right) b_i-\frac{A_i Q_i^2}{2}\frac{\left(1-(1-b_i)^3\right)}{3}$$

$$-\frac{B_i Q_i^3}{6}\frac{\left(1-(1-b_i)^4\right)}{4}-C_{Si}\Big[\Big\{ A_i\left(\frac{R_i^2 t_{2i}^2}{2T_i}-\frac{R_i^2 t_{2i}}{2}\right)$$

$$+B_i\left(\frac{R_i^3 t_{2i}^2}{3T_i}-\frac{R_i^3 t_{2i}}{3}\right)\Big\} b_i+\Big\{ A_i\left(\frac{R_i^3 t_{2i}^2}{6P_i T_i^2}-\frac{R_i^3}{6P_i}\right)+B_i\left(\frac{R_i^4 t_{2i}^2}{8P_i T_i^2}-\frac{R_i^4}{8P_i}\right)\Big\}$$

$$\log(1-b_i)+\Big\{ A_i\Big(\frac{R_i^4 t_{2i}^2}{4P_i^2 T_i^2}$$

$$-\frac{R_i^4}{8P_i^2 T_i}-\frac{R_i^4 t_{2i}^2}{8P_i^2 T_i^3}\Big)+B_i\left(\frac{R_i^5 t_{2i}^2}{5P_i^2 T_i^2}-\frac{R_i^5}{10P_i^2 T_i}-\frac{R_i^5 t_{2i}^2}{10P_i^2 T_i^3}\right)\Big\}$$

$$\left(\frac{b_i}{1-b_i}\right)+A_i\Big(\frac{Q_i^3 t_{2i}^2}{6P_i T_i^2}-\frac{Q_i^2 t_{2i}^2}{2T_i}+\frac{Q_i^2 t_{2i}}{2}-\frac{Q_i^3}{6P_i}-\frac{Q_i^4 t_{2i}^2}{4P_i^2 T_i^2}$$

$$+\frac{Q_i^4}{8P_i^2 T_i}+\frac{Q_i^4 t_{2i}^2}{8P_i^2 T_i^3}\Big)\frac{\left(1-(1-b_i)^3\right)}{3}+B_i\Big(\frac{Q_i^4 t_{2i}^2}{8P_i T_i^2}-\frac{Q_i^3 t_{2i}^2}{3T_i}$$

$$+\frac{Q_i^3 t_{2i}}{3}-\frac{Q_i^4}{8P_i}-\frac{Q_i^5 t_{2i}^2}{5P_i^2 T_i^2}+\frac{Q_i^5}{10P_i^2 T_i}+\frac{Q_i^5 t_{2i}^2}{10P_i^2 T_i^3}\Big)$$

$$\frac{\left(1-(1-b_i)^4\right)}{4}\Bigg]-C_{LSi}\Bigg[\Bigg\{A_i\left(\frac{R_i^2\delta T_i}{4}-\frac{R_i^2\delta t_{2i}^2}{4T_i}\right)$$

$$+B_i\left(\frac{R_i^3\delta T_i}{3}-\frac{R_i^3\delta t_{2i}^2}{6T_i}\right)\Bigg\}b_i+\Bigg\{A_i\left(\frac{R_i^3\delta}{3P_i}-\frac{R_i^3\delta t_{2i}}{3P_i\;T_i}\right)$$

$$+B_i\left(\frac{R_i^4\delta}{4P_i}-\frac{R_i^4\delta t_{2i}}{4P_i\;T_i}\right)\Bigg\}\log(1-b_i)+\Bigg\{A_i\left(\frac{R_i^4\delta t_{2i}^2}{8P_i^2T_i^3}+\frac{R_i^4\delta}{8P_i^2T_i}-\frac{R_i^4\delta t_{2i}}{4P_i^2T_i^2}\right)$$

$$+B_i\left(\frac{R_i^5\delta t_{2i}^2}{10P_i^2T_i^3}+\frac{R_i^5\delta}{10P_i^2\,T_i}-\frac{R_i^5\delta t_{2i}}{5P_i^2\,T_i^2}\right)\Bigg\}\left(\frac{b_i}{1-b_i}\right)$$

$$+A_i\left(\frac{Q_i^3\delta}{3P_i}-\frac{Q_i^4\delta t_{2i}^2}{8P_i^2T_i^3}-\frac{\delta T_i Q_i^2}{4}-\frac{\delta Q_i^4}{8P_i^2T_i}-\frac{Q_i^3\delta t_{2i}}{3P_i\;T_i}\right.$$

$$\left.+\frac{Q_i^4\delta t_{2i}}{4P_i^2T_i^2}+\frac{\delta Q_i^2\,t_{2i}^2}{4T_i}\right)\frac{\left(1-(1-b_i)^3\right)}{3}+B_i\left(\frac{Q_i^4\delta}{4P_i}-\frac{Q_i^5\delta t_{2i}^2}{10P_i^2T_i^3}\right.$$

$$\left.-\frac{\delta Q_i^3T_i}{6}-\frac{\delta Q_i^5}{10P_i^2T_i}-\frac{Q_i^4\delta t_{2i}}{4P_i\;T_i}+\frac{Q_i^5\delta t_{2i}}{5P_i^2T_i^2}+\frac{Q_i^3\delta t_{2i}^2}{6T_i}\right)\frac{\left(1-(1-b_i)^4\right)}{4}\Bigg]$$

$$-\frac{S_cQ_i}{d_i}-\frac{(NQ_i+G+HQ_i^2)}{d_i}\left(\frac{3x_i}{(1-b_i)P_i}-\frac{x_it_{2i}}{(1-b_i)P_iT_i}\right)\Bigg] \qquad \text{--------(31)}$$

Here,

$$Q_{Si}=\Bigg\{A_i\left(\frac{R_i^2t_{2i}^2}{2T_i}-\frac{R_i^2t_{2i}}{2}\right)+B_i\left(\frac{R_i^3t_{2i}^2}{3T_i}-\frac{R_i^3t_{2i}}{3}\right)\Bigg\}b_i$$

$$+\Bigg\{A_i\left(\frac{R_i^3t_{2i}^2}{6P_i\,T_i^2}-\frac{R_i^3}{6P_i^2}\right)+B_i\left(\frac{R_i^4\,t_{2i}^2}{8P_i\,T_i^2}-\frac{R_i^4}{8P_i}\right)\Bigg\}\log(1-b_i)$$

$$+\Bigg\{A_i\left(\frac{R_i^4t_{2i}^2}{4P_i^2T_i^2}-\frac{R_i^4}{8P_i^2T_i}-\frac{R_i^4t_{2i}^2}{8P_i^2T_i^3}\right)+B_i\left(\frac{R_i^5\,t_{2i}^2}{5P_i^2\,T_i^2}-\frac{R_i^5}{10P_i^2\,T_i}\right.$$

$$\left.-\frac{R_i^5\,t_{2i}^2}{10P_i^2\,T_i^3}\right)\Bigg\}\frac{b_i}{1-b_i}+A_i\left(\frac{Q_i^3t_{2i}^2}{6P_i\;T_i^2}-\frac{Q_i^2t_{2i}^2}{2T_i}+\frac{Q_i^2t_{2i}}{2}\right.$$

$$-\frac{Q_i^3}{6P_i}-\frac{Q_i^4 t_{2i}^2}{4P_i^2 T_i^2}+\frac{Q_i^4}{8P_i^2 T_i}+\frac{Q_i^4 t_{2i}^2}{8P_i^2 T_i^3})\frac{\left(1-(1-b_i)^3\right)}{3}+B_i(\frac{Q_i^4 t_{2i}^2}{8P_i T_i^2}-\frac{Q_i^3 t_{2i}^2}{3T_i}$$

$$+\frac{Q_i^3 t_{2i}}{3}-\frac{Q_i^4}{8P_i}-\frac{Q_i^5 t_{2i}^2}{5P_i^2 T_i^2}+\frac{Q_i^5}{10P_i^2 T_i}+\frac{Q_i^5 t_{2i}^2}{10P_i^2 T_i^3})\frac{\left(1-(1-b_i)^4\right)}{4} \quad \text{--------(32)}$$

Hence the problem given by (28) is reduced to ,
Max. EAP (Q_1, Q_2….Q_n) (from 31)

$$\text{S.t.,} \quad \sum_{i=1}^{n}(C_{Pi}+S_c)Q_i \le B \quad ,$$

$$\sum_{i=1}^{n} C_{Si}\,Q_{Si} \le S_M \quad , \; Q_i > 0 \, , \, i=1,2...n \qquad \text{-----------(33)}$$

Numerical Illustration:

To illustrate all results obtained in this paper, let us apply the proposed method to efficiently solve the following numerical examples in which we consider common input parameters are S_c =0.45, δ =0.04, α =0.5, G=2500, H=0.01, N=150:

From Tables 1 and 2, we can observe the optimal cycle time with various parameters of P_1,P_2, θ_1, & θ_2 respectively. The following inferences can be made based in Tables 1 and 2.

Item	A_i	B_i	b_i	P_i	T_i	S_i	L_i	Q_i	d_i	R_i	θ_i	K_i	C_S	C_{LS}
I	0.035	0.0045	0.04	50	10	20	12	100	30	700	0.005	6	3	12
II	0.03	0.0050	0.05	55	11	22	13	110	25	800	0.006	7	4	13

Table 1:

P_1=50, θ_1=.005, P_2=55			P_1=50, θ_1=.005, P_2=56			P_1=50, θ_1=.005, P_2=57		
θ_2	t_{22}	EAP	θ_2	t_{22}	EAP	θ_2	t_{22}	EAP

0.004	6.3987	28945100000	0.004	6.4928	28953800000	0.004	6.5845	28962300000
0.005	5.8147	22666100000	0.005	5.9111	22673200000	0.005	6.0056	22680100000
0.006	5.3352	19245400000	0.006	5.4320	19251300000	0.006	5.5271	19257000000
0.007	4.9346	17177400000	0.007	5.0305	17182400000	0.007	5.1250	17187300000
0.008	4.5947	15832100000	0.008	4.6892	15836300000	0.008	4.7823	15840500000

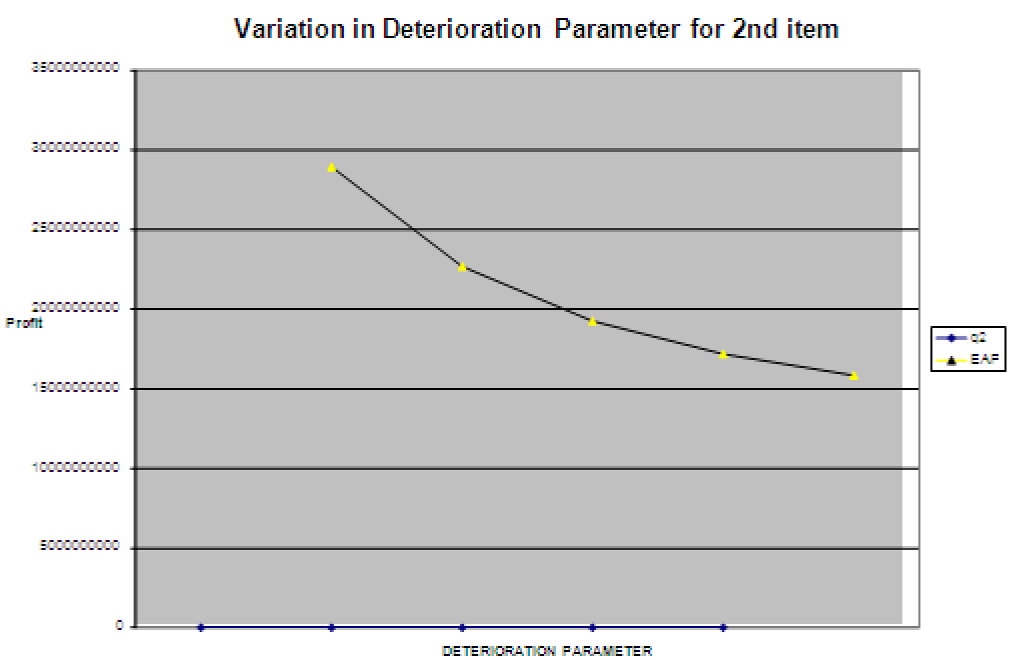
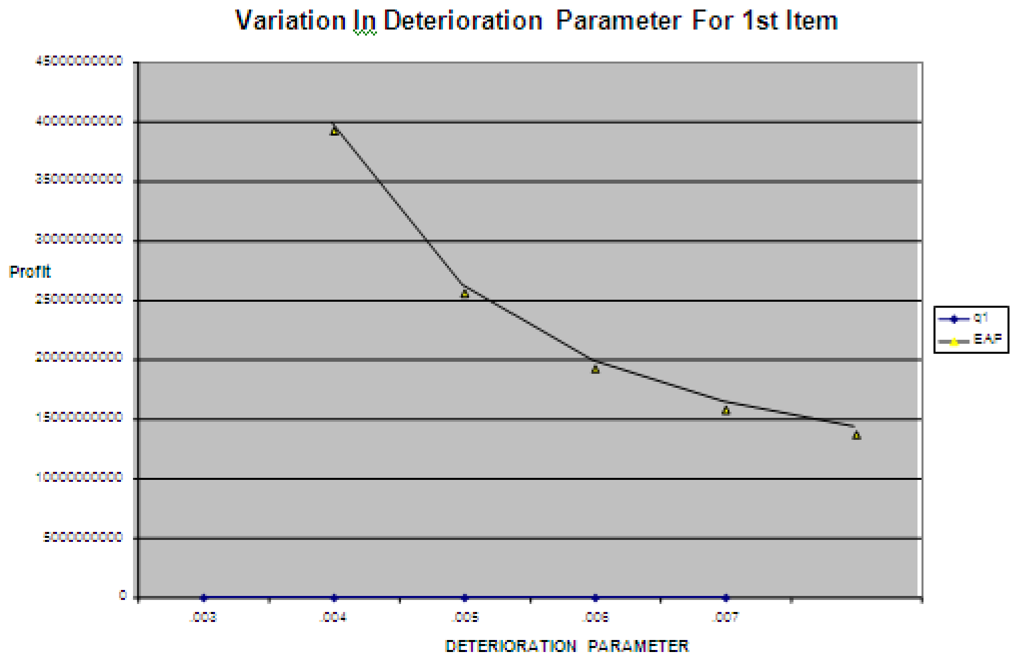


Table 2:

P_1=50, θ_2=.006, P_2=55			P_1=51,θ_2=.006, P_2=55			P_1=52, θ_2=.006, P_2=55		
θ_1	t_{21}	EAP	θ_1	t_{21}	EAP	θ_1	t_{21}	EAP
0.003	7.0871	39238000000	0.003	7.1665	39244700000	0.003	7.2431	39251200000
0.004	6.4745	25582800000	0.004	6.5625	25588100000	0.004	6.6476	25593400000
0.005	5.9649	19245400000	0.005	6.0579	19249900000	0.005	6.1482	19254200000
0.006	5.5346	15794500000	0.006	5.6301	15798300000	0.006	5.7233	15802000000
0.007	5.1663	13709100000	0.007	5.2629	13712400000	0.007	5.3574	13715500000



4. Conclusion

This paper presents the EPLS (Economic Production Lot Size) model which accounts for a production system producing perfect and imperfect quality items. Some defective products are suitable for sale at a reduced price and other remaining worst defective products are totally rejected. To maintain good quality and goodwill of the customers, the worst defective items are not accepted for sale. The defective items of many manufacturing system as textile industries, toy industries etc. are sold at a reduced priced rather than reworking them because such systems can't be restored to its original quality after reworking. And unit production cost depends upon machine production rate which can produce maximum units per unit time. We also considered the demand rate and percentage of defective items is stochastic under uncertain budget and shortage-constraints and the density function of demand and holding cost are linear. Numerical examples are used to illustrate all results obtained in this paper. In addition, we obtain a lot of managerial insights from numerical examples. Profit of the whole system is highly sensitivity towards the deterioration rate. It means as we increase the deterioration rate of the items then the profit decrease.

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